Notes on soft sets and aggregation operators

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Abstract

Under uncertainty, traditional sets may not be sufficient to represent real-world phenomena, and fuzzy sets can provide a more flexible and natural approach. The concept of fuzzy sets has been widely used in various fields, including artificial intelligence, control theory, decision-making, and pattern recognition. Fuzzy sets can also be combined with other mathematical tools, such as probability theory, to provide a more comprehensive approach to uncertainty management. In these notes, we explore the concept of fuzzy sets under uncertainty, and their applications in various fields. We discuss the fundamental concepts of fuzzy sets, including fuzzy membership functions, fuzzy operations, and fuzzy relations. We also examine different types of uncertainty, including epistemic and aleatory uncertainty, and how fuzzy sets can be used to model and manage uncertainty in these cases.

1 Fuzzy sets

Fuzzy sets provide a way to handle uncertainty in data by allowing membership of an element to a set to be represented by a degree of membership. This degree of membership is represented by a value between 0 and 1, where 0 indicates no membership and 1 indicates complete membership. Fuzzy sets are particularly useful in situations where data is imprecise, uncertain, or ambiguous.

Under uncertainty, traditional sets may not be sufficient to represent real-world phenomena, and fuzzy sets can provide a more flexible and natural approach. The concept of fuzzy sets has been widely used in various fields, including artificial intelligence, control theory, decision-making, and pattern recognition. Fuzzy sets can also be combined with other mathematical tools, such as probability theory, to provide a more comprehensive approach to uncertainty management.

In this abstract, we explore the concept of fuzzy sets under uncertainty, and their applications in various fields. We discuss the fundamental concepts of fuzzy sets, including fuzzy membership functions, fuzzy operations, and fuzzy relations. We also examine different types of uncertainty, including epistemic and aleatory uncertainty, and how fuzzy sets can be used to model and manage uncertainty in these cases.

Finally, we discuss some of the challenges and limitations of fuzzy sets under uncertainty, including the difficulty of selecting appropriate membership functions and the computational complexity of working with fuzzy sets. Despite these challenges, fuzzy sets remain a powerful tool for representing and managing uncertainty, and their application is likely to continue to grow in the coming years.

2 Soft sets and aggregation operators

Soft sets and aggregation operators are two concepts from the field of mathematics that are used in decision-making and data analysis. Here are some brief notes on each of them:

Soft sets were introduced by Molodtsov in 1999 as a tool for dealing with uncertain and vague information. A soft set is defined as a pair (X, Γ), where X is a non-empty set and Γ is a mapping from X to the power set of a universal set U. Γ assigns to each element of X a subset of U, which represents the possible values that the element may take. Soft sets are used to represent uncertain or incomplete information, where the values of certain attributes are not known with certainty. Soft set theory can be used to model decision-making problems, where the decision-maker has incomplete information about the options and their outcomes. Soft set theory has applications in fields such as data mining, machine learning, and decision support systems. Aggregation Operators:

Aggregation operators are functions that take multiple inputs and produce a single output. Aggregation operators are used to combine or summarize multiple pieces of information into a single value. Some common aggregation operators include min, max, sum, average, and weighted average. Aggregation operators are used in data analysis to summarize data and extract useful information from it. Aggregation operators can be used in decision-making to combine the opinions or preferences of multiple decision-makers into a single decision. There are different types of aggregation operators, such as t-norms, t-conorms, and fuzzy integrals, which are used in different contexts and for different purposes. Overall, soft sets and aggregation operators are important concepts in mathematics that have many applications in decision-making, data analysis, and other fields.

References

[1] Ghous Ali, Muhammad Akram, Ali N. A. Koam, and José Carlos R. Alcantud. Parameter reductions of bipolar fuzzy soft sets with their decision-making algorithms. *Symmetry*, 11(8), 2019.

[2] Ghous Ali, Muhammad Akram, and José Carlos R. Alcantud. Attributes reductions of bipolar fuzzy relation decision systems. *Neural Computing and Applications*, 32(14):10051–10071, 2020.

[3] José Carlos R. Alcantud and Carlos Rodriguez-Palmero. Characterization of the existence of semicontinuous weak utilities. *Journal of Mathematical Economics*, 32(4):503–509, 1999.

[4] Muhammad Akram, Ghous Ali, and José Carlos R. Alcantud. New decision-making hybrid model: intuitionistic fuzzy N-soft rough sets. *Soft Comput*, 23(20):9853–9868, 2019.

[5] Muhammad Akram, Arooj Adeel, and José Carlos R. Alcantud. Hesitant fuzzy N-soft sets: A new model with applications in decision-making. *Journal of Intelligent & Fuzzy Systems*, 36:6113–6127, 2019.

[6] Muhammad Akram, Ghous Ali, and José Carlos R. Alcantud. Parameter reduction analysis under interval-valued m-polar fuzzy soft information. *Artificial Intelligence Review*, 54(7):5541–5582, 2021.

[7] Muhammad Akram, Ghous Ali, and José Carlos R. Alcantud. Attributes reduction algorithms for m-polar fuzzy relation decision systems. *International Journal of Approximate Reasoning*, 140:232–254, 2022.

[8] Muhammad Akram, Ghous Ali, José Carlos R. Alcantud, and Fatia Fatimah. Parameter reductions in N-soft sets and their applications in decision-making. *Expert Systems*, 38(1):e12601, 2021.

[9] Muhammad Akram, Ghous Ali, José Carlos R. Alcantud, and Aneesa Riaz. Group decision-making with Fermatean fuzzy soft expert knowledge. *Artificial Intelligence Review*, 2022.

[10] Muhammad Akram, Amna Habib, and José Carlos R. Alcantud. An optimization study based on Dijkstra algorithm for a network with trapezoidal picture fuzzy numbers. *Neural Computing and Applications*, 33(4):1329–1342, 2021.

[11] José Carlos R Alcantud. Characterization of the existence of maximal elements of acyclic relations. *Economic Theory*, 19(2):407–416, 2002.

[12] A Luqman, M Akram, AN Al-Kenani, and JCR Alcantud. A study on hypergraph representations of complex fuzzy information. *Symmetry*, 11:1381, 2019.

[13] Neha Waseem, Muhammad Akram, and José Carlos R Alcantud. Multi-attribute decision-making based on m-polar fuzzy hamacher aggregation operators. *Symmetry*, 11(12):1498, 2019.

[14] Xiaoyan Liu, Hee Sik Kim, Feng Feng, and José Carlos R Alcantud. Centroid transformations of intuitionistic fuzzy values based on aggregation operators. *Mathematics*, 6(11):215, 2018.

[15] José Carlos R Alcantud. Revealed indifference and models of choice behavior. *Journal of Mathematical Psychology*, 46(4):418–430, 2002.

[16] Fatia Fatimah and José Carlos R Alcantud. The multi-fuzzy n-soft set and its applications to decision-making. *Neural Computing and Applications*, 33(17):11437–11446, 2021.

[17] Khizar Hayat, Muhammad Irfan Ali, José Carlos R Alcantud, Bing-Yuan Cao, and Kalim U Tariq. Best concept selection in design process: An application of generalized intuitionistic fuzzy soft sets. *Journal of Intelligent & Fuzzy Systems*, 35(5):5707–5720, 2018.

[18] Muhammad Akram, Ghous Ali, José CR Alcantud, and Fatia Fatimah. Parameter reductions in n-soft sets and their applications in decision-making. *Expert Systems*, 38(1):e12601, 2021.

[19] Ghous Ali, Muhammad Akram, and José Carlos R Alcantud. Attributes reductions of bipolar fuzzy relation decision systems. *Neural computing and applications*, 32(14):10051–10071, 2020.

[20] Muhammad Akram, Arooj Adeel, Ahmad N Al-Kenani, and José Carlos R Alcantud. Hesitant fuzzy n-soft electre-ii model: a new framework for decision-making. *Neural Computing and Applications*, 33(13):7505–7520, 2021.

[21] Muhammad Akram, Maria Shabir, Ahmad N Al-Kenani, and José Carlos R Alcantud. Hybrid decision-making frameworks under complex spherical fuzzy-soft sets. *Journal of Mathematics*, 2021, 2021.

[22] Muhammad Akram, Kiran Zahid, and José Carlos R Alcantud. A new outranking method for multicriteria decision making with complex pythagorean fuzzy information. *Neural Computing and Applications*, 34(10):8069–8102, 2022.

[23] Gustavo Santos-García and Miguel Palomino. Solving sudoku puzzles with rewriting rules. *Electronic Notes in Theoretical Computer Science*, 176(4):79–93, 2007.

[24] Emiliano Hernández Galilea, Gustavo Santos-García, and Inés Suárez-Barcena. Identification of glaucoma stages with artificial neural networks using retinal nerve fibre layer analysis and visual field parameters. In *Innovations in hybrid intelligent systems*, pages 418–424. Springer, 2007.

[25] José Carlos R. Alcantud, Gustavo Santos-García, and Emiliano Hernández-Galilea. Glaucoma diagnosis: A soft set based decision making procedure. In *Conference of the Spanish Association for Artificial Intelligence*, pages 49–60. Springer, 2015.

[26] José Carlos R. Alcantud and Gustavo Santos-García. A new criterion for soft set based decision making problems under incomplete information. *International Journal of Computational Intelligence Systems*, 10:394–404, 2017.

[27] Gustavo Santos-García, Las Rivas, Javier De, and Carolyn Talcott. A logic computational framework to query dynamics on complex biological pathways. In *8th International Conference on Practical Applications of Computational Biology & Bioinformatics (PACBB 2014)*, pages 207–214. Springer, Cham, 2014.

[28] José Carlos R Alcantud and Gustavo Santos-García. Incomplete soft sets: New solutions for decision making problems. In *Decision Economics, In Commemoration of the Birth Centennial of Herbert A. Simon 1916-2016 (Nobel Prize in Economics 1978)*, pages 9–17. Springer, Cham, 2016.

[29] Gustavo Santos-Garcıa, Gonzalo Varela, Nuria Novoa, and Marcelo F Jiménez. Prediction of postoperative morbidity after lung resection using an artificial neural network ensemble. *Artificial intelligence in medicine*, 30(1):61–69, 2004.

[30] Adrián Riesco, Beatriz Santos-Buitrago, Javier De Las Rivas, Merrill Knapp, Gustavo Santos-García, and Carolyn Talcott. Epidermal growth factor signaling towards proliferation: modeling and logic inference using forward and backward search. *BioMed research international*, 2017, 2017.

[31] José Carlos R. Alcantud and Gustavo Santos-García. Expanded hesitant fuzzy sets and group decision making. In *2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pages 1–6. IEEE, 2017.

[32] José Carlos R. Alcantud, Gonzalo Varela, Beatriz Santos-Buitrago, Gustavo Santos-García, and Marcelo F. Jiménez. Analysis of survival for lung cancer resections cases with fuzzy and soft set theory in surgical decision making. *PLoS ONE*, 14(6):e0218283, 2019.

[33] Beatriz Santos-Buitrago, Adrián Riesco, Merrill Knapp, José Carlos R. Alcantud, Gustavo Santos-García, and Carolyn L. Talcott. Soft set theory for decision making in computational biology under incomplete information. *IEEE Access*, 7:18183–18193, 2019.

[34] José Carlos R. Alcantud, Gustavo Santos-García, Xindong Peng, and Jianming Zhan. Dual extended hesitant fuzzy sets. *Symmetry*, 11(5):714, 2019.

[35] Gonzalo Varela, Nuria Novoa, Marcelo F Jiménez, and G Santos. Applicability of logistic regression (lr) risk modelling to decision making in lung cancer resection. *Interactive cardiovascular and thoracic surgery*, 2(1):12–15, 2003.

[36] Adrián Riesco, Beatriz Santos-Buitrago, Merrill Knapp, Gustavo Santos-García, Emiliano Hernández Galilea, and Carolyn L. Talcott. Fuzzy matching for cellular signaling networks in a choroidal melanoma model. In Gabriella Panuccio, Miguel Rocha, Florentino Fdez-Riverola, Mohd Saberi Mohamad, and Roberto Casado-Vara, editors, *Practical Applications of Computational Biology & Bioinformatics, 14th International Conference*

(PACBB 2020), L'Aquila, Italy, 17–19 June, 2020, volume 1240 of *Advances in Intelligent Systems and Computing*, pages 80–90. Springer, 2020.

[37] Beatriz Santos-Buitrago, Gustavo Santos-García, and Emiliano Hernández-Galilea. Artificial intelligence for modeling uveal melanoma. *Artificial Intelligence in Cancer*, 1(4):51–65, 2020.

[38] Muhammad Akram, Ayesha Khan, José Carlos R. Alcantud, and Gustavo Santos-García. A hybrid decision-making framework under complex spherical fuzzy prioritized weighted aggregation operators. *Expert Systems*, page e12712, 2021.

[39] Aiyared Iampan, Gustavo Santos García, Muhammad Riaz, Hafiz Muhammad Athar Farid, and Ronnason Chinram. Linear diophantine fuzzy einstein aggregation operators for multi-criteria decision-making problems. *Journal of Mathematics*, 2021, 2021.

[40] Zeeshan Ali, Tahir Mahmood, and Gustavo Santos-García. Heronian mean operators based on novel complex linear diophantine uncertain linguistic variables and their applications in multi-attribute decision making. *Mathematics*, 9(21):2730, 2021.

[41] Tahir Mahmood, Ubaid ur Rehman, Jabbar Ahmmad, and Gustavo Santos-García. Bipolar complex fuzzy hamacher aggregation operators and their applications in multi-attribute decision making. *Mathematics*, 10(1):23, 2021.

[42] Muhammad Akram, Umaira Amjad, José Carlos R. Alcantud, and Gustavo Santos-García. Complex Fermatean fuzzy n-soft sets: A new hybrid model with applications. *Journal of Ambient Intelligence and Humanized Computing*, pages 1–34, 2022.

[43] Muhammad Bilal Khan, Hatim Ghazi Zaini, Savin Treanţă, Gustavo Santos-García, Jorge E Macías-Díaz, and Mohamed S Soliman. Fractional calculus for convex functions in interval-valued settings and inequalities. *Symmetry*, 14(2):341, 2022.

[44] Muhammad Bilal Khan, Gustavo Santos-García, Muhammad Aslam Noor, and Mohamed S Soliman. Some new concepts related to fuzzy fractional calculus for up and down convex fuzzy-number valued functions and inequalities. *Chaos, Solitons & Fractals*, 164:112692, 2022.

[45] Muhammad Bilal Khan, Hatim Ghazi Zaini, Gustavo Santos-García, Pshtiwan Othman Mohammed, and Mohamed S Soliman. Riemann-Liouville fractional integral inequalities for generalized harmonically convex fuzzy-interval-valued functions. *International Journal of Computational Intelligence Systems*, 15(1):28, 2022.

[46] José Carlos R. Alcantud, Gustavo Santos-García, and Muhammad Akram. Owa aggregation operators and multi-agent decisions with n-soft sets. *Expert Systems with Applications*, 203:117430, 2022.

[47] Muhammad Akram, Ghulam Muhiuddin, and Gustavo Santos-Garcıa. An enhanced VIKOR method for multi-criteria group decision-making with complex Fermatean fuzzy sets. *Mathematical Biosciences and Engineering*, 19(7):7201–7231, 2022.

[48] Hafiz Muhammad Athar Farid, Harish Garg, Muhammad Riaz, and Gustavo Santos-García. Multi-criteria group decision-making algorithm based on single-valued neutrosophic einstein prioritized aggregation operators and its applications. *Management Decision*, (ahead-of-print), 2022.

[49] Gustavo Santos-García, Muhammad Bilal Khan, Hleil Alrweili, Ahmad Aziz Alahmadi, and Sherif SM Ghoneim. Hermite-Hadamard and Pachpatte type inequalities for coordinated preinvex fuzzy-interval-valued functions pertaining to a fuzzy-interval double integral operator. *Mathematics*, 10(15):2756, 2022.

[50] Muhammad Bilal Khan, Hatim Ghazi Zaini, Gustavo Santos-García, Muhammad Aslam Noor, and Mohamed S Soliman. New class up and down λ-convex fuzzy-number valued mappings and related fuzzy fractional inequalities. *Fractal and Fractional*, 6(11):679, 2022.

[51] Muhammad Bilal Khan, Gustavo Santos-García, Savin Treanţă, and Mohamed S Soliman. New class up and down pre-invex fuzzy number valued mappings and related inequalities via fuzzy riemann integrals. *Symmetry*, 14(11):2322, 2022.

[52] Muhammad Bilal Khan, Gustavo Santos-García, Hatim Ghazi Zaini, Savin Treanţă, and Mohamed S Soliman. Some new concepts related to integral operators and inequalities on coordinates in fuzzy fractional calculus. *Mathematics*, 10(4):534, 2022.

[53] Shazia Kanwal, Akbar Azam, Muhammad Gulzar, and Gustavo Santos-García. A fixed point approach to lattice fuzzy set via f-contraction. *Mathematics*, 10(19):3673, 2022.

[54] Muhammad Bilal Khan, Muhammad Aslam Noor, Hatim Ghazi Zaini, Gustavo Santos-García, and Mohamed S Soliman. The new versions of Hermite-Hadamard inequalities for pre-invex fuzzy-interval-valued mappings via fuzzy Riemann integrals. *International Journal of Computational Intelligence Systems*, 15(1):66, 2022.

[55] Muhammad Bilal Khan, Gustavo Santos-García, Muhammad Aslam Noor, and Mohamed S Soliman. New class of preinvex fuzzy mappings and related inequalities. *Mathematics*, 10(20):3753, 2022.

[56] Muhammad Akram, Anam Luqman, and José Carlos R. Alcantud. An integrated ELECTRE-I approach for risk evaluation with hesitant Pythagorean fuzzy information. *Expert Systems with Applications*, page 116945, 2022.

[57] Gustavo Santos-García, Abbas Ali, and Noor Rehman. Soft ordered double quantitative approximations based three-way decisions and their applications. *Scientific Reports*, 12(1):19211, 2022.

[58] Muhammad Bilal Khan, Gustavo Santos-García, Muhammad Aslam Noor, and Mohamed S Soliman. New Hermite-Hadamard inequalities for convex fuzzy-number-valued mappings via fuzzy Riemann integrals. *Mathematics*, 10(18):3251, 2022.

[59] Tahir Mahmood, Ubaid U. Rehman, and Gustavo Santos-García. The prioritization of solutions for reducing the influence of climate change on the environment by using the conception of bipolar complex fuzzy power Dombi aggregation operators. *Frontiers in Environmental Science. Sec. Environmental Economics and Management*, 11, 2023.

[60] Hafiz Muhammad Athar Farid, Harish Garg, Muhammad Riaz, and Gustavo Santos-García. Multi-criteria group decision-making algorithm based on single-valued neutrosophic Einstein prioritized aggregation operators and its applications. *Management Decision*, 61(2):382–420, 2023.

[61] Muhammad Akram, Naila Ramzan, Anam Luqman, and Gustavo Santos-García. An integrated MULTIMOORA method with 2-tuple linguistic Fermatean fuzzy sets: Urban quality of life selection application. *AIMS Mathematics*, 8(2):2798–2828, 2023.

[62] Muhammad Akram, Sumera Naz, Gustavo Santos-García, and Muhammad Ramzan Saeed. Extended CODAS method for MAGDM with 2-tuple linguistic T-spherical fuzzy sets. *AIMS Mathematics*, 8(2):3428–3468, 2023.

[63] Muhammad Bilal Khan, Gustavo Santos-García, Hüseyin Budak, Savin Treanţă, and Mohamed S Soliman. Some new versions of Jensen, Schur and Hermite-Hadamard type inequalities for $(p, \mathfrak{J})$-convex fuzzy-interval-valued functions. *AIMS Mathematics*, 8(3):7437–7470, 2023.

[64] Muhammad Bilal Khan, Hakeem A Othman, Gustavo Santos-García, Muhammad Aslam Noor, and Mohamed S Soliman. Some new concepts in fuzzy calculus for up and down λ-convex fuzzy-number valued mappings and related inequalities. *AIMS Mathematics*, 8(3):6777–6803, 2023.

[65] Sundas Shahzadi, Areen Rasool, and Gustavo Santos-García. Methods to find strength of job competition among candidates under single-valued neutrosophic soft model. *Mathematical Biosciences and Engineering*, 20(3):4609–4642, 2023.

[66] Muhammad Akram, Usman Ali, Gustavo Santos-García, and Zohra Niaz. 2-tuple linguistic Fermatean fuzzy MAGDM based on the WASPAS method for selection of solid waste disposal location. *Mathematical Biosciences and Engineering*, 20(2):3811–3837, 2023.

[67] Gustavo Santos-García and José Carlos R Alcantud. Ranked soft sets. *Expert Systems*, page e13231, 2023.

[68] Muhammad Bilal Khan, Hakeem A Othman, Gustavo Santos-García, Tareq Saeed, and Mohamed S Soliman. On fuzzy fractional integral operators having exponential kernels and related certain inequalities for exponential trigonometric convex fuzzy-number valued mappings. *Chaos, Solitons & Fractals*, 169:113274, 2023.

[69] Shazia Kanwal, Asif Ali, Abdullah Al Mazrooei, and Gustavo Santos-García. Existence of fuzzy fixed points of set-valued fuzzy mappings in metric and fuzzy metric spaces. *AIMS Mathematics*, 8(5):10095–10112, 2023.

[70] Hafiz Muhammad Athar Farid, Muhammad Riaz, and Gustavo Santos García. T-spherical fuzzy information aggregation with multi-criteria decision-making. *AIMS Mathematics*, 8(5):10113–10145, 2023.

[71] Shazia Kanwal, Abdullah Al Mazrooei, Gustavo Santos-García, and Muhammad Gulzar. Some fixed point results for fuzzy generalizations of nadler's contraction in b-metric spaces. *AIMS Mathematics*, 8(5):10177–10195, 2023.

[72] Muhammad Bilal Khan, Gustavo Santos-García, Savin Treanţă, Muhammad Aslam Noor, and Mohamed S Soliman. Perturbed mixed variational-like inequalities and auxiliary principle pertaining to a fuzzy environment. *Symmetry*, 14(12):2503, 2022.

[73] Shazia Kanwal, Akbar Azam, Muhammad Gulzar, and Gustavo Santos-García. A fixed point approach to lattice fuzzy set via f-contraction. *Mathematics*, 10(19):3673, 2022.

[74] Zeeshan Ali, Tahir Mahmood, and Gustavo Santos-García. Heronian mean operators based on novel complex linear Diophantine uncertain linguistic variables and their applications in multi-attribute decision making. *Mathematics*, 9(21):2730, 2021.

[75] Gustavo Santos-García and Rocío Santos-Buitrago. Soft computing under uncertain knowledge. *Journal of Advances in Applied & Computational Mathematics*, 2021.