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Research paper

An integrated ELECTRE method for selection of rehabilitation center with m-polar fuzzy N-soft information

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ABSTRACT

The primary goal of this research article is to apply ELECTRE I, a fundamental multi-criteria group decision-making technique, in an m-polar fuzzy N-soft environment. This new methodology helps us to pinpoint the best alternative(s) in the presence of multi-polar options with N-graded qualities. Its basic operational idea entails the comparison between any two alternatives by the assessment of score degrees. Concordance and discordance indices are then calculated to evaluate the alternatives' superiority and inferiority. We may disqualify the incompetent alternatives using concordance and discordance levels. An m-polar fuzzy N-soft dominance matrix can represent the combined effect of concordance and discordance dominance matrices. The steps of this new multi-criteria group decision making technique are summarized in a flowchart. In order to demonstrate its authenticity and applicability, we employ a case study involving the establishment of a rehabilitation facility for drug abusers. A comparison with the m-polar fuzzy PROMETHEE and m-polar fuzzy ELECTRE I methodologies establishes its validity. Finally, we conclude our study of the methodology proposed in this paper with a critical analysis of its benefits and drawbacks.

1. Introduction

Researchers disagree on the best method for selecting one option among a variety of alternatives, especially when those options have diverse features. Multi-criteria decision making (MCDM) first emerges in a situation like this. Sometimes, in addition to the presence of various attributes, we need to make use of the perspectives of a group of decision-makers. This situation is the beginning of multi-criteria group decision making (MCGDM).

We shall be preoccupied with opinions that are not necessarily crisp. Zadeh's fuzzy sets [1] reshaped the modelization of uncertain propositions and measurements. His models became remarkable functional tools in various fields of knowledge including medicine [2], industry [3,4] or technology [5]. As a reflect of their success, Atanassov extended the concept of fuzzy sets by splitting the notion of belongingness and non-belongingness of the attributes. The associated membership degree (μ) and non-membership degree (ν) of each alternative are subject to the condition $\mu + \nu \leq 1$, and the model was called intuitionistic fuzzy sets [6]. This constraint has been relaxed by Pythagorean fuzzy sets (a term coined by Yager [7]), that only require that the μ and ν degrees satisfy the condition $\mu^2 + \nu^2 \leq 1$. Nevertheless these models fail to encode multi-polar information, and thus the multi-polarity that we nowadays observe must be handled by m-polar fuzzy sets (mFS) [8]. They are designed to operate with m-poles, multi-polar agents or multi-polar attributes.

From an altogether different approach relating to non-numerical descriptions of the alternatives, soft set theory [9] utilizes a parameterized family of attributes. Still there is a strong crispness component in this approach: it is imposed that the attributes are either satisfied or not. To fine-tune this limited concept in a realistic manner, Fatimah [10] incorporated N-grading to crisp soft set theory, hence inaugurating N-soft set theory. The semantical interpretation of this generalization of the primitive soft set theory has been presented in [11]. N-soft sets add to a model that stems from the incorporation of fuzziness for similar purposes, since Maji et al. [12] had launched fuzzy soft set theory with these novelties. And valuation fuzzy soft sets [13] enhanced their idea with an applicable model that was tested with real-estate data. Akram et al. [14] were quick to combine fuzzy soft set theory with N-grading, and proposed fuzzy N-soft set theory. Its advancement can be demonstrated by the emergence of further hybrid models, to name but a few: hesitant fuzzy N-soft sets [15], m-polar fuzzy N-soft sets [16], N-soft rough sets [17] and N-soft sets approach to rough sets [18], multi-fuzzy N-soft sets [19], complex fuzzy N-soft sets [20], or picture fuzzy N-soft sets [21].

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Let us now turn our attention to MCDM and MCGDM. Many techniques have been developed in order to break the bottleneck of decision making hurdles with crisp values [22–25]. Furthermore, there are many MCGDM techniques [26–28] that were developed in the fuzzy environment and have been subsequently adapted to cope up with uncertainty, vagueness, multi-attributes and N-grading. Among all these techniques, ELECTRE I is regarded as a most reliable method that covers all the aspects of a suitable selection of alternatives according to the required conditions. It was proposed by Benayoun et al. [23] in the analysis of crisp settings. Sevkli [29] extended its scope to the fuzzy environment, and in this context Hatami-Marbini and Tavana [30] put forward an extension for group decision making. Later on, ELECTRE I has been adapted to operate under different informational structures, like hesitant fuzzy sets (Chen et al. [31]), m-polar fuzzy (Akram et al. [32]) and m-polar fuzzy linguistic environments (Adeel et al. [27]), or intuitionistic fuzzy sets (Rouyendegh [28]).

The following studies concern applications of the aforementioned ideas and related improvements. Intuitionistic and interval valued intuitionistic fuzzy parameterized soft set theory (and their decision making applications) were introduced by Deli and Çagman [33] and Deli and Karatas [34], respectively. Ali et al. [35] started the theory of neutrosophic cubic sets and their applications in pattern recognition. Deli and Çagman [36] presented similarity measures of intuitionistic fuzzy soft-sets and applied them in the medical field. Asghari et al. [37] used the fuzzy ELECTRE approach for evaluating mobile payment business models. Ayyildiz et al. [38] enhanced an individual credit ranking using an integrated interval type-2 trapezoidal fuzzy ELECTRE methodology. Aytac et al. [39] employed the fuzzy ELECTRE I method for evaluating catering firm alternatives. Devi and Yadav [40] resorted to a multicriteria intuitionistic fuzzy group decision making for plant location selection with the ELECTRE method. Kabak et al. [41] utilized a fuzzy hybrid MCDM approach for professional selection. Kheirkhah and Dehghani [42] took advantage of the group fuzzy ELECTRE method to evaluate the quality of public transportation service. Kirisci et al. [43] utilized a Fermatean fuzzy ELECTRE multi-criteria group decision-making for the selection of biomedical material. Hatami et al. [44] used the fuzzy group ELECTRE method for safety and health assessment in hazardous waste recycling facilities. Jagtap and Karande [45] studied the effect of normalization methods on rank performance in single valued m-polar fuzzy ELECTRE-I algorithm. Zeng et al. [46] utilized intuitionistic fuzzy social network hybrid MCDM model for the assessment of digital reforms of the manufacturing industry in China. Su et al. [47] evaluate online learning platforms based on probabilistic linguistic term sets with self-confidence multiple attribute group decision making method. Rouyendegh and Erkan [48] put forward an application of the fuzzy ELECTRE method for academic staff selection, and Rouyendegh and Erol [49] selected a best project using the fuzzy ELECTRE method. Akram et al. [50] enhanced the ELECTRE I method with the assistance of complex spherical fuzzy information.

To summarize, various MCGDM techniques [26,27,32] allow us to compute a best alternative among different choices in this multi-polar world. Particularly, the family of ELECTRE methods have found many applications in a variety of fields like medicine [43], business [37], transportation [42], or academia [48]. The following factors emboldened us to propose a novel MCGDM technique within this remarkable family, that we call mFNS-ELECTRE I:

- Zadeh's fuzzy sets are a breakthrough in the management of ambiguity and uncertainty, but they are unable to handle the multi-polarity that we commonly encounter in the modern world, where the options being considered are typically distinguished by a number of attributes.
- Although fuzzy soft sets [12] are not capable of capturing multiple grades, they can encapsulate the fuzzy fulfillment of characteristics from a parameterized family. However various scenarios are being faced in our daily life where we evaluate the alternatives by grading them using an N-ordered system. The MCGDM technique that we call mFNS-ELECTRE I addresses all facets of multi-polarity and N-grading.
- The mFNS-ELECTRE I strategy is able to overcome the difficulties of handling multi-polarity and N-ordered grading simultaneously in multi-criteria group decision making processes. It enables us to make the most practical decision in real-world situations since it may incorporate data directly from a wider range of problems.
- The novel mFNS-ELECTRE I technique refines the m-polar fuzzy ELECTRE I method [32] by infusing the concept of N-ordered grading that assists us to make the more suitable decision in real life goals.
- The novel mFNS ELECTRE I technique has the potential to be used in a variety of scientific and managerial fields, including information and technology, business administration, artificial intelligence, computational mathematics, industrial management, mechanics, and transportation, where we have multi-polar information as well as N-ordered grading.

The proposed methodology, mFNS-ELECTRE I, is based on the principle of comparison between any pair of alternatives, and at the end it extracts the most suitable alternative. Comparisons are made under the effectiveness of concordance and discordance sets, indices, levels, matrices and a dominance matrix.

The main contributions of this article are as follows:

- A very detailed step-by-step working procedure of mFNS-ELECTRE I technique is described, along with all the essential formulas and operators.
- To have a better understanding of the working order of this technique, a comprehensive flow chart is drawn picturing each and every step.
- A numerical case study of selection of the best rehabilitation center in the U.S. is included. This exhaustive analysis proves the practicality and rationality of the proposed mFNS-ELECTRE I technique.
- To prove accuracy and reliability, a comparison is made between the mFNS-ELECTRE I method with the mF-PROMETHEE and mF ELECTRE I techniques, indicating that both the techniques suggest the same best alternative.
- Finally, merits and demerits of mFNS-ELECTRE I method are listed in order to show its main advantages and limitations.

The remaining of this article proceeds as follows. Section 2 consists of the detailed step-by-step procedure of the novel MCGDM technique that we propose in our investigation. Section 3 contains the numerical case study (selection of the best rehabilitation center in the U.S.). Sections 4 and 5 incorporate the comparative analysis and comparative discussion between the mFNS-ELECTRE I method with the mF-PROMETHEE and mF ELECTRE I techniques. Section 6 discusses the pros and cons of the proposed methodology. Section 7 concludes the paper.

2. Structure of the m-polar fuzzy N-soft ELECTRE I method

The multi-polarity and the ordered grading abilities of m-polar fuzzy N-soft sets ($mFNSS$) will be merged with the fine features of the MCGDM technique known as ELECTRE I, that facilitates the comparison between pairs of alternatives with respect to the criteria under consideration through the concordance and discordance dominance matrices. Prior to the technical steps involved in the m-polar fuzzy N-soft ELECTRE I method, we recall the fundamental models that support its application:

Definition 2.1 (*[14]*)**.** Let $G_N = \{0, 1, 2, \dots, N-1\}$ be a set of N ordered grades, where $N > 1$ is a natural number. A triplet (Γ, A, N) is said to be a fuzzy N-soft set on U, where $F(U)$ consists of the set of all fuzzy sets on U and A is a set of attributes, if $\Gamma : A \to 2^{F(U)\times G_N}$ satisfies the condition for each $a \in A$ and $u \in F(U)$, there exists a unique $r \in G_N$ such that $(u, r) \in F(a)$.

Definition 2.2 ([16]). An m-polar fuzzy N-soft set (m, N) is a triple (f, D, m), where $D = (F, C, N)$ is an N-soft set on a universal set (U), and f maps any attribute in C with an m-polar fuzzy set (mF) A on $F(c_k)$, which is a convenient subset of $V \times M$ and $c_k \in C$. Therefore, the domain of f is of course C, and its codomain is $M'(V \times M)$, the family of all sub-mF sets over $V \times M$.

We address the reader to the founding Refs. [14,16] for examples and interpretations of the elements in the known models defined above.

2.1. Operations related to m-polar fuzzy N-soft sets

Let $B = \langle d_1, (\mu^1, \mu^2, \dots, \mu^m)\rangle$, $C = \langle d_2, (\nu^1, \nu^2, \dots, \nu^m)\rangle$ and $E = \langle d, (\alpha^1, \alpha^2, \dots, \alpha^m)\rangle$ be three m-polar fuzzy N-soft numbers, and let δ be a real constant. Some basic operations on these numbers are defined as follows:

1. $A \oplus B = \langle \max(d_1, d_2), (\mu^1+\nu^1-(\mu^1)(\nu^1)), (\mu^2+\nu^2-(\mu^2)(\nu^2)), \dots, (\mu^m+\nu^m-(\mu^m)(\mu^m))\rangle$
2. $A \otimes B = \langle \min(d_1, d_2), (\mu_1 * \nu_1), (\mu_2 * \nu_2), \dots, (\mu_m * \nu_m)\rangle$.
3. $\delta E = \langle d, (1-(1-\alpha^1)^\delta), (1-(1-\alpha^2)^\delta), \dots, (1-(1-\alpha^m)^\delta)\rangle$.
4. Score of E:
$$S(E) = (\frac{d}{N-1})^2 + (\frac{\alpha^1+\alpha^2+\cdots+\alpha^m}{m}). \tag{1}$$
5. **m-polar fuzzy N-soft weighted averaging operator (mFNSWA)**
$$\begin{aligned} mFNSWA(B,C) = {} & \langle max(d_1, d_2), (1-((1-\mu^1)^{\epsilon_1})(1-((1-\nu^1)^{\epsilon_2}))), \\ & \times(1-((1-\mu^2)^{\epsilon_1}) \\ = {} & (1-((1-\nu^2)^{\epsilon_2}))), \dots, (1-((1-\mu^m)^{\epsilon_1}) \\ & \times(1-((1-\nu^m)^{\epsilon_2})))\rangle. \end{aligned} \tag{2}$$
6. **Aggregated m-polar fuzzy N-soft averaging operator (AmFNSAO)**
$$AmFNSWAO(B,C) = \langle min(d_1, d_2), ((\mu^1 * \nu^1), (\mu^2 * \nu^2), \dots, (\mu^m * \nu^m))\rangle. \tag{3}$$
7. **Euclidean distance**
$$d(B,C) = \sqrt{\frac{(\frac{d_1-d_2}{N-1})^2 + \sum_{i=1}^{m}(\mu_i-\nu_i)^2}{m}}. \tag{4}$$

2.2. Steps involved in m-polar fuzzy N-soft ELECTRE I

To begin with, this section gives an account of the detailed steps conducive to the implementation of ELECTRE I. They involve evaluation of concordance and discordance levels, indices, and matrices by the utilization of Euclidean distance for $mFNSS$. This strategy consists of the following steps:

1. **Construction of independent decision matrices by each expert**:
ELECTRE I takes advantage of a group of decision making experts who, after examining the alternatives on the basis of their criteria, allocate membership degrees and ordered grades to the alternatives under the flexible structure of the mFNS environment. In formal terms, we consider a set of 'q' experts, $E = \{e_t\}$, $t = 1, 2, \dots, q$, who thoroughly analyze a set of 'r' alternatives, $A = \{a_l\}$, $l = 1, 2, \dots, r$, with respect to the 'p' criteria under consideration, $C = \{c_s\}$, $s = 1, 2, \dots, p$.
After a careful examination of the alternatives, the tth expert, from the panel of experts, assigns the independent membership degrees to the lth alternative with respect to the sth criterion in the form of m-polar fuzzy N-soft numbers $(mFNSNs)$ as $(d_{ls}, \rho_{ls}^{(t)^i} \circ \mu)$, $i = 1, 2, \dots, m$. These grades and membership degrees are represented in the form of an mFNS decision matrix as $\Gamma^{(t)^i} = (\beta_{ls}^{(t)})_{r\times p}$ where $\beta_{ls}^{(t)} = \langle d_{ls}, (b_{ls}^{(t)^1}, b_{ls}^{(t)^2}, \dots, b_{ls}^{(t)^m})\rangle$ and $b_{ls}^{(t)^i} = \rho_{ls}^{(t)^i} \circ \mu$. Matrix $\Gamma^{(t)^i}$ is given in Box I.
2. **Construction of aggregate m-polar fuzzy N-soft decision matrix:**
In most practical situations, the experts have different levels of importance in the decision making process, hence their recommendations should be weighed differently. Their individual evaluations, in the form of grades and membership degrees, must be combined to produce a joint decision in such way that it captures the relative impact of the recommendations made by each single expert. To this purpose, we resort to the m-polar fuzzy N-soft weighted averaging (or $mFNSWA$) operator defined in (2):
$$Y_{ls} = mFNSWA_\epsilon(\beta_{ls}^{(1)}, \beta_{ls}^{(2)}, \dots, \beta_{ls}^{(q)}) \tag{5}$$
$$= \epsilon_1\beta_{ls}^{(1)} \oplus \epsilon_2\beta_{ls}^{(2)} \oplus \dots \oplus \epsilon_2\beta_{ls}^{(q)} \tag{6}$$
$$= \langle d_{ls}^*, 1-\prod_{t=1}^{q}(1-b_{ls}^{(t)^i})^{\epsilon_t}\rangle, \tag{7}$$
where $b_{ls}^{(t)^i} = \rho_{ls}^{(t)^i} \circ \mu$, $i = 1, 2, \dots, m$ and,
$$d_{ls}^* = \max(d_{ls}^{(1)}, d_{ls}^{(2)}, \dots, d_{ls}^{(q)}). \tag{8}$$
This collective decision is arranged in an aggregate m-polar fuzzy N-soft decision matrix, denoted $AmFNSDM$, which gives a collective group satisfactory decision. The $AmFNSDM$ that stems from the application of the aforementioned $mFNSWA$ operator is as follows:
$$D = \begin{pmatrix} \langle d_{11}^*, (b_{11}^1, b_{11}^2, \dots, b_{11}^m)\rangle & \langle d_{12}^*, (b_{12}^1, b_{12}^2, \dots, b_{12}^m)\rangle & \dots & \langle d_{1p}^*, (b_{1p}^1, b_{1p}^2, \dots, b_{1p}^m)\rangle \\ \langle d_{21}^*, (b_{21}^1, b_{21}^2, \dots, b_{21}^m)\rangle & \langle d_{22}^*, (b_{22}^1, b_{22}^2, \dots, b_{22}^m)\rangle & \dots & \langle d_{2p}^*, (b_{2p}^1, b_{2p}^2, \dots, b_{2p}^m)\rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle d_{r1}^*, (b_{r1}^1, b_{r1}^2, \dots, b_{r1}^m)\rangle & \langle d_{r2}^*, (b_{r2}^1, b_{r2}^2, \dots, b_{r2}^m)\rangle & \dots & \langle d_{rp}^*, (b_{rp}^1, b_{rp}^2, \dots, b_{rp}^m)\rangle \end{pmatrix}.$$
3. **Allocation of weights to the decision criteria by each expert**:
In most real world problems, not only the experts but also the attributes should have unequal impact on the overall qualification of the elements under evaluation. Every attribute has its own influence on the value or adequacy of the alternatives. Definitely, this is another major factor affecting the decision of the experts. It is therefore essential to consider the relative importance of each and every attribute so that a most qualified decision can be made.
The relative significance of the criteria is expressed in the form of weights allocated by the experts. In the current mFNS environment, these weights are in the form of $mFNSNs$. The individual weight associated with each criterion is represented as $W_s^t = \langle d_s^{(t)}, (\omega_s^{(t)^1}, \omega_s^{(t)^2}, \dots, \omega_s^{(t)^m})\rangle$, where $d_s^{(t)}$ is the weight grade and $\omega_s^{(t)^i} = \rho_s^{(t)^i} \circ \mu$ is the weighting membership grade given to the criterion c_s by the expert e_t for the ith pole, respectively.
Now, these independent weights given by the team of experts are assembled to construct a vector of weights $W = (W_1, W_2, \dots, W_p)^T$. To this purpose we resort to the $mFNSWA$ operator considered above. The output is as follows:
$$W_s = mFNSWA_\epsilon(W_s^{(1)}, W_s^{(2)}, \dots, W_s^{(q)}) \tag{9}$$
$$= \epsilon_1 W_s^{(1)} \oplus \epsilon_2 W_s^{(2)} \oplus \dots \oplus \epsilon_2 W_s^{(q)} \tag{10}$$
$$= \langle d_s^*, 1-\prod_{t=1}^{q}(1-\rho_s^{(t)^i})^{\epsilon_t}\rangle, \tag{11}$$
where, $\omega_s^{*^i} = 1-\prod_{t=1}^{q}(1-\rho_s^{(t)^i})^{\epsilon_t}$ and,
$$d_s^* = \max(d_s^{(1)}, d_s^{(2)}, \dots, d_s^{(q)}). \tag{12}$$

$$\Gamma^{(t)^i} = \begin{array}{c} \\ a_1 \\ a_2 \\ \vdots \\ a_r \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_p \end{array} \\ \begin{pmatrix} (\langle d_{11}, (b_{11}^{(t)^1}, b_{11}^{(t)^2}, \ldots, b_{11}^{(t)^m})\rangle & \langle d_{12}, (b_{12}^{(t)^1}, b_{12}^{(t)^2}, \ldots, b_{12}^{(t)^m})\rangle & \ldots & \langle d_{1p}, (b_{1p}^{(t)^1}, b_{1p}^{(t)^2}, \ldots, b_{1p}^{(t)^m})\rangle) \\ (\langle d_{21}, (b_{21}^{(t)^1}, b_{21}^{(t)^2}, \ldots, b_{21}^{(t)^m})\rangle & \langle d_{22}, (b_{22}^{(t)^1}, b_{22}^{(t)^2}, \ldots, b_{22}^{(t)^m})\rangle & \ldots & \langle d_{2p}, (b_{2p}^{(t)^1}, b_{2p}^{(t)^2}, \ldots, b_{2p}^{(t)^m})\rangle) \\ \vdots & \vdots & \ddots & \vdots \\ (\langle d_{r1}, (b_{r1}^{(t)^1}, b_{r1}^{(t)^2}, \ldots, b_{r1}^{(t)^m})\rangle & \langle d_{r2}, (b_{r2}^{(t)^1}, b_{r2}^{(t)^2}, \ldots, b_{r2}^{(t)^m})\rangle & \ldots & \langle d_{rp}, (b_{rp}^{(t)^1}, b_{rp}^{(t)^2}, \ldots, b_{rp}^{(t)^m})\rangle) \end{pmatrix} \end{array}.$$

Box I.

Further, these aggregate weights are normalized so that their sum equals 1, and they all remain in $[0, 1]$. For normalization of the weights, the following formula is used.

$$N_s = \frac{[\frac{d_s^*}{N-1} + \omega_s^{*^1} + \omega_s^{*^2} + \cdots + \omega_s^{*^m}]}{\sum_{s=1}^{p}[\frac{d_s^*}{N-1} + \omega_s^{*^1} + \omega_s^{*^2} + \cdots + \omega_s^{*^m}]} \tag{13}$$

4. **Construction of aggregated weighted m-polar fuzzy N-soft decision matrix:**
The $AmFNSDM$ and the aggregated weights of the criteria assigned by the experts are merged to construct the aggregated weighted m-polar fuzzy N-soft decision matrix ($AWmFNSDM$) that embraces the collective decision of the experts and the relative importance of the criteria. The required $AWmFNSDM$, $D' = (F_{ls})_{r\times p}$, is generated as follows:

$$F_{ls} = \beta_{ls} \otimes W_s \tag{14}$$

$$= (d'_{ls}, h) \tag{15}$$

where $H = (h^1, h^2, \ldots, h^m)$ with

$$h^i = (\rho_{ls}^i \circ \mu)(\rho_s^i \circ \mu) \tag{16}$$

and

$$d'_{ls} = \min(d_{ls}, d_s). \tag{17}$$

The $AWmFNSDM$ constructed above can be presented in the following notation:

$$D' = \begin{pmatrix} \langle d'_{11}, (h_{11}^1, h_{11}^2, \ldots, h_{11}^m)\rangle & \langle d'_{12}, (h_{12}^1, h_{12}^2, \ldots, h_{12}^m)\rangle & \ldots & \langle d'_{1p}, (h_{1p}^1, h_{1p}^2, \ldots, h_{1p}^m)\rangle \\ \langle d'_{21}, (h_{21}^1, h_{21}^2, \ldots, h_{21}^m)\rangle & \langle d'_{22}, (h_{22}^1, h_{22}^2, \ldots, h_{22}^m)\rangle & \ldots & \langle d'_{2p}, (h_{2p}^1, h_{2p}^2, \ldots, h_{2p}^m)\rangle \\ \vdots & \vdots & \vdots & \vdots \\ \langle d'_{r1}, (h_{r1}^1, h_{r1}^2, \ldots, h_{r1}^m)\rangle & \langle d'_{r2}, (h_{r2}^1, h_{r2}^2, \ldots, h_{r2}^m)\rangle & \ldots & \langle d'_{rp}, (h_{rp}^1, h_{rp}^2, \ldots, h_{rp}^m)\rangle \end{pmatrix},$$

where $h_{ls}^i = \rho_{ls}^{(*)^i} \circ \mu,\ i = 1, 2, \ldots, m$.

5. **Construction of the score matrix**:
The ELECTRE I method works on the principle of comparison between the alternatives. In order to determine the superiority and inferiority of the alternatives, we rely on their respective score degrees. Alternatives having a higher score value are considered as dominant and superior to others with lower-level scores. The score value can be calculated by using (1), which gives

$$S(h_{ls}) = (\frac{d'_{ls}}{N-1}) + (\frac{h_{ls}^1 + h_{ls}^2 + \cdots + h_{ls}^m}{m}). \tag{18}$$

A matrix can capture all the score values in an organized and systematic manner. This score matrix is constructed as follows:

$$T = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1p} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ \gamma_{r1} & \gamma_{r2} & \cdots & \gamma_{rp} \end{pmatrix}.$$

6. **Construction of the m-polar fuzzy N-soft concordance and discordance sets:**
The main argument of ELECTRE I is the contrast or comparison between any pair of the alternatives, and for that purpose concordance and discordance sets are used. In an mFNS environment, one needs to use an mFNS concordance set which is the collection of all the indices of those criteria that suggest dominance of one alternative over another. The mFNS concordance set, evaluated on the basis of the score degree, is formulated as follows:

$$\Omega_{fd} = \{k : S(h_{fk}) \geq S(h_{dk}); f \neq d; f, d = 1, 2, \ldots, r; k = 1, 2, \ldots, p\}. \tag{19}$$

On the contrary, the mFNS discordance set speaks for the inferiority of one alternative over another with respect to the specified decision criteria. This set is formed by the collection of all the indices of those criteria that suggest the inferiority of an alternative over another. The mFNS discordance set, evaluated on the basis of the score degree, is formulated as follows:

$$\Omega'_{fd} = \{k : S(h_{fk}) < S(h_{dk}); f \neq d; f, d = 1, 2, \ldots, r; k = 1, 2, \ldots, p\}. \tag{20}$$

7. **Construction of the m-polar fuzzy N-soft concordance indices and matrix:**
The next step must be the determination of the levels of supremacy. Each mFNS concordance index captures the level of superiority of an alternative over others. It is actually the sum of the normalized weights of the indexed criteria responsible for the dominance of the alternative over others, therefore it is computed as follows:

$$\lambda_{fd} = \sum_{k \in \Omega_{fd}} N_k. \tag{21}$$

These values are arranged in a comprehensive matrix, called the mFNS concordance matrix. Hence the mFNS concordance matrix, $\mho = (\lambda_{fd})_{r\times r}$, is designed as follows:

$$\mho = \begin{pmatrix} - & \lambda_{12} & \cdots & \lambda_{1(r-1)} & \lambda_{1r} \\ \lambda_{21} & - & \cdots & \lambda_{2(r-1)} & \lambda_{2r} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_{(r-1)1} & \lambda_{(r-1)2} & \cdots & - & \lambda_{(r-1)r} \\ \lambda_{r1} & \lambda_{r1} & \cdots & \lambda_{r(r-1)} & - \end{pmatrix}.$$

8. **Construction of m-polar fuzzy N-soft discordance index and matrix:**
The next natural step must be the determination of the level of inferiority of an alternative over others according to the selected criteria. The mFNS discordance index does this job. It is actually the sum of the normalized weights of the indexed

criteria responsible for the dominance of the alternative over others, therefore it is computed as follows:

$$\lambda'_{fd} = \frac{\max\limits_{k\in\Omega'_{fd}} d(y_{fk}, y_{dk})}{\max\limits_{k} d(y_{fk}, y_{dk})}; f \neq d. \tag{22}$$

Here, $d(y_{fk}, y_{dk})$ is the Euclidean distance between the m-polar fuzzy N-soft numbers as defined in (4). Thus it can be calculated as follows:

$$d(y_{fk}, y_{dk}) = \sqrt{\frac{(\frac{d'_{fk}-d'_{dk}}{N-1})^2 + \sum\limits_{i=1}^{m}(h^i_{fk} - h^i_{dk})^2}{m}}. \tag{23}$$

These values are arranged in a comprehensive matrix, called the mFNS discordance matrix. Hence the mFNS discordance matrix, $\mho' = (\lambda'_{fd})_{r\times r}$, is designed as follows:

$$\mho' = \begin{pmatrix} - & \lambda'_{12} & \cdots & \lambda'_{1(r-1)} & \lambda'_{1r} \\ \lambda'_{21} & - & \cdots & \lambda'_{2(r-1)} & \lambda'_{2r} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda'_{(r-1)1} & \lambda'_{(r-1)2} & \cdots & - & \lambda'_{(r-1)r} \\ \lambda'_{r1} & \lambda'_{r1} & \cdots & \lambda'_{r(r-1)} & - \end{pmatrix}.$$

9. **Evaluation of the mFNS concordance level and dominance matrix:**
To check the effectiveness of the mFNS concordance indices and exclude the incompetent alternatives, we use a threshold. It is called the mFNS concordance level, which is compared with the mFNS concordance indices so that only the competent alternatives are preserved. The m-polar fuzzy concordance level L is the average of all m-polar fuzzy concordance indices, i.e.,

$$L = \frac{1}{r(r-1)} \sum_{f=1}^{r}\sum_{d=1}^{r} \lambda_{fd};\ f \neq d. \tag{24}$$

The comparison between the mFNS concordance indices and L gives rise to the mFNS concordance dominance matrix. It contains an organized view of the outranking competent alternatives. The entries of mFNS concordance dominance matrix $J = (j_{fd})_{r\times r}$ are computed as follows: for each $f \neq d$,

$$j_{fd} = \begin{cases} 1 & \text{when } \lambda_{fd} \geq L, \\ 0 & \text{when } \lambda_{fd} < L. \end{cases} \tag{25}$$

10. **Evaluation of the mFNS discordance level and dominance matrix:**
Similarly to the previous step, it is also convenient to outrank alternatives that are inferior enough. We do this by comparing their mFNS discordance indices with a threshold value or mFNS discordance level, L'. It is the average of all the values of mFNS discordance indices, i.e.,

$$L' = \frac{1}{r(r-1)} \sum_{f=1}^{r}\sum_{d=1}^{r} \lambda'_{fd};\ f \neq d. \tag{26}$$

The comparison between the mFNS discordance indices and L' gives rise to the mFNS discordance dominance matrix. The entries of mFNS discordance dominance matrix $J' = (j'_{fd})_{r\times r}$ can be evaluated by using the following formula: for each $f \neq d$,

$$j'_{fd} = \begin{cases} 0 & \text{when } \lambda'_{fd} \geq L; \\ 1 & \text{when } \lambda'_{fd} < L. \end{cases} \tag{27}$$

11. **Aggregated mFNS dominance matrix:**
The corresponding entries of the mFNS concordance/discordance dominance matrices are multiplied to construct an aggregated mFNS dominance matrix that captures their combined effect. It highlights the outranking relation among the alternatives under the selected criteria. Therefore the entries of the aggregated mFNS dominance matrix, $Q = (q_{fd})_{r\times r}$, can be calculated by the expression: for each $f \neq d$,

$$q_{fd} = j_{fd} \cdot j'_{fd}. \tag{28}$$

12. **Display of an outranking graph**:
In the last step of the ELECTRE I method, an outranking graph, $G = (V, E)$, shows the outranking relation among all the alternatives captured by the aggregated mFNS dominance matrix. It encapsulates information about the superior and inferior alternatives, based on the comparison of their mFNS concordance and discordance indices with the mFNS concordance and discordance levels. In an outranking graph, the set of vertices (V) represents the alternatives, whereas the set of edges (E) represents the relations among them. An edge between the two alternatives shows the superiority or inferiority of an alternative over the other. An arrow head from an alternative, a_f, towards an alternative, a_d, represents the supremacy of a_f over a_d. The outranking graph can show the following situations, as a function of the entries of the aggregated mFNS dominance matrix:

- If $q_{fd} = 1$ and $q_{df} = 0$, then an arc is drawn from a_f pointing towards a_d. This shows that alternative a_f is superior to a_d, i.e., $a_f \geq a_d$; and vice versa.
- If $q_{fd} = 1$ and $q_{df} = 1$, then a two way arc is drawn between a_f and a_d, showing the equal performance by both alternatives. Neither of them is superior or inferior over the other one.
- If $q_{fd} = 0$ and $q_{df} = 0$, then no arc is drawn between a_f and a_d. That means that both alternatives are incomparable. It remains undecided whether one is superior or inferior over the other.

3. Selection of a rehabilitation center in the United States

This section contains a detailed discussion of the statistics of illicit drug uses in the U.S, the factors responsible for the illegal use of drugs among teenagers, their health risks as a result of high consumption of drugs, how to save them from drug addictions, the features of **the** rehabilitation center, selection of the best rehabilitation centers for the treatment of the drug addicted people and the usage of the MCGDM method proposed in the previous section for the selection of a rehabilitation center.

3.1. Some scary facts about teenage drug abuse in the U.S.

People in developed and modernized states (such as Europe) are prone towards the illicit drug use, more likely in their teens. The statistics collected by the National Survey on Drug Use and Health (NSDUH) in 2019 have shocked the entire mankind. In 2019, the NSDUH survey found that 165.4 million people in the U.S. aged 12 and over had used substances, including tobacco, alcohol, or illicit drugs, in the past month. NSDUH also indicated that 20.8% of people aged 12 and over – which amounts to over one-fifth of the population – had used an illicit drug in the past year. This represents a notable increase from 2015, which is mainly due to an increase in the use of marijuana. In 2019, 17.5% of people aged 12 and over reported past-year marijuana use. This was a 4% increase from 2015 prevalence rates. Young adults aged 18 to 25 have the highest prevalence rates (35.4%). (Source: https://www.verywellmind.com).

3.2. Why do teenagers involve in drug abuse?

After a thorough examination of all the data on drug addicts in their teens, the topic of why young people become involved in such terrible addictions is frequently raised. Teenage drug abuse is caused

Fig. 1. Steps to help teenagers stay drug free.

by a variety of factors, such as the desire to fit in with their drug-using friends, the notion that addiction is a sign of coolness, depression and anxiety, poor mental health, frayed family ties, childhood trauma, bullying, and a host of other issues.

3.3. The chilling consequences of teenage drug abuse

There are negative effects on the body, mind, society, or soul. Due to a chemical alteration in the brain, drug addiction forces a person to remain in dark areas and erroneous thinking. Drug misuse often has physical side effects like cuts, bruises, track marks, burns to the lips, skin abscesses, infections (such HIV or Hepatitis C), physical dependence, and withdrawal symptoms. Abuse on a physical or sexual level is another effect of addiction. Depression, anxiety, mood swings, suicidal thoughts, and psychosis are examples of mental effects. Feeling helpless, lonely, afraid, and restless for no apparent cause are some examples of the spiritual repercussions.

3.4. How to help teenagers stay drug free

To stop this captivity, the drug user may need assistance from parents, relatives, instructors, and the community. Parents and teachers should establish a close relationship with children, show them how much you value their lives and how much you care about them, and inform them of the harmful effects of drug use on their mental, physical, and social well-being. Parents should also be aware of their children's social environment, including who their children are friends with, who they are not, and who are their competitors in school, college, or the workplace. You should seek professional assistance for them and yourself because experts can not only advise you on how to treat them but can also help you cope with that difficult time, etc. All of these efforts can be combined in one location, and that is a rehabilitation center where a completely relaxing and soothing environment is being provided for the assistance of the teenage drug users (see Fig. 1).

3.5. Selection of a rehabilitation center

U.S., being a developed country, has constructed many rehabilitation centers in their cities. Still, there are some cities that lack facilities due to financial and economical crisis. *San Juan* is one of them, with 41.4% of its population living below the poverty line, 58.8% of its population under 18 living below the poverty line, and 31.1% of its population over 65 living below the poverty line (Source: https://www.neoch.org/top-poorest-cities-in-us). With such immense poverty and drug addiction rate, we have considered the case of *San Juan* city for the construction of a rehabilitation center. The main aim is to select a most appropriate model among various rehabilitation centers of the developed states of the U.S., so the construction of the most suitable rehabilitation center facility can be made possible for the San Juan city. With this motivation, we apply mFNS-ELECTRE I strategy proposed above is useful in selecting a best rehabilitation center in *San Juan*.

To select a best rehabilitation center in *San Juan*, suppose the city's higher authorities have short-listed a set of five alternatives (rehabilitation centers), $\Phi = \{\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5\}$, and a group of experts, $\zeta = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\}$, who will select the best rehabilitation center from the lists on the basis of the set of criteria $\xi = \{\eta_1, \eta_2, \eta_3, \eta_4\}$ that further depend on four poles. The set of experts consists of:

- Project Supervisor (ς_1):
  The project supervisor will be in charge of the whole project who will keep an eye on the budget of the selection, quality of installed equipments, behavior of the staff, structure of the rehabilitation center and programs offered.
- Finance Supervisor (ς_2):
  He will keep check on the budget of the project, import or export of the equipments, installation of the devices. In addition to this, he will administer the fee packages of all the offered programs according to the financial condition of the surrounding inhabitants.
- Health Supervisor (ς_3):
  His main priority will be to provide a healthy, nurturing, and safe environment for the patients. Everything in the center, including beds, dinner tables, crockery, kitchen items, testing machines, and so on, should be sanitized and hygienic.
- Chief of the staff (ς_4):
  He will guarantee the hiring of the trained and professional staff ensuring their exceptional performance and their respectful behavior towards the patients.

These experts make the decision after a thorough analysis of the considered criteria that are displayed in Fig. 4 (see Fig. 2).

The given set of criteria comprises the following details:

- Structure of a rehabilitation center (η_1):
  The anatomy of the rehabilitation center is very essential component fostering healing and health to the drug abusers. Its structure further depends on 3-poles, named as interior, location and the neighboring of the center.

Fig. 2. Figure of decision criteria.

Table 1
Weightage of experts.

$\alpha\backslash\zeta$	ς_1	ς_2	ς_3	ς_4
α_n	0.25	0.25	0.25	0.25

- Offered programs (η_2):
  There should be a very wise selection of the offered programs for the drug abusers comprises of 3-poles as whole-bodied approach, faith-based approach and gender specific approach that ensure both mental health of the patient and after-care treatment.
- Quality of staff (η_3):
  The appointed staff must be highly trained and professional who must be available 24/7 to the patients. The quality of staff rests on 3-poles, namely, highly trained professionals, individual designed care and the dedicated staff.
- Availability of the rehabilitation center (η_4):
  Easy to access, 24/7 service and guarantee of the privacy are the 3-poles on which the availability of the rehabilitation center rely. There must be distribution of pamphlets of the center, advertisement on the internet, active call and mailing service and assurance of the patient's privacy so that he would be benefitted by the treatment at his best.

3.6. Steps of selection of a rehabilitation center by using mFNS-ELECTRE I

The data for the calculations are taken as the test data. To get a clear and comprehensive view of the steps followed in performing our proposed MCGDM strategy, *m*F*N*S-ELECTRE I, an inclusive pictorial diagram is being drawn in Fig. 3. The step-by-step procedure of evaluation of *m*F*N*S-ELECTRE I with a set of alternatives (Φ), experts (ζ) and criteria (ξ) is as follows:

1 The higher authorities of *San Juan* assign weights to each decision making expert according to the worth and credibility of their positions in the selection of the best rehabilitation center to be constructed. The weights granted to each expert is arranged in Table 1.

2 Each decision making expert, after a thorough analysis of the alternatives with respect to the criteria under consideration, submits their individual assessed data in the form of $3F6S$ numbers that are arranged in the $3F6SDMs$ in Tables 2, 3, 4, 5.

Table 2
$3F6SDM$ of the project supervisor (ς_1).

ς_1	η_1	η_2	η_3	η_4
φ_1	(5, 0.82, 0.95, 0.89)	(3, 0.47, 0.53, 0.59)	(2, 0.28, 0.35, 0.38)	(0, 0.11, 0.13, 0.12)
φ_2	(4, 0.66, 0.78, 0.79)	(1, 0.15, 0.19, 0.16)	(2, 0.25, 0.29, 0.36)	(3, 0.47, 0.59, 0.57)
φ_3	(5, 0.98, 0.85, 0.92)	(4, 0.67, 0.79, 0.68)	(4, 0.69, 0.75, 0.77)	(5, 0.88, 0.94, 0.85)
φ_4	(0, 0.11, 0.14, 0.13)	(2, 0.25, 0.31, 0.24)	(1, 0.18, 0.16, 0.15)	(2, 0.21, 0.33, 0.23)
φ_5	(4, 0.67, 0.73, 0.79)	(1, 0.17, 0.16, 0.19)	(4, 0.67, 0.78, 0.79)	(3, 0.47, 0.55, 0.59)

Table 3
$3F6SDM$ of the finance supervisor (ς_2).

ς_2	η_1	η_2	η_3	η_4
φ_1	(3, 0.47, 0.53, 0.58)	(2, 0.28, 0.33, 0.29)	(0, 0.12, 0.14, 0.11)	(1, 0.19, 0.16, 0.17)
φ_2	(4, 0.69, 0.74, 0.77)	(5, 0.89, 0.93, 0.96)	(3, 0.46, 0.52, 0.58)	(2, 0.37, 0.29, 0.24)
φ_3	(5, 0.87, 0.95, 0.84)	(5, 0.82, 0.99, 0.87)	(4, 0.78, 0.75, 0.63)	(5, 0.97, 0.92, 0.88)
φ_4	(1, 0.16, 0.18, 0.17)	(0, 0.11, 0.12, 0.14)	(2, 0.25, 0.38, 0.29)	(2, 0.21, 0.33, 0.23)
φ_5	(3, 0.49, 0.57, 0.52)	(2, 0.37, 0.32, 0.29)	(3, 0.46, 0.57, 0.59)	(4, 0.66, 0.77, 0.76)

Table 4
$3F6SDM$ of the health supervisor (ς_3).

ς_3	η_1	η_2	η_3	η_4
φ_1	(2, 0.39, 0.23, 0.38)	(1, 0.18, 0.15, 0.19)	(3, 0.59, 0.44, 0.51)	(2, 0.38, 0.27, 0.39)
φ_2	(3, 0.47, 0.55, 0.58)	(4, 0.78, 0.69, 0.72)	(5, 0.83, 0.98, 0.88)	(2, 0.37, 0.36, 0.29)
φ_3	(4, 0.79, 0.69, 0.75)	(5, 0.97, 0.85, 0.92)	(5, 0.89, 0.92, 0.97)	(5, 0.87, 0.89, 0.99)
φ_4	(2, 0.21, 0.34, 0.32)	(1, 0.19, 0.18, 0.17)	(3, 0.47, 0.51, 0.48)	(0, 0.09, 0.11, 0.08)
φ_5	(2, 0.38, 0.35, 0.29)	(1, 0.18, 0.16, 0.15)	(2, 0.38, 0.29, 0.36)	(3, 0.48, 0.58, 0.54)

Table 5
$3F6SDM$ of the staff supervisor (ς_4).

ς_4	η_1	η_2	η_3	η_4
φ_1	(4, 0.66, 0.77, 0.76)	(3, 0.48, 0.57, 0.54)	(1, 0.18, 0.16, 0.19)	(2, 0.28, 0.35, 0.27)
φ_2	(3, 0.58, 0.44, 0.57)	(4, 0.68, 0.79, 0.74)	(2, 0.28, 0.34, 0.29)	(0, 0.06, 0.09, 0.08)
φ_3	(5, 0.94, 0.88, 0.99)	(4, 0.69, 0.79, 0.76)	(5, 0.92, 0.87, 0.88)	(4, 0.68, 0.79, 0.74)
φ_4	(1, 0.19, 0.15, 0.16)	(2, 0.36, 0.25, 0.38)	(0, 0.04, 0.09, 0.12)	(1, 0.16, 0.15, 0.18)
φ_5	(4, 0.68, 0.79, 0.66)	(1, 0.18, 0.19, 0.15)	(2, 0.28, 0.39, 0.27)	(0, 0.05, 0.07, 0.11)

3 This discrete assessment from each decision maker is assembled in one matrix that have collective effect of the decision of the experts by using $mFNSWA$ operator, given in Eqs. (7) and (8), represented as $A3F6SDM$ in Table 6.

4 Each decision making expert allots some weights to each decision criterion according to its relative importance in the considered project. The weights assigned by the experts in $3F6S$ environment are displayed in Table 7.

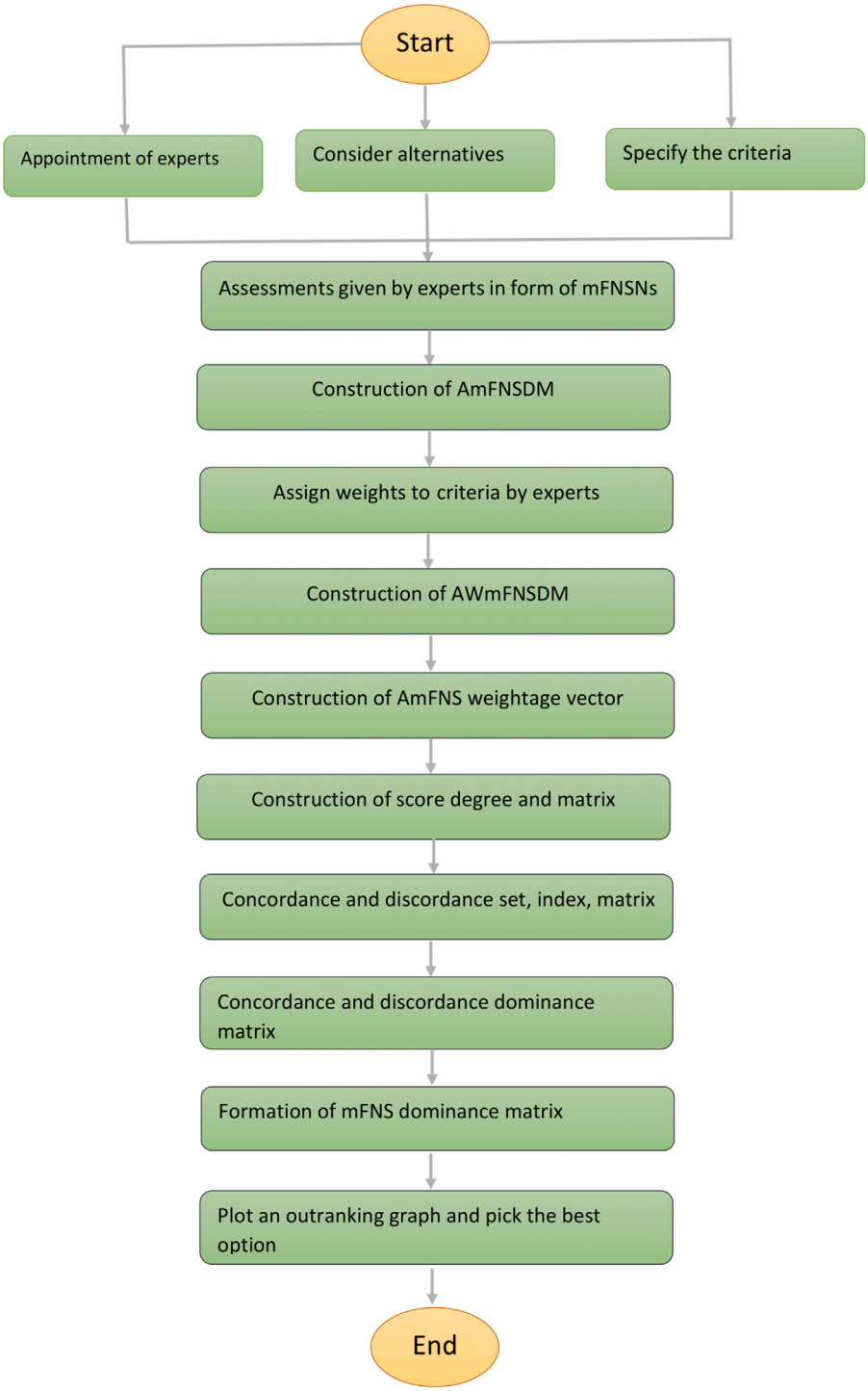

Fig. 3. Figure of algorithm of $mFNS$-ELECTRE I.

5 The assigned $3F6S$ weights by the experts are now aggregated by using $3F6SWA$ operator given in Eqs. (11) and (12), to construct an $A3F6S$ weightage vector of the decision criteria displayed in Table 8.

6 These aggregated weights are normalized so that the sum of the weights is equal to 1 and the calculated weights remain in $[0, 1]$. The normalized weights are calculated by Eq. (13) and shown in Table 9.

7 The $A3F6SDM$ and the weightage vector of each decision criterion are merged to form an $AW3F6SDM$, as specified in Eqs. (16) and (17). The formulated $AW3F6SDM$ is presented in Table 10.

8 The entries of the score matrix in Table 11 contain the score degree of each entry of $A3F6SDM$, computed by Eq. (18).

9 The main theme of ELECTRE I is the comparison of the alternatives based on the score degrees so, the next step is the formulation

Table 6
Aggregated $3F6SDM$.

D	η_1	η_2	η_3	η_4
φ_1	(5, 0.62, 0.75, 0.71)	(3, 0.36, 0.42, 0.43)	(3, 0.32, 0.28, 0.32)	(2, 0.25, 0.23, 0.24)
φ_2	(4, 0.61, 0.65, 0.69)	(5, 0.72, 0.75, 0.78)	(5, 0.53, 0.74, 0.61)	(3, 0.33, 0.36, 0.32)
φ_3	(5, 0.92, 0.87, 0.92)	(5, 0.85, 0.91, 0.83)	(5, 0.84, 0.84, 0.87)	(5, 0.89, 0.90, 0.92)
φ_4	(2, 0.17, 0.21, 0.20)	(2, 0.23, 0.22, 0.24)	(3, 0.25, 0.31, 0.28)	(2, 0.17, 0.24, 0.18)
φ_5	(4, 0.57, 0.65, 0.61)	(2, 0.23, 0.21, 0.20)	(4, 0.47, 0.55, 0.55)	(4, 0.45, 0.55, 0.55)

Table 7
Weightage of criteria by experts.

	ς_1	ς_2	ς_3	ς_4
η_1	(4, 0.69, 0.78, 0.77)	(3, 0.47, 0.56, 0.59)	(4, 0.71, 0.64, 0.68)	(2, 0.37, 0.22, 0.27)
η_2	(5, 0.87, 0.94, 0.99)	(4, 0.62, 0.73, 0.68)	(5, 0.91, 0.94, 0.88)	(3, 0.41, 0.54, 0.47)
η_3	(3, 0.51, 0.46, 0.55)	(3, 0.48, 0.52, 0.59)	(4, 0.77, 0.66, 0.69)	(5, 0.98, 0.89, 0.99)
η_4	(4, 0.79, 0.68, 0.69)	(4, 0.62, 0.77, 0.79)	(5, 0.86, 0.92, 0.95)	(5, 0.92, 0.83, 0.96)

Table 8
Aggregated weightage of criteria.

	$3F6S$ aggregated weightage of criteria
η_1	4, 0.58, 0.59, 0.61
η_2	5, 0.77, 0.85, 0.88
η_3	5, 0.81, 0.69, 0.86
η_4	5, 0.83, 0.82, 0.89

Table 9
Normalized weights.

	Normalized weights
η_1	0.20
η_2	0.27
η_3	0.26
η_4	0.27

Table 10
Aggregated weighted $3F6SDM$.

	η_1	η_2	η_3	η_4
φ_1	(4, 0.36, 0.44, 0.43)	(3, 0.28, 0.36, 0.38)	(3, 0.26, 0.19, 0.28)	(2, 0.21, 0.19, 0.21)
φ_2	(4, 0.35, 0.38, 0.42)	(5, 0.55, 0.64, 0.69)	(5, 0.43, 0.51, 0.52)	(3, 0.27, 0.30, 0.28)
φ_3	(4, 0.53, 0.51, 0.56)	(5, 0.65, 0.77, 0.73)	(5, 0.68, 0.58, 0.75)	(5, 0.74, 0.74, 0.82)
φ_4	(2, 0.10, 0.12, 0.12)	(2, 0.18, 0.19, 0.21)	(3, 0.20, 0.21, 0.24)	(2, 0.14, 0.20, 0.16)
φ_5	(4, 0.33, 0.38, 0.37)	(2, 0.18, 0.18, 0.18)	(4, 0.38, 0.38, 0.47)	(4, 0.37, 0.45, 0.49)

Table 11
Score degree matrix.

	η_1	η_2	η_3	η_4
φ_1	1.21	0.94	0.84	0.60
φ_2	1.18	1.63	1.49	0.88
φ_3	1.33	1.72	1.67	1.77
φ_4	0.51	0.59	0.82	0.57
φ_5	1.16	0.58	1.21	1.24

Table 12
$3F6S$ concordance set.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	{1}	{}	{1, 2, 3, 4}	{1, 2}
φ_2	{2, 3, 4}	–	{}	{1, 2, 3, 4}	{1, 2}
φ_3	{1, 2, 3, 4}	{1, 2, 3, 4}	–	{1, 2, 3, 4}	{1, 2, 3, 4}
φ_4	{}	{}	{}	–	{}
φ_5	{3, 4}	{4}	{}	{1, 3, 4}	–

of the concordance and discordance set by comparing the score degrees, given in Eqs. (19) and (20), respectively. The concordance and discordance sets are presented in Tables 12 and 13, respectively.

10 To check the level of superiority, $3F6S$ concordance indices are formulated with the assistance of Eq. (21). To give the concordance indices in an organized view, a matrix is formed. It is named as $3F6S$ concordance matrix, and it is given in Table 14.

Table 13
$3F6S$ discordance set.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	{2, 3, 4}	{1, 2, 3, 4}	{}	{3, 4}
φ_2	{1}	–	{1, 2, 3, 4}	{}	{3, 4}
φ_3	{}	{}	–	{}	{}
φ_4	{1, 2, 3, 4}	{1, 2, 3, 4}	{1, 2, 3, 4}	–	{1, 2, 3, 4}
φ_5	{1, 2}	{1, 2, 3}	{1, 2, 3, 4}	{2}	–

Table 14
$3F6S$ concordance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0.2	0	1	0.47
φ_2	0.8	–	0	1	0.47
φ_3	1	1	–	1	1
φ_4	0	0	0	–	0
φ_5	0.53	0.27	0	0.73	–

Table 15
Euclidean distance with respect to criterion η_1 and η_2.

	y_{11}	y_{21}	y_{31}	y_{41}	y_{51}		y_{12}	y_{22}	y_{32}	y_{42}	y_{52}
y_{11}	–	0.04	0.13	0.38	0.05	y_{12}	–	0.37	0.44	0.19	0.20
y_{21}	0.04	–	0.15	0.36	0.03	y_{22}	0.37	–	0.10	0.56	0.57
y_{31}	0.13	0.13	–	0.48	0.18	y_{32}	0.44	0.10	–	0.63	0.64
y_{41}	0.38	0.36	0.48	–	0.34	y_{42}	0.19	0.56	0.63	–	0.02
y_{51}	0.34	0.05	0.18	0.34	–	y_{52}	0.20	0.57	0.64	0.02	–

Table 16
Euclidean distance with respect to criterion η_3 and η_4.

	y_{13}	y_{23}	y_{33}	y_{43}	y_{53}		y_{14}	y_{24}	y_{34}	y_{44}	y_{54}
y_{13}	–	0.34	0.49	0.04	0.21	y_{14}	–	0.14	0.66	0.05	0.33
y_{23}	0.34	–	0.20	0.36	0.14	y_{24}	0.14	–	0.54	0.16	0.20
y_{33}	0.49	0.20	–	0.51	0.29	y_{34}	0.66	0.54	–	0.69	0.35
y_{43}	0.04	0.36	0.51	–	0.23	y_{44}	0.05	0.16	0.69	–	0.36
y_{53}	0.21	0.14	0.19	0.23	–	y_{54}	0.33	0.20	0.35	0.36	–

Table 17
$3F6S$ discordance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	1	1	0	1
φ_2	0.108	–	1	0	0
φ_3	0	0	–	0	0
φ_4	1	1	1	–	0.003
φ_5	1	1	1	0.056	–

11 To examine the level of inferiority, the Euclidean distances between the entries of aggregated weighted m-polar decision matrix, evaluated by Eq. (23), are given in Tables 15 and 16. Further, these discordance indices are arranged in Table 17 to construct the m-polar discordance matrix.

12 To check the effectiveness of $3F6S$ concordance and discordance indices, a threshold value is evaluated, named as $3F6S$ concordance and discordance level. Using Eqs. (24) and (26), the concordance level is $L = 0.47$ and the discordance level is $L' = 0.51$.

13 The comparison between $3F6S$ concordance indices and $3F6S$ concordance level gives rise to the $3F6S$ concordance dominance matrix that puts forward the competent alternatives, where the alternatives having higher $3F6S$ concordance index than $3F6S$ concordance level are preferable. The $3F6S$ concordance dominance matrix in Table 18 is computed by Eq. (25).

Table 18
$3F6S$ concordance dominance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0	0	1	1
φ_2	1	–	0	1	1
φ_3	1	1	–	1	1
φ_4	0	0	0	–	0
φ_5	1	0	0	1	–

Table 19
$3F6S$ discordance dominance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0	0	1	0
φ_2	1	–	0	1	1
φ_3	1	1	–	1	1
φ_4	0	0	0	–	1
φ_5	0	0	0	1	–

Table 20
Aggregated $3F6S$ dominance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0	0	1	0
φ_2	1	–	0	1	1
φ_3	1	1	–	1	1
φ_4	0	0	0	–	0
φ_5	0	0	0	1	–

Table 21
Analysis drawn from outranking graph.

	Outranked alternatives	Incomparable alternatives
φ_1	φ_4	φ_5
φ_2	φ_1, φ_4 and φ_5	
φ_3	φ_1, φ_2, φ_4, φ_5	
φ_4	None	
φ_5	φ_4	φ_1

14 On the same lines, the comparison between $3F6S$ discordance indices and $3F6S$ discordance level gives rise to the $3F6S$ discordance dominance matrix that puts forward the competent alternatives, where the alternatives having lower $3F6S$ discordance index than $3F6S$ discordance level are preferable. The $3F6S$ discordance dominance matrix in Table 19 is computed by Eq. (27).

15 The $3F6S$ concordance dominance matrix and $3F6S$ discordance dominance matrix are assembled in aggregated $3F6S$ dominance matrix, that exhibits the combined effect of $3F6S$ concordance dominance matrix and $3F6S$ discordance dominance matrix which highlights the outranking relation among the alternatives, arranged in Table 20 by Eq. (28).

16 The outranking graph plotted with respect to the outranking relation among the alternatives is displayed in Fig. 4. The information, inferred from the outranking graph, is organized in Table 21. According to outranking relations, φ_3 is the most preferred alternative because it outranks all the remaining alternatives, $\varphi_1, \varphi_2, \varphi_4$ *and* φ_5, whereas φ_1 *and* φ_5 are the non-comparable alternatives.

The partial outranking of the alternatives is shown in a directed graph (Fig. 4) where an arrow head is drawn from each superior to inferior option.

4. Comparative analysis

In this section, we compare the technique proposed in this paper with existing alternatives. We use the same application (selection of best rehabilitation center) that illustrated our methodology in the previous section. Two alternative methods, namely, PROMETHEE [26] and ELECTRE I [2], adapted to the m-polar fuzzy environment, will be considered. We shall observe that the computations coincide to select the same best alternative, which will validate our technique and will prove its credibility.

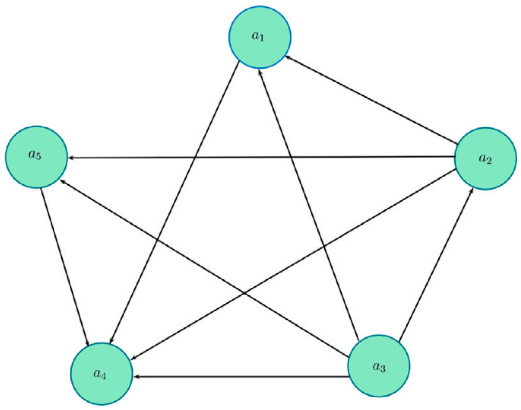

Fig. 4. Outranking graph of the alternatives.

Table 22
Comparison table for the criteria.

C	η_1	η_2	η_3	η_4
η_1	1	0.143	0.20	0.143
η_2	7	1	3	1
η_3	5	0.333	1	0.333
η_4	7	1	3	1

Table 23
Normalized weights.

	η_1	η_2	η_3	η_4
η_1	0.05	0.058	0.028	0.058
η_2	0.35	0.404	0.417	0.404
η_3	0.25	0.135	0.139	0.135
η_4	0.35	0.404	0.417	0.404

Table 24
Criteria weights (W).

	W
w_1	0.049
w_2	0.394
w_3	0.165
w_4	0.394

4.1. *m-polar fuzzy PROMETHEE*

The calculations corresponding to the application of the PROMETHEE technique on m-polar fuzzy environment [26] proceed as follows. First, we compute the required elements in Tables 22–24.

Now, after multiplication of the comparison table of criteria and criteria weights, the CW matrix is as follows:

$$CW = \begin{pmatrix} 0.195 \\ 1.626 \\ 0.673 \\ 1.626 \end{pmatrix}. \tag{29}$$

The maximum eigenvalue is calculated by applying the following formula:

$$\lambda_{max} = \frac{1}{4} \times [\frac{0.195}{0.049} + \frac{1.626}{0.394} + \frac{0.673}{0.165} + \frac{1.626}{0.394}] \tag{30}$$

$$= 4.078 \tag{31}$$

Table 25
$3FDM$ of the project supervisor (ς_1).

ς_1	η_1	η_2	η_3	η_4
φ_1	(0.82, 0.95, 0.89)	(0.47, 0.53, 0.59)	(0.28, 0.35, 0.38)	(0.11, 0.13, 0.12)
φ_2	(0.66, 0.78, 0.79)	(0.15, 0.19, 0.16)	(0.25, 0.29, 0.36)	(0.47, 0.59, 0.57)
φ_3	(0.98, 0.85, 0.92)	(0.67, 0.79, 0.68)	(0.69, 0.75, 0.77)	(0.88, 0.94, 0.85)
φ_4	(0.11, 0.14, 0.13)	(0.25, 0.31, 0.24)	(0.18, 0.16, 0.15)	(0.21, 0.33, 0.23)
φ_5	(0.67, 0.73, 0.79)	(0.17, 0.16, 0.19)	(0.67, 0.78, 0.79)	(0.47, 0.55, 0.59)

Table 26
$3FDM$ of the finance supervisor (ς_2).

ς_2	η_1	η_2	η_3	η_4
φ_1	(0.47, 0.53, 0.58)	(0.28, 0.33, 0.29)	(0.12, 0.14, 0.11)	(0.19, 0.16, 0.17)
φ_2	(0.69, 0.74, 0.77)	(0.89, 0.93, 0.96)	(0.46, 0.52, 0.58)	(0.37, 0.29, 0.24)
φ_3	(0.87, 0.95, 0.84)	(0.82, 0.99, 0.87)	(0.78, 0.75, 0.63)	(0.97, 0.92, 0.88)
φ_4	(0.16, 0.18, 0.17)	(0.11, 0.12, 0.14)	(0.25, 0.38, 0.29)	(0.21, 0.33, 0.23)
φ_5	(0.49, 0.57, 0.52)	(0.37, 0.32, 0.29)	(0.46, 0.57, 0.59)	(0.66, 0.77, 0.76)

Table 27
$3FDM$ of the health supervisor (ς_3).

ς_3	η_1	η_2	η_3	η_4
φ_1	(0.39, 0.23, 0.38)	(0.18, 0.15, 0.19)	(0.59, 0.44, 0.51)	(0.38, 0.27, 0.39)
φ_2	(0.47, 0.55, 0.58)	(0.78, 0.69, 0.72)	(0.83, 0.98, 0.88)	(0.37, 0.36, 0.29)
φ_3	(0.79, 0.69, 0.75)	(0.97, 0.85, 0.92)	(0.89, 0.92, 0.97)	(0.87, 0.89, 0.99)
φ_4	(0.21, 0.34, 0.32)	(0.19, 0.18, 0.17)	(0.47, 0.51, 0.48)	(0.09, 0.11, 0.08)
φ_5	(0.38, 0.35, 0.29)	(0.18, 0.16, 0.15)	(0.38, 0.29, 0.36)	(0.48, 0.58, 0.54)

Table 28
$3FDM$ of the staff supervisor (ς_4).

ς_4	η_1	η_2	η_3	η_4
φ_1	(0.66, 0.77, 0.76)	(0.48, 0.57, 0.54)	(0.18, 0.16, 0.19)	(0.28, 0.35, 0.27)
φ_2	(0.58, 0.44, 0.57)	(0.68, 0.79, 0.74)	(0.28, 0.34, 0.29)	(0.06, 0.09, 0.08)
φ_3	(0.94, 0.88, 0.99)	(0.69, 0.79, 0.76)	(0.92, 0.87, 0.88)	(0.68, 0.79, 0.74)
φ_4	(0.19, 0.15, 0.16)	(0.36, 0.25, 0.38)	(0.04, 0.09, 0.12)	(0.16, 0.15, 0.18)
φ_5	(0.68, 0.79, 0.66)	(0.18, 0.19, 0.15)	(0.28, 0.39, 0.27)	(0.05, 0.07, 0.11)

Table 29
Aggregated $3FDM$.

	η_1	η_2	η_3	η_4
φ_1	(0.585, 0.620, 0.653)	(0.353, 0.395, 0.403)	(0.293, 0.273, 0.298)	(0.240, 0.228, 0.238)
φ_2	(0.60, 0.628, 0.678)	(0.625, 0.650, 0.645)	(0.455, 0.533, 0.528)	(0.318, 0.333, 0.295)
φ_3	(0.895, 0.843, 0.875)	(0.788, 0.855, 0.808)	(0.820, 0.823, 0.813)	(0.850, 0.885, 0.865)
φ_4	(0.168, 0.203, 0.195)	(0.228, 0.215, 0.233)	(0.235, 0.285, 0.260)	(0.168, 0.230, 0.180)
φ_5	(0.555, 0.610, 0.565)	(0.225, 0.208, 0.195)	(0.448, 0.508, 0.503)	(0.415, 0.493, 0.50)

The consistency index is calculated as

$CI = 0.026$

by using the following formula:

$$CI = \frac{(\lambda_{max} - n)}{n - 1}, \tag{32}$$

where n is the total number of criteria. A consistency index near zero shows that the comparative values are consistent. After evaluating this index, we calculate the consistency ratio by dividing the consistency index by the random index as follows:

$$CR = \frac{CI}{RI}. \tag{33}$$

Here RI is the random index which is defined for different values of n.

In this case we have considered $n = 4$ and $RI = 0.90$. The consistency ratio is calculated as $CR = 0.0289$. As $0.0289 < 0.10$, it is acceptable. Let us consider the case study of the selection of the best rehabilitation center, but within the m-polar fuzzy environment. The assessments given by the experts and all the related calculations are given in Tables 25–31:

Now, the types of criteria are specified by the generalized preference functions given in Table 32. Further, the preference degree of every pair of alternative with respect to every criteria is calculated, and it is displayed in Table 33.

Table 30
Score matrix.

	η_1	η_2	η_3	η_4
φ_1	0.619	0.384	0.288	0.235
φ_2	0.635	0.640	0.505	0.315
φ_3	0.871	0.817	0.819	0.867
φ_4	0.189	0.225	0.260	0.193
φ_5	0.577	0.209	0.486	0.469

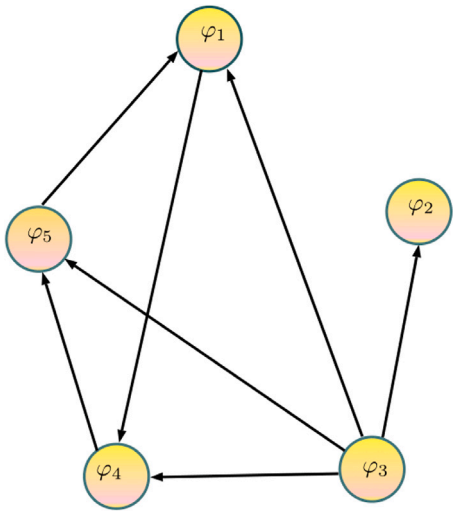

Fig. 5. Outranking graph of the alternatives.

The weighted averages of these preference functions are known as the respective multi-criteria preference index of alternatives. They are shown in Table 34.

The outgoing and incoming flows of the alternatives are presented in Table 35. The partial ranking of the alternatives is determined by considering the intersection of the positive and negative flows as follows:

$$\varphi_1 \hat{P} \varphi_4, \varphi_3 \hat{P} \varphi_1, \varphi_3 \hat{P} \varphi_2, \varphi_3 \hat{P} \varphi_4, \varphi_3 \hat{P} \varphi_5, \varphi_5 \hat{P} \varphi_1, \varphi_5 \hat{P} \varphi_4.$$

However, φ_2 and φ_1, φ_2 and φ_4, φ_2 and φ_5, and φ_2 and φ_3 are incomparable. This partial relation of PROMETHEE I is shown in Fig. 5. Now, the complete outranking of the alternatives by PROMETHEE II is shown in Table 36. From all the above calculations, we come to know that alternative φ_3 is selected as the best rehabilitation center to be constructed in the $U.S$, and the ordering of the alternatives is as follows:

$$\varphi_3 > \varphi_2 > \varphi_5 > \varphi_1 > \varphi_4.$$

4.2. *m-polar fuzzy ELECTRE I*

For the purpose of comparison, we consider the m-polar fuzzy environment and apply ELECTRE I technique on it [2]. Recall that the individual assessments of the experts with respect to each criterion for each alternative are considered in Tables 25, 26, 27, 28. The following calculations are being made in the selection of best rehabilitation center in the U.S.: (see Tables 37–52).

The partial outranking of the alternatives is shown in a directed graph (Fig. 6), where arrows point at inferior from superior alternatives.

Table 31
Deviation of the alternatives with respect to the criteria.

Alternatives	η_1	η_2	η_3	η_4	Alternatives	η_1	η_2	η_3	η_4
$\varphi_1\varphi_2$	−0.016	−0.256	−0.217	−0.08	$\varphi_1\varphi_3$	−0.252	−0.433	−0.531	−0.632
$\varphi_1\varphi_4$	0.430	0.159	0.028	0.042	$\varphi_1\varphi_5$	0.042	0.175	−0.198	−0.234
$\varphi_2\varphi_1$	0.016	0.256	0.217	0.08	$\varphi_2\varphi_3$	−0.236	−0.177	−0.314	−0.552
$\varphi_2\varphi_4$	0.446	0.415	0.245	0.112	$\varphi_2\varphi_5$	0.058	0.431	0.019	−0.154
$\varphi_3\varphi_1$	0.252	0.433	0.531	0.632	$\varphi_3\varphi_2$	0.236	0.177	0.314	0.552
$\varphi_3\varphi_4$	0.682	0.592	0.559	0.674	$\varphi_3\varphi_5$	0.294	0.608	0.333	0.398
$\varphi_4\varphi_1$	−0.430	−0.159	−0.028	0.042	$\varphi_4\varphi_2$	−0.446	−0.415	−0.245	−0.122
$\varphi_4\varphi_3$	0.682	−0.592	−0.559	−0.674	$\varphi_4\varphi_5$	−0.388	0.016	−0.226	−0.276
$\varphi_5\varphi_1$	−0.042	−0.175	0.198	0.234	$\varphi_5\varphi_2$	0.058	−0.431	−0.019	0.154
$\varphi_5\varphi_3$	−0.294	−0.608	−0.333	−0.398	$\varphi_5\varphi_4$	0.388	−0.016	0.226	0.276

Table 32
Type of criteria and preference functions.

Criteria	Max or Min	Type of criteria
φ_1	Max	I
φ_2	Max	I
φ_3	Max	I
φ_4	Max	I

Table 33
Generalized criteria preference function.

Alternatives	η_1	η_2	η_3	η_4	Alternatives	η_1	η_2	η_3	η_4
$\varphi_1\varphi_2$	0	0	0	0	$\varphi_1\varphi_3$	0	0	0	0
$\varphi_1\varphi_4$	1	1	1	1	$\varphi_1\varphi_5$	1	1	0	0
$\varphi_2\varphi_1$	1	1	1	1	$\varphi_2\varphi_3$	0	0	0	0
$\varphi_2\varphi_4$	1	1	1	1	$\varphi_2\varphi_5$	1	1	1	0
$\varphi_3\varphi_1$	1	1	1	1	$\varphi_3\varphi_2$	1	1	1	1
$\varphi_3\varphi_4$	1	1	1	1	$\varphi_3\varphi_5$	1	1	1	1
$\varphi_4\varphi_1$	0	0	0	1	$\varphi_4\varphi_2$	0	0	0	0
$\varphi_4\varphi_3$	1	0	0	0	$\varphi_4\varphi_5$	0	1	0	0
$\varphi_5\varphi_1$	0	0	1	1	$\varphi_5\varphi_2$	1	0	0	1
$\varphi_5\varphi_3$	0	0	0	o	$\varphi_5\varphi_4$	1	0	1	1

Table 34
Multi-criteria preference index.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0	0	1	0.443
φ_2	1	–	0	1	0.608
φ_3	1	1	–	1	
φ_4	0.394	0	0.049	–	0.394
φ_5	0.559	0.443	0	0.608	–

Table 35
Positive and negative flows of the alternatives (PROMETHEE I).

Alternatives	Outgoing flow (Y^+)	Ingoing flow (Y^-)
φ_1	0.361	0.738
φ_2	0.652	0.361
φ_1	1	0.012
φ_1	0.209	0.902
φ_1	0.403	0.611

Table 36
Net outranking flow of the alternatives (PROMETHEE II).

Alternatives	Net flow (Y)
φ_1	−0.378
φ_2	0.291
φ_1	0.988
φ_1	−0.693
φ_1	−0.209

5. Comparative discussion

This section is concerned with a detailed discussion of the comparative results extracted from application of mFNS-ELECTRE I, mF-PROMETHEE [26] and mF-ELECTRE I [2] methods, including their working mechanisms, their application on the numerical case study

Table 37
Aggregated $3FDM$.

	η_1	η_2	η_3	η_4
φ_1	(0.62, 0.75, 0.71)	(0.36, 0.42, 0.43)	(0.32, 0.28, 0.32)	(0.25, 0.23, 0.24)
φ_2	(0.61, 0.65, 0.69)	(0.72, 0.75, 0.78)	(0.53, 0.74, 0.61)	(0.33, 0.36, 0.32)
φ_3	(0.92, 0.87, 0.92)	(0.85, 0.91, 0.83)	(0.84, 0.84, 0.87)	(0.89, 0.90, 0.82)
φ_4	(0.17, 0.21, 0.20)	(0.23, 0.22, 0.24)	(0.25, 0.31, 0.28)	(0.17, 0.24, 0.18)
φ_5	(0.57, 0.65, 0.61)	(0.23, 0.21, 0.20)	(0.47, 0.55, 0.55)	(0.45, 0.55, 0.55)

Table 38
Weightage of criteria by experts.

	ς_1	ς_2	ς_3	ς_4
η_1	(0.69, 0.78, 0.77)	(0.47, 0.56, 0.59)	(0.71, 0.64, 0.68)	(0.37, 0.22, 0.27)
η_2	(0.87, 0.94, 0.99)	(0.62, 0.73, 0.68)	(0.91, 0.94, 0.88)	(0.41, 0.54, 0.47)
η_3	(0.51, 0.46, 0.55)	(0.48, 0.52, 0.59)	(0.77, 0.66, 0.69)	(0.98, 0.89, 0.99)
η_4	(0.79, 0.68, 0.69)	(0.62, 0.77, 0.79)	(0.86, 0.92, 0.95)	(0.92, 0.83, 0.96)

Table 39
Aggregated weightage of criteria.

	$3F6S$ aggregated weightage of criteria
η_1	4, 0.58, 0.59, 0.61
η_2	5, 0.77, 0.85, 0.88
η_3	5, 0.81, 0.69, 0.86
η_4	5, 0.83, 0.82, 0.89

Table 40
Normalized weights.

	Normalized weights
η_1	0.19
η_2	0.27
η_3	0.26
η_4	0.28

Table 41
Aggregated weighted $3F6SDM$.

	η_1	η_2	η_3	η_4
φ_1	(0.36, 0.44, 0.43)	(0.28, 0.36, 0.38)	(0.26, 0.19, 0.28)	(0.21, 0.19, 0.21)
φ_2	(0.35, 0.38, 0.42)	(0.55, 0.64, 0.69)	(0.43, 0.51, 0.52)	(0.27, 0.30, 0.28)
φ_3	(0.53, 0.51, 0.56)	(0.65, 0.77, 0.73)	(0.68, 0.58, 0.75)	(0.74, 0.74, 0.82)
φ_4	(0.10, 0.12, 0.12)	(0.18, 0.19, 0.21)	(0.20, 0.21, 0.24)	(0.14, 0.20, 0.16)
φ_5	(0.33, 0.38, 0.37)	(0.18, 0.18, 0.18)	(0.38, 0.38, 0.47)	(0.37, 0.45, 0.49)

Table 42
Score degree matrix.

	η_1	η_2	η_3	η_4
φ_1	0.41	0.34	0.24	0.20
φ_2	0.38	0.63	0.49	0.28
φ_3	0.53	0.72	0.67	0.77
φ_4	0.11	0.19	0.22	0.17
φ_5	0.36	0.18	0.41	0.44

Table 43
$3F6S$ concordance set.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	{1}	{}	{1, 2, 3, 4}	{1, 2}
φ_2	{2, 3, 4}	–	{}	{1, 2, 3, 4}	{1, 2, 3}
φ_3	{1, 2, 3, 4}	{1, 2, 3, 4}	–	{1, 2, 3, 4}	{1, 2, 3, 4}
φ_4	{}	{}	{}	–	{2}
φ_5	{3, 4}	{4}	{}	{1, 3, 4}	–

Table 44
$3F6S$ discordance set.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	{2, 3, 4}	{1, 2, 3, 4}	{}	{3, 4}
φ_2	{1}	–	{1, 2, 3, 4}	{}	{4}
φ_3	{}	{}	–	{}	{}
φ_4	{1, 2, 3, 4}	{1, 2, 3, 4}	{1, 2, 3, 4}	–	{1, 2, 3, 4}
φ_5	{1, 2}	{1, 2, 3}	{1, 2, 3, 4}	{2}	–

Table 45
$3F6S$ concordance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0.2	0	1	0.46
φ_2	0.81	–	0	1	0.72
φ_3	1	1	–	1	1
φ_4	0	0	0	–	0.27
φ_5	0.54	0.28	0	0.73	–

Table 46
Euclidean distance with respect to criteria η_1 and η_2.

	y_{11}	y_{21}	y_{31}	y_{41}	y_{51}		y_{12}	y_{22}	y_{32}	y_{42}	y_{52}
y_{11}	–	0.04	0.13	0.3	0.06	y_{12}	–	0.28	0.38	0.15	0.17
y_{21}	0.04	–	0.15	0.27	0.03	y_{22}	0.28	–	0.10	0.44	0.45
y_{31}	0.13	0.15	–	0.42	0.18	y_{32}	0.38	0.10	–	0.53	0.54
y_{41}	0.3	0.27	0.42	–	0.25	y_{42}	0.15	0.44	0.53	–	0.02
y_{51}	0.06	0.03	0.18	0.18	–	y_{52}	0.17	0.45	0.54	0.02	–

Table 47
Euclidean distance with respect to criteria η_3 and η_4.

	y_{13}	y_{23}	y_{33}	y_{43}	y_{53}		y_{14}	y_{24}	y_{34}	y_{44}	y_{54}
y_{13}	–	0.25	0.43	0.04	0.17	y_{14}	–	0.08	0.56	0.05	0.24
y_{23}	0.25	–	0.20	0.27	0.09	y_{24}	0.08	–	0.49	0.18	0.16
y_{33}	0.43	0.20	–	0.46	0.26	y_{34}	0.56	0.49	–	0.60	0.33
y_{43}	0.04	0.27	0.46	–	0.20	y_{44}	0.05	0.18	0.60	–	0.27
y_{53}	0.17	0.09	0.26	0.26	–	y_{54}	0.24	0.16	0.33	0.27	–

Table 48
$3F6S$ discordance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	1.12	1	0	1
φ_2	0.14	–	1	0	0.36
φ_3	0	0	–	0	0
φ_4	1	1	1	–	1
φ_5	0.7	1	1	0.07	–

Table 49
$3F6S$ concordance dominance matrix with concordance index (0.5).

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0	0	1	0
φ_2	1	–	0	1	1
φ_3	1	1	–	1	1
φ_4	0	0	0	–	0
φ_5	1	0	0	1	–

of the selection of the best rehabilitation center in the $U.S.$, their final results and the ranking among the alternatives. For this purpose, Table 53 encapsulates the respective outcomes and rankings of the alternatives by these methods.

Table 50
$3F6S$ discordance dominance matrix with discordance index (0.57).

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0	0	1	0
φ_2	1	–	0	1	1
φ_3	1	1	–	1	1
φ_4	0	0	0	–	1
φ_5	0	0	0	1	–

Table 51
Aggregated $3F6S$ dominance matrix.

	φ_1	φ_2	φ_3	φ_4	φ_5
φ_1	–	0	0	1	0
φ_2	1	–	0	1	1
φ_3	1	1	–	1	1
φ_4	0	0	0	–	0
φ_5	0	0	0	1	–

Table 52
Analysis drawn from outranking graph.

	Outranked alternatives	Incomparable alternatives
φ_1	φ_4	φ_5
φ_2	φ_1, φ_4 and φ_5	
φ_3	φ_1, φ_2, φ_4, φ_5	
φ_4	None	
φ_5	φ_4	φ_1

Table 53
Elements for a comparative analysis.

Method	Ranking	Best alternative
$mFNS-ELECTRE\ I\ (proposed)$	$\varphi_3\ outranks\ \varphi_1, \varphi_2, \varphi_4, \varphi_5$	φ_3
$mF-PROMETHEE$ [26]	$\varphi_3 > \varphi_2 > \varphi_5 > \varphi_1 > \varphi_4$	φ_3
$mF-ELECTRE\ I$ [2]	$\varphi_3\ outranks\ \varphi_1, \varphi_2, \varphi_4, \varphi_5$	φ_3

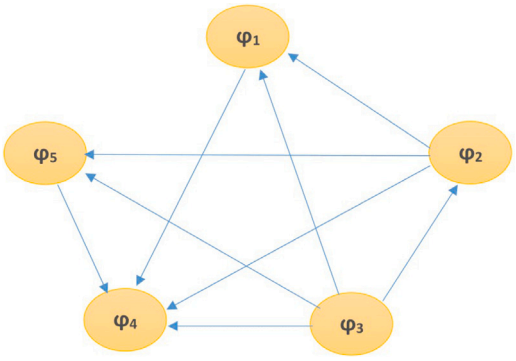

Fig. 6. Outranking graph of the alternatives.

- Our technique, $mFNS$ ELECTRE I, is being compared to mF PROMETHEE [26] and mF ELECTRE I [2] on the same numerical case study (selection of the best rehabilitation center). The fact that two alternative methodologies suggest the same choice supports the validity and precision of our approach.
- Both approaches have completely distinct functioning principles. While $mFNS$-ELECTRE I technique compares any pair of alternatives based on the concordance and discordance sets, indices and dominance matrix, the mF-PROMETHEE method [26] calculates the deviation of the alternatives, defines suitable preference function and preference ranking. Despite the fact that the latter result is more advantageous, we want to stress that both strategies advise choosing the same best alternative.
- In contrast to mF ELECTRE I method, which can only handle multi-polar fuzzy information, $mFNS$-ELECTRE I can handle multi-polar information combined with N-ordered grading.

This characteristic emphasizes the greater applicability of *m*F*N*S-ELECTRE I technique in real world ranking problems. Nevertheless in both cases, the application of ELECTRE I technique put forward the same best alternative.

- The outranking graphs for both techniques are being drawn, indicating that despite the different working mechanisms, the same best alternative is drawn out that outranks all the other elements.
- As a result, ELECTRE method's underlying theory enables the suggested technique to surpass the limitations of the conventional techniques in *m*F environments. These were designed to use multi-polar information, but their structures lack the ability to adjust the gradings.

6. Merits and demerits of the model

In view of the discussion above, we can summarize the pros and cons of our novel methodology as follows:

- *m*F*N*S-ELECTRE I technique has been designed to address the multi-criteria group decision hurdles by entertaining with multi-polar information and graded features with the broad structure of *m*-polar fuzzy *N*-soft information.
- *m*F*N*S-ELECTRE I technique has been of great assistance for us to select the best alternative among various choices.
- A numerical case study has been studied to illustrate the computations required to reach a solution and manifest validity of our technique.
- The outranking graph is drawn in such way that not only it shows the outranking of the alternatives, but also puts forward the best alternative.
- Without a doubt, *m*F*N*S ELECTRE I method manages to overcome every obstacle associated with handling multi-polar data with *N*-grading, but one significant flaw that limits its efficacy is that it does not produce a comprehensive rating of the alternatives and some of the alternatives appear to be incomparable.
- *m*F*N*S-ELECTRE I method, unquestionably, has proved to be a very efficient tool in multi-criteria group decision making in the presence of multi-polar information, however this outstanding characteristic is useless when we are dealing with complex two-dimensional data. When working with complicated two-dimensional data, this technique's productivity suffers.

7. Conclusions

Several MCGDM techniques have been developed to assist us in the difficult task of selecting the optimal option in today's complicated multi-polar environment. The ELECTRE I MCGDM approach was used in this investigation. In this field, the MCGDM approach of ELECTRE has been redefined as a brand-new *m*F*N*S-ELECTRE I methodology that covers all facets of multi-polar characteristics with *N*-soft grading. Along with the detailed procedure of *m*F*N*S-ELECTRE I approach, this technique is succinctly outlined in a comprehensive flowchart. Its usefulness has been proved by a numerical case study of selecting the best rehabilitation institution in the United States. We discovered that the *m*F*N*S-ELECTRE I approach can work effectively in both a multi-polar scenario and the *N*-ordering. A comparison of *m*F*N*S-ELECTRE I approach with the *m*F-PROMETHEE and *m*F ELECTRE I methods has been conducted to demonstrate the competency of the *m*F*N*S-ELECTRE I strategy. The outcome of the comparison confirms the authenticity of our proposed technique along with its superiority, which owes to assigning *N*-ordered grading, over the already existing techniques. All three strategies choose the same optimal option for a common case study, despite the differences in their perspectives and methodologies. An overview of the advantages and disadvantages of the *m*F*N*S-ELECTRE I approach concludes this research article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data were used to support this study.

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