N-soft sets: OWA aggregation operators and multi-agent decisions — Slides in 22nd IPMC 2022 (3/3)

Gustavo Santos-García

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Abstract

The 22nd International Pure Mathematics Conference 2022 (**22nd IPMC 2022**) on Algebra, Analysis and Geometry, was held in Islamabad (Pakistan) from August 21–23, 2022. It provides a stimulating opportunity to interact with experts from various countries in a variety of branches of pure mathematics. The conference is organized in hybrid mode, with a first day face-to-face and the other two days online.

The emeritus professor Qaiser Mushtaq, Department of Mathematics, Quaid-i-Azam University, Islamabad and the Organizing Committee has been organizing the International Pure Mathematics Conference (IPMC) annually in Islamabad since 2000.

Here are the slides of the lecture given by the author.

N-soft sets: OWA aggregation operators and multi-agent decisions

Gustavo Santos-García

August 25, 2022 at Islamabad, Pakistan

Multidisciplinary Institute of Enterprise (IME)
University of Salamanca, Spain
santos@usal.es

Aggregation of N-soft sets

The problem

A list of N-soft sets $\{(F_1, T, N), \ldots, (F_k, T, N)\}$ on $O = \{o_1, \ldots, o_p\}$ with a common set of attributes $T = \{t_1, \ldots, t_q\}$.

(F_1, T, N)	t_1	$\ldots\ldots$	t_q
o_1	r^1_{11}	$\ldots\ldots$	r^1_{1q}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_p	r^1_{p1}	$\ldots\ldots$	r^1_{pq}

$\ldots\ldots$

(F_k, T, N)	t_1	$\ldots\ldots$	t_q
o_1	r^k_{11}	$\ldots\ldots$	r^k_{1q}
$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_p	r^k_{p1}	$\ldots\ldots$	r^k_{pq}

Question. What is a sensible aggregate N-soft set of this information?

1st semantical interpretation of grades: Levels I

Procedure: Cell-by-cell application of an ordinal version of the OWA operator (Lizasoain and Moreno, 2013) on the grades.

The general expression needs the utilization of a t-norm and a t-conorm plus the definition of 'distributive weighting vector'.

A particular expression (standard t-norm and t-conorm) is:

for any distributive weighting vector $(\alpha_1, \ldots, \alpha_k) \in G^k$, cell-by-cell aggregation with

$F_\alpha(r_{ij}^1, \ldots, r_{ij}^k) = \max\left(\min(r_{ij}^{\sigma(1)}, \alpha_1), \ldots, \min(r_{ij}^{\sigma(k)}, \alpha_k)\right)$ for every i, j.

The permutation σ of $\{1, \ldots, k\}$ guarantees $r_{ij}^{\sigma(1)} \geqslant \ldots \geqslant r_{ij}^{\sigma(k)}$.

Examples of this operator: max, min, median.

1st semantical interpretation of grades: Levels II

Example. Tabular representation of three 4-soft sets.

Distributive weighting vector $(2, 3, 0)$.

$(F_1, T, 4)$	t_1	t_2	t_3
o_1	1	1	2
o_2	3	2	0
o_3	0	1	2
o_4	2	3	2
o_5	1	0	3

$(F_2, T, 4)$	t_1	t_2	t_3
o_1	1	0	3
o_2	2	3	0
o_3	0	0	3
o_4	2	1	2
o_5	2	0	2

$(F_3, T, 4)$	t_1	t_2	t_3
o_1	1	1	3
o_2	3	3	0
o_3	0	1	3
o_4	2	2	2
o_5	2	0	3

To aggregate emphasized values: We order values $(3, 1, 2)$ as $(3, 2, 1)$.

$\max(\min(3, 2), \min(2, 3), \min(1, 0)) = 2.$

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2nd semantical interpretation of grades: Many-valued logic I

Procedure: Aggregation of values of truth with conjunctive / disjunctive connective in Łukasiewicz N-valued logic.

Truth values $\{0, 1, \ldots, N-1\}$.

Negation is computed by subtraction from $N-1$:

$\neg 0 = N-1$, $\neg 1 = N-2$, $\ldots$, $\neg(N-2) = 1$, $\neg(N-1) = 0$.

The truth value of $a \to b$ is $a \to b = \min(N-1, N-1+b-a)$.

The other logical connectives are derived from these by rules inclusive of the following instances:

$a \vee b = (a \to b) \to b = \max(a, b)$

$a \wedge b = \neg(\neg a \vee \neg b) = \min(a, b)$

$a \leftrightarrow b = (a \to b) \wedge (b \to a) = N-1-|a-b|$

Particular examples may call for the utilization of alternative logics.

2nd semantical interpretation of grades: Many-valued logic II

Example. A special session of a conference receives two sets of reports on five articles.

$O = \{o_1, \ldots, o_5\}$ is the universe of articles.

$T = \{t_1, t_2, t_3\}$ is the set of attributes that a perfect candidate paper should meet: "enough scientific quality", "suitable for the special session", and "adequate quality of presentation".

The reports use 4 values of truth to declare whether it is 'true' that an article satisfies each of the desirable properties.

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2nd semantical interpretation of grades: Many-valued logic III

Tabular representation of two 4-soft sets and their aggregate output (conjunction operator - conservative position).

$(F_1, T, 4)$	t_1	t_2	t_3
o_1	1	1	2
o_2	3	2	0
o_3	0	1	2
o_4	2	3	2
o_5	1	0	3

$(F_2, T, 4)$	t_1	t_2	t_3
o_1	1	0	3
o_2	2	3	0
o_3	0	0	3
o_4	2	1	2
o_5	2	0	2

$(F, T, 4)$	t_1	t_2	t_3
o_1	1	0	2
o_2	2	2	0
o_3	0	0	2
o_4	2	1	2
o_5	1	0	2

2nd semantical interpretation of grades: Many-valued logic IV

Tabular representation of two 4-soft sets and their aggregate output (disjunction operator - optimistic position).

$(F_1, T, 4)$	t_1	t_2	t_3
o_1	1	1	2
o_2	3	2	0
o_3	0	1	2
o_4	2	3	2
o_5	1	0	3

$(F_2, T, 4)$	t_1	t_2	t_3
o_1	1	0	3
o_2	2	3	0
o_3	0	0	3
o_4	2	1	2
o_5	2	0	2

$(F', T, 4)$	t_1	t_2	t_3
o_1	1	1	3
o_2	3	3	0
o_3	0	1	3
o_4	2	3	2
o_5	2	0	3

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Conclusions

Conclusions

▷ The semantical analysis of N-soft sets is quite rich (both in terms of the ‘attributes’ and ‘grades’) and interacts with the field of logics.

▷ The aggregation of N-soft sets allows for various interesting approaches.

Also with the help of other models like hesitant N-soft sets or fuzzy N-soft sets.

▷ Many other issues have been explored in the aforementioned papers, like the implications for decision-making, the construction of WAOWA scores, or the embedding of incomplete soft sets into 3-soft sets (under three-valued semantics of the grades).

▷ Other related topics like N-soft topology (Riaz, Çağman, Zareef, Aslam, 2019) might benefit from these insights in the future.

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