N-soft sets: OWA aggregation operators and multi-agent decisions — Slides in 22nd IPMC 2022 (2/3)

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Abstract

The 22nd International Pure Mathematics Conference 2022 (**22nd IPMC 2022**) on Algebra, Analysis and Geometry, was held in Islamabad (Pakistan) from August 21–23, 2022. It provides a stimulating opportunity to interact with experts from various countries in a variety of branches of pure mathematics. The conference is organized in hybrid mode, with a first day face-to-face and the other two days online.

The emeritus professor Qaiser Mushtaq, Department of Mathematics, Quaid-i-Azam University, Islamabad and the Organizing Committee has been organizing the International Pure Mathematics Conference (IPMC) annually in Islamabad since 2000.

Here are the slides of the lecture given by the author.

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August 25, 2022 at Islamabad, Pakistan

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Semantics of N-soft sets

Structure of the discussion

Two levels of discussion
- The semantics of the attributes.
- The semantics of the grades.

Two interpretations for each level.

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First semantical interpretation of attributes: Multi-context

The original interpretation of soft sets (replicated for N-soft sets).

An N-soft set offers a taxonomy: it classifies, describes or categorizes the alternatives based on their characteristic features.

N-soft sets are distinguished by their ability to rate the level of satisfaction of the attributes.

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Second semantical interpretation of attributes: Possible worlds

Due to Yang and Yao (2020) for soft sets, it can also be replicated for N-soft sets.

The set of attributes is formed by possible worlds for the interpretation of a partially-known concept.

Also here, N-soft sets allow us to rate the level of achievement under each possible world.

Example. In a gala dinner, the suitability of the dishes on a menu depends on the list of guests. If we do not know exactly who will show up, the situation is described by a soft set or an N-soft set.

In an MSc program, the adequacy of the elective courses depends on the list of students. What type of students will be enrolled?

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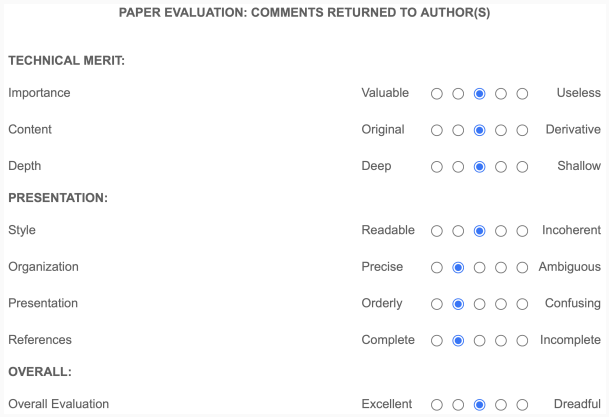
First semantical interpretation of grades: Levels or ratings

This is the original interpretation of N-soft sets.

Grades are labels representing a "level of fulfilment", like hotel stars, referee reports, language skills, or student's marks.

Heterogeneity is allowed.



PAPER EVALUATION: COMMENTS RETURNED TO AUTHOR(S)

TECHNICAL MERIT:			
Importance	Valuable	○ ○ ◉ ○ ○	Useless
Content	Original	○ ○ ◉ ○ ○	Derivative
Depth	Deep	○ ○ ◉ ○ ○	Shallow
PRESENTATION:			
Style	Readable	○ ○ ◉ ○ ○	Incoherent
Organization	Precise	○ ◉ ○ ○ ○	Ambiguous
Presentation	Orderly	○ ◉ ○ ○ ○	Confusing
References	Complete	○ ◉ ○ ○ ○	Incomplete
OVERALL:			
Overall Evaluation	Excellent	○ ○ ◉ ○ ○	Dreadful

Second semantical interpretation of grades: Many-valued logic

This interpretation owes to Alcantud (2022).

Exclusive for N-soft sets ($N \geqslant 3$), meaningless for soft sets.

Basic assumption of the soft set model: every object can be unequivocally associated to each characteristic that it possesses.

Already in 3-valued systems of propositional logic, propositions must not be either true or false.

The rejection of the law of excluded middle means: there are objects that neither satisfy nor do not satisfy a property.

They are within the scope of N-soft set theory with $N \geqslant 3$. 'Grades' become values of truth — both under multi-context and possible worlds semantics.

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The semantical interpretation in logical terms: An example

Four values of truth: 0 for "totally false", 3 for "totally true".

And 1 and 2 represent "more false than true" and "more true than false", respectively.

$(F, T, 4)$	t_1	t_2	t_3
o_1	1	1	2
o_2	3	2	0
o_3	0	1	2
o_4	2	3	2
o_5	1	0	3

For example: 1 means that the statement "o_1 satisfies property t_1" is more false than true (first semantic interpretation of attributes).

Alternatively, that the claim "o_1 is suitable under possible world t_1" is more false than true (second semantic interpretation of attributes).

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Three-valued logic: Some examples

Two-valued logic can be extended to three-valued logics in various reasonable ways. Pioneered by Łukasiewicz (1920).

The next table shows the primitives of some three-valued logics: $\frac{1}{2}$ denotes indeterminacy or possibility (1 holds for truth, 0 for falsehood).

		Łukasiewicz				Bochvar				Kleene				Heyting				Reichenbach			
a	b	$\wedge$	$\vee$	$\rightarrow$	$\leftrightarrow$	$\wedge$	$\vee$	$\rightarrow$	$\leftrightarrow$	$\wedge$	$\vee$	$\rightarrow$	$\leftrightarrow$	$\wedge$	$\vee$	$\rightarrow$	$\leftrightarrow$	$\wedge$	$\vee$	$\rightarrow$	$\leftrightarrow$
0	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	1	$\frac{1}{2}$
0	1	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1
$\frac{1}{2}$	1	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$
1	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

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Three-valued logics in practice

Structured Query Language (SQL) has become the standard language for retrieving, updating, and removing information from relational databases.

SQL implements three logical results, and there is a state or marker identified by the reserved word NULL, indicating that a data value is not found in the database.

The truth tables that SQL applies for the combination of logical states (AND or $\wedge$, OR or $\vee$, and NOT or $\neg$) correspond to the Kleene and Łukasiewicz three-valued logics.

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Four-valued logics in practice

IEEE established a four-valued logic with the standard IEEE 1364 (Verilog) in order to model signal values in digital circuits.

Truth and falseness are retrieved from various sources (like databases or multi-person inputs).

Incomplete information happens when no answer is found.

Simultaneous false and true answers produce contradictory information.

Truth values in Belnap's four-valued logic (1977): $\{T, F, N, B\}$ (true, false, none, both).

$\wedge$	T	B	N	F
T	T	B	N	F
B	B	B	F	F
N	N	F	N	F
F	F	F	F	F

$\vee$	T	B	N	F
T	T	T	T	T
B	T	B	T	B
N	T	T	N	N
F	T	B	N	F

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