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Doctorado en Estadística Multivariante Aplicada

Tesis Doctoral

Contribuciones al Análisis de la Sostenibilidad Internacional, desde una Perspectiva Algebraica Multivariante Comparada

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CERTIFICAN que **D. Miguel Rodríguez Rosa** ha realizado en la Universidad de Salamanca, bajo su dirección, el trabajo que para optar al título de Doctor en Estadística Multivariante Aplicada, presenta con el título ***Contribuciones al Análisis de la Sostenibilidad Internacional, desde una Perspectiva Algebraica Multivariante Comparada***, autorizando expresamente su lectura y defensa.

Y para que conste, firman el presente certificado en Salamanca a 18 de julio de 2016.

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CONTRIBUCIONES AL ANÁLISIS DE LA SOSTENIBILIDAD INTERNACIONAL, DESDE UNA PERSPECTIVA ALGEBRAICA MULTIVARIANTE COMPARADA

Dpto. de Estadística
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Notación

$X = (x_1, \ldots, x_p), Y = (y_1, \ldots, y_p)$	puntos de $\mathbb{R}^p$
π	subespacio de $\mathbb{R}^p$
$V = (v_{ij})_{p\times r} =$	base de π
$= (v_{11}, \ldots, v_{p1}, \ldots, v_{1r}, \ldots, v_{pr}) =$	
$= (v_1 \vert \ldots \vert v_r)$	
$U = (u_{ij})_{n\times r} =$	base de un subespacio de $\mathbb{R}^n$
$= (u_{11}, \ldots, u_{n1}, \ldots, u_{1r}, \ldots, u_{nr}) =$	
$= (u_1 \vert \ldots \vert u_r)$	
$D_p = (d_{ij})_{p\times p}$	métrica simétrica de $\mathbb{R}^p$
$X = (x_{ij})_{n\times p}$	matriz
n	número de filas de X
p	número de columnas de X
X_j	j-ésima columna de X
$D_n(n \times n)$	matriz diagonal con pesos para las filas de X
$\omega_1, \ldots, \omega_n$	pesos para las filas de X
$A = (a_1, \ldots, a_r)$	si Y pertenece a π, coordenadas de Y respecto a V
$B = (b_1, \ldots, b_r)$	si Y pertenece a un subespacio de $\mathbb{R}^n$, coordenadas de Y respecto a U
$\Lambda = \text{diag}(\lambda_{gf})_{r\times r}$	multiplicadores de Lagrange

δ_{ef}	función delta de Kronecker
$U_{n \times n}$	base de vectores propios para las filas
$V_{p \times p}$	base de vectores propios para las columnas
$U_r(n \times r)$	primeras r columnas de U
$V_r(p \times r)$	primeras r columnas de V
$\Lambda = diag(\lambda_{gf})_{n \times n}$	valores propios para las filas
$\Lambda = diag(\lambda_{gf})_{p \times p}$	valores propios para las columnas

Notación para el Análisis Entre-Grupos:

$G_1, \ldots, G_g$	grupos a los que pertenecen las filas
g	número de grupos
$n_1, \ldots, n_g$	tamaños de los grupos
$D(g \times g)$	matriz diagonal con pesos para los grupos
$D_g(g \times g)$	matriz diagonal con pesos para los grupos con suma unidad
$X_B(g \times p)$	matriz con las medias de la matriz X por grupos
$\overline{x}_j{}^k$	media de la columna j en el grupo k
$B = (b_{ik})_{n \times g}$	matriz de los indicadores de clase

Notación para el Análisis de Co-Inercia:

$X(n \times p)$	primera tabla de la co-inercia
$Y(n \times q)$	segunda tabla de la co-inercia
n	número de filas de X e Y
p	número de columnas de X
q	número de columnas de Y
$D_p(p \times p)$	métrica simétrica de $\mathbb{R}^p$
$D_q(q \times q)$	métrica simétrica de $\mathbb{R}^q$

Notación para los Análisis Parcial Triádico y Tucker:

$X_{I \times J \times K}$	tensor, cubo de datos
I	número de filas de X
J	número de columnas de X
K	número de repeticiones de X
P	número de filas de un tensor simplificado
Q	número de columnas de un tensor simplificado
R	número de repeticiones de un tensor simplificado
$S = P + Q + R$	suma del número de componentes
$X_c(n \times p)$	matriz compromiso
$X_k(n \times p)$	matriz de X fijada una repetición k para la tercera dimensión
$D_K(K \times K)$	matriz diagonal con pesos para las repeticiones de X
$\Omega_1, \ldots, \Omega_K$	pesos para las repeticiones de X
$\alpha = (\alpha_1, \ldots, \alpha_K)^t$	vector de $\mathbb{R}^K$ combinación lineal para las X_k en X_c
$Covv_{K \times K}$	matriz de varianzas-covarianzas vectoriales
$X_{(1)}(I \times JK)$	desplegamiento de X fijando la primera dimensión
$X_{(2)}(J \times IK)$	desplegamiento de X fijando la segunda dimensión
$X_{(3)}(K \times IJ)$	desplegamiento de X fijando la tercera dimensión
$A_{I \times P}$	matriz para la primera dimensión en la descomposición Tucker
$B_{J \times Q}$	matriz para la segunda dimensión en la descomposición Tucker
$C_{K \times R}$	matriz para la tercera dimensión en la descomposición Tucker
$G_{P \times Q \times R}$	tensor core en la descomposición Tucker
$A_n(I \times P)$	matriz para la primera dimensión obtenida en la iteración n-ésima en el algoritmo Tucker3
$B_n(J \times Q)$	matriz para la segunda dimensión obtenida en la iteración n-ésima en el algoritmo Tucker3
$C_n(K \times R)$	matriz para la tercera dimensión obtenida en la iteración n-ésima en el algoritmo Tucker3
$G_n(P \times Q \times R)$	tensor core obtenido en la iteración n-ésima en el algoritmo Tucker3

Notación para el Análisis BGCOIA:

$X_{n \times p \times g}$	primer cubo de datos
$Y_{n \times q \times g}$	segundo cubo de datos
n	número de filas de X e Y
p	número de columnas de X
q	número de columnas de Y
g	número de repeticiones (grupos) de X e Y
$X_{ng \times p}$	primer cubo de datos interpretado como tabla sin especificar a qué grupo pertenecen las filas
$Y_{ng \times q}$	segundo cubo de datos interpretado como tabla sin especificar a qué grupo pertenecen las filas
$X_B(g \times p)$	matriz con las medias de X por grupos
$Y_B(g \times q)$	matriz con las medias de Y por grupos
$B_X(ng \times g)$	matriz de los indicadores de clase
$B_Y(ng \times g)$	matriz de los indicadores de clase

Notación para el Análisis STATICO:

$X_{n \times p \times K}$	primer cubo de datos
$Y_{n \times q \times K}$	segundo cubo de datos
K	número de repeticiones de X e Y
$X_k(n \times p)$	matriz de X fijada una repetición k para la tercera dimensión
$Y_k(n \times q)$	matriz de Y fijada una repetición k para la tercera dimensión
$D_{n_k}(n_k \times n_k)$	matriz diagonal con pesos para las filas de X_k e Y_k
$Z_k(q \times p)$	k-ésima tabla de productos cruzados
$Z(q \times p)$	matriz compromiso de las tablas de productos cruzados

Notación para el Análisis COSTATIS:

$X_{n\times p\times K_1}$	primer cubo de datos
$Y_{n\times q\times K_2}$	segundo cubo de datos
K_1	número de repeticiones de X
K_2	número de repeticiones de Y
$X_c(n \times p)$	matriz compromiso de las X_k
$Y_c(n \times p)$	matriz compromiso de las Y_k
$D_{K_1}(K_1 \times K_1)$	matriz diagonal con pesos para las repeticiones de X
$D_{K_2}(K_2 \times K_2)$	matriz diagonal con pesos para las repeticiones de Y
$\alpha = (\alpha_1, \ldots, \alpha_{K_1})^t$	vector de $\mathbb{R}^K$ combinación lineal para las X_k en X_c
$\beta = (\beta_1, \ldots, \beta_{K_2})^t$	vector de $\mathbb{R}^K$ combinación lineal para las Y_k en Y_c

Notación para el Co-Tucker3:

$X_{I_1\times J_1\times K_1}$	primer cubo de datos
$Y_{I_2\times J_2\times K_2}$	segundo cubo de datos
I_1	número de filas de X
I_2	número de filas de Y
J_1	número de columnas de X
J_2	número de columnas de Y
K_1	número de repeticiones de X
K_2	número de repeticiones de Y
P	número de filas de los cubos simplificados
Q	número de columnas de los cubos simplificados
R	número de repeticiones de los cubos simplificados
$A_X(I_1 \times P)$	matriz para la primera dimensión en la descomposición de X con el Co-Tucker3
$B_X(J_1 \times Q)$	matriz para la segunda dimensión en la descomposición de X con el Co-Tucker3

$C_X(K_1 \times R)$	matriz para la tercera dimensión en la descomposición de X con el Co-Tucker3
$G_X(P \times Q \times R)$	tensor core en la descomposición de X con el Co-Tucker3
$A_Y(I_2 \times P)$	matriz para la primera dimensión en la descomposición de Y con el Co-Tucker3
$B_Y(J_2 \times Q)$	matriz para la segunda dimensión en la descomposición de Y con el Co-Tucker3
$C_Y(K_2 \times R)$	matriz para la tercera dimensión en la descomposición de Y con el Co-Tucker3
$G_Y(P \times Q \times R)$	tensor core en la descomposición de Y con el Co-Tucker3
$A_{X_n}(I_1 \times P)$	matriz para la primera dimensión obtenida en la iteración n-ésima en la descomposición de X
$B_{X_n}(J_1 \times Q)$	matriz para la segunda dimensión obtenida en la iteración n-ésima en la descomposición de X
$C_{X_n}(K_1 \times R)$	matriz para la tercera dimensión obtenida en la iteración n-ésima en la descomposición de X
$G_{X_n}(P \times Q \times R)$	tensor core obtenido en la iteración n-ésima en la descomposición de X
$A_{Y_n}(I_2 \times P)$	matriz para la primera dimensión obtenida en la iteración n-ésima en la descomposición de Y
$B_{Y_n}(J_2 \times Q)$	matriz para la segunda dimensión obtenida en la iteración n-ésima en la descomposición de Y
$C_{Y_n}(K_2 \times R)$	matriz para la tercera dimensión obtenida en la iteración n-ésima en la descomposición de Y
$G_{Y_n}(P \times Q \times R)$	tensor core obtenido en la iteración n-ésima en la descomposición de Y
I	número de filas de X e Y en el caso en que sean iguales
J	número de columnas de X e Y en el caso en que sean iguales
K	número de repeticiones de X e Y en el caso en que sean iguales
$D_I(I \times I)$	matriz diagonal con pesos para las filas de X e Y en el caso en que sean iguales

$D_J(J \times J)$	matriz diagonal con pesos para las columnas de X e Y en el caso en que sean iguales
$D_K(K \times K)$	matriz diagonal con pesos para las repeticiones de X e Y en el caso en que sean iguales
$D_P(P \times P)$	métrica simétrica de $\mathbb{R}^P$ en el caso en que las filas de X e Y sean iguales
$D_Q(Q \times Q)$	métrica simétrica de $\mathbb{R}^Q$ en el caso en que las columnas de X e Y sean iguales
$D_R(R \times R)$	métrica simétrica de $\mathbb{R}^R$ en el caso en que las repeticiones de X e Y sean iguales
D_P	matriz diagonal con pesos para las componentes de las filas de X e Y en el caso en que sean distintas
D_Q	matriz diagonal con pesos para las componentes de las columnas de X e Y en el caso en que sean distintas
D_R	matriz diagonal con pesos para las componentes de las repeticiones de X e Y en el caso en que sean distintas
$D_{I_1}(I_1 \times I_1)$	métrica simétrica de $\mathbb{R}^{I_1}$ en el caso en que las filas de X e Y sean distintas
$D_{I_2}(I_2 \times I_2)$	métrica simétrica de $\mathbb{R}^{I_2}$ en el caso en que las filas de X e Y sean distintas
$D_{J_1}(J_1 \times J_1)$	métrica simétrica de $\mathbb{R}^{J_1}$ en el caso en que las columnas de X e Y sean distintas
$D_{J_2}(J_2 \times J_2)$	métrica simétrica de $\mathbb{R}^{J_2}$ en el caso en que las columnas de X e Y sean distintas
$D_{K_1}(K_1 \times K_1)$	métrica simétrica de $\mathbb{R}^{K_1}$ en el caso en que las repeticiones de X e Y sean distintas
$D_{K_2}(K_2 \times K_2)$	métrica simétrica de $\mathbb{R}^{K_2}$ en el caso en que las repeticiones de X e Y sean distintas

Resumen

Para establecer estructuras en diferentes tablas de datos, anteriormente se analizaban por separado y luego se intentaban deducir las relaciones entre ellas. Pero actualmente existen muchas técnicas para estudiar de forma conjunta estas relaciones entre las estructuras, pero apenas se utilizan. En este trabajo se estudian tres métodos existentes: el Análisis de Co-Inercia Entre Grupos (BGCOIA), el STATICO y el COSTATIS, estos dos últimos basados en los llamados métodos STATIS (Structuration des Tableaux À Trois Indices de la Statistique). Además se presenta una nueva propuesta, el Co-Tucker3. Se analiza de manera comparada el álgebra que subyace bajo estos métodos, explicando los conceptos previos correspondientes a técnicas para tablas individuales (Análisis de Componentes Principales, Biplots, Análisis Entre Grupos) o a varias tablas de datos (Análisis de Co-Inercia, Análisis Parcial Triádico, Tucker). Para demostrar la validez de estas técnicas se han aplicado a datos reales de sostenibilidad social, medioambiental y económica de países de todo el mundo. Este tema ha sido últimamente de gran trascendencia a nivel internacional y preocupa cada vez más a la sociedad, ya que afecta tanto a la generación actual como a las generaciones futuras. Los principales resultados que se obtienen son que los países de Europa y América presentan mayor interés por temas sociales y económicos, como una sanidad segura, suficiencia de comida y bebida, la calidad de vida, un buen gobierno y una buena educación y el Producto Interior Bruto, mientras que los países africanos se decantan por aspectos medioambientales como la emisión de gases, el uso de energías renovables y la calidad del aire.

Palabras clave: co-estructuras, BGCOIA, STATICO, COSTATIS, Co-Tucker3, Índice de Sociedad Sostenible.

Summary in English

To establish structures in different data tables, previously they were analyzed separately and then one tried to infer the relations between them. But now there are many techniques to jointly study these relations between the structures, but they are hardly used. In this paper three existing methods are studied: the Between-Groups Co-Inertia Analysis (BGCOIA), the STATICO and the COSTATIS, the latter two based on the methods called STATIS (Structuration des Tableaux À Trois Indices de la Statistique). A new proposal, the Co-Tucker3, is also presented. The algebra underlying these methods, explaining the previous concepts corresponding to techniques for individual tables (Principal Components Analysis, Biplots, Between-Groups Analysis) or to several data tables (Co-Inertia Analysis, Partial Triadic Analysis, Tucker), is analyzed in a comparative way. To proof the validity of these techniques they have been applied to real data of social, environmental and economic sustainability of countries around the world. This topic has recently been of a great importance internationally and it is concerned more and more to society, since it affects both the current generation and future generations. The main results that have been obtained are that the countries of Europe and America have a greater interest in social and economic issues, such as safe sanitation, sufficiency of food and drink, quality of life, good governance and a good education and Gross Domestic Product, whereas the countries of Africa choose environmental aspects such as emission of greenhouse gases, use of renewable energy and quality of the air.
Keywords: co-structures, BGCOIA, STATICO, COSTATIS, Co-Tucker3, Śustainable Society Index.

Capítulo 1

Introducción

Durante muchos años, para comparar estructuras se analizaban por separado las diferentes tablas de datos y el investigador “elucubraba” sobre las posibles relaciones entre las estructuras encontradas en cada una de las matrices.
Hoy en día existen muchos métodos estadísticos para estudiar, de manera objetiva, las relaciones entre las estructuras; sin embargo, su uso no está generalizado. Tampoco existen estudios que analicen de manera comparada el álgebra en la que se asientan estos métodos. Por esta razón se considera de gran interés, tanto teórico como práctico, el realizar un estudio exhaustivo de las diferentes técnicas existentes en la literatura, tanto en el análisis de dos tablas de datos, como en tablas de tres vías.
Las diferentes técnicas se aplicarán a datos de sostenibilidad en distintos países, cedidos por la Dra. Isabel Gallego, del Departamento de Administración y Economía de la Empresa de la Universidad de Salamanca:
En los últimos años uno de los temas que más transcendencia ha tenido a nivel internacional es el que se refiere a la sostenibilidad de los distintos países y áreas geográficas que conforman nuestro planeta, especialmente desde que en 1987 y en el Informe Brundtland se diera una definición que ha calado muy profundamente en toda la sociedad. En dicho informe se pone de manifiesto la necesidad de satisfacer las necesidades actuales pero, algo muy importante, es que no se comprometa la capacidad de que las generaciones futuras satisfagan sus

propias necesidades.

De esta acepción de sostenibilidad se deduce que tiene que existir un equilibrio actual y futuro de tres aspectos que afectan a toda la humanidad: por una parte el económico, con una combinación óptima entre el desarrollo económico y la conservación de los medios naturales; por otra parte el social, que implica la necesidad de garantizar la equidad intergeneracional en aspectos sociales y calidad de vida; y por último el medioambiental, que implica la necesidad de mantener la continuidad de los recursos medioambientales a lo largo del tiempo, lo cual se puede lograr a través de la limitación del consumo de los recursos y productos fácilmente agotables, la reducción de los residuos y la contaminación en todas sus vertientes, la conservación de la energía y el reciclaje.

Todos estos aspectos son importantes para conseguir una sociedad sostenible donde cada ser humano sea capaz de: desarrollarse por sí mismo de manera saludable y obtener una educación adecuada; vivir en un medio ambiente limpio; vivir en una sociedad segura y bien equilibrada; usar recursos no renovables de manera responsable; y contribuir a un mundo sostenible (Van de Kerk and Manuel, 2008).

Se ha implementado un número de indicadores para ayudar a entender y para gestionar los temas sobre sostenibilidad. Unos de los más importantes han sido: Human Development Index (HDI), Millennium Development Indicators, Indicators for the EU Sustainable Development Strategy e Index for Sustainability Economic Welfare. En este trabajo, se usa el Índice de Sociedad Sostenible (Sustainable Society Index - SSI), usado en análisis previos (por ejemplo Van de Kerk and Manuel (2008)).

Este índice incluye un conjunto de indicadores sobre bienestar económico, medioambiental y social, y ha sido recientemente auditado por el Joint Research Center de la Comisión Europea, que lo considera un método integral y cuantitativo para medir y vigilar la salud de los sistemas humanos y medioambientales a nivel mundial, además, los considera una herramienta conceptual y estadísticamente sólida que es ampliamente aplicable para la evaluación continua de los sistemas humanos y ambientales y un punto de referencia clave con el que comparar el progreso futuro e informar sobre la sociedad actual (Saisana and Philippas, 2012).

El trabajo está estructurado como sigue: después de los objetivos del trabajo, en el capítulo 3, se analiza el marco teórico del desarrollo sostenible y los métodos de la investigación, incluyendo la muestra y las técnicas de análisis. En el capítulo 4, se da un estudio exhaustivo de las técnicas existentes para analizar pares de cubos de datos. A continuación, en el capítulo 5, se presenta la nueva propuesta, el Co-Tucker3, y luego se incluye una discusión. En el siguiente capítulo 6 se presentan los programas creados por el autor. En el capítulo 7 se presentan los resultados de los análisis empíricos. En los últimos apartados se resumen las principales conclusiones. Finalmente, se incluye la bibliografía revisada. Además se incluyen cuatro apéndices con, respectivamente, los artículos publicados por el autor, la muestra, los datos numéricos y los resultados gráficos.

Capítulo 2

Objetivos

Objetivo general

Realizar un estudio comparado de las técnicas existentes en la actualidad para el estudio de co-estructuras y probar su interés en análisis de datos reales.

Objetivos específicos

1. Realizar una exhaustiva revisión bibliográfica de las técnicas existentes en la bibliografía.
2. Realizar un desarrollo del álgebra subyacente.
3. Realizar un estudio comparativo que ponga de manifiesto la similitud y las diferencias entre las distintas técnicas.
4. Aplicar las diferentes técnicas a datos de sostenibilidad en diferentes países de Europa, Asia, África, América y Oceanía.
    - Determinar si todos los aspectos sociales, medioambientales y económicos preocupan por igual en todos los países objeto de estudio.
    - Analizar si a lo largo de los años se han producido variaciones significativas en los distintos aspectos que componen el Sustainable Society Index.
    - Analizar en qué medida los indicadores sociales pueden influir en medioambientales y económicos y viceversa.

Capítulo 3

Material y Métodos

3.1. Marco teórico

3.1.1. Desarrollo Sostenible e indicadores de sostenibilidad

En los últimos años se ha producido en todo el mundo un gran interés por los temas relacionados con el desarrollo sostenible o sostenibilidad, tanto a nivel micro como macroeconómico. El nivel microeconómico se refiere a las sostenibilidad en el ámbito empresarial, que se recoge en los informes de sostenibilidad que presentan las empresas, cada vez más desarrollados a nivel internacional; el nivel macroeconómico se refiere a la sostenibilidad de los diferentes países, tema que quizás está menos desarrollado que a nivel empresarial pero que sin duda es muy importante.

La presente investigación se centra en la sostenibilidad de los países, la cual ha cobrado gran relevancia a partir de la Conferencia de las Naciones Unidas sobre Medio Ambiente y Desarrollo Sostenible celebrada en Rio de Janeiro en 1992, que incorpora el Desarrollo Sostenible en la agenda política mundial y reafirma el concepto introducido en 1987 en el Informe Brundtland. En dicho informe se recoge la primera formulación, en un documento oficial, del concepto de Desarrollo Sostenible, el cual se define como “el desarrollo que satisface las necesidades presentes sin comprometer la capacidad de que las generaciones futuras satisfagan sus propias necesidades” (World Commission on Environment and Development, 1987).

De este modo, se definen los principios generales que deben orientar a nivel internacional las relaciones entre la economía y el medio ambiente, donde se destaca la necesidad de buscar estrategias que permitan compatibilizar los procesos de crecimiento con la sostenibilidad (Erias Rey, 2003).
El logro del Desarrollo Sostenible supone avanzar en tres pilares fundamentales: el desarrollo económico, la cohesión social y la protección del medio ambiente. Es decir, el Desarrollo Sostenible presupone la integración de tres dimensiones:

- Social, a través de la sostenibilidad social. Implica la necesidad de garantizar la equidad intergeneracional, es decir, satisfacer las necesidades básicas actuales de todas las personas, garantizando al mismo tiempo el que, llegado el momento, las generaciones futuras puedan igualmente satisfacer las suyas.
- Medioambiental (ecológica), a través de la sostenibilidad medioambiental. Se define como la necesidad de mantener la continuidad de los recursos medioambientales a lo largo del tiempo. Se puede lograr a través de la limitación del consumo de los recursos y productos fácilmente agotables, la reducción de los residuos y la contaminación en todas sus vertientes, la conservación de la energía y el reciclaje.
- Económico, a través de la sostenibilidad económica. Supone la búsqueda del equilibrio económico mediante una combinación óptima entre el desarrollo económico y la conservación de los recursos naturales.

Alcanzar la sostenibilidad implica, en primer lugar, definir sus componentes en términos medibles (Hales and Prescott-Allen, 2002).
Sin embargo, la noción de lo que se entiende por sostenibilidad varía considerablemente y su definición sigue siendo ambigua (Mori and Christodoulou, 2012). Ni que decir tiene que la literatura al respecto con estudios sobre la sostenibilidad es abundante (Guy and Kibert, 1998; Meadows, 1998; Hák et al., 2007; Arezki and Van der Ploeg, 2007; Bell and Morse, 2008; Betsill and Rabe, 2009).
Según Van de Kerk and Manuel (2008) una sociedad sostenible es una sociedad en la que

cada ser humano es capaz de: desarrollarse por sí mismo de manera saludable y obtener una educación adecuada; vivir en un medio ambiente limpio; vivir en una sociedad segura y bien equilibrada; usar recursos no renovables de manera responsable para que futuras generaciones no se queden con las manos vacías; y contribuir a un mundo sostenible.

Para Saisana and Philippas (2012) el término de sostenibilidad también ha sido utilizado por políticos y economistas para declarar que una sociedad es económicamente viable, medioambientalmente racional y socialmente responsable, si bien los grandes cambios que han experimentado los temas sociales y económicos hacen que la medida de la sostenibilidad sea muy complicada a pesar de los grandes avances manifestados en este tema.

Si se consideran estas premisas, cada vez más han surgido nuevos indicadores que tratan de medir estos tres aspectos de la sostenibilidad, por una parte el aspecto medioambiental, por otra parte el aspecto social y por otra parte el aspecto económico. Algunos de ellos han sido establecidos por la OECD, ONU, EPI y de una forma más completa, puesto que abarcan los tres aspectos, por el SSI (Sustainable Society Index) que se elabora desde el 2006 y por ello puede considerarse más novedoso (Van de Kerk and Manuel, 2012).

Pero además de ser novedoso, este Índice de Sociedad Sostenible ha sido recientemente auditado por el Joint Research Centre de la Comisión Europea que lo considera un método integral y cuantitativo para medir y vigilar la salud de los sistemas humanos y medioambientales a nivel mundial. Además, lo considera una herramienta conceptual y estadísticamente sólida que es ampliamente aplicable para la evaluación continua de los sistemas humanos y ambientales y un punto de referencia clave con el que comparar el progreso futuro e informar sobre la sociedad actual (Saisana and Philippas, 2012).

Estos indicadores de sostenibilidad pueden ser útiles a título individual para ver cómo está cada uno de los países respecto a temas de sostenibilidad, cuáles son las carencias y los aspectos más relevantes, comparar la sostenibilidad de cada país con respecto a otros del mismo área geográfica e identificar los aspectos más eficaces. Para el gobierno, estos indicadores de sostenibilidad le servirán para mostrar de forma transparente y efectiva al público en general la situación de la sostenibilidad en cada país y tomar decisiones sobre políticas, proyectos y estrategias sociales, medioambientales o económicas a adoptar; a nivel educativo, poder incluir materias de sostenibilidad en las enseñanzas medias y universitarias para que los alumnos puedan conocer la situación del mundo que nos rodea; a nivel empresarial, conocer los indicadores de sostenibilidad de los distintos países donde las empresas realizan su actividad para observar si puede tener algún tipo de ventaja competitiva y realizar innovaciones empresariales.

3.1.2. SSI - Índice de Sociedad Sostenible

Recientemente (Karavanas et al., 2009), se han usado varios indicadores para temas como calidad de vida y el medioambiente, principalmente para ordenar el nivel de actuación de un país.

Además, proporcionan información del status del medioambiente y evalúa el impacto económico, social y medioambiental en el desarrollo.

Hablando en términos generales, los indicadores tienen tres funciones principales. Primero, reducen el número de instrumentos de medida necesarios para dar una descripción de una situación (Organization of Economic Co-operation and Development, 2003).

Como tal, son indispensables para medir el progreso hacia los objetivos políticos y para evaluar la efectividad de las políticas (Dalal-Clayton and Krikhaar, 2007).

Hansen (1996), Jasch (2000) y Perotto et al. (2008) sostienen que el desarrollo de indicadores a nivel nacional, regional o local ha llegado a ser una aproximación comúnmente usada para conocer la necesidad crucial por herramientas de evaluación. Tales herramientas son un prerrequisito para la implementación del concepto de sostenibilidad.

Con el fin de estudiar la sostenibilidad a nivel internacional, se han revisado muchos de los índices e indicadores actuales relacionados con la sostenibilidad de tal forma que los buenos indicadores, es decir, aquellos que dan un punto de vista completo de todos los aspectos relevantes de sostenibilidad de forma transparente y fácilmente comprensible, deben cumplir los siguientes criterios (Guy and Kibert, 1998; Meadows, 1998; Bell and Morse, 2008; Van de Kerk and Manuel, 2008):

- Deben ser relevantes para alguno de los temas relativos a la definición de sostenibilidad anterior.
- Deben cubrir el campo completo de sostenibilidad, en la línea de la definición usada.
- Tienen que ser independientes unos de otros y no deben solaparse mutuamente.
- Deben ser medibles.
- Deben ser fácilmente accesibles, también para el público general. Esto significa que, además, el número de indicadores debe ser limitado.
- Los datos para construir los indicadores deben estar disponibles públicamente.
- Los datos deben estar disponibles para todos los países, al menos para todos excepto los más pequeños.
- Los datos deben ser fiables.
- Los datos deben ser recientes y ser actualizados regularmente.
- El conjunto total de indicadores debe dar un buen punto de vista de la situación presente de sostenibilidad e indicar la diferencia entre la situación presente y la situación de sostenibilidad completa.
- Deben permitir la comparación entre países.

La conclusión general es que ninguno de los índices existentes parece ajustarse completamente a nuestras necesidades, ya que, o ninguno es completamente adecuado, o cada conjunto sirve más o menos a distintos objetivos. Seguidamente se exponen algunos de los indicadores más relevantes en el ámbito de sostenibilidad (Saisana and Philippas, 2012; Van de Kerk and Manuel, 2012).

- Human Development Index (HDI): Cubre solo una pequeña parte de todos los aspectos del desarrollo sostenible e incluso ha sido considerado un indicador redundante que proporciona poca información adicional a nivel de desarrollo internacional.

- Environmental Sustainability Index (ESI-2005): Falta un índice relacionado con la igualdad de género y el “Buen Gobierno” recibe una menor atención. No es muy transparente debido al enorme conjunto de datos. Se duda si se hacen actualizaciones.

- Environmental Perfomance Index (EPI-2006): Solo cubre parcialmente el desarrollo sostenible en su contexto más amplio.

- Commitment to Development Index (CDI-2006): Cubre el desarrollo sostenible solo parcialmente y ofrece información relativa a no más de 21 países.

- Index for Sustainability Economic Welfare (ISEW): No incluye los principales aspectos de calidad de vida y no ofrece un punto de vista claro del nivel de sostenibilidad de un país. Está disponible solo para un número limitado de países.

- Genuine Progress Indicator (GPI): Los mismos defectos del ISEW se pueden aplicar al GPI.

- Ecological Footprint: Solo cubre parcialmente la sostenibilidad en su sentido más amplio. Hay bastante discusión sobre el método de cálculo utilizado.

- Well-being of Nations: Da un enorme conjunto de información, que lo hace bastante complicado. Ha sido publicado solo una vez.

- Millennium Development Indicators: Utilidad limitada a la hora de visualizar el nivel de sostenibilidad de un país. No cubre el concepto entero de sociedad sostenible.

- Indicators for the EU Sustainable Development Strategy: Incluye un número de indicadores que no están muy relacionados con la sostenibilidad, mientras que a otros temas se les presta menor atención o faltan, como los relacionados con la igualdad de género y el acceso a agua potable. Se limita a los países miembros de la Unión Europea.

- CSD Indicators: El conjunto comprende muchos indicadores y ofrece demasiada información. No cubre la sostenibilidad en su sentido más amplio.

Considerando las limitaciones existentes a la hora de establecer un índice a aplicar de forma general en todos los países, este trabajo se ha inclinado por el Índice de Sociedad Sostenible, establecido por Van de Kerk and Manuel (2012), puesto que dicho índice ha sido recientemente auditado por el Joint Research Centre de la Comisión Europea, que lo considera un método integral y cuantitativo para medir y vigilar la salud de los sistemas humanos y medioambientales a nivel mundial. Además se le considera una herramienta conceptual y estadísticamente sólida que es ampliamente aplicable para la evaluación continua de los sistemas humanos y ambientales y un punto de referencia clave con el que comparar el progreso futuro e informar sobre la sociedad actual (Saisana and Philippas, 2012).
El Índice de Sociedad Sostenible -Sustainable Society Index (SSI)- consiste en 21 indicadores agrupados en tres categorías, bienestar humano, ambiental y económico y que se pasa a detallar seguidamente (Van de Kerk and Manuel, 2012).

- Social well-being
  - Sufficient Food: Number of undernourished people in percentage of total population.
  - Sufficient to Drink: Number of people as percentage of the total population, with sustainable access to an improved water source.
  - Safe Sanitation: Number of people in percentage of total population, with sustainable access to improved sanitation.
  - Healthy Life: Life expectancy at birth in number of healthy life years (HALE - Health Adjusted Life Expectancy).
  - Clean Air: Air pollution in its effects on humans.
  - Clean Water: Surface water quality.
  - Education: Combined gross enrolment ratio for primary, secondary and tertiary schools.
  - Gender Equality: Gender Gap Index.

- Income Distribution: Ratio of income of the richest 10 % to the poorest 10 % of the people in a country.
- Good Governance: The average of values of the six Governance Indicators of the World Bank.

- Environmental well-being
  - Air Quality: Air pollution in its effects on nature.
  - Biodiversity: Size of protected areas (in percentage of land area).
  - Renewable Water Resources: Annual water withdrawals (m^2 per capita) as percentage of renewable water resources.
  - Consumption: Ecological Footprint minus Carbon Footprint.
  - Renewable Energy: Renewable energy as percentage of total energy consumption.
  - Greenhouse Gases: CO_2 emissions per capita per year.

- Economic well-being
  - Organic Farming: Area for organic farming in percentage of total agricultural area of a country.
  - Genuine Savings: Genuine Savings (Adjusted Net Savings) as percentage of Gross National Income (GNI).
  - Gross Domestic Product: GDP, per capita, in Purchasing Power Parity, in current international dollars.
  - Employment: Unemployment as percentage of total labour force.
  - Public Debt: The level of public debt of a country as percentage of GDP.

3.2. Método de la investigación

3.2.1. Población y muestra

Los datos de los indicadores estaban disponibles para 151 de los 194 países existentes. Así, se ha podido calcular el SSI para la mayoría de los países de medio o gran tamaño. Las excepciones de países más grandes son Afghanistan, Djibouti, Eritrea, Somalia y Surinam. Aparte de estos, la mayoría de los estados insulares se han tenido que dejar fuera por falta de datos. De esta forma, el Índice de Sociedad Sostenible se ha conseguido para tantos países como fue posible. Esto permite comparar entre países desde varios puntos de vista: países limítrofes o de la misma región, países más o menos similares, comparar entre países ricos, como los miembros de la Organización para la Cooperación y el Desarrollo Económico (OECD), comparar entre los hemisferios norte y sur, etcétera.

Con los objetivos de este trabajo en mente, se seleccionan la mayoría de países de todo el mundo como población objetivo. Esta población fue escogida con el interés de extender y generalizar los resultados obtenidos en estudios previos, y superar dos de sus limitaciones: los países estudiados y las técnicas usadas en el análisis de datos.

Los estudios previos se enfocaron normalmente en contextos de áreas geográficas específicas, tales como países occidentales industrializados (Crepaz, 1995; Jahn, 1998; Scruggs, 2003); 21 países del OECD (Neumayer, 2003); 17 países democráticos industrializados (Scruggs, 1999, Scruggs, 2001); 14 países del OECD y cinco medidas del bienestar (Giles and Feng, 2005); y 131 países (Hosseini and Kaneko, 2011).

La muestra usada comprende los países seleccionados por Van de Kerk and Manuel (2008) (ver Apéndice B), e incorpora las ventajas derivadas de considerar diferentes contextos económicos: países con ingresos de $1035 o menos, países con ingresos entre $1036 y $4085, países con ingresos entre $4086 y $12615, y países con ingresos de $12616 o más.

3.2.2. Técnicas de análisis

En este trabajo, se consideran los 151 países de todo el mundo presentados en el apéndice B, formando 4 grupos según su nivel de ingresos; las 21 características numéricas son las puntuaciones obtenidas por los países elegidos relativas a las categorías políticas propuestas en el SSI en los últimos bienios disponibles (2006, 2008, 2010 y 2012), básicamente: Sufficient Food, Sufficient to Drink, Safe Sanitation, Healthy Life, Clean Air, Clean Water, Education, Gender Equality, Income Distribution, Good Governance, Air Quality, Biodiversity, Renewable Water Resources, Consumption, Renewable Energy, Greenhouse Gases, Organic Farming, Genuine Savings, Gross Domestic Product, Employment y Public Debt (véase apéndice C).
Así que, en este trabajo, los datos consisten en las puntuaciones del SSI para cada país en cada periodo de tiempo, esto es una matriz tridimensional 151 filas $\times$ 21 columnas $\times$ 4 repeticiones.
El análisis de muchos problemas de sostenibilidad de una sola vez requiere un gran volumen de datos. Para explorar los datos para conseguir una mejor comprensión del comportamiento de muchos procesos, es importante identificar sus principales características subyacentes. La reducción en la dimensionalidad del problema ayuda a resumir la información capturada en un gran número de variables mediante un número más pequeño de variables latentes. Los gráficos que muestran tanto los países como los índices simultáneamente, pueden ser de gran ayuda a este respecto. Estos gráficos se usan en este trabajo.
Estos métodos permitirán comprobar si los indicadores propuestos por el SSI son similares a lo largo de los diferentes países (por ejemplo, si los temas sociales, medioambientales o económicos son similares en diferentes zonas geográficas) o a lo largo de los diferentes años, encontrar áreas geográficas con perfiles de sostenibilidad similares, identificar los más diferenciados y ordenarlos todos según un gradiente de sostenibilidad. También se podrá identificar cuáles son las componentes de sostenibilidad más importantes en cada zona geográfica y en cada año.
El software usado para implementar el HJ-Biplot, desarrollado por Vicente-Villardón (2010), está disponible para ser descargado gratuitamente en:

http://biplot.usal.es/ClassicalBiplot/index.html
mientras que el resto de programas han sido creados por el autor para su empleo en este trabajo (véase capítulo 6).

Capítulo 4

Desarrollo

4.1. Consideraciones algebraicas previas

4.1.1. Proyecciones ortogonales

Sea $X = (x_1, \ldots, x_p)$ un punto del espacio $\mathbb{R}^p$, π un subespacio de $\mathbb{R}^p$ de dimensión r con $\{v_1 = (v_{11}, \ldots, v_{p1})^t, \ldots, v_r = (v_{1r}, \ldots, v_{pr})^t\}$ base de π, esto es, la matriz

$$V = (v_{ij})_{p\times r} = \begin{pmatrix} v_{11} & & v_{1r} \\ \vdots & \ldots & \vdots \\ v_{p1} & & v_{pr} \end{pmatrix} = (v_1 | \ldots | v_r)$$

es una base (ortonormal o no) de π; y sea $D_p(p \times p)$ una métrica simétrica de $\mathbb{R}^p$, esto es, una matriz simétrica (${D_p}^t = D_p$) definida positiva ($v^t D_p v \geq 0$ para todo $v \in \mathbb{R}^p$ y $v^t D_p v = 0$ si, y solo si, $v = (0, \ldots, 0)^t$).

Calculemos el punto $Y = (y_1, \ldots, y_p)$ de $\mathbb{R}^p$ que sea la proyección ortogonal de X sobre π, para lo cual se imponen dos condiciones:

- $Y \in \pi$, entonces existe $A = (a_1, \ldots, a_r)$ tal que $Y^t = a_1 \cdot v_1 + \ldots + a_r \cdot v_r$, o lo que es lo mismo, $Y^t = VA^t$.
- El vector $\overrightarrow{XY}$ es ortogonal a π (según la métrica D_p), es decir, a cada uno de los vectores de V, entonces $(Y - X)D_pV = (0, \ldots, 0)$, o lo que es lo mismo, $YD_pV = XD_pV$.

Por tanto se ha de resolver el sistema

$$\begin{cases} Y^t = VA^t \\ YD_pV = XD_pV \end{cases}$$

con incógnitas Y y A.

Se resuelve por cualquier método y se obtiene

$$\begin{cases} Y^t = V(V^tD_pV)^{-1}V^tD_pX^t & \Leftrightarrow & Y = XD_pV((V^tD_pV)^{-1})^tV^t = XD_pV(V^tD_pV)^{-1}V^t \\ A^t = (V^tD_pV)^{-1}V^tD_pX^t & \Leftrightarrow & A = XD_pV((V^tD_pV)^{-1})^t = XD_pV(V^tD_pV)^{-1} \end{cases},$$

(las dos últimas igualdades porque la traspuesta de la inversa es igual a la inversa de la traspuesta y porque V^tD_pV es simétrica por serlo D_p), que se comprueba fácilmente que es solución.

Por lo tanto, se ha obtenido Y, las coordenadas de la proyección ortogonal de X sobre π, y A, las coordenadas de Y considerándolo como elemento de π en la base V.

Como se han obtenido factores $(V^tD_pV)^{-1}$, a partir de este momento se va a suponer que las bases de los subespacios son ortonormales (dada una base cualquiera, siempre se puede encontrar una base ortonormal, por ejemplo, mediante el proceso de ortonormalización de Gram-Schmidt), esto es $V^tD_pV = Id_{p\times p}$, y por tanto

$$\begin{cases} Y = XD_pVV^t \\ A = XD_pV \end{cases}.$$

Supongamos ahora que se tienen n puntos del espacio $\mathbb{R}^p$ dados por las filas de una matriz $X_{n\times p}$. La solución anterior permite calcular las proyecciones ortogonales de los n puntos en el subespacio π: la matriz $Y_{n\times p}$ con las coordenadas de las proyecciones en filas, y $A_{n\times r}$, con las coordenadas de las proyecciones como elementos de π en la base V.

4.1.2. Reducción de la dimensionalidad

Dada una matriz $X = (x_{ij})_{n\times p}$, una métrica simétrica $D_p = (d_{ij})_{p\times p}$ en $\mathbb{R}^p$ y una matriz diagonal $D_n(n \times n)$ con pesos para las filas de X, esto es, n coeficientes $\omega_1, \ldots, \omega_n$ cuya suma sea 1:

$$D_n = \begin{pmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \omega_n \end{pmatrix},$$

se quiere encontrar un subespacio π de $\mathbb{R}^p$ de dimensión r dado por la base ortonormal $V = (v_{ij})_{p\times r}$, tal que las diferencias entre las filas de X y sus proyecciones ortogonales sobre π sean mínimas, es decir, se quiere encontrar una matriz V, y por tanto $A = XD_pV$ con solo r columnas y las n filas proyecciones ortogonales de las de X, que aproximen lo mejor posible a X, lo que se denomina reducir la dimensionalidad de X.

Se entiende por diferencia entre una fila de X y su proyección sobre π como el cuadrado de la norma de cada vector fila de $X - XD_pVV^t$ (ya se ha visto que la proyección de X sobre π en $\mathbb{R}^p$ es XD_pVV^t), esto es, los elementos de la diagonal de $(X - XD_pVV^t)D_p(X - XD_pVV^t)^t$.

Enunciemos nuestro problema según el método de los multiplicadores de Lagrange: se quiere minimizar la suma ponderada de las diferencias entre las filas de X y sus proyecciones sobre π (o V) sujeta a que V sea una base ortonormal.

$$F(v_{ij}, \lambda_{ef}) = \sum_{a=1}^{n} \omega_a \left[(X - XD_pVV^t)D_p(X - XD_pVV^t)^t\right]_{aa} - \sum_{e=1}^{r}\sum_{f=1}^{r} \lambda_{ef} \left(\sum_{g=1}^{p}\sum_{h=1}^{p} v_{ge}d_{eh}v_{hf} - \delta_{ef}\right)$$

siendo δ_{ef} la función delta de Kronecker, y con $\Lambda = (\lambda_{ef})_{r\times r}$ la matriz con los multiplicadores de Lagrange, la cual a partir de este momento, por simplicidad, se supone diagonal.

Desarrollando el término entre corchetes y el primer sumando se obtiene que el problema de minimización equivale al siguiente de maximización:

$$F(v_{ij}, \lambda_{ef}) = \mathrm{Tr}\left[D_nXD_pVV^tD_pX^t\right] - \sum_{e=1}^{r}\sum_{f=1}^{r} \lambda_{ef} \left(\sum_{g=1}^{p}\sum_{h=1}^{p} v_{ge}d_{eh}v_{hf} - \delta_{ef}\right)$$

siendo Tr el operador traza de una matriz. Esta equivalencia se tiene porque hay sumandos que no dependen de v_{ij} ni de λ_{ef} y se pueden suprimir, y porque todos los restantes sumandos tienen signo negativo y se pueden cambiar a positivo (por eso se pasa de un problema de minimización a otro de maximización).

Desarrollando el primer sumatorio a términos con factores x_{ij}, d_{ij}, v_{ij}, derivando F respecto de v_{ij} para todo $i = 1, \ldots, n$, $j = 1, \ldots, p$ y respecto de λ_{ef} para todo $e, f = 1, \ldots, r$ igualando a cero dichas derivadas, y reagrupando de nuevo las sumas en forma de matriz se obtiene el siguiente sistema de ecuaciones con incógnitas V y Λ:

$$\begin{cases} X^t D_n X D_p V = V\Lambda \\ V^t D_p V = Id_r \end{cases}.$$

Si $r = 1$, la primera ecuación equivale a que v_1 sea vector propio de $X^t D_n X D_p$ de valor propio λ_{11}, y la segunda, a que v_1 esté normalizado según la métrica D_p. Pero, de todos los vectores propios de distintos valores propios asociados, ¿con cuál nos quedamos?

Sustituyamos en F que v_1 es vector propio de valor propio λ_{11} y veamos para cuál es mayor el valor de F:

$$Tr\left[D_n X D_p V V^t D_p X^t\right] = Tr\left[\underbrace{X^t D_n X D_p V}\, V^t D_p\right] = Tr\left[V\Lambda V^t D_p\right] = Tr\left[\Lambda\, \underbrace{V^t D_p V}\right] = Tr\left[\Lambda\right] = \lambda_{11},$$

entonces el máximo se alcanza si v_1 es el vector propio de $X^t D_n X D_p$ normalizado según D_p asociado al mayor valor propio.

Por recurrencia, si $r > 1$ se obtiene

$$V = (v_1 | \ldots | v_r)$$

con $v_1, \ldots, v_r$ los vectores propios de $X^t D_n X D_p$ normalizados según D_p cuyos valores propios asociados están ordenados descendentemente.

Como la matriz D_n es simétrica y definida positiva, esta define una métrica simétrica y se puede reducir la dimensionalidad de X a lo largo de las filas análogamente a como se ha hecho para las columnas: $B = X^t D_n U$, siendo

$$U = (u_1 | \ldots | u_r)$$

con $u_1, \ldots, u_r$ los vectores propios de $X D_p X^t D_n$ normalizados según D_n cuyos valores propios asociados están ordenados descendentemente.

Si se quere reducir la dimensionalidad de X tanto para las filas como para las columnas, se deberían entonces realizar las descomposiciones espectrales de dos matrices:

$$X D_p X^t D_n = U \Lambda_{n\times n} U^{-1} \quad \text{y} \quad X^t D_n X D_p = V \Lambda_{p\times p} V^{-1},$$

con $U_{n\times n}$ y $V_{p\times p}$; y después calcular

$$A = XD_pV \quad y \quad B = X^tD_nU$$

y de A y B retener r columnas.

Pero vamos a demostrar que U y V se pueden calcular a partir de otras matrices obtenidas al realizar la descomposición en valores singulares de una única matriz, la matriz $\overline{X} = \overline{U}\overline{\Lambda}_{n\times p}{}^{1/2}\overline{V}^t$ con $\overline{U}^t\overline{U} = Id_n$ y $\overline{V}^t\overline{V} = Id_p$, si se define

$$\begin{aligned}
\overline{X} &= D_n{}^{1/2}XD_p{}^{1/2} &\Leftrightarrow\quad X &= D_n{}^{-1/2}\overline{X}D_p{}^{-1/2}\\
\overline{U} &= D_n{}^{1/2}U &\Leftrightarrow\quad U &= D_n{}^{-1/2}\overline{U}\\
\overline{V} &= D_p{}^{1/2}V &\Leftrightarrow\quad V &= D_p{}^{-1/2}\overline{V}\\
\overline{\Lambda}^{1/2} &= \Lambda^{1/2}.
\end{aligned}$$

En efecto:

$$\begin{aligned}
U^tD_nU &= U^tD_n{}^{1/2}D_n{}^{1/2}U = \overline{U}^t\overline{U} = Id_n\\
V^tD_pV &= V^tD_p{}^{1/2}D_p{}^{1/2}V = \overline{V}^t\overline{V} = Id_p,
\end{aligned}$$

además

$$XD_pX^tD_n = D_n{}^{-1/2}\overline{X}\underbrace{D_p{}^{-1/2}D_pD_p{}^{-1/2}}\overline{X}^tD_n{}^{-1/2}D_n = D_n{}^{-1/2}\overline{X}\overline{X}^tD_n{}^{-1/2}D_n =$$

$$= \underbrace{D_n{}^{-1/2}\overline{U}}\overline{\Lambda}^{1/2}\underbrace{\overline{V}^t\overline{V}}\overline{\Lambda}^{1/2}\underbrace{\overline{U}^tD_n{}^{-1/2}}D_n = U\Lambda U^tD_n = U\Lambda U^{-1}$$

$$X^tD_nXD_p = D_p{}^{-1/2}\overline{X}^t\underbrace{D_n{}^{-1/2}D_nD_n{}^{-1/2}}\overline{X}D_p{}^{-1/2}D_p = D_p{}^{-1/2}\overline{X}^t\overline{X}D_p{}^{-1/2}D_p =$$

$$= \underbrace{D_p{}^{-1/2}\overline{V}}\overline{\Lambda}^{1/2}\underbrace{\overline{U}^t\overline{U}}\overline{\Lambda}^{1/2}\underbrace{\overline{V}^tD_p{}^{-1/2}}D_p = V\Lambda V^tD_p = V\Lambda V^{-1}.$$

Este método para reducir la dimensionalidad, es el más sencillo, el que tiene las restricciones más simples, pero existen otros métodos para reducir la dimensionalidad con restricciones adaptadas a la estructura y los tipos de datos, por ejemplo el método llamado Sparse Principal Component Analysis (SPCA) (Zou et al., 2006), la descomposición CUR (Mahoney and Drineas, 2009; Bodor et al., 2012; Mitrovic et al., 2013), o el Disjoint Biplot (Nieto, 2015).

4.1.3. Matriz compromiso

Sea $X_{I\times J\times K}$ un tensor, un cubo de datos. Se define el producto tensorial entre una matriz bidimensional $U_{I\times P}$ (o un vector unidimensional tomando $P = 1$) por el tensor X a lo largo de la primera dimensión de la siguiente manera:

$$[X \times_1 U]_{pjk} = \sum_{i=1}^{I} x_{ijk}u_{ip},$$

es decir, cada elemento p que tendrá el nuevo tensor en cada columna fijadas j y k es igual al producto escalar de la columna p-ésima de U y la columna de X fijadas j y k.

De la misma manera se definen los productos tensoriales de $V_{J\times Q}$ y $W_{K\times R}$ por X a lo largo de la segunda y la tercera dimensión:

$$[X \times_2 V]_{iqk} = \sum_{j=1}^{J} x_{ijk}v_{jq}$$

$$[X \times_3 W]_{ijr} = \sum_{k=1}^{K} x_{ijk}w_{kr}.$$

Se quiere ahora, al igual que para el caso bidimensional, reducir la dimensionalidad de un tensor. Para ello, primero se reducirá el tensor a una matriz a lo largo de la tercera dimensión, y en segundo lugar se reducirá la dimensionalidad de dicha matriz aplicando el anterior apartado. A la matriz $I \times J$ a la que se reducirá el tensor se la llamará matriz compromiso y se representará por X_c. Además se representará cada matriz del tensor fijada una k en la tercera dimensión como X_k.

Junto con la métrica simétrica para las columnas, y la matriz de los pesos para las filas (que ya se ha visto que también es una métrica simétrica), se tendrá en cuenta otra matriz diagonal

$$D_K = \mathrm{diag}(\Omega_1, \ldots, \Omega_K) \text{ tal que } \Omega_1 + \ldots + \Omega_K = 1,$$

con los pesos para las repeticiones a lo largo de la tercera dimensión, que de la misma manera que la matriz de los pesos para las filas, también es una métrica simétrica.

La forma de hallar la matriz compromiso de un tensor es encontrar un vector

$$\alpha = (\alpha_1, \ldots, \alpha_K)^t \in \mathbb{R}^K,$$

tal que el cuadrado de la norma de la matriz combinación lineal de las X_k según los α_k y D_K sea máxima sujeta a que el vector $D_K\alpha$ tenga suma uno en sus componentes, porque se considera $D_K\alpha$ como el vector con los pesos. A tal combinación lineal es a la que se llamará matriz compromiso:

$$[X_c]_{ij} = \sum_{k=1}^{K} x_{ijk}\Omega_k\alpha_k.$$

Así que el cuadrado de la norma (de Hilbert-Schmidt) de X_c vale:

$$\|X_c\|^2 = \mathrm{Tr}[X_c{}^t D_n X_c D_p] = \sum_{a=1}^{I}\sum_{b=1}^{J}\sum_{c=1}^{J}\left(\sum_{d=1}^{K} x_{abd}\Omega_d\alpha_d\right)\left(\sum_{e=1}^{K} x_{ace}\Omega_e\alpha_e\right)\omega_a d_{cb}.$$

Definamos nuestro problema según el método de los multiplicadores de Lagrange, imponiendo que la norma de α según D_K sea la unidad (más tarde se impondrá la restricción de que la suma de los componentes de $D_K\alpha$ es la unidad), es decir, maximizar

$$F(\alpha_k, \lambda) = \sum_{a=1}^{I}\sum_{b=1}^{J}\sum_{c=1}^{J}\left(\sum_{d=1}^{K} x_{abd}\Omega_d\alpha_d\right)\left(\sum_{e=1}^{K} x_{ace}\Omega_e\alpha_e\right)\omega_a d_{cb} - \lambda\left(\sum_{f=1}^{K}\alpha_f{}^2\Omega_f - 1\right).$$

Derivando F respecto de α_k para todo $k = 1, \ldots, K$ y respecto de λ igualando a cero dichas derivadas, y reagrupando en forma de matriz se obtiene el siguiente sistema de ecuaciones con incógnitas α y λ:

$$\begin{cases} \mathrm{Tr}[X_k{}^t D_n X_c D_p] = \lambda\alpha_k & \text{para todo } k = 1, \ldots, K \\ \alpha^t D_K \alpha = 1 & \end{cases}.$$

Desarrollemos la primera ecuación:

$$\mathrm{Tr}[X_k{}^t D_n X_c D_p] = \mathrm{Tr}[D_p X_k{}^t D_n X_c] = \mathrm{Tr}\left[D_p X_k{}^t D_n\left(\sum_{a=1}^{K}\Omega_a\alpha_a X_a\right)\right] = \sum_{a=1}^{K}\mathrm{Tr}[D_p X_k{}^t D_n X_a]\Omega_a\alpha_a =$$

$$= \sum_{a=1}^{K}\mathrm{Tr}[X_k{}^t D_n X_a D_p]\Omega_a\alpha_a,$$

por simplicidad en la notación, se construye la matriz llamada de varianzas-covarianzas vectoriales, $\mathrm{Covv}_{K\times K}$, tal que $[\mathrm{Covv}]_{ab} = \mathrm{Tr}[X_a{}^t D_n X_b D_p]$, así $\mathrm{Tr}[X_k{}^t D_n X_c D_p]$ es igual al producto de la fila k-ésima de Covv por $D_K\alpha$, y el sistema de ecuaciones queda:

$$\begin{cases} \mathrm{Covv}D_K\alpha = \lambda\alpha \\ \alpha^t D_K \alpha = 1 \end{cases}.$$

La primera ecuación equivale a que α sea vector propio de $\mathrm{Covv}D_K$ de valor propio λ, y la segunda, a que α esté normalizado según la métrica D_K. Pero, de nuevo, veamos con cuál de todos los vectores propios de distintos valores propios asociados nos quedamos.

Sustituyamos en F que α es vector propio de valor propio λ y veamos para cuál es mayor el valor de F:

$$\mathrm{Tr}[X_c{}^t D_n X_c D_p] = \sum_{a=1}^{K} \Omega_a \alpha_a \mathrm{Tr}[X_a{}^t D_n X_c D_p] = \sum_{a=1}^{K} \Omega_a \alpha_a \mathrm{Tr}[D_p X_a{}^t D_n X_c] =$$

$$= \sum_{a=1}^{K} \sum_{b=1}^{K} \Omega_a \alpha_a \mathrm{Tr}[D_p X_a{}^t D_n X_b] \Omega_b \alpha_b = \alpha^t D_K \mathrm{Covv} D_K \alpha = \lambda \alpha^t D_K \alpha = \lambda,$$

entonces el máximo se alcanza si α es el vector propio de $\mathrm{Covv}D_K$ normalizado según D_K asociado al mayor valor propio.

Finalmente, se impone que la suma de los componentes de $D_K\alpha$ sea la unidad. Para ello, se divide cada elemento de $D_K\alpha$ entre la suma de los componentes de $D_K\alpha$, así el nuevo α verificará que al multiplicarlo por D_K sus componentes suman la unidad.

4.2. Inspección de matrices de datos multivariantes

4.2.1. Diagramas de dualidad. Diagrama de dualidad del PCA

Sea $X = (x_{ij})_{n\times p}$ una matriz de datos con n filas y p columnas. Sin pérdida de la generalidad se puede suponer que la matriz X está centrada por columnas, esto es, la media de los datos de cada columna es 0:

$$\frac{1}{n}\sum_{i=1}^{n} x_{ij} = 0, \quad \forall j = 1, \ldots, p,$$

en otro caso, se resta a cada dato la media de la columna a la que pertenece.
Sea D_n la matriz diagonal $n \times n$ con los pesos para las filas, esto es, n coeficientes cuya suma sea 1:

$$D_n = \mathrm{diag}(\omega_1, \ldots, \omega_n) = \begin{pmatrix} \omega_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \omega_n \end{pmatrix},$$

y sea D_p una métrica simétrica en $\mathbb{R}^p$. El diagrama de dualidad del análisis general de la tabla de datos X se define como sigue:

$$\begin{tikzcd} \mathbb{R}^p \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[d, "X"] \\ \mathbb{R}^{n*} \arrow[u, "X^t"] & \mathbb{R}^n \arrow[l, "D_n"] \end{tikzcd}$$

Esto se denomina "diagrama de dualidad" porque $\mathbb{R}^{p*}$ y $\mathbb{R}^{n*}$ son los espacios duales de $\mathbb{R}^p$ y $\mathbb{R}^n$, y porque los llamados operadores duales

$$X^tD_nXD_p \quad \text{y} \quad XD_pX^tD_n \tag{4.1}$$

comparten el mismo espectro (los mismos valores propios).
Las cuatro matrices que forman este diagrama de dualidad son las que se obtuvieron en el

apartado 4.1.2 y teniendo en cuenta que la métrica D_n, por ejemplo, se puede ver como una aplicación de $\mathbb{R}^n \times \mathbb{R}^n$ en $\mathbb{R}$ o, canónicamente, como una aplicación de $\mathbb{R}^n$ en $\mathbb{R}^{n*}$. Así, para diagramas de dualidad en análisis explicados posteriormente, si se quiere analizar una matriz cualquiera, siempre el centro del diagrama de dualidad, es decir, las cuatro matrices de los cuatro tramos que forman el cuadrado principal de un diagrama de dualidad, tendrán este aspecto.

Este diagrama está completamente definido por la "notación de triplete":

$$(X, D_p, D_n),$$

y la inercia total de este triplete estadístico es

$$Iner_X = Tr[XD_pX^tD_n] = Tr[X^tD_nXD_p].$$

En el caso particular en que D_n sea la matriz con pesos uniformes para las filas ($\omega_i = 1/n$), y D_p es la identidad (métrica euclídea), entonces este análisis es un PCA simple, y como las variables están centradas, la inercia es la suma de sus varianzas:

$$Iner_X = Tr[X^tD_nXD_p] = \sum_{j=1}^{p} \frac{1}{n} \left[X^tX\right]_{jj} = \sum_{j=1}^{p} \frac{1}{n} \sum_{i=1}^{n} \left[X^t\right]_{ji} [X]_{ij} = \sum_{j=1}^{p} \frac{1}{n} \sum_{i=1}^{n} {x_{ij}}^2 = \sum_{j=1}^{p} Var(X_j)$$

con X_j la j-ésima columna de X.

El Análisis de Componentes Principales generalizado (generalized Principal Component Analysis gPCA) de este triplete (X, D_p, D_n) corresponde a la descomposición espectral de 4.1.

Así, de $X^tD_nXD_p$ ó $XD_pX^tD_n$ se obtiene Λ, la matriz en cuya diagonal principal están los valores propios de $X^tD_nXD_p$ ó $XD_pX^tD_n$ ordenados descendentemente:

$$\lambda_1 \geq \ldots \geq \lambda_p \quad \text{ó} \quad \lambda_1 \geq \ldots \geq \lambda_n,$$

pero ocurre que si $n > p$,

$$\lambda_{p+1} = \lambda_{p+2} = \ldots = \lambda_n = 0$$

(si $n < p$ es $\lambda_{n+1} = \lambda_{n+2} = \ldots = \lambda_p = 0$). También se pueden calcular las bases ortonormales de vectores propios de $XD_pX^tD_n$ y $X^tD_nXD_p$, respectivamente $U_{n\times n}$ y $V_{p\times p}$, que tienen por columnas los vectores propios en el mismo orden en que sus valores propios asociados están en Λ.

Teóricamente, se sabe que se puede representar la matriz X en un espacio de dimensión p mediante un diagrama de dispersión con n puntos correspondientes a las filas y haciendo corresponder a cada eje coordenado una de las variables columna, pero en la práctica p puede tomar valores superiores a 3 y habría que graficar el diagrama de dispersión en planos que fueran representando de dos en dos las columnas, que no es lo que se busca, puesto que interesa representar gráficamente a todos los datos conjuntamente.

Si se quiere representar la matriz X en un subespacio de dimensión menor, digamos $r < p$ y $r < n$ (en la práctica normalmente r = 2 ó 3), se buscará una matriz también $n \times p$ pero de orden r, que sea aproximadamente igual a la X con pérdida mínima de información en sus datos, esto es, que la inercia de la nueva matriz sea lo más cercana posible a la de X. Esto es lo que ya se ha denominado como reducción de la dimensionalidad de X.

Mediante el procedimiento estudiado en el apartado 4.1.2 ya se sabe obtener U y V, y al quedarnos con sus primeras r columnas, se pueden calcular las proyecciones de X y X^t en un subespacio de dimensión r menor. El diagrama de dualidad es, por tanto, el siguiente:

$$\begin{tikzcd}
\mathbb{R}^p \arrow[rrr, "D_p"] & & & \mathbb{R}^{p*} \arrow[ddd, "X"] \\
& \bullet \arrow[ul, "V_r"] & & \\
& & \bullet \arrow[dr, "U_r"] & \\
\mathbb{R}^{n*} \arrow[uuu, "X^t"] & & & \mathbb{R}^n \arrow[lll, "D_n"]
\end{tikzcd}$$

La matriz que dará las nuevas coordenadas de las filas en ese subespacio de dimensión r es

$$XD_pV_r,$$

y las coordenadas de las columnas vendrán en las filas de

$$X^tD_nU_r.$$

En general, las matrices para calcular las coordenadas de las filas y las columnas en cualquier diagrama de dualidad están formadas por el producto de tres factores, y para deducirlas se deben seguir las siguientes normas:

- La primera norma es que solo hay que tener en cuenta dos métricas, las del mismo orden que U_r y V_r (en el gPCA solo existen D_n y D_p, pero más adelante se tendrán casos en los que intervienen más métricas).

- El primer factor del producto es: la propia matriz X, si se quieren calcular las nuevas coordenadas para las filas; o la matriz traspuesta X^t, si se quieren calcular las coordenadas para las columnas.

- El segundo factor y el tercero son respectivamente una métrica (de las que se tienen en cuenta según la primera norma) y una base ortonormal de entre U_r y V_r. La métrica es la que tenga el <u>mismo</u> orden que el número de <u>columnas</u> de la matriz primer factor: número de columnas de X si se quieren filas y número de columnas de X^t si se quieren columnas; y la base ortonormal es la que tenga el mismo orden que la métrica.

- Puede ocurrir que el número de columnas de la matriz primer factor no coincida con el orden de ninguna de las métricas (de las de la primera norma), así que la métrica y la base que hay que considerar son las que tengan un orden <u>distinto</u> al del número de <u>filas</u> de la matriz primer factor.

- La última norma es que el producto de estas tres matrices dará efectivamente las nuevas coordenadas siempre y cuando estén concatenadas en el diagrama de dualidad. En caso de que en el diagrama no exista ningún camino que concatene las tres matrices, la matriz que dará las nuevas coordenadas será la dada por el producto de todas las matrices que formen el camino más corto posible que contenga a las tres necesarias.

Se pueden ver ejemplos de las distintas aplicaciones de estas normas en los apartados sucesivos.

4.2.2. Biplots clásicos

Un biplot es una representación gráfica de datos multivariantes: una representación conjunta, en un espacio euclídeo de menor dimensión (normalmente un plano), de una matriz $X_{n\times p}$, mediante marcadores $a_1,\ldots,a_n$ para sus filas (puntos) y marcadores $b_1,\ldots,b_p$ para sus columnas (vectores), escogidos de tal manera que el producto interno (o escalar) $a_i{}^t \cdot b_j$ represente el elemento x_{ij} de la matriz X (Gabriel, 1971).

El biplot es una potente herramienta de visualización de datos, debido a las propiedades que posee el producto escalar. Gabriel (1971) propuso varios biplots con diferentes propiedades, el JK-Biplot, en el que las filas están representadas con alta calidad de representación, y el GH-Biplot, donde las columnas están representadas con alta calidad de representación.

Galindo (1986) propuso una nueva forma de representación, el HJ-Biplot, en la cual las coordenadas de las columnas coinciden con los marcadores columna del GH-Biplot, y las coordenadas de las filas coinciden con los marcadores fila del JK-Biplot, pero de tal forma que estas coordenadas pueden ser representadas en el mismo sistema de representación. Esta alternativa permite, como los biplots clásicos de Gabriel, interpretar cercanía entre puntos y proximidad entre los vectores mediante sus ángulos como similaridad/correlación, pero con una nueva ventaja porque es posible interpretar cercanía entre puntos y variables.

Todas estas representaciones (GH, JK y HJ-Biplot) son solo técnicas exploratorias; no se está considerando ninguna suposición paramétrica.

Las normas para la interpretación del HJ-Biplot son una combinación de las normas usadas para otras técnicas de escalamiento multidimensional, análisis de correspondencias, análisis factorial y biplots clásicos:

- La distancia entre marcadores fila (puntos) se interpreta como una función inversa de la similaridad, de tal forma que marcadores cercanos son más similares. Esta propiedad permite la identificación de agrupaciones de puntos con perfiles similares.

- Las longitudes de los marcadores columna (vectores) aproximan la desviación típica de las variables.

- Los cosenos de los ángulos entre los marcadores columna aproximan las correlaciones entre variables, de manera que ángulos agudos pequeños se asocian con variables con alta correlación positiva, ángulos obtusos cercanos a llanos se asocian con variables con alta correlación negativa y ángulos rectos se asocian con variables incorreladas.

- El orden de las proyecciones ortogonales de los marcadores fila en un marcador columna aproxima el orden de los valores de las filas en esa columna. La mayor de las proyecciones de un punto en una variable es el punto que más se desvía de la media de esa variable.

Desde un punto de vista analítico, sea X la matriz de datos con n filas y p columnas. Sea D_n la matriz diagonal $n \times n$ de los pesos uniformes de las filas: $D_n = \mathrm{diag}(1/n, \ldots, 1/n)$, y sea D_p la métrica euclídea en $\mathbb{R}^p$. El diagrama de dualidad del HJ-Biplot es

$$\begin{tikzcd}
\mathbb{R}^p \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[d, "X"] \\
\mathbb{R}^{n*} \arrow[u, "X^t"] & \mathbb{R}^n \arrow[l, "D_n"]
\end{tikzcd}$$

El triplete de este diagrama es (X, D_p, D_n), y el análisis HJ-Biplot de este triplete correspondería a la descomposición espectral de $X^tD_nXD_p$ y $XD_pX^tD_n$, pero basta con realizar la descomposición en valores singulares de X.
Así, se obtiene

$$X = U\Lambda V^t,$$

siendo ahora Λ la matriz $n \times p$ en cuya diagonal principal se encuentran los valores singulares ordenados descendentemente; U, que tiene por columnas una base de vectores singulares por la izquierda de X, o, equivalentemente, vectores propios ortonormales de XX^t, en el mismo orden en que están sus valores singulares asociados en Λ; y V, una base de vectores singulares por la derecha de X, o, de forma análoga, vectores propios ortonormales de X^tX, de nuevo en el mismo orden que sus valores singulares.

Si se quiere representar la matriz X en un subespacio de dimensión r menor se puede usar la aproximación

$$X \approx U_r \Lambda_r {V_r}^t,$$

con U_r y V_r las matrices obtenidas al quedarnos con las primeras r columnas de U y V, y Λ_r la matriz diagonal de dimensión r $\times$ r con los r primeros valores singulares.

Los marcadores fila en ese subespacio de dimensión r son las filas de

$$XD_p V_r = U\Lambda V^t V_r = U\Lambda \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \\ 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix}_{p\times r} = U \begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_r \\ 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix}_{n\times r} = U_r \Lambda_r$$

y los marcadores columna tienen por coordenadas las filas de

$$X^t D_n U_r = \text{ salvo un factor escalar } =$$
$$= X^t U_r = V \Lambda^t U^t U_r = V_r \Lambda_r.$$

Estas dos nubes de puntos pueden ser superpuestas en el mismo sistema de referencia (si se multiplican convenientemente por un factor escalar), pero no se verifica que el producto escalar de los marcadores fila y columna coincida con la aproximación de X:

$$(U_r \Lambda_r)\,(V_r \Lambda_r)^t = U_r\,(\Lambda_r)^2\,{V_r}^t \neq U_r \Lambda_r {V_r}^t.$$

El caso del GH-Biplot se correspondería a considerar como marcadores filas U_r en vez de $U_r\Lambda_r$, y el caso del JK-Biplot, con marcadores columna V_r en vez de $V_r\Lambda_r$, de esta forma el producto escalar en cada uno de ellos sí reproduce la matriz X.

Además sería posible construir una familia de factorizaciones de X tomando distintos exponentes para Λ_r:

$$X \approx U_r \Lambda_r {V_r}^t = (U_r {\Lambda_r}^{\alpha}) \left(V_r {\Lambda_r}^{1-\alpha}\right)^t$$

con α entre 0 y 1. En particular, si $\alpha = 0$ es el caso del GH-Biplot y si $\alpha = 1$ es el JK-Biplot. Pero para un α cualquiera distinto de $0, 1$, los marcadores fila y columna correspondientes perderían varias propiedades importantes, como la posibilidad de poder representarse en el mismo sistema de referencia o la obtención de buenas calidades de representación aun representándolas por separado.

4.2.3. Análisis Entre-Grupos

Si en la matriz de datos X, los individuos a los que representan las filas pertenecen a distintos grupos definidos en el estudio, mediante el Análisis Entre-Grupos (Between-Group Analysis BGA) es posible representar, por ejemplo, mediante un PCA, la tabla de las medias de los valores de los individuos en cada uno de los grupos. Y en un segundo paso, las filas de la tabla inicial se proyectan en este PCA para conseguir las coordenadas de las filas para todos los individuos. Hay muchos tipos de análisis entre-grupos, dependiendo del análisis inicial después del cual se computará el Análisis Entre-Grupos. Este puede ser, por ejemplo, un PCA, un Análisis de Correspondencias, o un Análisis de Correspondencias Múltiple.

En el Análisis Entre-Grupos, los individuos pertenecen a g grupos, digamos, $G_1, \ldots, G_g$, con tamaños $n_1, \ldots, n_g$, y

$$n = \sum_{k=1}^{g} n_k.$$

A partir de los cuales se puede construir la matriz auxiliar D:

$$D = \mathrm{diag}(1/n_k),$$

y la métrica simétrica que contiene los pesos para los grupos (posiblemente no uniformes), como un múltiplo de D tal que la suma de sus elementos sea la unidad:

$$D_g = \mathrm{diag}\left(\frac{1/n_k}{\sum_{k=1}^{g} 1/n_k}\right) = \begin{pmatrix} \frac{1/n_1}{\sum_{k=1}^{g} 1/n_k} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1/n_g}{\sum_{k=1}^{g} 1/n_k} \end{pmatrix},$$

que verifican que tienen suma unidad.

El Análisis Entre-Grupos es el análisis del triplete (X_B, D_p, D_g), donde X_B es la matriz $g \times p$ de las medias por grupos:

$$(X_B)_{kj} = \overline{x}_j^{\,k},$$

el término $\overline{x}_j^{\,k}$ es la media de la variable j en el grupo k

$$\overline{x}_j^{\,k} = \frac{1}{n_k} \sum_{i \in G_k} x_{ij}.$$

En notación matricial, si B es la matriz de los indicadores de clase,

$$B = (b_{ik})_{n \times g}, \quad \text{con } b_{ik} = \begin{cases} 1 & \text{si } i \in G_k \\ 0 & \text{si } i \notin G_k \end{cases},$$

entonces se tiene

$$X_B = DB^tX.$$

El correspondiente diagrama de dualidad es el siguiente:

$$\begin{tikzcd}
\mathbb{R}^{n*} \arrow[r, "X^t"] & \mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[r, "X"] \arrow[ddd, "X_B"] & \mathbb{R}^{n} \arrow[ddd, "B^t"] \\
 & & \bullet \arrow[ul, "V_r"'] & \\
 & \bullet \arrow[dr, "U_r"] & & \\
\mathbb{R}^{g} \arrow[uuu, "B"] & \mathbb{R}^{g*} \arrow[l, "D"] \arrow[uuu, "X_B{}^t"] & \mathbb{R}^{g} \arrow[l, "D_g"] & \mathbb{R}^{g*} \arrow[l, "D"]
\end{tikzcd}$$

En este caso, en el digrama de dualidad, en su parte central, están representados los cuatro tramos que forman el análisis de la matriz X_B, mientras que a derecha e izquierda se encuentran las definiciones de X_B y $X_B{}^t$ respectivamente como producto de matrices.

El Análisis Entre-Grupos es por tanto el análisis de la tabla de las medias de grupos, que conduce a la descomposición de las matrices $X_B{}^tD_gX_BD_p$ y $X_BD_pX_B{}^tD_g$, o a la descomposición de una única matriz según se ha desarrollado en el apartado 4.1.2 y se pueden representar las filas y las columnas de X_B de forma similar al PCA:

$$\begin{array}{ll} \text{filas de } X_B: & X_BD_pV_r \\ \text{columnas de } X_B: & X_B{}^tD_gU_r. \end{array}$$

El objetivo de este análisis es destacar las diferencias entre grupos, y las coordenadas de las filas maximizan la varianza entre-grupos. Las coordenadas de las filas y las columnas de la tabla de datos inicial pueden ser computadas proyectando las filas y columnas de la tabla X en el subespacio de componentes principales:

filas de X : XD_pV_r

columnas de X : $X^tBDD_gU_r = {X_B}^tD_gU_r$ (igual a las columnas de X_B).

Esto último es un ejemplo de aplicación de las normas para la deducción de la matriz que da las coordenadas de las columnas de X:

- Las dos únicas métricas que se deben tener en cuenta son D_p y D_g, no así D_n.
- La primera matriz factor es X^t porque se quieren las coordenadas de las columnas de X.
- Como el número de columnas de X^t (n) no es el orden de ninguna métrica, se toma la métrica que tenga un orden distinto al número de filas de X^t (p), así la métrica y la base son D_g y U_r.
- Por último, como no existe ningún camino en el diagrama de dualidad en el que los tres factores X^t, D_g y U_r sean consecutivos, se toma el camino más corto que los una, $X^tBDD_gU_r$.

El método BGA así explicado es el más sencillo, el más simple de aplicar, pero existen otros métodos para analizar una matriz en el que las filas pertenecen naturalmente a varios grupos definidos, ejemplos son el Biplot Canónico (Amaro et al., 2004), el Clustering and disjoint principal component analysis (Vichi and Saporta, 2009), o el Clustering and Disjoint Biplot (Nieto, 2015).

4.2.4. Análisis de Co-Inercia

En un PCA simple, así como la inercia es la suma de varianzas, la co-inercia es una suma de cuadrados de covarianzas, y el Análisis de Co-Inercia (Co-Inertia Analysis CoIA) describe la co-estructura entre dos tablas de datos representando tan bien como sea posible los cuadrados de las covarianzas entre una tabla y otra.

Sea X la primera tabla, con n filas y p columnas, y sea Y la segunda tabla, con las mismas n filas y q columnas. Sea D_n la matriz diagonal $n \times n$ de los pesos de las filas:

$$D_n = \mathrm{diag}(\omega_1, \ldots, \omega_n),$$

y sean D_p y D_q dos métricas en $\mathbb{R}^p$ y $\mathbb{R}^q$ respectivamente.

Antes de hacer el Análisis de Co-Inercia, se necesita analizar cada tabla separadamente. Los diagramas de dualidad de los análisis separados de las dos tablas de datos son como sigue:

$$\begin{tikzcd}
\mathbb{R}^p \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[d, "X"] & \mathbb{R}^q \arrow[r, "D_q"] & \mathbb{R}^{q*} \arrow[d, "Y"] \\
\mathbb{R}^{n*} \arrow[u, "X^t"] & \mathbb{R}^n \arrow[l, "D_n"] & \mathbb{R}^{n*} \arrow[u, "Y^t"] & \mathbb{R}^n \arrow[l, "D_n"]
\end{tikzcd}$$

Un gPCA de estos tripletes se corresponde a la descomposición explicada en el apartado 4.1.2 para X y para Y. Si D_n es la matriz de pesos de filas uniforme ($\omega_i = 1/n$), y D_p y D_q son identidades (métricas euclídeas), entonces estos análisis son PCAs simples.

El Análisis de Co-Inercia está definido por el diagrama de dualidad obtenido al mezclar los dos diagramas separados. Esto será posible cuando tengan los mismos espacios $\mathbb{R}^n$ y $\mathbb{R}^{n*}$ en común, lo que implica que las filas de las dos tablas deben ser idénticas. El “diagrama conjunto” del Análisis de Co-Inercia es por tanto

$$
\begin{tikzcd}
\mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[d, "X"] \\
\mathbb{R}^{n*} \arrow[u, "X^t"] \arrow[d, "Y^t"'] & \mathbb{R}^{n} \arrow[l, shift right, "D_n"'] \arrow[l, shift left, "D_n"] \\
\mathbb{R}^{q} \arrow[r, "D_q"'] & \mathbb{R}^{q*} \arrow[u, "Y"']
\end{tikzcd}
\equiv
\begin{tikzcd}
\mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[d, "X"] \\
\mathbb{R}^{n*} \arrow[u, "X^t"] & \mathbb{R}^{n} \arrow[dl, "D_n"'] \\
\mathbb{R}^{n*} \arrow[d, "Y^t"'] & \mathbb{R}^{n} \arrow[ul, "D_n"] \\
\mathbb{R}^{q} \arrow[r, "D_q"'] & \mathbb{R}^{q*} \arrow[u, "Y"']
\end{tikzcd}
\equiv
\begin{tikzcd}
\mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[d, "X"] \\
\mathbb{R}^{n*} \arrow[u, "X^t"] & \mathbb{R}^{n} \arrow[d, "D_n"] \\
\mathbb{R}^{n} \arrow[u, "D_n"] & \mathbb{R}^{n*} \arrow[d, "Y^t"] \\
\mathbb{R}^{q*} \arrow[u, "Y"] & \mathbb{R}^{q} \arrow[l, "D_q"]
\end{tikzcd}
\tag{4.2}
$$

El Análisis de Co-Inercia es el análisis de vectores y valores propios de $X^tD_nYD_qY^tD_nXD_p$ y de $Y^tD_nXD_pX^tD_nYD_q$, o la explicada en el apartado 4.1.2 para Y^tD_nX. Esto es equivalente al siguiente “diagrama cruzado”:

$$
\begin{tikzcd}
\mathbb{R}^{n*} \arrow[r, "X^t"] & \mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[r, "X"] \arrow[d, "Y^tD_nX"] & \mathbb{R}^{n} \arrow[d, "D_n"] \\
\mathbb{R}^{n} \arrow[u, "D_n"] & \mathbb{R}^{q*} \arrow[l, "Y"] \arrow[u, "X^tD_nY"] & \mathbb{R}^{q} \arrow[l, "D_q"] & \mathbb{R}^{n*} \arrow[l, "Y^t"]
\end{tikzcd}
$$

En este caso, la parte central del diagrama de dualidad representa el análisis de la matriz Y^tD_nX mientras que a derecha y a izquierda están las definiciones de la matriz de productos cruzados y su traspuesta.

Este diagrama destaca el hecho de que el Análisis de Co-Inercia es el análisis de una tabla de productos cruzados, y su notación triplete es (Y^tD_nX, D_p, D_q). Si se está en el caso en que D_n sea la matriz con pesos uniformes para las filas y las métricas D_p y D_q sean euclídeas, como las columnas de ambas tablas están centradas, entonces la inercia total de cada tabla es simplemente la suma de varianzas:

$$Iner_X = \sum_{j=1}^{p} Var(X_j) \quad e \quad Iner_Y = \sum_{k=1}^{q} Var(Y_k).$$

Y la co-inercia entre X e Y es, en este caso, una suma de cuadrados de covarianzas:

$$CoIner_{XY} = Tr[Y^tD_nXD_pX^tD_nYD_q] = \sum_{k=1}^{q}(Y^tD_nXX^tD_nY)_{kk} = \sum_{k=1}^{q}\sum_{j=1}^{p}(Y^tD_nX)_{kj}(X^tD_nY)_{jk} =$$

$$= \sum_{j=1}^{p}\sum_{k=1}^{q}((X^tD_nY)_{jk})^2 = \sum_{j=1}^{p}\sum_{k=1}^{q}\left(\frac{1}{n}\sum_{i=1}^{n}(X^t)_{ji}(Y)_{ik}\right)^2 = \sum_{j=1}^{p}\sum_{k=1}^{q}\left(\frac{1}{n}\sum_{i=1}^{n}x_{ij}y_{ik}\right)^2 =$$

$$= \sum_{j=1}^{p}\sum_{k=1}^{q}(Cov(X_j, Y_k))^2.$$

El Análisis de Co-Inercia maximiza las covarianzas entre las coordenadas de las filas de las dos tablas. La co-inercia es alta cuando los valores en ambas tablas son altos simultáneamente (o cuando varían inversamente) y baja cuando varían independientemente o cuando no varían. Esto es el significado de la co-estructura entre las dos tablas de datos.

Se pueden representar gráficamente las filas y las columnas de las dos matrices originales en el subespacio de dimensión r obtenido en el Análisis de Co-Inercia, calculando distintas coordenadas:

$$
\begin{tikzcd}
\mathbb{R}^{n*} \arrow[r, "X^t"] & \mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[r, "X"] \arrow[ddd, "Y^tD_nX"] & \mathbb{R}^{n} \arrow[ddd, "D_n"] \\
 & & \bullet \arrow[ul, "V_r"'] & \\
 & \bullet \arrow[dr, "U_r"] & & \\
\mathbb{R}^{n} \arrow[uuu, "D_n"] & \mathbb{R}^{q*} \arrow[l, "Y"] \arrow[uuu, "X^tD_nY"] & \mathbb{R}^{q} \arrow[l, "D_q"] & \mathbb{R}^{n*} \arrow[l, "Y^t"]
\end{tikzcd}
$$

las filas y las columnas de la matriz X tienen por coordenadas

$$
\begin{array}{ll}
\text{filas de X :} & XD_pV_r \\
\text{columnas de X :} & X^tD_nYD_qU_r
\end{array}
$$

y las de Y

$$
\begin{array}{ll}
\text{filas de Y :} & YD_qU_r \\
\text{columnas de Y :} & Y^tD_nXD_pV_r
\end{array}.
$$

El anterior "diagrama conjunto" 4.2 muestra la similaridad del Análisis de Co-Inercia con el Análisis de Correlación Canónica (Canonical Correlation Analysis CCA). En efecto, la única diferencia entre los diagramas de dualidad del Análisis de Co-Inercia y del Análisis de Correlación Canónica viene por las métricas de $\mathbb{R}^p$ y $\mathbb{R}^q$:

$$
\begin{tikzcd}
\mathbb{R}^{p} \arrow[r, "{V_X}^{-1}"] & \mathbb{R}^{p*} \arrow[d, "X"] \\
\mathbb{R}^{n*} \arrow[u, "X^t"] \arrow[d, "Y^t"] & \mathbb{R}^{n} \arrow[l, shift right, "D_n"'] \arrow[l, shift left, "D_n"] \\
\mathbb{R}^{q} \arrow[r, "{V_Y}^{-1}"'] & \mathbb{R}^{q*} \arrow[u, "Y"']
\end{tikzcd}.
$$

El Análisis de Correlación Canónica usa la métrica de Mahalanobis en $\mathbb{R}^p$ y $\mathbb{R}^q$, cuyas matrices son las inversas de las matrices de varianzas-covarianzas

$$V_X = X^t D_n X \quad y \quad V_Y = Y^t D_n Y.$$

Esto conduce al triplete del Análisis de Correlación Canónica:

$$(Y^t D_n X, (X^t D_n X)^{-1}, (Y^t D_n Y)^{-1}).$$

En el Análisis de Correlación Canónica las coordenadas de las filas maximizan sus correlaciones, pero esto puede ser alcanzado con varianzas muy pequeñas. Maximizando las covarianzas en vez de las correlaciones, el Análisis de Co-Inercia asegura que las puntuaciones no tienen varianzas muy pequeñas, y por tanto tienen un buen porcentaje de varianza explicada en cada espacio.

4.3. Análisis de una sucesión de matrices de datos

4.3.1. Análisis Parcial Triádico

El Análisis Parcial Triádico (Partial Triadic Analysis PTA), también denominado X-STATIS, pertenece a la familia de métodos STATIS de análisis de k-tablas (Structuration des Tableaux À Trois Indices de la Statistique). La familia de STATIS puede ser pensada como proporcionar un PCA de un conjunto de PCAs. El Análisis Parcial Triádico es el más simple de estos métodos, pero también es el más restrictivo. Su objetivo es analizar una serie de K tablas que tengan las mismas filas y las mismas columnas. Esto significa que las mismas variables deben ser medidas a los mismos individuos, varias veces. Sin embargo existen otros métodos STATIS en función de los objetivos o el diseño del estudio que se esté realizando, por ejemplo, los STATIS y STATIS DUAL (L'Hermier des Plantes, 1976), el COVSTATIS (Thioulouse, 2011), el DISTATIS (Abdi et al., 2007), el Power-STATIS (Benasseni and Dosse, 2012), el CANOSTATIS (Vallejo-Arboleda et al., 2007), el k + 1-STATIS (Sauzay et al., 2006), el DO-ACT (Vivien and Sabatier, 2004), o el STATIS-4 (Sabatier and Vivien, 2008).

El Análisis Parcial Triádico, como cualquier método STATIS, sigue tres pasos: interestructura, compromiso e intraestructura (también llamada "trayectorias").

El paso interestructura proporciona los coeficientes de una combinación lineal especial de las tablas de datos, lo que conduce a una representación óptima llamada "compromiso". El segundo paso computa el PCA de esta combinación lineal. El paso intraestructura es una proyección de las filas y columnas de cada tabla de la serie en el espacio multidimensional del análisis del compromiso.

La ventaja del Análisis Parcial Triádico es que destaca la "estructura estable" de una sucesión de tablas. El paso del compromiso representa esta estructura estable (cuando existe), y el paso de la intraestructura muestra cómo cada tabla se aleja de ella.

La interestructura está basada en los conceptos de "varianza vectorial" y "covarianza vectorial". Se construye una matriz de productos escalares entre tablas, la matriz de varianzas-covarianzas vectoriales, Covv, que puede ser escrita simplemente

$$\mathrm{Covv}_{k_1 k_2} = \mathrm{Covv}(X_{k_1}, X_{k_2}) = \left\langle X_{k_1}, X_{k_2} \right\rangle = \text{ por definición } =$$
$$= \mathrm{Tr}[X_{k_1}{}^t D_n X_{k_2} D_p],$$

donde X_{k_1} y X_{k_2} son la k_1-ésima y la k_2-ésima tabla de la serie. El análisis de vectores y valores propios de esta matriz de varianzas-covarianzas vectoriales (ponderada según los pesos para la tercera dimensión) da un primer vector propio, y las coordenadas α_k de este primer vector propio (con las mismas ponderaciones) son usadas como pesos para computar el compromiso. Además, puede representarse gráficamente esta interestructura, tradicionalmente en un subespacio bidimensional, con vectores desde el origen y extremo en los puntos dados por las filas de $\mathrm{Covv}D_K V_2$ siendo V_2 los dos primeros vectores propios de la matriz de varianzas-covarianzas vectoriales y D_K la matriz diagonal con los pesos para la tercera dimensión.
Alternativamente, puede ser usada una matriz de "correlaciones vectoriales", que reescala las tablas:

$$\mathrm{Rv}(X_{k_1}, X_{k_2}) = \frac{\mathrm{Covv}(X_{k_1}, X_{k_2})}{\sqrt{\mathrm{Varv}(X_{k_1})}\sqrt{\mathrm{Varv}(X_{k_2})}},$$

donde $\mathrm{Varv}(X_k)$ es la varianza vectorial de la tabla k:

$$\mathrm{Varv}(X_k) = \mathrm{Tr}[X_k{}^t D_n X_k D_p]$$

que, en el caso particular en que D_p sea la identidad, es simplemente la suma de las varianzas de cada una de las columnas de X_k.
El compromiso X_c es una combinación lineal de las tablas iniciales, ponderadas por las coordenadas del primer vector propio de la interestructura y por D_K (al ser la matriz de varianzas-covarianzas vectoriales simétrica de elementos positivos, si D_K es la matriz de pesos uniformes para la tercera dimensión, el primer vector propio tiene todas sus coordenadas del mismo signo, que se supone positivo):

$$X_c = \sum_{k=1}^{K} \alpha_k \Omega_k X_k.$$

La inercia de este compromiso es máxima, y su principal propiedad es que maximiza la similaridad con todas las tablas iniciales, medida mediante la suma de sus productos,

$$\sum_{k=1}^{K} \langle X_c, X_k \rangle .$$

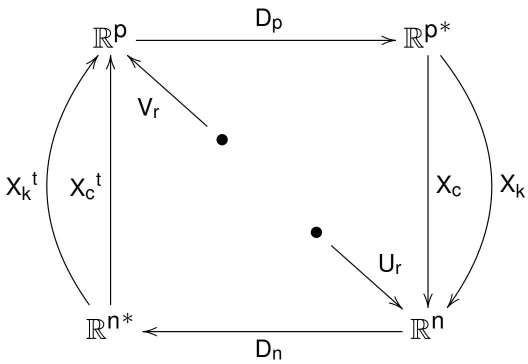
Cuando las tablas están estandarizadas, es decir, la varianza vectorial de cada tabla es la unidad (lo cual no implica que cada una de las varianzas de sus columnas sea la unidad) el producto escalar es el coeficiente Rv

$$\left\langle X_{k_1}, X_{k_2} \right\rangle = \mathrm{Covv}(X_{k_1}, X_{k_2}) = \mathrm{Rv}(X_{k_1}, X_{k_2})\sqrt{\mathrm{Varv}(X_{k_1})}\sqrt{\mathrm{Varv}(X_{k_2})} = \mathrm{Rv}(X_{k_1}, X_{k_2}).$$

El peso de cada tabla es proporcional a su inercia, así que tablas que son diferentes de las otras serán ponderadas por defecto. Esta propiedad asegura que el compromiso se parezca a todas las tablas de la sucesión “lo mejor posible” en el sentido de los mínimos cuadrados.

A la vista del diagrama de dualidad, primero, el análisis del compromiso, por ejemplo, mediante un PCA, da representaciones de baja dimensión (gráficos de los ejes principales) que pueden ser usados para interpretar su estructura.

En este caso, la parte central del diagrama de dualidad representa el análisis de la matriz X_c.



En este diagrama se representan las matrices X_k con tramos curvos con extremos en los mismos que los de X_c por ser X_c combinación lineal de las X_k.

Por tanto, las filas y las columnas de X_c, es decir, el análisis del compromiso, tienen por coordenadas:

$$\begin{array}{ll} \text{filas de } X_c : & X_c D_p V_r \\ \text{columnas de } X_c : & X_c^t D_n U_r \end{array}$$

con U_r y V_r las primeras r columnas de la base de vectores propios de las descomposiciones de $X_cD_pX_c^tD_n$ y $X_c^tD_nX_cD_p$ respectivamente, o la descomposición explicada en el apartado 4.1.2 para X_c.

La intraestructura se obtiene proyectando las filas y columnas de cada tabla de la serie en el análisis del compromiso. Las coordenadas de las filas y las columnas de cada tabla X_k son

filas de X_k : $X_kD_pV_r$

columnas de X_k : $X_k^tD_nU_r$.

4.3.2. Métodos Tucker

Antes del descubrimiento de las técnicas de análisis de datos tridimensionales (o multidimensionales), este tipo de datos se analizaban desplegando el contenido del cubo. La idea es construir una matriz de datos bidimensional a partir del cubo tridimensional eliminando una de las dimensiones, solo se buscarán interacciones entre dos tipos de unidades en vez de entre tres tipos. Se entiende por desplegar el construir una matriz a partir de un cubo concatenando las matrices del cubo en una única matriz alta, de manera que ahora las relaciones entre la primera y la tercera dimensión se pierden. A veces este procedimiento puede no funcionar, el desplegamiento es una simplificación inaceptable.

Otra de las técnicas era tratar el cubo de datos como un conjunto de matrices y aplicar un Análisis de Componentes Principales a la matriz de cada repetición del cubo tridimensional.

$$X_k \approx A_k G_k {B_k}^t$$

con A_k y B_k vectores singulares por la izquierda y la derecha respectivamente de X_k, y G_k la matriz diagonal con los valores singulares de X_k (figura 4.1).

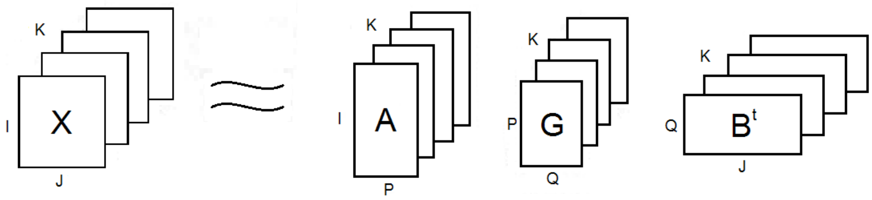


Figura 4.1: *Análisis de Componentes Principales separados*

Esta técnica es poco recomendable porque se estaría suponiendo que todos los análisis son independientes y, por tanto, ninguna repetición estaría relacionada con cualquiera de las otras. Hoy en día, una de las formas de analizar un cubo de datos $X_{I \times J \times K}$ es llevar a cabo uno de los llamados métodos Tucker. Su objetivo es reducir la dimensionalidad del problema, $I \times J \times K$, para resumir la información, construyendo un modelo simplificado, $P \times Q \times R$, para facilitar la descripción de los datos. Además, gráficos que muestren simultáneamente las tres dimensiones pueden ser muy útiles a este respecto.

Estos métodos permitirán responder a preguntas como: ¿Qué grupos de individuos (primera dimensión) se comportan de manera diferente en qué variables (segunda dimensión) durante qué repeticiones (tercera dimensión)?, ¿qué relaciones hay entre las variables?, ¿qué tendencia se puede descubrir a lo largo del tiempo?, ¿hay diferentes tipos de individuos?, y cuestiones más complejas, como si las relaciones entre variables varían en el tiempo, o si la estructura de las variables cambia en el tiempo de forma diferente para diferentes grupos de individuos.

Antes de hablar del punto de vista algebraico de los métodos Tucker conviene definir matemáticamente lo que se quiere decir con desplegamiento. Dado un cubo $X_{I\times J\times K}$ se definen los desplegamientos a lo largo de cada una de las dimensiones, y se denotan como $X_{(1)}, X_{(2)}, X_{(3)}$ de órdenes respectivos $I \times JK, J \times IK, K \times IJ$, como las matrices bidimensionales:

$$\left[X_{(1)}\right]_{i,j+J\cdot(k-1)} = x_{ijk}$$

$$\left[X_{(2)}\right]_{j,i+I\cdot(k-1)} = x_{ijk}$$

$$\left[X_{(3)}\right]_{k,i+I\cdot(j-1)} = x_{ijk}.$$

Desde el punto de vista algebraico, el objetivo de los métodos Tucker es encontrar una descomposición del cubo de datos $X_{I\times J\times K}$ en matrices ortogonales (o matriz) y otros cubos de datos (o cubo); en el caso más utilizado, tres matrices ortogonales $A_{I\times P}, B_{J\times Q}$ y $C_{K\times R}$, y otro cubo de datos $G_{P\times Q\times R}$ llamado core array, con $P \times Q \times R$ más simple que $I \times J \times K$, tales que el producto tensorial de G, A^t, B^t y C^t, cuyo elemento ijk-ésimo es

$$[((G\times_1 A^t)\times_2 B^t)\times_3 C^t]_{ijk} = \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{r=1}^{R} a_{ip}b_{jq}c_{kr}g_{pqr}$$

siendo $A = (a_{ip}), B = (b_{jq}), C = (c_{kr}), G = (g_{pqr})$, sea la mejor aproximación de X.

En función de las demandas del problema que se esté estudiando se distinguen tres modelos Tucker:

1. Tucker1: Una restricción que se puede pedir a los resultados es que la matriz de componentes para una de las dimensiones, de los Análisis de Componentes Principales por separado, sea la misma para todas las repeticiones, esto es, por ejemplo, $A_k = A_{I\times P}$ para

$k = 1 \ldots, K$ (figura 4.2):

$$X_k \approx AG_kB_k{}^t.$$

Se le denomina Tucker1 porque el 1 se refiere al numero de dimensiones para las que la matriz de componentes está determinada independientemente de k.

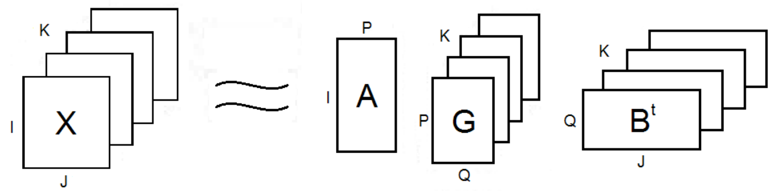


Figura 4.2: *Modelo Tucker1*

2. Tucker2: Análogamente a como se ha definido el Tucker1, el método Tucker2 es aquel en el que se restringen simultáneamente las matrices de componentes para las dos dimensiones, de los Análisis de Componentes Principales de cada repetición, esto es, $A_k = A_{I \times P}$ y $B_k = B_{J \times Q}$ para $k = 1, \ldots, K$ (figura 4.3):

$$X_k \approx AG_kB^t.$$

De nuevo, se le denomina Tucker2 por ser 2 el número de dimensiones para las que se fijan las componentes.

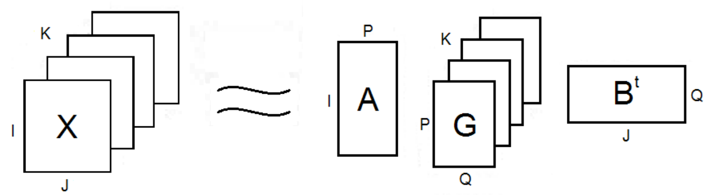


Figura 4.3: *Modelo Tucker2*

3. Tucker3: Este método tiene las matrices de las dos primeras dimensiones fijas para todas las repeticiones y, además, las matrices G_k se pueden resumir en un único cubo de datos

$G_{P\times Q\times R}$ y una matriz para la tercera dimensión $C_{K\times R}$ (figura 4.4):

$$X \approx ((G\times_1 A^t)\times_2 B^t)\times_3 C^t.$$

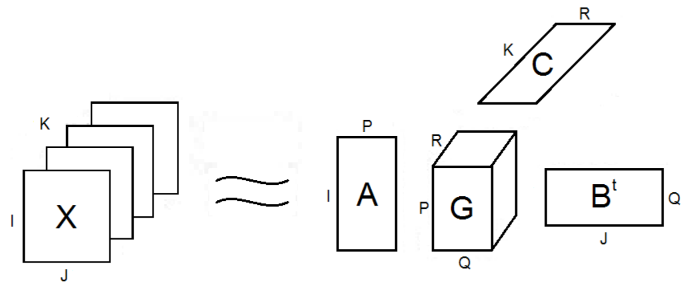


Figura 4.4: *Modelo Tucker3*

Este último es el método Tucker más utilizado de los tres, por tener en cuenta las tres dimensiones simultáneamente de forma independiente para las repeticiones, así que es el que pasa a ser descrito más detalladamente de forma algebraica.

El proceso para encontrar la mejor aproximación de X con $P \times Q \times R$ componentes es el siguiente algoritmo iterativo (Kiers et al., 1992). En este algoritmo, el término A_n representa la matriz para la primera dimensión obtenida en la iteración n-ésima, en particular, A_1 es la matriz calculada en la iteración inicial; B_n, C_n y G_n se definen análogamente:

1. A_1 se define como el conjunto de los P primeros vectores singulares por la izquierda de $X_{(1)}$. Análogamente, se definen B_1 y C_1 como los primeros Q y R vectores singulares por la izquierda de $X_{(2)}$ y $X_{(3)}$.
   Expliquemos mejor lo que significa este paso: dado el cubo de datos X, se despliega a lo largo de la primera dimensión, es decir, se obtiene una matriz bidimensional en la que se dejan fijas las filas y en columnas se tienen todas las columnas originales de cada una de las repeticiones, una a continuación de otra. Y de esta matriz se retienen los P primeros vectores singulares por la izquierda. Después se hace lo mismo desplegando

el cubo a lo largo de la segunda y la tercera dimensión, reteniendo los primeros Q y R vectores singulares por la izquierda respectivamente.

2. G_1 se calcula como el producto tensorial de X, A_1, B_1 y C_1:

$$[G_1]_{pqr} = [((X \times_1 A_1) \times_2 B_1) \times_3 C_1]_{pqr} = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} [A_1]_{ip} [B_1]_{jq} [C_1]_{kr} x_{ijk}.$$

   En este paso, lo que se busca es encontrar el core array, que es lo que le falta a A_1, B_1 y C_1 para que se obtenga una aproximación de X.

3. Paso de iteración, $n = 1, \ldots$:

   a) Se calcula la matriz óptima A_{n+1}, dejando fijas B_n y C_n.
   Se denomina matriz óptima en este caso a los P primeros vectores singulares por la izquierda de $((X \times_2 B_n) \times_3 C_n)_{(1)}$.
   Es decir, dados los <u>B_n</u> y <u>C_n</u> de la <u>iteración</u> anterior, se reduce X a lo largo de la segunda y la tercera dimensión, obteniendo un cubo con las filas originales, pero con las columnas y las repeticiones reducidas. Ahora este cubo reducido se despliega a lo largo de la primera dimensión y se retienen los primeros P vectores singulares por la izquierda de la matriz desplegada, que formarán A_{n+1}.

   b) Se calcula la matriz óptima B_{n+1}, dejando fijas A_{n+1} y C_n: los Q primeros vectores singulares por la izquierda de $((X \times_1 A_{n+1}) \times_3 C_n)_{(2)}$.
   Esto quiere decir que, dados los <u>A_{n+1}</u> de la <u>instrucción</u> anterior y los <u>C_n</u> de la <u>iteración</u> anterior, se realiza lo mismo para la segunda dimensión: se reduce X a lo largo de la primera y la tercera dimensión, se despliega este cubo a lo largo de la segunda dimensión, y se retienen los primeros Q vectores singulares por la izquierda de esta matriz, que formarán B_{n+1}.

   c) Se calcula la matriz óptima C_{n+1}, dejando fijas A_{n+1} y B_{n+1}: los R primeros vectores singulares por la izquierda de $((X \times_1 A_{n+1}) \times_2 B_{n+1})_{(3)}$.
   Esto quiere decir que, dados los <u>A_{n+1}</u> y <u>B_{n+1}</u> de las <u>instrucciones</u> anteriores, se reduce X a lo largo de la primera y la segunda dimensión, se despliega este cubo

a lo largo de la tercera dimensión y se retienen los primeros R vectores singulares por la izquierda de esta matriz, que formarán C_{n+1}.

d) Se calcula el core array G_{n+1}, como el producto tensorial de X, A_{n+1}, B_{n+1} y C_{n+1}. Es decir, de nuevo, se calcula el core array, que es lo que le falta a A_{n+1}, B_{n+1} y C_{n+1} para que se obtenga una aproximación de X.

e) Este paso se para cuando las diferencias entre $A_{n+1}, B_{n+1}, C_{n+1}, G_{n+1}$ y A_n, B_n, C_n, G_n sean más pequeñas que un valor establecido de inicio.

4. Las matrices A, B y C y el core array G se definen entonces como los obtenidos tras la iteración n-ésima: $A = A_{n+1}, B = B_{n+1}, C = C_{n+1}$ y $G = G_{n+1}$.

La manera de escoger qué modelo $P \times Q \times R$ será considerado es computar la descomposición previa para todas las combinaciones $P \times Q \times R$ con $P \leq I, Q \leq J$ y $R \leq K$; pero además se debe tener en cuenta la llamada regla del máximo producto, que dice que los casos en que el número de elementos de una dimensión es mayor que el producto de los otros dos, $P > QR, Q > PR$ o $R > PQ$, pueden ignorarse, porque en estos casos el modelo es equivalente a otro más reducido:

$$\text{Si } P > QR \text{ el modelo } P \times Q \times R \text{ es equivalente al modelo } QR \times Q \times R$$
$$\text{Si } Q > PR \text{ el modelo } P \times Q \times R \text{ es equivalente al modelo } P \times PR \times R$$
$$\text{Si } R > PQ \text{ el modelo } P \times Q \times R \text{ es equivalente al modelo } P \times Q \times PQ.$$

Cuando se haya hecho esto, se estudia cuál es la combinación más simple de entre las más estables y las que alcancen un porcentaje de varianza explicada suficientemente alto.

Para cada modelo se calcula la suma del número de componentes, $S = P + Q + R$, y para cada valor de S se elige aquel modelo que tenga un menor valor de la suma de cuadrados residual, o equivalentemente, un mayor valor de la varianza explicada. Así se tiene una lista de modelos, uno para cada valor de S. A continuación, se calcula el cociente incremental entre la suma de cuadrados residual y S para cada uno de los modelos anteriores en orden creciente de S, y nos quedaremos con aquellos modelos para los que el cociente incremental sea similar al del siguiente, esto es, los modelos más estables. Finalmente el modelo que será escogido para el análisis será aquel de los estables que tenga un menor valor de S, esto es, el más simple de entre los más estables.

> *Ejemplo: En la figura 4.5 están representados los posibles modelos para un cubo de datos particular (para facilitar la visualización, no se muestran todas las combinaciones, aquellas que no aparecen tienen coordenadas similares a alguna de las representadas, además solo se ha considerado hasta* $\mathrm{P}, \mathrm{Q} \leq 5$ *y* $\mathrm{R} \leq 4$*). Los modelos pertenecientes a la línea poligonal inferior son los que tienen menor suma de cuadrados residual (eje vertical) para cada valor de* S *(eje horizontal), y la línea vertical separa los modelos más estables (derecha) de los menos estables (izquierda). Así que en este caso se elegiría el modelo* $4 \times 4 \times 1$ *(rodeado) por ser el más simple de entre los más estables y por tener una mayor varianza explicada,* 95.415 %*, para* S = 9.

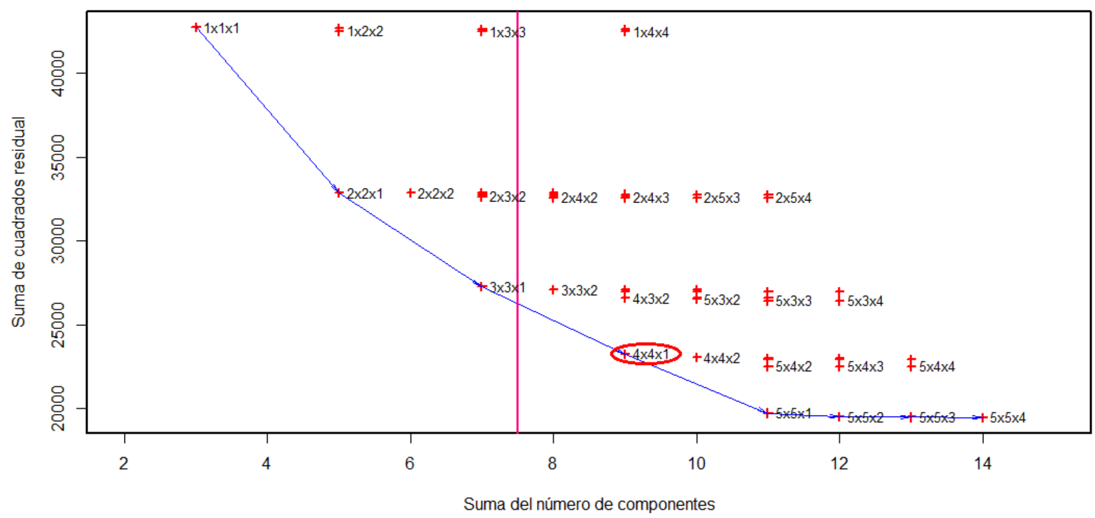


Figura 4.5: *Suma del número de componentes vs. Suma de cuadrados residual en el Tucker*

El core array se puede interpretar como la fuerza de las relaciones entre los componentes de las diferentes dimensiones, así como el peso de las combinaciones de componentes, o como una medida de las interacciones, y el cuadrado de cada elemento como la varianza explicada.

Pero la interpretación final de los individuos, las variables y las repeticiones para una combinación de componentes, $p \times q \times r$, no solo depende de si el elemento de G tiene un valor alto, sino también de la combinación de los signos de los cuatro factores del término $a_{ip}b_{jq}c_{kr}g_{pqr}$. Por ejemplo, si g_{pqr} tiene signo positivo, el individuo i-ésimo tiene signo positivo en la componente p, la variable j-ésima tiene signo positivo en la componente q y la repetición k-ésima tiene signo positivo en la componente r, la interacción entre el individuo i, la variable j y la repetición k será positiva: durante la repetición k, el individuo i toma un valor alto en la variable j. Si un elemento del core array es pequeño, la interpretación de tal combinación no es necesaria.

La posibilidad de un gran número de combinaciones y sus interpretabilidades con la técnica Tucker3 para este tipo de datos tridimensionales, lo hacen un método indudablemente atractivo a tener en cuenta.

La mayoría de los resultados del Tucker3 se enfocan en las interpretaciones de las interacciones de acuerdo con los signos de los cuatro factores, como se acaba de explicar; sin embargo, estos resultados pueden ser difíciles de interpretar. Los gráficos para las tres dimensiones, llamados biplots conjuntos, son una manera fácil de comprender visualmente estas interacciones. Se interpretan como los biplots clásicos (Gabriel, 1971), excepto que cada biplot se construye para dos combinaciones de componentes, una en horizontal y otra en vertical.

Por lo tanto, cada uno de los gráficos tiene tres subgráficos, uno para cada dimensión, con dos componentes en todos, y en ellos se representan las correspondientes columnas de las matrices A, B y C de la descomposición de X. Al representar conjuntamente las tres dimensiones en un único gráfico se pueden hacer interpretaciones visuales de las relaciones.

4.4. Análisis simultáneo de una sucesión de pares de tablas

Se presentan cuatro métodos para analizar un par de cubos de datos: BGCOIA, STATICO, COSTATIS y Co-Tucker3.
BGCOIA es un análisis de co-inercia entre grupos. Es por tanto simplemente computado haciendo un Análisis de Co-Inercia a las dos tablas de las medias por grupos, considerando cada tabla como un grupo. En STATICO, primero se usa un Análisis de Co-Inercia K veces para computar la sucesión de K tablas de covarianzas cruzadas, y luego un Análisis Parcial Triádico para analizar esta nueva k-tabla. En COSTATIS, primero se usan dos Análisis Triádicos Parciales para computar los compromisos de las dos k-tablas, y luego un Análisis de Co-Inercia para analizar las relaciones entre estos dos compromisos.

4.4.1. BGCOIA

Sea g el número de grupos. En el Análisis de Co-Inercia Entre-Grupos (Between-Group Co-Inertia Analysis BGCOIA) se parte de los dos cubos de datos $X_{n\times p\times g}$ e $Y_{n\times q\times g}$, pero se transforman en tablas concatenando una debajo de la otra las g tablas $n \times p$ ó $n \times q$. Así, se interpretarán X e Y indistintamente como cubos o como tablas de la siguiente forma:

$$\text{como cubos:} \quad \begin{array}{l} X_{ijk} \quad \forall i = 1, \ldots, n, \quad \forall j = 1, \ldots, p, \quad \forall k = 1, \ldots, g \\ Y_{ijk} \quad \forall i = 1, \ldots, n, \quad \forall j = 1, \ldots, q, \quad \forall k = 1, \ldots, g \end{array}$$

es el valor que toma el individuo i-ésimo en la variable j-ésima en el grupo k-ésimo,

$$\text{como tablas:} \quad \begin{array}{l} X_{ij} \quad \forall i = 1, \ldots, n, n+1, \ldots, 2n, \ldots, n \cdot g, \quad \forall j = 1, \ldots, p \\ Y_{ij} \quad \forall i = 1, \ldots, n, n+1, \ldots, 2n, \ldots, n \cdot g, \quad \forall j = 1, \ldots, q \end{array}$$

es el valor que toma el individuo i-ésimo en la variable j-ésima sin especificar el grupo al que pertenece.
La tabla de las medias de grupos para la primera tabla se obtiene computando las medias de cada variable dentro de cada grupo. Esto da una nueva tabla X_B, con g filas y p columnas. Las mismas computaciones se hacen para la segunda tabla, conduciendo a una segunda nueva tabla con g filas y q columnas, Y_B. Después se lleva a acabo un Análisis de Co-Inercia simple

a estas dos tablas.

Así, se pueden representar las filas y las columnas tanto de las matrices X_B e Y_B, como de las matrices originales X e Y para cada grupo, de la siguiente manera:

El diagrama de dualidad es el siguiente, en el que la parte central representa el análisis de la matriz ${Y_B}^t D_g X_B$, mientras que a derecha y a izquierda están las definiciones de la matriz de productos cruzados y su traspuesta, y hacia ariba y abajo están las definiciones de las matrices X_B, Y_B y sus traspuestas como productos de matrices.

$$
\begin{tikzcd}
\mathbb{R}^g \arrow[r,"B_X"] & (\mathbb{R}^{n\cdot g})^* \arrow[d,"X^t"] & & \mathbb{R}^{n\cdot g} \arrow[r,"{B_X}^t"] & \mathbb{R}^{g*} \arrow[d,"D"] \\
\mathbb{R}^{g*} \arrow[u,"D"] \arrow[r,"{X_B}^t"] & \mathbb{R}^p \arrow[rr,"D_p"] & & \mathbb{R}^{p*} \arrow[u,"X"'] \arrow[r,"X_B"] \arrow[ddd,"{Y_B}^t D_g X_B"] & \mathbb{R}^g \arrow[ddd,"D_g"] \\
& & \bullet \arrow[ul,"V_r"] & & \\
& & \bullet \arrow[dr,"U_r"] & & \\
\mathbb{R}^g \arrow[uuu,"D_g"] & \mathbb{R}^{q*} \arrow[l,"Y_B"] \arrow[uuu,"{X_B}^t D_g Y_B"] \arrow[d,"Y"] & & \mathbb{R}^q \arrow[ll,"D_q"] & \mathbb{R}^{g*} \arrow[l,"{Y_B}^t"] \arrow[d,"D"] \\
\mathbb{R}^{g*} \arrow[u,"D"] & \mathbb{R}^{n\cdot g} \arrow[l,"{B_Y}^t"] & & (\mathbb{R}^{n\cdot g})^* \arrow[u,"Y^t"] & \mathbb{R}^g \arrow[l,"B_Y"]
\end{tikzcd}
$$

las filas de X_B e Y_B tienen por coordenadas respectivamente las filas de

$$X_B D_p V_r \quad \text{e} \quad Y_B D_q U_r$$

siendo D_p y D_q las métricas para las columnas de ambas tablas, D_g la matriz diagonal con los pesos para los grupos, y U_r y V_r las matrices de las descomposiciones espectrales de

${Y_B}^t D_g X_B D_p {X_B}^t D_g Y_B D_q$ y ${X_B}^t D_g Y_B D_q {Y_B}^t D_g X_B D_p$, o de la descomposición explicada en el apartado 4.1.2 para la matriz ${Y_B}^t D_g X_B$.

Las columnas de X_B e Y_B vienen dadas por las matrices

$$ {X_B}^t D_g Y_B D_q U_r \quad \text{e} \quad {Y_B}^t D_g X_B D_p V_r. $$

Las filas de las tablas iniciales X e Y tienen por coordenadas las filas de

$$ X D_p V_r \quad \text{e} \quad Y D_q U_r, \tag{4.3} $$

así que si se quieren las filas de las tablas X_k e Y_k se debe quedarse con las filas correspondientes a X_k ó Y_k de 4.3. Mientras que las columnas de X e Y tienen como coordenadas

$$ X^t B_X D D_g Y_B D_q U_r \quad \text{e} \quad Y^t B_Y D D_g X_B D_p V_r $$

las mismas coordenadas que las columnas de X_B e Y_B.

La principal ventaja de este método es su simplicidad, desde los puntos de vista tanto teórico como práctico. Los dos cubos de datos se reducen a dos tablas tomando las medias de las columnas de cada tabla individual de los cubos. Entonces se aplica un Análisis de Co-Inercia a las dos tablas resultantes.

El hecho es que un Análisis de Co-Inercia Entre-Grupos puede usarse para dar más importancia a los efectos espaciales o temporales en un diseño experimental espacio-temporal. Las tablas pueden corresponderse a fechas o a lugares muestrales. Dependiendo de la importancia que debería darse a los procesos espaciales o temporales, una u otra de las posibilidades se puede usar.

4.4.2. STATICO

El método X-STATICO (X-STATIS + CoIA), o STATICO, se basa en el Análisis Parcial Triádico de una sucesión de tablas de productos cruzados. Empezando por la sucesión de pares de tablas, cada tabla de producto cruzado se computa usando el par de tablas de cada repetición. Todas las tablas de la sucesión no necesitan tener las mismas de filas en cada repetición. Esto significa que el número de filas puede variar de una repetición a otra, pero el número de variables (p y q) debe ser el mismo para todas las repeticiones. Por tanto, todas las tablas de productos cruzados tienen el mismo número de filas (p) y columnas (q). Contienen las covarianzas entre las columnas de las dos tablas.
Sean (X_k, D_p, D_{n_k}) e (Y_k, D_q, D_{n_k}) los tripletes para la repetición k. X_k es la primera tabla medida en la repetición k, e Y_k es la segunda tabla observada en la misma repetición. D_p y D_q son las mismas métricas para todas las repeticiones y D_{n_k} es la misma matriz con los pesos (uniformes o no) para las filas para ambas tablas. Sea Z_k la k-ésima tabla de productos cruzados:

$$Z_k = {Y_k}^t D_{n_k} X_k.$$

El triplete para el Análisis de Co-Inercia para la repetición k es (Z_k, D_p, D_q) y el método de STATICO es el Análisis Parcial Triádico de la k-tabla hecha a partir de esta serie de tablas de productos cruzados.
El paso de interestructura da pesos óptimos α_k tales que la inercia del triplete

$$\left(\sum_{k=1}^{K} \alpha_k \Omega_k Z_k, D_p, D_q \right)$$

es máxima con la restricción

$$\sum_{k=1}^{K} {\alpha_k}^2 \Omega_k = 1,$$

esto es que el vector $(\alpha_1, \ldots, \alpha_K)^t$ tenga módulo unidad según la métrica para la tercera dimensión D_K.
El compromiso del método STATICO (Z) es una media ponderada de las tablas de productos

cruzados usando los pesos de $D_K\alpha$:

$$Z = \sum_{k=1}^{K} \alpha_k \Omega_k Z_k.$$

El análisis (PCA) de este compromiso da una representación gráfica de las variables de ambos cubos (filas y columnas de Z). El diagrama de dualidad es el siguiente

$$\begin{tikzcd}
\mathbb{R}^{n*} \arrow[r, "X_k^t"] & \mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[r, "X_k"] \arrow[dd, "Z"'] \arrow[dd, bend left, "Z_k"] & \mathbb{R}^{n} \arrow[dd, "D_{n_k}"] \\
 & \bullet \arrow[u, "V_r"'] & \bullet \arrow[d, "U_r"] & \\
\mathbb{R}^{n} \arrow[uu, "D_{n_k}"] & \mathbb{R}^{q*} \arrow[l, "Y_k"] \arrow[uu, "Z^t"'] \arrow[uu, bend left, "Z_k^t"] & \mathbb{R}^{q} \arrow[l, "D_q"] & \mathbb{R}^{n*} \arrow[l, "Y_k^t"]
\end{tikzcd}$$

donde ahora la parte central representa el análisis de la matriz Z, las matrices de productos cruzados Z_k y $Z_k{}^t$ están representadas con tramos curvos, y los cuadros a derecha e izquierda son las definiciones de las matrices de productos cruzados y sus traspuestas.
Así, las coordenadas de las filas del compromiso son las filas de

$$ZD_pV_r$$

siendo V_r los primeros r vectores de una base de vectores propios ortonormales de $Z^tD_qZD_p$, y las coordenadas de las columnas de Z son las filas de la matriz

$$Z^tD_qU_r$$

donde U_r son los primeros r vectores propios de la matriz $ZD_pZ^tD_q$. Equivalentemente, U_r y V_r se pueden obtener según lo explicado en el apartado 4.1.2 para la matriz Z.
Finalmente, el paso de intraestructura proyecta las filas y columnas de cada tabla de la sucesión en el análisis del compromiso. Esto da una representación de las filas de cada repetición

$$X_kD_pV_r \quad \text{e} \quad Y_kD_qU_r$$

y las dos representaciones de las variables de cada repetición, una desde el punto de vista de las matrices de productos cruzados:

columnas de Z_k (columnas de X_k):	$Z_k{}^t D_q U_r$
filas de Z_k (columnas Y_k):	$Z_k D_p V_r$

y otra desde el punto de vista de las matrices originales:

columnas de X_k:	$X_k{}^t D_{n_k} Y_k D_q U_r$
columnas de Y_k:	$Y_k{}^t D_{n_k} X_k D_p V_r$

que son iguales a las anteriores.

4.4.3. COSTATIS

CO-X-STATIS (CoIA + X-STATIS), o COSTATIS, es un análisis que también está basado en métodos para k-tablas y Co-Inercia, pero se beneficia de las ventajas del STATICO y del BGCOIA. En efecto, tienen las mismas propiedades de optimalidad del análisis de k-tablas como el STATICO (i.e. las propiedades maximizantes del compromiso), pero tiene la simplicidad del BGCOIA.

COSTATIS es un Análisis de Co-Inercia simple del compromiso de dos análisis de k-tablas. El primer paso del COSTATIS consiste en llevar a cabo dos Análisis Parciales Triádicos: uno para cada k-tabla. El segundo paso es un Análisis de Co-Inercia simple de los compromisos de estos dos Análisis Parciales Triádicos. Esto significa que el número de tablas no tiene que ser el mismo para las dos series de datos (K_1 y K_2), pero el número de filas y columnas de los dos cubos deben ser los mismos para todas las tablas.

Los compromisos de los Análisis Parciales Triádicos del primer y el segundo cubo son

$$(X_c)_{(n\times p)} = \sum_{k=1}^{K_1} \alpha_k \Omega_k X_k, \text{ e } (Y_c)_{(n\times q)} = \sum_{k=1}^{K_2} \beta_k \Omega_k Y_k$$

(se llamará indistintamente Ω_k a los pesos de las métricas D_{K_1} y D_{K_2}).

Estos compromisos son medias ponderadas de las tablas de las sucesiones originales, con pesos iguales a las métricas D_{K_1} y D_{K_2} y las coordenadas del primer vector propio de la interestructura de los dos Análisis Parciales Triádicos. Las inercias de los tripletes (X_c, D_p, D_n) e (Y_c, D_q, D_n) son máximas bajo las restricciones

$$\sum_{k=1}^{K_1} {\alpha_k}^2 \Omega_k = 1 \text{ y } \sum_{k=1}^{K_2} {\beta_k}^2 \Omega_k = 1,$$

esto es, que los vectores $(\alpha_1, \ldots, \alpha_{K_1})^t, (\beta_1, \ldots, \beta_{K_2})^t$ tengan módulo unidad cada uno según su métrica.

El Análisis de Co-Inercia de estos dos compromisos:

$$\mathrm{CoIner}_{X_c Y_c} = \mathrm{Tr}[{Y_c}^t D_n X_c D_p {X_c}^t D_n Y_c D_q]$$

(que ya se ha demostrado que, en el caso en que D_n sea la matriz de pesos uniformes para las filas y las métricas D_p y D_q sean las euclídeas, es igual a la suma de los cuadrados de las covarianzas de las columnas de X_c e Y_c) maximiza la co-inercia entre las puntuaciones de las variables de los dos cubos.

Se puede implementar un paso adicional, como en el método de STATICO: es posible proyectar las filas y columnas de todas las tablas de las dos sucesiones en el espacio multidimensional de este Análisis de Co-Inercia.

El diagrama de dualidad del COSTATIS es el siguiente, con el Análisis de Co-Inercia en el centro, es decir, el análisis de $Y_c{}^tD_nX_c$ y su traspuesta, a derecha y a izquierda las definiciones de la matriz de productos cruzados y su traspuesta, y las matrices que forman los dos compromisos y sus traspuestas en curvas

$$\begin{tikzcd}
\mathbb{R}^{n*} \arrow[r, "X_c{}^t"] \arrow[r, bend left, "X_k{}^t"] & \mathbb{R}^{p} \arrow[r, "D_p"] & \mathbb{R}^{p*} \arrow[r, "X_c"] \arrow[r, bend left, "X_k"] \arrow[dd, "Y_c{}^tD_nX_c"] & \mathbb{R}^{n} \arrow[dd, "D_n"] \\
 & \bullet \arrow[u, "V_r"] & \bullet \arrow[d, "U_r"] & \\
\mathbb{R}^{n} \arrow[uu, "D_n"] & \mathbb{R}^{q*} \arrow[l, "Y_c"] \arrow[l, bend left, "Y_k"] \arrow[uu, "X_c{}^tD_nY_c"'] & \mathbb{R}^{q} \arrow[l, "D_q"] & \mathbb{R}^{n*} \arrow[l, "Y_c{}^t"] \arrow[l, bend left, "Y_k{}^t"]
\end{tikzcd}$$

Así, se pueden representar los siguientes gráficos: los compromisos de X e Y

filas de X_c:	$X_cD_pV_r$
columnas de X_c:	$X_c{}^tD_nY_cD_qU_r$
filas de Y_c:	$Y_cD_qU_r$
columnas de Y_c:	$Y_c{}^tD_nX_cD_pV_r$

y las filas y columnas de las matrices X_k e Y_k

filas de X_k:	$X_kD_pV_r$
columnas de X_k:	$X_k{}^tD_nY_cD_qU_r$

filas de Y_k: $Y_k D_q U_r$

columnas de Y_k: $Y_k{}^t D_n X_c D_p V_r$

donde V_r son los primeros r vectores propios de $X_c{}^t D_n Y_c D_q Y_c{}^t D_n X_c D_p$, y U_r los primeros r vectores propios de $Y_c{}^t D_n X_c D_p X_c{}^t D_n Y_c D_q$, o lo explicado en el apartado 4.1.2 para $Y_c{}^t D_n X_c$. Cada compromiso representa la "estructura estable" de la correspondiente sucesión: X_c es la estructura estable de la primera sucesión de tablas, e Y_c es la estructura estable de la segunda sucesión de tablas. COSTATIS destaca las relaciones entre estas dos estructuras estables, y descarta las variaciones conflictivas entre las sucesiones completas. Es por tanto muy fácil de interpretar (como en un Análisis de Co-Inercia estándar), porque retiene las propiedades de optimalidad de los compromisos de los dos Análisis Parciales Triádicos.

Capítulo 5

Co-Tucker3: Un nuevo método para analizar simultáneamente una sucesión de pares de tablas

Se presenta una nueva propuesta para analizar un par de cubos de datos, el método Co-Tucker3 (CoIA + Tucker3), llamado así para seguir con la terminología del resto de métodos, colocando en primer lugar el nombre del segundo método que se usa, por lo tanto, Co-Tucker3 quiere decir que primero se usa el Tucker3 y luego el Análisis de Co-Inercia.

Está basado en el método Tucker3 y en el Análisis de la Co-Inercia, y se beneficia de las ventajas de ellos. Ya se ha hablado de las ventajas que tiene la aplicación de un análisis del método Tucker3: resuelve el problema de describir no solo la parte estable de una estructura de datos, sino también la posibilidad de extraer la estructura latente, así como las interacciones entre las tres dimensiones; sin embargo, solo tiene en cuenta una único conjunto de datos, esto es, un cubo de datos. Por lo tanto, cuando se debe estudiar un par de cubos de datos se puede combinar con el Análisis de la Co-Inercia, tal como se lleva a cabo en el Análisis COSTATIS, que es una combinación del Análisis de Co-Inercia con un método para un cubo de datos (Análisis Parcial Triádico).

El primer paso del Co-Tucker3 consiste en llevar a cabo dos análisis Tucker3: uno para cada cubo de datos. Y el segundo paso es tres Análisis de Co-Inercia simples de las componentes retenidas para cada una de las dimensiones de ambos cubos (ver figura 5.1). Esto significa que el número de componentes retenidas (P, Q y R) para cada dimensión ha de ser igual para las dos series de datos, pero el número de filas, columnas y repeticiones de los dos cubos puede ser diferente.

También existe otro método para analizar un par de cubos de datos, el Tucker-Co (Mendes, 2011). Es similar al Co-Tucker3 en el sentido de que está basado en uno de los métodos Tucker y en el Análisis de la Co-Inercia, pero difiere de aquel en el orden en el que se llevan a cabo los pasos. En el Tucker-Co primero se construye un cubo de datos formado por tablas de productos cruzados a partir de cada par de matrices para cada repetición, y en segundo lugar se realiza un análisis con un método Tucker. El modo de elegir cuál de los dos métodos, Co-Tucker3 o Tucker-Co, es el necesario para analizar un conjunto de datos en particular, sigue el mismo criterio que el que servía para elegir entre el COSTATIS o el STATICO respectivamente.

El objetivo del método Co-Tucker3 es, primero, descomponer los dos cubos de datos en matrices ortogonales y dos cubos de datos (los core arrays) de orden $P \times Q \times R$ más simple que los originales $X_{I_1 \times J_1 \times K_1}$ e $Y_{I_2 \times J_2 \times K_2}$; y después descubrir las relaciones entre estas filas, columnas y repeticiones reducidas a un número más pequeño de componentes.

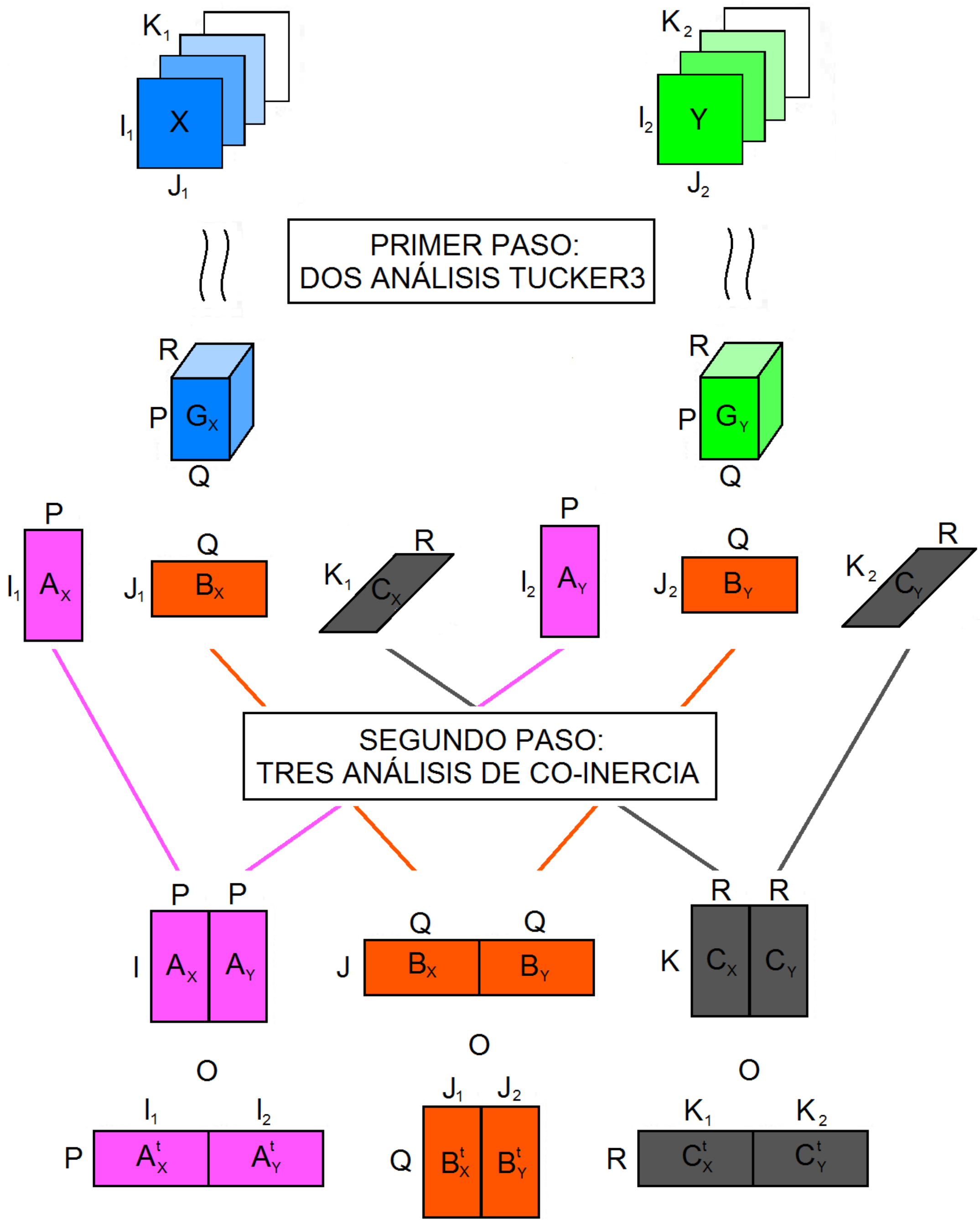


Figura 5.1: *Esquema del Análisis Co-Tucker3*

5.1. Primer paso: dos análisis Tucker

Con esta propuesta, primero se analizará cada cubo de datos $X_{I_1 \times J_1 \times K_1}$ e $Y_{I_2 \times J_2 \times K_2}$ llevando a cabo uno de los métodos Tucker. Su objetivo es reducir la dimensionalidad del problema, $I_1 \times J_1 \times K_1$ e $I_2 \times J_2 \times K_2$, para resumir la información, construyendo un modelo simplificado para cada uno, $P \times Q \times R$, para facilitar la descripción de los datos. Además, gráficos que muestren simultáneamente las tres dimensiones pueden ser muy útiles a este respecto.
Esta primera parte del método Co-Tucker permitirá responder a preguntas como: ¿Qué grupos de individuos (primera dimensión de ambos cubos) se comportan de manera diferente en qué variables de cada uno de los cubos (segunda dimensión de cada cubo) durante qué repeticiones (tercera dimensión de ambos cubos)?, ¿qué relaciones hay entre las variables individualmente dentro de cada cubo?, ¿qué tendencia se puede descubrir a lo largo del tiempo?, ¿hay diferentes tipos de individuos?, y cuestiones más complejas, como si las relaciones entre variables individualmente dentro de cada cubo varían en el tiempo, o si la estructura de las variables cambia en el tiempo de forma diferente para diferentes grupos de individuos.
Desde el punto de vista algebraico, el objetivo de los dos métodos Tucker (uno para cada cubo) es encontrar una descomposición de los cubos de datos X e Y en matrices ortogonales y otros cubos de datos; tres pares de matrices ortogonales $A_X(I_1 \times P)$ y $A_Y(I_2 \times P)$, $B_X(J_1 \times Q)$ y $B_Y(J_2 \times Q)$ y $C_X(K_1 \times R)$ y $C_Y(K_2 \times R)$, y otros dos cubos de datos $G_X(P \times Q \times R)$ y $G_Y(P \times Q \times R)$, los core arrays, con $P \times Q \times R$ más simple que $I_1 \times J_1 \times K_1$ e $I_2 \times J_2 \times K_2$, tales que el producto tensorial de $G_X, {A_X}^t, {B_X}^t$ y ${C_X}^t$, y $G_Y, {A_Y}^t, {B_Y}^t$ y ${C_Y}^t$ sean las mejores aproximaciones de X e Y (ver figura 5.2).

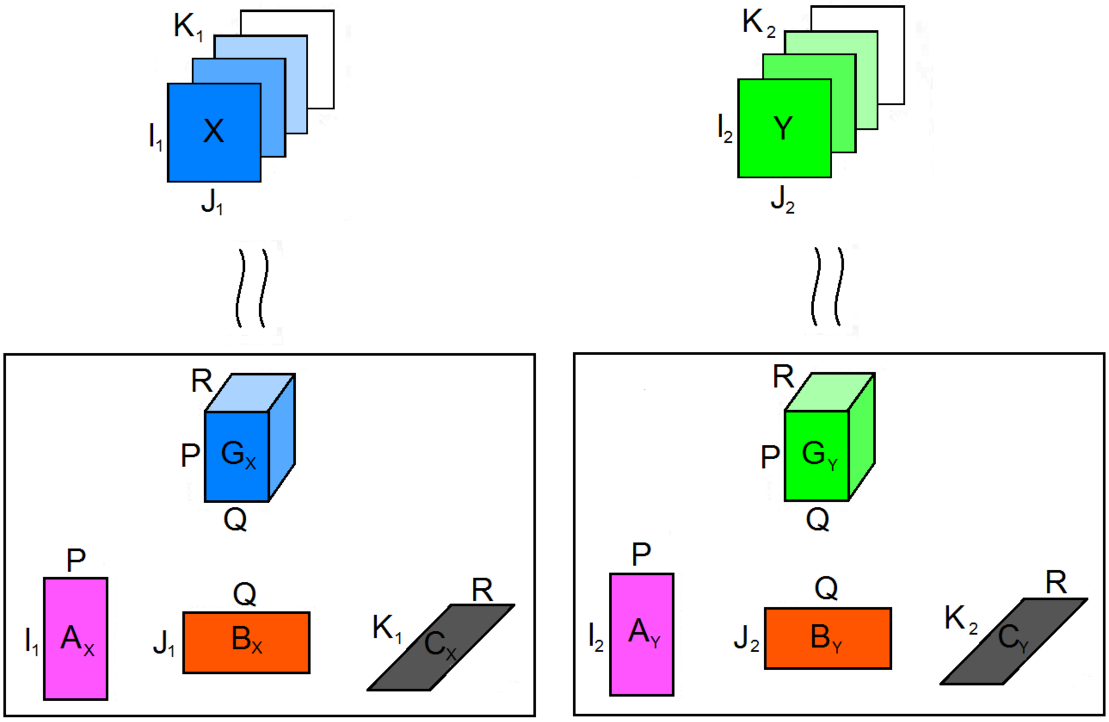


Figura 5.2: *Esquema del Análisis Tucker3 del Co-Tucker3*

El método Tucker más utilizado, por tener en cuenta las tres dimensiones simultáneamente de forma independiente para las repeticiones es el Tucker3, así que es el que se implementará para los dos cubos para el Co-Tucker, así que el Co-Tucker pasa a ser denominado más detalladamente Co-Tucker3. Pero pudiera ser que en función de las demandas del problema que se esté estudiando se utilizara cualquiera de los otros modelos Tucker, solo que el siguiente paso del Co-Tucker, los Análisis de la Co-Inercia solo se podrían realizar para aquellas dimensiones para las que se hayan calculado matrices y no sucesiones de matrices.

El proceso para encontrar la mejor aproximación de X e Y con $P \times Q \times R$ componentes es el mismo algoritmo iterativo que para el Tucker3 (Kiers et al., 1992). La manera de escoger qué modelo $P \times Q \times R$ será considerado es computar la descomposición dada por el algoritmo para todas las combinaciones $P \times Q \times R$ con $P \leq \min(I_1, I_2)$, $Q \leq \min(J_1, J_2)$ y $R \leq \min(K_1, K_2)$;

y también se debe tener en cuenta la regla del máximo producto.

En este algoritmo, el término A_{X_n} representa la matriz para la primera dimensión del primer cubo obtenida en la iteración n-ésima, en particular, A_{X_1} es la matriz calculada en la iteración inicial; $B_{X_n}, C_{X_n}, G_{X_n}$ y $A_{Y_n}, B_{Y_n}, C_{Y_n}, G_{Y_n}$ se definen análogamente:

1. A_{X_1} se define como el conjunto de los P primeros vectores singulares por la izquierda de $X_{(1)}$. Análogamente, se definen B_{X_1} y C_{X_1} como los primeros Q y R vectores singulares por la izquierda de $X_{(2)}$ y $X_{(3)}$ (ídem para A_{Y_1}, B_{Y_1} y C_{Y_1}).

   Expliquemos mejor lo que significa este paso: dado el cubo de datos X, se despliega a lo largo de la primera dimensión, es decir, se obtiene una matriz bidimensional en la que se dejan fijas las filas y en columnas se tienen todas las columnas originales de cada una de las repeticiones, una a continuación de otra, y de esta matriz se retienen los P primeros vectores singulares por la izquierda. Después se hace lo mismo desplegando el cubo a lo largo de la segunda y la tercera dimensión, reteniendo los primeros Q y R vectores singulares por la izquierda respectivamente (ídem para el cubo Y).

2. G_{X_1} se calcula como el producto tensorial de X, A_{X_1}, B_{X_1} y C_{X_1} (ídem para G_{Y_1}):

$$[G_{X_1}]_{pqr} = \left[\left(\left(X \times_1 A_{X_1}\right) \times_2 B_{X_1}\right) \times_3 C_{X_1}\right]_{pqr} = \sum_{i=1}^{I_1} \sum_{j=1}^{J_1} \sum_{k=1}^{K_1} [A_{X_1}]_{ip} [B_{X_1}]_{jq} [C_{X_1}]_{kr} x_{ijk}.$$

   En este paso, lo que se busca es encontrar el core array, que es lo que le falta a A_{X_1}, B_{X_1} y C_{X_1} para que se obtenga una aproximación de X (ídem para G_{Y_1}).

3. Paso de iteración, $n = 1, \ldots$:

   a) Se calcula la matriz óptima $A_{X_{n+1}}$, dejando fijas B_{X_n} y C_{X_n} (ídem para $A_{Y_{n+1}}$).
   Se denomina matriz óptima en este caso a los P primeros vectores singulares por la izquierda de $\left(\left(X \times_2 B_{X_n}\right) \times_3 C_{X_n}\right)_{(1)}$.
   Es decir, dados los $\underline{B_{X_n}}$ y $\underline{C_{X_n}}$ de la <u>iteración</u> anterior, se reduce X a lo largo de la segunda y la tercera dimensión, obteniendo un cubo con las filas originales, pero con las columnas y las repeticiones reducidas. Ahora este cubo reducido se despliega

a lo largo de la primera dimensión y se retienen los primeros P vectores singulares por la izquierda de la matriz desplegada, que formarán $A_{X_{n+1}}$ (ídem para $A_{Y_{n+1}}$).

b) Se calcula la matriz óptima $B_{X_{n+1}}$, dejando fijas $A_{X_{n+1}}$ y C_{X_n}: los Q primeros vectores singulares por la izquierda de $\left(\left(X \times_1 A_{X_{n+1}}\right) \times_3 C_{X_n}\right)_{(2)}$ (ídem para $B_{Y_{n+1}}$).
Esto quiere decir que, dados los $\underline{A_{X_{n+1}}}$ de la instrucción anterior y los $\underline{C_{X_n}}$ de la iteración anterior, se realiza lo mismo para la segunda dimensión: se reduce X a lo largo de la primera y la tercera dimensión, se despliega este cubo a lo largo de la segunda dimensión, y se retienen los primeros Q vectores singulares por la izquierda de esta matriz, que formarán $B_{X_{n+1}}$ (ídem para $B_{Y_{n+1}}$).

c) Se calcula la matriz óptima $C_{X_{n+1}}$, dejando fijas $A_{X_{n+1}}$ y $B_{X_{n+1}}$: los R primeros vectores singulares por la izquierda de $\left(\left(X \times_1 A_{X_{n+1}}\right) \times_2 B_{X_{n+1}}\right)_{(3)}$ (ídem para $C_{Y_{n+1}}$).
Esto quiere decir que, dados los $\underline{A_{X_{n+1}}}$ y $\underline{B_{X_{n+1}}}$ de las instrucciones anteriores, se reduce X a lo largo de la primera y la segunda dimensión, se despliega este cubo a lo largo de la tercera dimensión y se retienen los primeros R vectores singulares por la izquierda de esta matriz, que formarán $C_{X_{n+1}}$ (ídem para $C_{Y_{n+1}}$).

d) Se calcula el core array $G_{X_{n+1}}$, como el producto tensorial de $X, A_{X_{n+1}}, B_{X_{n+1}}$ y $C_{X_{n+1}}$ (ídem para $G_{Y_{n+1}}$).
Es decir, de nuevo, se calcula el core array, que es lo que le falta a $A_{X_{n+1}}, B_{X_{n+1}}$ y $C_{X_{n+1}}$ para que se obtenga una aproximación de X (ídem para $G_{Y_{n+1}}$).

e) Este paso se para cuando las diferencias entre $A_{X_{n+1}}, B_{X_{n+1}}, C_{X_{n+1}}, G_{X_{n+1}}$ y $A_{X_n}, B_{X_n}, C_{X_n}, G_{X_n}$ sean más pequeñas que un valor establecido de inicio (ídem para las correspondientes matrices de Y).

4. Las matrices A_X, B_X y C_X y el core array G_X se definen entonces como los obtenidos tras la iteración n-ésima: $A_X = A_{X_{n+1}}, B_X = B_{X_{n+1}}, C_X = C_{X_{n+1}}$ y $G_X = G_{X_{n+1}}$ (ídem para A_Y, B_Y, C_Y y G_Y).

La manera de escoger qué modelo $P \times Q \times R$ será considerado es computar la descomposición previa para todas las combinaciones $P \times Q \times R$ con $P \leq \text{mín}(I_1, I_2), Q \leq \text{mín}(J_1, J_2)$ y $R \leq \text{mín}(K_1, K_2)$; pero además se debe tener en cuenta la llamada regla del máximo producto, que dice que los casos en que el número de elementos de una dimensión es mayor que el producto de los otros dos, $P > QR, Q > PR$ o $R > PQ$, pueden ignorarse, porque en estos casos el modelo es equivalente a otro más reducido:

$$\text{Si } P > QR \text{ el modelo } P \times Q \times R \text{ es equivalente al modelo } QR \times Q \times R$$
$$\text{Si } Q > PR \text{ el modelo } P \times Q \times R \text{ es equivalente al modelo } P \times PR \times R$$
$$\text{Si } R > PQ \text{ el modelo } P \times Q \times R \text{ es equivalente al modelo } P \times Q \times PQ.$$

Cuando se haya hecho esto, se estudia cuál es la combinación más simple de entre las más estables y las que alcancen un porcentaje de varianza explicada suficientemente alto, tal como se ha explicado para el Tucker3, siendo la varianza explicada de un modelo la combinación de las obtenidas para cada uno de los dos cubos. Así, si el porcentaje de varianza explicada para un modelo dado según el primer cubo es

$$x\,\% = \frac{x_1}{x_2} \times 100$$

y el porcentaje para el mismo modelo según el segundo cubo es

$$y\,\% = \frac{y_1}{y_2} \times 100,$$

se llama porcentaje de varianza explicada combinado para ese modelo según los dos cubos a

$$\frac{x_1 + x_2}{y_1 + y_2} \times 100.$$

Para cada modelo se calcula la suma del número de componentes, $S = P + Q + R$, y para cada valor de S se elige aquel modelo que tenga un menor valor de la suma de cuadrados residual, o equivalentemente, un mayor valor de la varianza explicada. Así se tiene una lista de modelos, uno para cada valor de S. A continuación, se calcula el cociente incremental entre la suma de cuadrados residual y S para cada uno de los modelos anteriores en orden creciente de S, y nos quedaremos con aquellos modelos para los que el cociente incremental sea similar al del siguiente, esto es, los modelos más estables. Finalmente el modelo que será escogido para el análisis será aquel de los estables que tenga un menor valor de S, esto es, el más simple de entre los más estables.

Este modelo más simple de entre los más estables puede ser escogido a la vista de los resultados numéricos o del diagrama de sedimentación para los dos cubos conjuntamente.

Una vez elegido el modelo $P \times Q \times R$, tras la ejecución del algoritmo para ese modelo para cada uno de los dos cubos, se obtendrán las matrices $A_X(I_1 \times P), B_X(J_1 \times Q), C_X(K_1 \times R)$ y $A_Y(I_2 \times P), B_Y(J_2 \times Q), C_Y(K_2 \times R)$, y los core arrays G_X y G_Y, ambos de orden $P \times Q \times R$.

Los core arrays se pueden interpretar como la fuerza de las relaciones entre los componentes de las diferentes dimensiones de cada uno de los cubos, así como el peso de las combinaciones de componentes, o como una medida de las interacciones, y el cuadrado de cada elemento como la varianza explicada.

Pero la interpretación final de los individuos, las variables de ambos cubos y las repeticiones para una combinación de componentes, $p \times q \times r$, no solo depende de si el elemento de G_X o G_Y tiene un valor alto, sino también de la combinación de los signos de los cuatro factores del término del producto tensorial. Por ejemplo, si $g_{X_{pqr}}$ tiene signo positivo, el individuo i-ésimo tiene signo positivo en la componente p, la variable j-ésima del primer cubo tiene signo positivo en la componente q y la repetición k-ésima tiene signo positivo en la componente r, la interacción entre el individuo i, la variable j del primer cubo y la repetición k será positiva: durante la repetición k, el individuo i toma un valor alto en la variable j del primer cubo. Si un elemento de alguno de los core arrays es pequeño, la interpretación de tal combinación no es necesaria.

La mayoría de los resultados del primer paso del Co-Tucker3 se enfocan en las interpretaciones de las interacciones dentro de cada uno de los cubos por separado, de acuerdo con los signos de los cuatro factores, como se acaba de explicar; sin embargo, estos resultados pueden ser difíciles de interpretar. Los pares de gráficos para las tres dimensiones de ambos cubos, los biplots conjuntos, son una manera fácil de comprender visualmente estas interacciones. Se interpretan como los biplots clásicos (Gabriel, 1971), excepto que cada biplot se construye para dos combinaciones de componentes, una en horizontal y otra en vertical.

En este punto podrían estudiarse los resultados obtenidos como si se tratasen de dos Tucker3 individuales, al igual que, por ejemplo, en el primer paso del Análisis de la Co-Inercia se podían estudiar los dos PCAs individualmente antes de realizar el propio Análisis de la Co-Inercia. Los dos Tucker3 por separado se pueden interpretar, tal como se ha explicado anteriormente, mediante los llamados core arrays, que se definían como la fuerza de las relaciones entre los componentes de las tres dimensiones, en este caso, para aquellas de cada uno de los dos cubos de datos por separado, antes de estudiar las relaciones entre las de un cubo y las del otro. Además, se pueden interpretar las interacciones entre las tres dimensiones de cada uno de los cubos por separado mediante los distintos gráficos conjuntos subdivididos en tres, y teniendo en cuenta el producto de los cuatro signos, positivos o negativos, de los individuos, las variables, las repeticiones y los elementos del core array correspondientes.

Por lo tanto, cada uno de los pares de gráficos tiene tres subgráficos, uno para cada dimensión, con dos componentes en todos, y en ellos se representan las correspondientes columnas de las matrices A_X, B_X, C_X y A_Y, B_Y, C_Y de las descomposiciones de X e Y. Al representar conjuntamente las tres dimensiones en un único gráfico se pueden hacer interpretaciones visuales de las relaciones de cada cubo individualmente.

5.2. Segundo paso: tres Análisis de Co-Inercia

El segundo paso, entonces, del Co-Tucker3 es llevar a cabo los tres Análisis de Co-Inercia para cada par de matrices para cada una de las tres dimensiones: un CoIA entre ${A_X}^t$ y ${A_Y}^t$, uno entre ${B_X}^t$ y ${B_Y}^t$ y otro entre ${C_X}^t$ y ${C_Y}^t$, puesto que cada par tiene el mismo número de columnas, P, Q y R respectivamente y para poder llevar a cabo un Análisis de Co-Inercia han de tener las mismas filas. Pero en el caso en que los dos cubos X e Y tuvieran las mismas filas ($I_1 = I_2$), columnas ($J_1 = J_2$) o repeticiones ($K_1 = K_2$), se podría aprovechar este hecho para realizar los Análisis de Co-Inercia directamente entre A_X y A_Y, B_X y B_Y o C_X y C_Y.
Se hablará de los dos diferentes casos, el primero suponiendo que las dos matrices del mismo par tienen las mismas filas ($I_1 = I_2$), columnas ($J_1 = J_2$) o repeticiones ($K_1 = K_2$), y el segundo, en el que las dos matrices del mismo par tienen distintas filas, columnas o repeticiones. Sin pérdida de la generalidad, se hablará del Análisis de la Co-Inercia para el par de matrices correspondientes a las filas obtenidas tras las descomposiciones Tucker3 para cada uno de los cubos, para las otras dos dimensiones el desarrollo es análogo.

1. Caso en que X e Y tengan las mismas filas (figura 5.3):

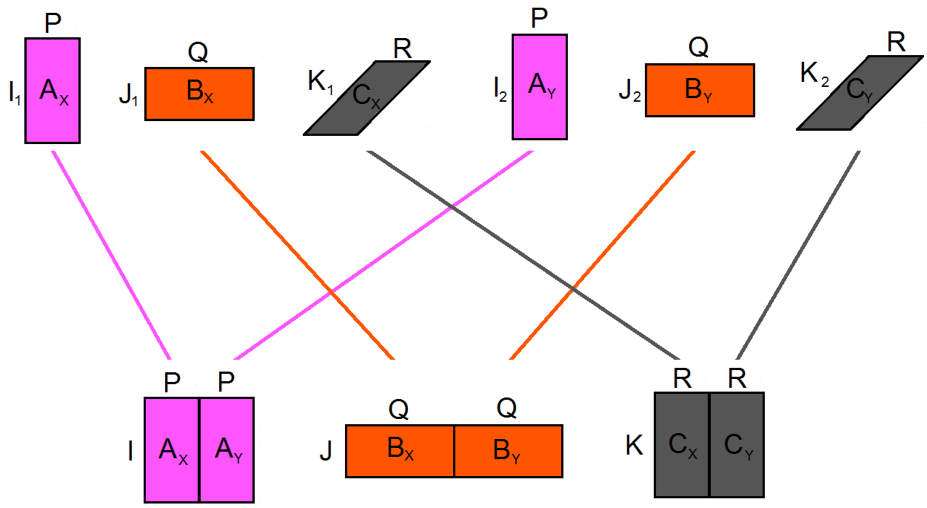


Figura 5.3: *Esquema del Análisis de Co-Inercia del Co-Tucker3 para el caso en que* X *e* Y *tengan las mismas filas, columnas o repeticiones*

Sea A_X la primera tabla, con $I = I_1$ filas y P columnas, y sea A_Y la segunda tabla, con las mismas $I_2 = I_1 = I$ filas y P columnas.

Sea D_I la matriz diagonal $I \times I$ de los pesos de las filas:

$$D_I = \mathrm{diag}(\omega_1, \ldots, \omega_I),$$

y sea D_P una métrica en $\mathbb{R}^P$.

Antes de hacer los dos Análisis de Co-Inercia, se podrían analizar las dos tablas del mismo par separadamente mediante dos gPCAs.

El Análisis de Co-Inercia está definido por el diagrama de dualidad obtenido al mezclar los dos diagramas de los gPCA por separado. Esto es posible porque las filas de las dos matrices son idénticas. El Análisis de Co-Inercia es la descomposición explicada en el apartado 4.1.2 para $A_Y{}^t D_I A_X$. Esto es, el siguiente "diagrama cruzado":

$$\begin{tikzcd}
\mathbb{R}^{I*} \arrow[r, "A_X{}^t"] & \mathbb{R}^{P} \arrow[r, "D_P"] & \mathbb{R}^{P*} \arrow[r, "A_X"] \arrow[d, "A_Y{}^t D_I A_X"] & \mathbb{R}^{I} \arrow[d, "D_I"] \\
\mathbb{R}^{I} \arrow[u, "D_I"] & \mathbb{R}^{P*} \arrow[l, "A_Y"] \arrow[u, "A_X{}^t D_I A_Y"'] & \mathbb{R}^{P} \arrow[l, "D_P"] & \mathbb{R}^{I*} \arrow[l, "A_Y{}^t"]
\end{tikzcd}$$

Por lo tanto, este Análisis de Co-Inercia es el análisis del triplete $(A_Y{}^t D_I A_X, D_P, D_P)$. Si se está en el caso en que D_I sea la matriz con pesos uniformes para las filas y la métrica D_P sea euclídea, como las columnas de ambas tablas están centradas, entonces la inercia total de cada tabla es simplemente la suma de varianzas. Y la co-inercia entre A_X y A_Y es, en este caso, una suma de cuadrados de covarianzas.

El Análisis de Co-Inercia maximiza las covarianzas entre las coordenadas de las filas de las dos tablas. La co-inercia es alta cuando los valores en ambas tablas son altos simultáneamente (o cuando varían inversamente) y baja cuando varían independientemente o cuando no varían. Esto es el significado de la co-estructura entre las dos tablas de datos.

Ahora, como parte del Co-Tucker3, se pueden representar gráficamente las filas de las dos matrices A_X y A_Y en el subespacio de dimensión r obtenido en el Análisis de Co-Inercia, calculando distintas coordenadas:

$$\begin{tikzcd}
\mathbb{R}^{I*} \arrow[r, "{A_X}^t"] & \mathbb{R}^{P} \arrow[r, "D_P"] & \mathbb{R}^{P*} \arrow[r, "A_X"] \arrow[dd, "{A_Y}^t D_I A_X"] & \mathbb{R}^{I} \arrow[dd, "D_I"] \\
& \bullet \arrow[dr, "U_r"] & \bullet \arrow[ul, "V_r"'] & \\
\mathbb{R}^{I} \arrow[uu, "D_I"] & \mathbb{R}^{P*} \arrow[l, "A_Y"] \arrow[uu, "{A_X}^t D_I A_Y"'] & \mathbb{R}^{P} \arrow[l, "D_P"] & \mathbb{R}^{I*} \arrow[l, "{A_Y}^t"]
\end{tikzcd}$$

las filas de la matriz A_X, es decir, las filas según el cubo X tienen por coordenadas

$$\text{filas de X} : \quad A_X D_P V_r$$

y las de A_Y, las mismas filas pero según el cubo Y

$$\text{filas de Y} : \quad A_Y D_P U_r \ .$$

2. Caso en que X e Y tengan distintas filas (figura 5.4):

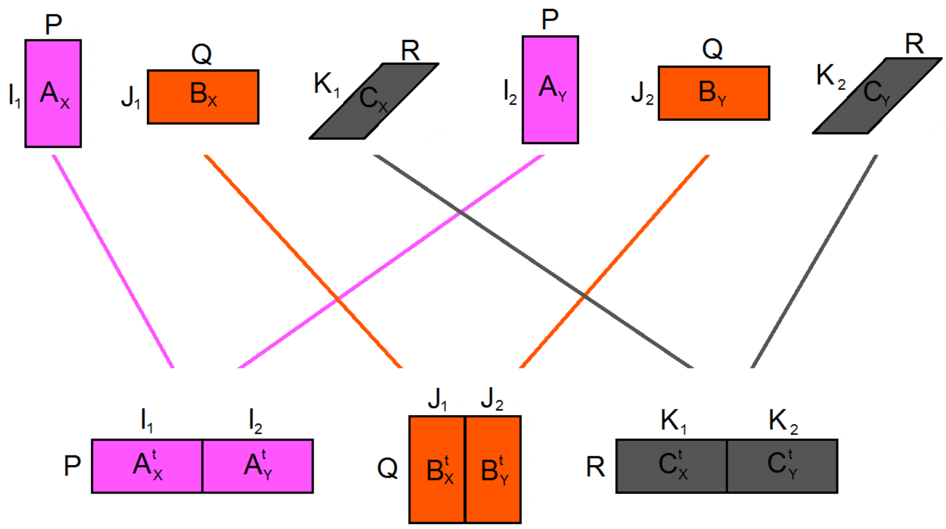


Figura 5.4: *Esquema del Análisis de Co-Inercia del Co-Tucker3 para el caso en que* X *e* Y *tengan distintas filas, columnas o repeticiones*

Sea ${A_X}^t$ la primera tabla, con P filas e I_1 columnas (puesto que A_X tiene I_1 filas y P columnas), y sea ${A_Y}^t$ la segunda tabla, con las mismas P filas e I_2 columnas distintas a las de ${A_X}^t$.

Como en realidad las columnas de ${A_X}^t$ y ${A_Y}^t$ representan a las filas (de A_X y A_Y), con el fin de aclarar el desarrollo, a partir de ahora se denominará "filas" a las I_1 columnas de ${A_X}^t$ y a las I_2 columnas de ${A_Y}^t$, y se llamará "componentes" a las P filas de ${A_X}^t$ y ${A_Y}^t$.

Sea D_P la matriz diagonal $P \times P$ de los pesos de las "componentes":

$$D_P = \text{diag}(\omega_1, \ldots, \omega_P),$$

y sean D_{I_1} y D_{I_2} dos métricas en $\mathbb{R}^{I_1}$ y $\mathbb{R}^{I_2}$ respectivamente.

De nuevo, antes de hacer los dos Análisis de Co-Inercia, se podrían analizar las dos tablas del mismo par separadamente mediante dos gPCAs.

Ahora la mezcla de los diagramas de dualidad para el Análisis de Co-Inercia es posible porque las "componentes" de las dos matrices son idénticas. El Análisis de Co-Inercia es la descomposición explicada en el apartado 4.1.2 para $A_Y D_P {A_X}^t$. Esto es, el siguiente "diagrama cruzado":

$$\begin{tikzcd}
\mathbb{R}^{P*} \arrow[r, "A_X"] & \mathbb{R}^{I_1} \arrow[r, "D_{I_1}"] & \mathbb{R}^{I_1*} \arrow[r, "{A_X}^t"] \arrow[d, "A_Y D_P {A_X}^t"] & \mathbb{R}^{P} \arrow[d, "D_P"] \\
\mathbb{R}^{P} \arrow[u, "D_P"] & \mathbb{R}^{I_2*} \arrow[l, "{A_Y}^t"] \arrow[u, "A_X D_P {A_Y}^t"'] & \mathbb{R}^{I_2} \arrow[l, "D_{I_2}"] & \mathbb{R}^{P*} \arrow[l, "A_Y"]
\end{tikzcd}$$

Por lo tanto, ahora el Análisis de Co-Inercia es el análisis del triplete $(A_Y D_P {A_X}^t, D_{I_1}, D_{I_2})$. Si se está en el caso en que D_P sea la matriz con pesos uniformes para las "componentes" y las métricas D_{I_1} y D_{I_2} sean euclídeas, como las "filas" de ambas tablas están centradas, entonces la inercia total de cada tabla es simplemente la suma de varianzas. Y la co-inercia entre ${A_X}^t$ y ${A_Y}^t$ es, en este caso, una suma de cuadrados de covarianzas. Ahora, como parte del Co-Tucker3, se pueden representar gráficamente las "filas" de las dos matrices ${A_X}^t$ y ${A_Y}^t$ en el subespacio de dimensión r obtenido en el Análisis de Co-Inercia, calculando distintas coordenadas:

$$\begin{tikzcd}
\mathbb{R}^{P*} \arrow[r, "A_X"] & \mathbb{R}^{I_1} \arrow[rr, "D_{I_1}"] & & \mathbb{R}^{I_1*} \arrow[r, "{A_X}^t"] \arrow[ddd, "A_Y D_P {A_X}^t"] & \mathbb{R}^{P} \arrow[ddd, "D_P"] \\
& & \bullet \arrow[ul, "V_r"'] & & \\
& & \bullet \arrow[dr, "U_r"] & & \\
\mathbb{R}^{P} \arrow[uuu, "D_P"] & \mathbb{R}^{I_2*} \arrow[l, "{A_Y}^t"] \arrow[uuu, "A_X D_P {A_Y}^t"'] & & \mathbb{R}^{I_2} \arrow[ll, "D_{I_2}"] & \mathbb{R}^{P*} \arrow[l, "A_Y"]
\end{tikzcd}$$

las "filas" de la matriz ${A_X}^t$, es decir, las filas del cubo X tienen por coordenadas

$$\text{filas de X}: \quad A_X D_P {A_Y}^t D_{I_2} U_r$$

y las de ${A_Y}^t$, las filas del cubo Y

$$\text{filas de Y}: \quad A_Y D_P {A_X}^t D_{I_1} V_r \ .$$

Así, teniendo en cuenta los dos tipos de Análisis de la Co-Inercia con los que uno se puede encontrar, la segunda parte del análisis Co-Tucker3 consiste en el análisis de tres diagramas de dualidad, uno para las filas, otro para las columnas y el último para las repeticiones. Cualquier combinación de los dos tipos es válida, un ejemplo podrían ser:

$$
\begin{tikzcd}
\mathbb{R}^{I*} \arrow[r, "A_X^t"] & \mathbb{R}^{P} \arrow[rr, "D_P"] & & \mathbb{R}^{P*} \arrow[r, "A_X"] \arrow[ddd, "A_Y^t D_I A_X"] & \mathbb{R}^{I} \arrow[ddd, "D_I"] \\
& & \bullet \arrow[ul, "V_r"'] & & \\
& & \bullet \arrow[dr, "U_r"] & & \\
\mathbb{R}^{I} \arrow[uuu, "D_I"] & \mathbb{R}^{P*} \arrow[l, "A_Y"] \arrow[uuu, "A_X^t D_I A_Y"] & & \mathbb{R}^{P} \arrow[ll, "D_P"] & \mathbb{R}^{I*} \arrow[l, "A_Y^t"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathbb{R}^{Q*} \arrow[r, "B_X"] & \mathbb{R}^{J_1} \arrow[rr, "D_{J_1}"] & & \mathbb{R}^{J_1*} \arrow[r, "B_X^t"] \arrow[ddd, "B_Y D_Q B_X^t"] & \mathbb{R}^{Q} \arrow[ddd, "D_Q"] \\
& & \bullet \arrow[ul, "V_r"'] & & \\
& & \bullet \arrow[dr, "U_r"] & & \\
\mathbb{R}^{Q} \arrow[uuu, "D_Q"] & \mathbb{R}^{J_2*} \arrow[l, "B_Y^t"] \arrow[uuu, "B_X D_Q B_Y^t"] & & \mathbb{R}^{J_2} \arrow[ll, "D_{J_2}"] & \mathbb{R}^{Q*} \arrow[l, "B_Y"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathbb{R}^{K*} \arrow[r, "C_X^t"] & \mathbb{R}^{R} \arrow[rr, "D_R"] & & \mathbb{R}^{R*} \arrow[r, "C_X"] \arrow[ddd, "C_Y^t D_K C_X"] & \mathbb{R}^{K} \arrow[ddd, "D_K"] \\
& & \bullet \arrow[ul, "V_r"'] & & \\
& & \bullet \arrow[dr, "U_r"] & & \\
\mathbb{R}^{K} \arrow[uuu, "D_K"] & \mathbb{R}^{R*} \arrow[l, "C_Y"] \arrow[uuu, "C_X^t D_K C_Y"] & & \mathbb{R}^{R} \arrow[ll, "D_R"] & \mathbb{R}^{K*} \arrow[l, "C_Y^t"]
\end{tikzcd}
$$

Que se correspondería con el caso en que los dos cubos de datos X e Y tuvieran las mismas filas y repeticiones, pero difirieran en las columnas.

El algoritmo completo del Co-Tucker3 quedaría resumido en la siguiente Tabla 5.1:

Tabla 5.1: Algoritmo del Co-Tucker3

Se centran X e Y por columnas y, si se quiere, se normalizan por capas laterales.		
Se realiza el siguiente algoritmo para todas las combinaciones $P \times Q \times R$ con $P \leq \min(I_1, I_2), Q \leq \min(J_1, J_2)$ y $R \leq \min(K_1, K_2)$, y también se debe tener en cuenta la regla del máximo producto:		
	A_{X_1} se define como el conjunto de los P primeros vectores singulares por la izquierda de $X_{(1)}$. Análogamente, se definen B_{X_1} y C_{X_1} como los primeros Q y R vectores singulares por la izquierda de $X_{(2)}$ y $X_{(3)}$ (ídem para A_Y, B_Y y C_Y).	
	G_{X_1} se calcula como el producto tensorial de X, A_{X_1}, B_{X_1} y C_{X_1} (ídem para G_{Y_1}): $[G_{X_1}]_{pqr} = [((X \times_1 A_{X_1}) \times_2 B_{X_1}) \times_3 C_{X_1}]_{pqr} = \sum_{i=1}^{I_1} \sum_{j=1}^{J_1} \sum_{k=1}^{K_1} [A_{X_1}]_{ip} [B_{X_1}]_{jq} [C_{X_1}]_{kr} x_{ijk}$.	
	Paso de iteración, $n = 1, \ldots$:	
		Se calcula la matriz óptima $A_{X_{n+1}}$, dejando fijas B_{X_n} y C_{X_n} (ídem para $A_{Y_{n+1}}$). Se denomina matriz óptima en este caso a los P primeros vectores singulares por la izquierda de $((X \times_2 B_{X_n}) \times_3 C_{X_n})_{(1)}$.
		Se calcula la matriz óptima $B_{X_{n+1}}$, dejando fijas $A_{X_{n+1}}$ y C_{X_n}: los Q primeros vectores singulares por la izquierda de $((X \times_1 A_{X_{n+1}}) \times_3 C_{X_n})_{(2)}$ (ídem para $B_{Y_{n+1}}$).
		Se calcula la matriz óptima $C_{X_{n+1}}$, dejando fijas $A_{X_{n+1}}$ y $B_{X_{n+1}}$: los R primeros vectores singulares por la izquierda de $((X \times_1 A_{X_{n+1}}) \times_2 B_{X_{n+1}})_{(3)}$ (ídem para $C_{Y_{n+1}}$).
		Se calcula el core array $G_{X_{n+1}}$, como el producto tensorial de $X, A_{X_{n+1}}, B_{X_{n+1}}$ y $C_{X_{n+1}}$ (ídem para $G_{Y_{n+1}}$).
		Este paso se para cuando las diferencias entre $A_{X_{n+1}}, B_{X_{n+1}}, C_{X_{n+1}}, G_{X_{n+1}}$ y $A_{X_n}, B_{X_n}, C_{X_n}, G_{X_n}$ sean más pequeñas que un valor establecido de inicio (ídem para las correspondientes matrices de Y).
	Las matrices A_X, B_X y C_X y el core array G_X se definen entonces como los obtenidos tras la iteración n-ésima: $A_X = A_{X_{n+1}}, B_X = B_{X_{n+1}}, C_X = C_{X_{n+1}}$ y $G_X = G_{X_{n+1}}$ (ídem para A_Y, B_Y, C_Y y G_Y).	
Se calcula la varianza explicada combinada para cada combinación.		
Se eligen las combinaciones con un mejor ajuste para cada valor da la suma de componentes $S = P + Q + R$.		
Se eligen las combinaciones que pertenecen a la envolvente convexa de entre todas.		
Se eliminan todas las combinaciones más estables menos la más simple de entre estas.		
Se elige una combinación de componentes.		
Se realizan los tres Análisis de la Co-Inercia para las tres dimensiones:		
	Se centran las dos matrices de datos por columnas y, si se quiere, se normalizan por columnas.	
	Se extraen los vectores singulares por la izquierda y la derecha para la Co-Inercia según el método explicado en el apartado 4.1.2.	
	Se calculan las coordenadas para las filas y las columnas para la Co-Inercia mediante los diagramas de dualidad.	

5.3. Discusión

Los cuatro métodos usados para analizar, incluso un conjunto de datos con estructura clara, pueden tener ventajas e inconvenientes (Thioulouse, 2011). Las ventajas de estos métodos se pueden resumir como sigue:
BGCOIA: Es el método más sencillo. Es simple de aplicar y los resultados son fáciles de interpretar. Puede usarse para favorecer un punto de vista (por ejemplo, espacio o tiempo), escogiendo el factor del Análisis Entre-Grupos.
STATICO: La principal ventaja de este método es la optimalidad del compromiso (maximización de la similaridad con todas las tablas iniciales). Da un compromiso de co-estructuras, lo que significa que representa la componente estable de las variaciones en las relaciones entre las variables de los dos cubos. Se beneficia del esquema de computación en tres pasos de los métodos STATIS (interestructura, compromiso, intraestructura), y los resultados gráficos pueden ser muy detallados.
COSTATIS: Este método se beneficia de las ventajes de los dos primeros: optimalidad de los compromisos de los Análisis Parciales Triádicos, facilidad de uso y simplicidad de los resultados gráficos del Análisis de Co-Inercia. COSTATIS es el Análisis de Co-Inercia de dos compromisos, así que busca las relaciones entre dos estructuras estables. Esto es diferente del punto de vista del STATICO (co-estructura de dos compromisos vs. compromiso de una serie de co-estructuras).
Co-Tucker3: Este método comparte las mismas ventajas del Análisis de Co-Inercia que se daban en el análisis COSTATIS, pero con la diferencia que existe entre el análisis Tucker3 y el Análisis Parcial Triádico: los métodos STATIS (entre los que se encuentra el Análisis Parcial Triádico) sirven para realzar la parte estable de la estructura de un cubo de datos, mientras que con el método Tucker3 se pueden descubrir interacciones más profundas que las estables obtenidas con los STATIS.

Los cuatro métodos pueden también compararse desde la perspectiva de los posibles objetivos del motivo para analizar un par de cubos de datos. El primer objetivo es encontrar un "consenso" en las relaciones entre las variables de los dos cubos. Este consenso debería ser independiente de las repeticiones (tiempo o espacio), y los cuatro métodos alcanzan esto de diferentes maneras.

En COSTATIS, primero se extrae un consenso, separada e independientemente para las variables de los dos cubos. Las relaciones entre estos dos resúmenes son luego investigadas por un Análisis de Co-Inercia. En STATICO, las relaciones entre las variables de los dos cubos se analizan primero para cada repetición, y luego se computa un resumen estable de estas relaciones. En el Co-Tucker3, también se puede extraer la parte estable de cada uno de los cubos, pero de forma diferente al STATICO y el COSTATIS, puesto que aquel computa un consenso para cada una de las tres dimensiones del cubo, uno para las filas, otro para las columnas y otro para las repeticiones, y después se analizarán las relaciones entre los tres pares de consensos.

Si las relaciones entre las variables de los dos cubos son débiles, o presentan solo algunas repeticiones, pueden desaparecer después del primer paso del COSTATIS o del Co-Tucker3 (los dos Análisis Parciales Triádicos o los dos Tucker3 por separado) y el Análisis de Co-Inercia final puede no ser significativo. Inversamente, si las relaciones entre las variables de los dos cubos son muy fuertes, la estructura cronológica puede desaparecer en el STATICO. El COSTATIS o el Co-Tucker3 deberían preferirse por tanto cuando las relaciones entre las variables de los dos cubos sean fuertes y las estructuras cronológicas no sean de principal importancia.

Otro objetivo del motivo de analizar un par de cubos de datos, complementario al primero, puede ser buscar una descripción de la evolución de las relaciones entre las variables de los dos cubos, en vez de una descripción de la parte estable de estas relaciones. En este caso, el STATICO puede ser más apropiado que el COSTATIS o el Co-Tucker3, porque computa un consenso de las relaciones entre las variables de los dos cubos para cada repetición, y solo después construye un consenso para el tiempo.

BGCOIA es ligeramente diferente, ya que facilita una opción en el análisis inicial entre una configuración espacial o temporal. Esto debería ser usado solo cuando haya buenas razones para dar prioridad al espacio o al tiempo.

Además, con el método Co-Tucker3, aparte de la posibilidad de utilizarlo para buscar una descripción de la parte estable o de la evolución de las dimensiones o las relaciones de los dos cubos, que es lo que llevan a cabo los otros tres métodos, sirve para encontrar interacciones más profundas que las estables obtenidas con los STATIS, es decir, se puede utilizar para describir las interacciones entre las distintas filas, columnas y repeticiones de una forma más especializada que la alcanzada visualizando e interpretando las agrupaciones obtenidas en los análisis de los consensos o las evoluciones, esto es, se puede encontrar y estudiar una interacción para la que se hayan retenido diferentes números de componentes para las tres dimensiones, por ejemplo, la combinación 1 componente para las filas, 3 para las columnas, y 2 para las repeticiones.

Para los métodos de k-tablas, la forma de organizar la k-tabla en una serie de k tablas es también importante. Para una matriz de tres dimensiones (filas $\times$ columnas $\times$ repeticiones), hay tres formas de cortar el cubo de datos en una serie de tablas. Sin embargo, solo dos son realmente interesantes. En efecto, la opción “una tabla = una variable” no es coherente con el objetivo del análisis: un compromiso entre variables no tendría significado. Así que se escoge entre “una tabla = una repetición” o “una tabla = un individuo”. Esta decisión está dictada por los objetivos del estudio y también por el hecho de que el método intentará computar un compromiso como combinación lineal de las tablas. Esto significa que este compromiso debería tener sentido.

Un tercer punto de vista de comparación entre los métodos de pares de cubos de datos es las restricciones numéricas exigidas a los parámetros de las k-tablas, esto es, el número de filas, columnas de ambos cubos y repeticiones. Desde este punto de vista, BGCOIA, STATICO, COSTATIS y Co-Tucker3 comparten las mismas restricciones para las columnas de los dos cubos, que deberían ser siempre idénticas: las mismas variables de un cubo y de otro para todas las repeticiones.

Pero las restricciones son diferentes para las repeticiones y las filas. En COSTATIS, las dos series de tablas pueden tener diferente número de repeticiones (e incluso diferentes repeticiones), mientras que las filas deben ser las mismas para todas las tablas y todas las repeticiones. En STATICO, las dos series de tablas deben tener las mismas repeticiones, pero las filas pueden diferir de una repetición a otra (aunque deben ser iguales para las dos tablas de un par). Y para el Co-Tucker3 existe una mayor flexibilidad, los dos cubos pueden tener distinto número de filas, columnas y repeticiones, siempre y cuando cada uno de los cubos sea perfecto, es decir, todas las capas horizontales, frontales o laterales han de tener la misma dimensión. Las restricciones del diseño experimental por tanto pueden influir a la elección del método.

Además, las restricciones en las variables de los dos cubos vienen de la elección del análisis de k-tablas en COSTATIS y STATICO (un Análisis Parcial Triádico). Pueden imaginarse extensiones de estos métodos, que podrían utilizar otras variantes de análisis STATIS en vez de Análisis Parcial Triádico. Esta posibilidad de tener varias filas, variables de los dos cubos y repeticiones hace el uso de los cuatro métodos mucho más flexible, sin embargo, debería ser usado con cuidado, porque esta flexibilidad puede obtenerse a expensas de perder algo de estructura en el conjunto de datos.

Un inconveniente común de todos estos métodos es la relativa complejidad de realizar análisis de datos multivariantes. En esta área, el paquete "ade4" para el entorno R ha intentado hacer las cosas más fáciles. Si bien ha sido correcto el uso de objetos estructurados y que además está disponible una interfaz gráfica de usuario, el paquete "ade4TkGUI", no todos los métodos de k-tablas están implementados en esta interfaz. Todos los cálculos y representaciones gráficas (excepto los de los métodos Tucker y los del método Co-Tucker3) se pueden hacer interactivamente online, gracias a un ejemplo de su página: `http://pbil.univ-lyon1.fr/SAOASOPET/`, pero no mediante la interfaz gráfica de usuario, si no mediante el uso de múltiples funciones sintácticas relativamente complejas.

Por ello se ha creído conveniente crear un compendio que incluya todos los programas para todas las técnicas descritas en este trabajo, el cual aventaja al paquete "ade4" en el sentido que todos los cálculos y las representaciones gráficas se llevan a cabo con una única instrucción para cada análisis, no como en el caso anterior que, para algunas de las técnicas, creaba una serie de objetos, útiles solo de un paso para el siguiente, desde que se leen los datos hasta que se presentan los resultados gráficos.

Además, los programas diseñados en este trabajo (véase capítulo 6) incluyen las siguientes posibilidades: se pueden usar etiquetas por defecto para los elementos de las distintas dimensiones si en los datos no están incluidas; se pueden suprimir aquellas filas con datos faltantes

(el paquete “ade4” lo que realizaba cuando se encontraba con datos faltantes era sustituirlos por el valor 0, o por el valor medio de la variable correspondiente, o incluso parar y no permitir la ejecución del programa); se pueden dar colores a las filas o a las columnas; se pueden pedir como resultados tablas con las calidades de representación de todos los elementos de cada una de las dimensiones; y se puede obtener, para cada gráfico, otro igual pero con aquellas filas o columnas con la mejor calidad de representación.
Por último, entre los programas de este trabajo también se encuentran: aquel para el método Tucker3, que no existía en el paquete “ade4”, y para el Co-Tucker3, que es el método nuevo propuesto para analizar simultáneamente una sucesión de pares de tablas.

La disponibilidad de métodos capaces de analizar conjuntos de datos con una organización compleja, como pares de cubos de datos, es importante porque permite tener en cuenta esta organización y analiza el conjunto de datos globalmente. Hay alternativas a estos métodos, como analizar tablas apiladas, o llevar a cabo varios análisis separados, como análisis de las series para cada variable, o análisis de datos para cada tabla. Pero el análisis está facilitado al tener en cuenta la estructura de datos como dicta el diseño experimental. Explorar relaciones entre variables de dos tablas no es una tarea fácil, y añadir repeticiones lo hace incluso más difícil, pero es un paso necesario.

Capítulo 6

Software

6.1. Programas

Implementación de los algoritmos de los análisis de Componentes Principales (PCA), Entre-Grupos (BGA), de Co-Inercia (CoIA), Parcial Triádico (PTA), de la Co-Inercia Entre-Grupos (BGCOIA), STATICO, COSTATIS y Tucker3 en R.

Se ha creído conveniente crear un compendio que incluya todos los programas para todas las técnicas descritas en este trabajo, el cual aventaja a otros en el sentido que todos los cálculos y las representaciones gráficas se llevan a cabo con una única instrucción para cada análisis, no como en los otros que, para algunas de las técnicas, creaba una serie de objetos, útiles solo de un paso para el siguiente, desde que se leen los datos hasta que se presentan los resultados gráficos.

Además, los programas diseñados en este trabajo incluyen las siguientes posibilidades: se pueden usar etiquetas por defecto para los elementos de las distintas dimensiones si en los datos no están incluidas; se pueden suprimir aquellas filas con datos faltantes (otros programas lo que realizaban cuando se encontraban con datos faltantes era sustituirlos por el valor 0, o por el valor medio de la variable correspondiente, o incluso parar y no permitir la ejecución del programa); se pueden dar colores a las filas o a las columnas; se pueden pedir como resultados tablas con las calidades de representación de todos los elementos de cada una de las dimensiones; y se puede obtener, para cada gráfico, otro igual pero con aquellas filas o columnas con la mejor calidad de representación.

Por último, entre los programas de este trabajo también se encuentra aquel para el método Tucker3, que aunque ya existe algún programa que lo ejecute, estos han quedado obsoletos porque solo se pueden ejecutar en versiones anteriores de Microsoft Windows, o con ayuda de máquinas virtuales; o presentan las desventajas ya explicadas.

6.1.1. Funciones auxiliares para todos los análisis

```
read<-function(X,Y=NULL)
{
 filas<-dim(X)[1]
 columnas<-dim(X)[2]
 if((length(dim(X)))==3){repeticiones<-dim(X)[3]}else{repeticiones<-NULL}
 if(!is.null(dimnames(X)[[1]])){namesf<-dimnames(X)[[1]]}else{namesf<-paste(1:filas,sep="")}
 if(!is.null(dimnames(X)[[2]])){namesc<-dimnames(X)[[2]]}else{namesc<-paste("V",1:columnas,sep="")}
 if((length(dim(X)))==3)
 {
  if(!is.null(dimnames(X)[[3]])){namesr<-dimnames(X)[[3]]}else{namesr<-paste("R",1:repeticiones,sep="")}
 }else{namesr<-NULL}
 nas<-apply(is.na(X),1,function(v){return(all(!v))})
 columnas2<-NULL
 namesc2<-NULL
 conf<-FALSE
 if(!is.null(Y))
```

```
{
 columnas2<-dim(Y)[2]
 if(!is.null(dimnames(Y)[[2]])){namesc2<-dimnames(Y)[[2]]} else{
    namesc2<-paste("V",1:columnas2,sep="")}
 if(filas!=dim(Y)[1])
 {
  conf<-TRUE
  print("te has confundido, el numero de filas de las dos matrices
     es distinto")
 }
 if((length(dim(X)))==3)
 {
  if(repeticiones!=dim(Y)[3])
  {
   conf<-TRUE
   print("te has confundido, el numero de repeticiones de los dos
      cubos es distinto")
  }
 }
 nasY<-apply(is.na(Y),1,function(v){return(all(!v))})
 nas<-nas&nasY
}
if(!all(nas))
{
 if((length(dim(X)))==2){X<-X[nas,]} else{X<-X[nas,,]}
 if((length(dim(Y)))==2){Y<-Y[nas,]} else{Y<-Y[nas,,]}
 namesf<-namesf[nas]
 print(paste("se han suprimido algunas filas (",filas-sum(nas),")
    con datos faltantes",sep=""))
```

```
    filas <-sum(nas)
  }
  return(list(X,filas,columnas,repeticiones,namesf,namesc,namesr,Y,
      columnas2,namesc2,conf))
}

colores<-function(filas,columnas,coloresf,coloresc,conf,columnas2=
     NULL,coloresc2=NULL)
{
  if(is.null(coloresf)){coloresf<-rep("black",filas)}
  if(is.null(coloresc)){coloresc<-rep("black",columnas)}
  if((length(coloresf))!=filas)
  {
   conf<-TRUE
   print("te has confundido, el numero de etiquetas de colores de las
       filas es distinto del numero de filas")
  }
  if((length(coloresc))!=columnas)
  {
   conf<-TRUE
   print("te has confundido, el numero de etiquetas de colores de las
       columnas es distinto del numero de columnas")
  }
  if(is.null(columnas2))
  {
   return(list(coloresf,coloresc,conf))
  } else {
   if(is.null(coloresc2)){coloresc2<-rep("black",columnas2)}
   if((length(coloresc2))!=columnas2)
```

```
  {
   conf<-TRUE
   print("te has confundido, el numero de etiquetas de colores de las columnas de la segunda matriz
      es distinto del numero de columnas de la segunda matriz")
  }
  return(list(coloresf,coloresc,coloresc2,conf))
  }
}

preproc<-function(X,norm,cubo=FALSE,capas=FALSE)
{
 if(!cubo)
 {
  X<-apply(X,2,function(v){return(v-mean(v))})
  if(norm){X<-apply(X,2,function(v){return(v/sqrt(mean(v^2)))})}
 } else {
  filas<-dim(X)[1]
  columnas<-dim(X)[2]
  repeticiones<-dim(X)[3]
  X<-apply(X,2:3,function(v){return(v-mean(v))})
  if(norm)
  {
   if(!capas){X<-apply(X,2:3,function(v){return(v/sqrt(mean(v^2)))})
   } else {
    X<-aperm(array(apply(X,2,function(m){return(m/sqrt(mean(m^2)))}),
              dim=c(filas,repeticiones,columnas)),c(1,3,2))
   }
  }
```

```
  }
  X[is.nan(X)]<-0
  return(X)
}

pesos<-function(t)
{
  Dn<-diag((1/t)/(sum(1/t)))
  Dn2<-diag(sqrt(diag(Dn)))
  return(list(Dn,Dn2))
}

contributions<-function(M,names,title,first=FALSE)
{
  A<-t(round(1000*apply(M,1,function(v){return((v^2)/(sum(v^2)))})))
  if(dim(M)[2]==1){A<-t(A)}
  rownames(A)<-names
  if(!first){write(paste("\n",paste(rep("-",100),collapse=""),"\n",sep
      =""),file="results.txt",append=TRUE)}
  write(paste("\n",paste(rep("  ",20),collapse=""),"Contribuciones de "
      ,title,"\n",sep=""),file="results.txt",
      append=!first)
  write(paste("Axis",paste(1:(dim(M)[2]),collapse="\t"),sep="\t"),file
      ="results.txt",append=TRUE)
  write.table(A,file="results.txt",append=TRUE,sep="\t",col.names=
      FALSE)
}

contributions2<-function(M,dimension)
```

```
{
 A<-t(round(1000*apply(M,1,function(v){return((v^2)/(sum(v^2)))})))
  if(dim(M)[2]==1){A<-t(A)}
 A<-matrix(A[,dimension],ncol=length(dimension))
  return(apply(A,1,sum))
}

screeplot<-function(d)
{
 e<-100*d^2/sum(d^2)
  barplot(e,space=0.2,names.arg=1:length(d),cex.names=0.7)
  text((1:length(d))+0.2*(1:length(d))-0.5,0.5,paste(round(e,3),"%",sep=""),adj=0,srt=90)
}

compr<-function(dimX,dimY,d,c=NULL,tucker=FALSE)
{
  if(!tucker)
  {
   if((dimX!=(floor(dimX)))||(dimY!=(floor(dimY)))||(dimX>(length(d)))||(dimY>(length(d)))||(dimX>=dimY))
   {
     print("te has confundido, ejes incorrectos")
     return(FALSE)
   } else {return(TRUE)}
  } else {
   if((dimX!=(floor(dimX)))||(dimY!=(floor(dimY)))||(dimX>d)||(dimY>d)||((c!=1)&&(dimX>=dimY))||
     ((c==1)&&((dimX!=1)||(dimY!=1))))
```

```
   {
    print("te has confundido, ejes incorrectos")
    return(FALSE)
   } else {return(TRUE)}
  }
}

# funcion general para representar todos los graficos de los distintos analisis, excepto para el Tucker3 y el CoTucker3

plotm<-function(dim,d,M1,M2,M3=NULL,M4=NULL,M5=NULL,M6=NULL,lim1,lim2,names1,names2,colores1,colores2,contf,contc,
           cotucker=FALSE)
{
 contf[is.nan(contf)]<-0
 contc[is.nan(contc)]<-0
 mcontf<-max(contf)/2
 mcontc<-max(contc)/2
 s<-function(i,M,colores){segments(M[,1,i],M[,2,i],M[,1,i+1],M[,2,i+1],col=colores)}
 windows(width=14,height=7)
 layout(matrix(1:2,nrow=1))
 layout.show(2)
 e<-round((100*d^2/sum(d^2))[dim],3)
 plot(M1[,1],M1[,2],type="n",xlim=c(min(0,lim1[,1]),max(0,lim1[,1])),ylim=c(min(0,lim1[,2]),max(0,lim1[,2])),
    xlab=paste("Axis ",dim[1]," (",e[1],"%)",sep=""),ylab=paste("Axis ",dim[2]," (",e[2],"%)",sep=""))
 text(M1[,1],M1[,2],names1,cex=0.8,col=colores1)
```

```
box(lwd=2)
abline(h=0,lwd=2)
abline(v=0,lwd=2)
if(!cotucker)
{
  if(!is.null(M3)){arrows(M1[,1],M1[,2],M3[,1],M3[,2],col=colores1,angle=10,length=0.1)}
} else {
  if(!is.null(M3))
  {
    cual1<-cbind(M1[,1],M3[,1])
    cual2<-cbind(M1[,2],M3[,2])
    cual<-(apply(cual1,1,function(v){return(abs(v[1]-v[2])>1e-15)}))|
      (apply(cual2,1,function(v){return(abs(v[1]-v[2])>1e-15)}))
     if(sum(cual)!=0){arrows(M1[cual,1],M1[cual,2],M3[cual,1],M3[cual,2],col=colores1[cual],angle=10,length=0.1)}
  }
}
if(!is.null(M4)){lapply(1:(dim(M4)[3]-1),s,M=M4,colores=colores1)}
if(!is.null(M6))
{
 lcolores1<-colores1[!apply(M6==0,1,all)]
 lM6<-M6[!apply(M6==0,1,all),]
 arrows(0,0,lM6[,1],lM6[,2],col=lcolores1,angle=10,length=0.1)
}
plot(M2[,1],M2[,2],type="n",xlim=c(min(0,lim2[,1]),max(0,lim2[,1])),
    ylim=c(min(0,lim2[,2]),max(0,lim2[,2])),
    xlab=paste("Axis ",dim[1]," (",e[1],"%)",sep=""),ylab=paste("Axis ",dim[2]," (",e[2],"%)",sep=""))
```

```
text(M2[,1],M2[,2],names2,cex=0.8,col=colores2)
box(lwd=2)
abline(h=0,lwd=2)
abline(v=0,lwd=2)
lcolores2<-colores2[!apply(M2==0,1,all)]
if (!cotucker){lM2<-M2[!apply(M2==0,1,all),]} else {lM2<-M2[!apply(
    abs(M2)<1e-15,1,all),]}
arrows(0,0,lM2[,1],lM2[,2],col=lcolores2,angle=10,length=0.1)
if (!is.null(M5)){lapply(1:(dim(M5)[3]-1),s,M=M5,colores=colores2)}
h<-function(m){return(abs(c(min(0,m[,1]),max(0,m[,1]),min(0,m[,2]),
    max(0,m[,2]))))}
k<-h(lim1)/h(lim2)
k<-min(k[k!=0])
windows()
plot(M1[,1],M1[,2],type="n",xlim=c(min(0,lim1[,1],k*lim2[,1]),max(0,
    lim1[,1],k*lim2[,1])),
    ylim=c(min(0,lim1[,2],k*lim2[,2]),max(0,lim1[,2],k*lim2[,2])),
    xlab=paste("Axis ",dim[1]," (",e[1],"%)",sep=""),ylab=paste("Axis
        ",dim[2]," (",e[2],"%)",sep=""))
lcolores1<-colores1[contf>=mcontf]
lnames1<-names1[contf>=mcontf]
lM1<-matrix(M1[contf>=mcontf,],ncol=2)
box(lwd=2)
abline(h=0,lwd=2)
abline(v=0,lwd=2)
title(main=paste("Elementos con mayor contribucion.\nVariables
    multiplicadas por una constante (",round(k,3),
        ")\npara que sean representables conjuntamente.",sep=""),
            cex.main=0.75,font.main=1)
```

```
if(!cotucker)
{
 if(!is.null(M3))
 {
  lM3<-matrix(M3[contf>=mcontf,],ncol=2)
  arrows(lM1[,1],lM1[,2],lM3[,1],lM3[,2],col=lcolores1,angle=10,
      length=0.1)
 }
} else {
 if(!is.null(M3))
 {
  lM3<-matrix(M3[contf>=mcontf,],ncol=2)
  cual1<-cbind(lM1[,1],lM3[,1])
  cual2<-cbind(lM1[,2],lM3[,2])
  cual<-(apply(cual1,1,function(v){return(abs(v[1]-v[2])>1e-15)}))|
   (apply(cual2,1,function(v){return(abs(v[1]-v[2])>1e-15)}))
  if(sum(cual)!=0)
  {
   arrows(lM1[cual,1],lM1[cual,2],lM3[cual,1],lM3[cual,2],col=
       lcolores1[cual],angle=10,length=0.1)
  }
 }
}
if(!is.null(M4))
{
 lM4<-array(M4[contf>=mcontf,,],dim=c(length(lcolores1),2,dim(M4)[3
     ]))
 lapply(1:(dim(lM4)[3]-1),s,M=lM4,colores=lcolores1)
}
```

```
 if (!is.null(M6))
 {
  lM6<-matrix(lM6[contf>=mcontf,], ncol=2)
  lcolores1<-lcolores1[!apply(lM6==0,1,all)]
  lM6<-matrix(lM6[!apply(lM6==0,1,all),], ncol=2)
  arrows(0,0,lM6[,1],lM6[,2],col=lcolores1,angle=10,length=0.1)
 }
 lcolores2<-colores2[contc>=mcontc]
 lnames2<-names2[contc>=mcontc]
 lM2<-matrix(M2[contc>=mcontc,], ncol=2)
 lcolores2<-lcolores2[!apply(lM2==0,1,all)]
 lnames2<-lnames2[!apply(lM2==0,1,all)]
 lM2<-matrix(lM2[!apply(lM2==0,1,all),], ncol=2)
 text(k*lM2[,1],k*lM2[,2],lnames2,cex=0.8,col=lcolores2)
 arrows(0,0,k*lM2[,1],k*lM2[,2],col=lcolores2,angle=10,length=0.1)
 if (!is.null(M5))
 {
  lM5<-array(M5[contc>=mcontc,,],dim=c(length(lcolores2),2,dim(M5)[3
     ]))
  lapply(1:(dim(lM5)[3]-1),s,M=k*lM5,colores=lcolores2)
 }
 if (is.null(M6))
 {
  points(lM1[,1],lM1[,2],pch=19,col=lcolores1,cex=0.75)
  identify(lM1[,1],lM1[,2],lnames1,cex=0.8,tolerance=3)
 }else{text(lM1[,1],lM1[,2],lnames1,cex=0.8,col=lcolores1)}
}

tensorial<-function(X,dimension,U)
```

```
{
 m<-matrix(c(1,2,3,2,1,3,3,2,1),nrow=3,byrow=TRUE)
 X<-aperm(X,m[dimension,])
 return(aperm(array(apply(X,2:3,function(v,b){return(t(b) %*% v)},b=
    U),dim=c(dim(U)[2],dim(X)[2:3])),
          m[dimension,]))
}

desplegar<-function(X,dimension)
{
 m<-matrix(c(1,2,3,2,1,3,3,1,2),nrow=3)
 return(matrix(aperm(X,m[,dimension]),nrow=dim(X)[dimension]))
}

results<-function(X,title,names=NULL,axis=FALSE,first=FALSE)
{
 if(!is.null(names))
 {
  X<-t(X)
  colnames(X)<-names
  X<-t(X)
 }
 if(!first){write(paste("\n",paste(rep("-",100),collapse=""),"\n",sep
    =""),file="results.txt",append=TRUE)}
 write(paste("\n",paste(rep(" ",20),collapse=""),title,"\n",sep=""),
    file="results.txt",append=!first)
 if(axis){write(paste("Axis",paste(1:(dim(X)[2]),collapse="\t"),sep="
    \t"),file="results.txt",append=TRUE)}
```

```
 write.table(X, file="results.txt",append=TRUE,sep="\t",col.names=
    FALSE)
}

# funcion auxiliar para realizar el TUCKER3 fijado el numero de
   componentes para cada dimension

every<-function(p,q,r,T,iter,tol,mas=FALSE)
{

 # calcula los vectores singulares por la izquierda de los
    desplegamientos del tensor a lo largo de cada una
 # de las dimensiones

 A1<-(svd(desplegar(T,1),nu=p,nv=0))$u
 A2<-A1
 B1<-(svd(desplegar(T,2),nu=q,nv=0))$u
 B2<-B1
 C1<-(svd(desplegar(T,3),nu=r,nv=0))$u
 C2<-C1

 # calcula el tensor core

 G1<-tensorial(tensorial(tensorial(T,1,A1),2,B1),3,C1)
 G2<-G1

 # paso de iteracion:

 i<-function(l)
```

```
{
 seguir<-l[[14]]
 rep<-l[[15]]
 iter<-l[[16]]
 if((seguir)&&(rep<iter))
 {
  T<-l[[1]]
  p<-l[[2]]
  q<-l[[3]]
  r<-l[[4]]
  A1<-l[[5]]
  B1<-l[[6]]
  C1<-l[[7]]
  G1<-l[[8]]
  A2<-l[[9]]
  B2<-l[[10]]
  C2<-l[[11]]
  G2<-l[[12]]
  tol<-l[[13]]

  # calcula la matriz optima fijando los vectores para las
     dimensiones dos y tres

  A<-(svd(desplegar(tensorial(tensorial(T,2,B1),3,C1),1),nu=p,nv=0))
     $u

  # calcula la matriz optima fijando los vectores para las
     dimensiones uno y tres

```

```
B<-(svd(desplegar(tensorial(tensorial(T,1,A),3,C1),2),nu=q,nv=0))$u

# calcula la matriz optima fijando los vectores para las dimensiones uno y dos

C<-(svd(desplegar(tensorial(tensorial(T,1,A),2,B),3),nu=r,nv=0))$u

# calcula el tensor core

G<-tensorial(tensorial(tensorial(T,1,A),2,B),3,C)

# calcula la norma de los vectores diferencias entre una iteracion y las dos anteriores

normA1<-sum((A-A1)^2)
normA2<-sum((A-A2)^2)
normB1<-sum((B-B1)^2)
normB2<-sum((B-B2)^2)
normC1<-sum((C-C1)^2)
normC2<-sum((C-C2)^2)
normG1<-sum((G-G1)^2)
normG2<-sum((G-G2)^2)

# si los pares de normas anteriores son menores que la tolerancia definida desde un principio se detienen las iteraciones

if(((normA1<=tol)||(normA2<=tol))&&((normB1<=tol)||(normB2<=tol))&&((normC1<=tol)||(normC2<=tol))&&
```

```
      ((normG1<=tol)||(normG2<=tol)))
    {
     seguir<-FALSE
    }
    rep<-rep+1
    return(list(T,p,q,r,A,B,C,G,A1,B1,C1,G1,tol,seguir,rep,iter))
   } else {return(l)}
  }
  l<-Reduce(function(i,...){return(i(...))},rep.int(list(i),iter),
        list(T,p,q,r,A1,B1,C1,G1,A2,B2,C2,G2,tol,TRUE,0,iter),right=
           TRUE)
  p<-l[[2]]
  q<-l[[3]]
  r<-l[[4]]
  A<-l[[5]]
  B<-l[[6]]
  C<-l[[7]]
  G<-l[[8]]
  rep<-l[[15]]

  # calcula el error: la suma de los cuadrados de las diferencias
     entre los tensores original y aproximacion

  E<-sum((T-tensorial(tensorial(tensorial(G,1,t(A)),2,t(B)),3,t(C)))^2
     )

  # finalmente, la funcion devuelve la suma de las componentes, el
     error, el ajuste en forma de porcentaje y
  # el numero de iteraciones llevadas a cabo
```

```
 # cuando se haya elegido la combinacion de componentes devuelve las matrices A,B,C y el tensor core

 if(!mas){return(c(p+q+r,E,100-((100*E)/(sum(T^2))),rep))}else{return(list(A,B,C,G))}
}

# funcion para crear los graficos para el Tucker3, divididos en tres para representar cada una de las tres dimensiones,
# incluso si en alguna se ha retenido solo una componente

plotmt<-function(dim1,dim2,dim3,sos,M1,M2,M3,lim1,lim2,lim3,names1,names2,names3,colores1,colores2,
             titles=FALSE)
{
 sos<-round(sos,3)
 windows(width=14,height=4.666)
 layout(matrix(1:3,nrow=1))
 layout.show(3)
 if(dim1[2]!=1)
 {
  plot(M1[,1],M1[,2],type="n",xlim=c(min(0,lim1[,1]),max(0,lim1[,1])),
      ylim=c(min(0,lim1[,2]),max(0,lim1[,2])),
      xlab=paste("Component ",dim1[1]," (",sos[dim1[1],1],"%)",sep=""),
      ylab=paste("Component ",dim1[2]," (",sos[dim1[2],1],"%)",sep=""))
  text(M1[,1],M1[,2],names1,cex=0.8,col=colores1)
```

```
  abline(v=0,lwd=2)
} else {
  plot(1:(dim(M1)[1]),M1[,1],type="n",xlim=c(0,dim(M1)[1]+1),
      ylim=c(min(0,M1[,1]),max(0,M1[,1])),xlab="",ylab=paste("
         Component 1 (",sos[1,1],"%)",sep=""))
  text(1:(dim(M1)[1]),M1[,1],names1,cex=0.8,col=colores1)
}
box(lwd=2)
abline(h=0,lwd=2)
if(titles){title(main="Elementos con mayor contribucion.",cex.main=0
     .75,font.main=1)}
if(dim2[2]!=1)
{
  plot(M2[,1],M2[,2],type="n",xlim=c(min(0,lim2[,1]),max(0,lim2[,1]))
      ,
      ylim=c(min(0,lim2[,2]),max(0,lim2[,2])),
      xlab=paste("Component ",dim2[1]," (",sos[dim2[1],2],"%)",sep="")
         ,
      ylab=paste("Component ",dim2[2]," (",sos[dim2[2],2],"%)",sep="")
         )
  text(M2[,1],M2[,2],names2,cex=0.8,col=colores2)
  abline(v=0,lwd=2)
  arrows(0,0,M2[,1],M2[,2],col=colores2,angle=10,length=0.1)
} else {
  plot(1:(dim(M2)[1]),M2[,1],type="n",xlim=c(0,dim(M2)[1]+1),
      ylim=c(min(0,M2[,1]),max(0,M2[,1])),xlab="",ylab=paste("
         Component 1 (",sos[1,2],"%)",sep=""))
  text(1:(dim(M2)[1]),M2[,1],names2,cex=0.8,col=colores2)
}
```

```
box(lwd=2)
abline(h=0,lwd=2)
if(titles){title(main="Elementos con mayor contribucion.",cex.main=0
    .75,font.main=1)}
if(dim3[2]!=1)
{
 plot(M3[,1],M3[,2],type="n",xlim=c(min(0,lim3[,1]),max(0,lim3[,1]))
    ,
    ylim=c(min(0,lim3[,2]),max(0,lim3[,2])),
    xlab=paste("Component ",dim3[1]," (",sos[dim3[1],3],"%)",sep="")
      ,
    ylab=paste("Component ",dim3[2]," (",sos[dim3[2],3],"%)",sep="")
      )
 text(M3[,1],M3[,2],names3,cex=0.8)
 abline(v=0,lwd=2)
 arrows(0,0,M3[,1],M3[,2],angle=10,length=0.1)
} else {
 plot(1:(dim(M3)[1]),M3[,1],type="n",xlim=c(0,dim(M3)[1]+1),ylim=c(
    min(0,M3[,1]),max(0,M3[,1])),
    xlab="",ylab=paste("Component 1 (",sos[1,3],"%)",sep=""))
 text(1:(dim(M3)[1]),M3[,1],names3,cex=0.8)
}
box(lwd=2)
abline(h=0,lwd=2)
if(titles){title(main="Elementos con mayor contribucion.",cex.main=0
    .75,font.main=1)}
}
```

6.1.2. Programa para realizar un PCA

```
PCA<-function(X,dimX=NULL,dimY=NULL,coloresf=NULL,coloresc=NULL,norm=FALSE,contr=FALSE)
{

# lee la matriz de datos con las etiquetas de las filas y las columnas (si no estan incluidas, se nombran por
# defecto) y suprime las filas que tengan datos faltantes

l<-read(X)
X<-l[[1]]
filas<-l[[2]]
columnas<-l[[3]]
namesf<-l[[5]]
namesc<-l[[6]]

# lee las etiquetas de los colores para las filas y columnas y comprueba que hay tantas como filas y columnas
# (si no se dan, se asignan por defecto de color negro)

l<-colores(filas,columnas,coloresf,coloresc,FALSE)
coloresf<-l[[1]]
coloresc<-l[[2]]
conf<-l[[3]]

# centra la matriz de datos por columnas y si se introdujo TRUE en normalizacion, normaliza por columnas

```

```
if (!conf)
{
 X<-preproc(X,norm)

 # crea la matriz con los pesos uniformes para las filas y su raiz
    cuadrada

 l<-pesos(rep(1,filas))
 Dn<-l[[1]]
 Dn2<-l[[2]]

 # extrae los vectores singulares por la izquierda y la derecha y
    los valores singulares segun el metodo explicado

 c<-svd(Dn2 %*% X)
 d<-c$d
 u<-c$u
 v<-c$v

 # calcula las coordenadas para las filas y las columnas mediante el
    diagrama de dualidad

 F<-X %*% v %*% diag(1/sqrt(diag(t(v) %*% v)))
 C<-t(X) %*% Dn %*% (solve(Dn2) %*% u %*% diag(1/sqrt(diag(t(u)
    %*% u))))

 # si se ha elegido, calcula las contribuciones para las filas y las
    columnas

```

```
if(contr)
{
 contributions(F,namesf,"las filas",TRUE)
 contributions(C,namesc,"las columnas")
}

# si no se ha elegido representar los ejes, representa los valores
   singulares en un diagrama de sedimentacion

if((is.null(dimX))||(is.null(dimY)))
{
 windows()
 layout(matrix(c(1,1,2,1),ncol=2),widths=c(4.375,2.625),heights=c(3
   .5,3.5))
 layout.show(2)
 screeplot(d)

 # representa un esquema con la forma de la matriz de datos

 plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,1.25),ylim=c(-1.25,1
   .5),bty="n",xaxt="n",yaxt="n")
 rect(-1,-1,1,1)
 text(0,0,"X")
 text(-1.25,0,filas)
 text(0,1.25,columnas)

 # si se ha elegido representar los ejes, comprueba que los ejes a
   representar en los graficos sean correctos

```

```
    } else {
      if(compr(dimX,dimY,d))
      {
        dimension<-c(dimX,dimY)
        Fd<-F[,dimension]
        Cd<-C[,dimension]

        # representa el grafico para las filas en la izquierda con las etiquetas de colores
        # representa las columnas en la derecha con las etiquetas y vectores desde el origen de colores

        plotm(dim=dimension,d=d,M1=Fd,M2=Cd,lim1=Fd,lim2=Cd,names1=namesf,names2=namesc,colores1=coloresf,
            colores2=coloresc,contf=contributions2(F,dimension),contc=contributions2(C,dimension))
      }
    }
  }
}
```

6.1.3. Programa para realizar un BGA

```
BGA<-function(X,gruposf,dimX=NULL,dimY=NULL,coloresf=NULL,coloresc=
    NULL,norm=FALSE,contr=FALSE)
{

 # lee la matriz de datos con las etiquetas de las filas y las
     columnas (si no estan incluidas, se nombran por
 # defecto) y suprime las filas que tengan datos faltantes

 l<-read(X)
 X<-l[[1]]
 filas<-l[[2]]
 columnas<-l[[3]]
 namesf<-l[[5]]
 namesc<-l[[6]]

 # lee los grupos para las filas y las etiquetas (mas de uno)

 namesg<-unique(gruposf)
 conf<-FALSE
 if(length(namesg)<2)
 {
  conf<-TRUE
  print("te has confundido, el numero de grupos para las filas tiene
     que ser mayor que uno")
 }

```

```
# lee las etiquetas de los colores para las filas y columnas y comprueba que hay tantas como filas y columnas
# (si no se dan, se asignan por defecto de color negro)

if(!conf)
{
 l<-colores(filas,columnas,coloresf,coloresc,conf)
 coloresf<-l[[1]]
 coloresc<-l[[2]]
 conf<-l[[3]]
}

# centra la matriz de datos por columnas y si se introdujo TRUE en normalizacion, normaliza por columnas

if(!conf)
{
 X<-preproc(X,norm)

 # crea la matriz con los pesos para las filas segun los tamanos de los grupos y su raiz cuadrada

 l<-pesos(unlist(lapply(1:(length(namesg)),function(x,g,n){return(sum(g==n[x]))},g=gruposf,n=namesg)))
 Dg<-l[[1]]
 Dg2<-l[[2]]

 # calcula la matriz con las medias para los grupos

```

```
XB<-matrix(unlist(lapply(1:length(namesg),function(x,X1,g1,n){
    return(apply(X1[g1==n[x],],2,mean))},X1=X,
                   g1=gruposf,n=namesg)),ncol=columnas,byrow=TRUE)

# centra la matriz de las medias por columnas y si se introdujo
   TRUE en normalizacion normaliza la matriz de medias
# por columnas

XB<-preproc(XB,norm)

# extrae los vectores singulares por la izquierda y la derecha y
    los valores singulares segun el metodo explicado

c<-svd(Dg2 %*% XB)
d<-c$d
u<-c$u
v<-c$v
Vr<-v %*% diag(1/sqrt(diag(t(v) %*% v)))

# calcula las coordenadas para los grupos, las filas y las columnas
     mediante el diagrama de dualidad

FB<-XB %*% Vr
F<-X %*% Vr
C<-t(XB) %*% Dg %*% solve(Dg2) %*% u %*% diag(1/sqrt(diag(t(u)
    %*% u)))

# si se ha elegido, calcula las contribuciones para los grupos, las
     filas y las columnas
```

```
if(contr)
{
 contributions(FB,namesg,"los grupos",TRUE)
 contributions(F,namesf,"las filas")
 contributions(C,namesc,"las columnas")
}

# si no se ha elegido representar los ejes, representa los valores
    singulares en un diagrama de sedimentacion

if((is.null(dimX))||(is.null(dimY)))
{
 windows()
 layout(matrix(c(1,1,2,1),ncol=2),widths=c(4.375,2.625),heights=c(3
     .5,3.5))
 layout.show(2)
 screeplot(d)

 # representa un esquema con la forma de las matrices de datos

 plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,1.25),ylim=c(-2.25,1
     .5),bty="n",xaxt="n",yaxt="n")
 rect(-1,-1,1,1)
 rect(-1,-2,1,-1)
 text(0,0,"X")
 text(0,-1.5,"XB")
 text(-1.25,0,filas)
 text(0,1.25,columnas)
```

```
    text(-1.25,-1.5,length(namesg))

    # si se ha elegido representar los ejes, comprueba que los ejes a representar en los graficos sean correctos

  } else {
    if(compr(dimX,dimY,d))
    {
      dimension<-c(dimX,dimY)
      FBd<-FB[,dimension]
      Fd<-F[,dimension]
      Cd<-C[,dimension]
      F2<-rbind(FBd,Fd)

      # representa el grafico para los grupos en la izquierda con las etiquetas
      # representa las columnas en la derecha con las etiquetas y vectores desde el origen segun los colores de los grupos
      # a los que pertenecen

      plotm(dim=dimension,d=d,M1=FBd,M2=Cd,lim1=F2,lim2=Cd,names1=namesg,names2=namesc,
          colores1=rep("black",length(namesg)),colores2=coloresc,contf=contributions2(FB,dimension),
          contc=contributions2(C,dimension))

      # representa el grafico para las filas en la izquierda con las etiquetas segun los colores de los grupos a los que
      # pertenecen
```

```
      # representa las columnas en la derecha con las etiquetas y
          vectores desde el origen segun los colores de los grupos
      # a los que pertenecen

      plotm(dim=dimension,d=d,M1=Fd,M2=Cd,lim1=F2,lim2=Cd,names1=namesf
          ,names2=namesc,colores1=coloresf,
          colores2=coloresc,contf=contributions2(F,dimension),contc=
              contributions2(C,dimension))
    }
  }
 }
}
```

6.1.4. Programa para realizar un CoIA

```
COIA<-function(X,Y,dimXx=NULL,dimYx=NULL,dimXy=NULL,dimYy=NULL,dimX=NULL,dimY=NULL,coloresf=NULL,coloresc1=NULL,
          coloresc2=NULL,norm=FALSE,contr=FALSE,cotucker=FALSE,tcotucker=FALSE)
{

 # lee las dos matrices de datos con las etiquetas de las filas y de las columnas de ambas matrices (si no estan
 # incluidas, se nombran por defecto), suprime las filas que tengan datos faltantes y comprueba que las dos matrices
 # tengan las mismas filas

 l<-read(X,Y)
 X<-l[[1]]
 filas<-l[[2]]
 columnas1<-l[[3]]
 namesf<-l[[5]]
 namesc1<-l[[6]]
 Y<-l[[8]]
 columnas2<-l[[9]]
 namesc2<-l[[10]]
 conf<-l[[11]]

 # lee los colores de las filas y de las columnas de las dos matrices y comprueba que hay tantos como filas y como
 # columnas (si no se dan, se asignan por defecto en negro)

```

```
 if (!conf)
 {
  l<-colores(filas,columnas1,coloresf,coloresc1,conf,columnas2,
     coloresc2)
  coloresf<-l[[1]]
  coloresc1<-l[[2]]
  coloresc2<-l[[3]]
  conf<-l[[4]]
 }

# centra las dos matrices de datos por columnas y si se introdujo
   TRUE en normalizacion, normaliza por columnas

 if (!conf)
 {
  X<-preproc(X,norm)
  Y<-preproc(Y,norm)

  # crea la matriz con los pesos uniformes para las filas y su raiz
     cuadrada

  l<-pesos(rep(1,filas))
  Dn<-l[[1]]
  Dn2<-l[[2]]

  # extrae los vectores singulares por la izquierda y la derecha y
     los valores singulares para las dos matrices y para
  # la coinercia segun el metodo explicado

```

```
cX<-svd(Dn2 %*% X)
dx<-cX$d
ux<-cX$u
vx<-cX$v
cY<-svd(Dn2 %*% Y)
dy<-cY$d
uy<-cY$u
vy<-cY$v
c<-svd(t(Y) %*% Dn %*% X)
d<-c$d
u<-c$u
v<-c$v
Ur<-u %*% diag(1/sqrt(diag(t(u) %*% u)))
Vr<-v %*% diag(1/sqrt(diag(t(v) %*% v)))

# calcula las coordenadas para las filas y las columnas por
    separado y para la coinercia mediante los diagramas de
# dualidad

FX<-X %*% vx %*% diag(1/sqrt(diag(t(vx) %*% vx)))
CX<-t(X) %*% Dn %*% solve(Dn2) %*% (ux) %*% diag(1/sqrt(diag(t(
    ux) %*% ux)))
FY<-Y %*% vy %*% diag(1/sqrt(diag(t(vy) %*% vy)))
CY<-t(Y) %*% Dn %*% solve(Dn2) %*% uy %*% diag(1/sqrt(diag(t(uy
    ) %*% uy)))
FXc<-X %*% Vr
CXc<-t(X) %*% Dn %*% Y %*% Ur
FYc<-Y %*% Ur
CYc<-t(Y) %*% Dn %*% X %*% Vr
```

```
# si se ha elegido, calcula las contribuciones para las filas y las
    columnas de ambas matrices por separado y para la
# coinercia

if(contr)
{
  if(!cotucker)
  {
    contributions(FX,namesf,"las filas segun la primera matriz",TRUE)
    contributions(CX,namesc1,"las columnas de la primera matriz")
    contributions(FY,namesf,"las filas segun la segunda matriz")
    contributions(CY,namesc2,"las columnas de la segunda matriz")
  }
  if(!cotucker||!tcotucker){contributions(FXc,namesf,"las filas
      segun la primera matriz en la co-inercia",cotucker)}
  if(!cotucker||tcotucker){contributions(CXc,namesc1,"las columnas
      de la primera matriz en la co-inercia",tcotucker)}
  if(!cotucker||!tcotucker){contributions(FYc,namesf,"las filas
      segun la segunda matriz en la co-inercia")}
  if(!cotucker||tcotucker){contributions(CYc,namesc2,"las columnas
      de la segunda matriz en la co-inercia")}
}

# si no se ha elegido representar ninguno de los ejes, representa
    los valores singulares en tres diagramas de
# sedimentacion

```

```
if(((is.null(dimXx))||(is.null(dimYx)))&&((is.null(dimXy))||(is.null(dimYy)))&&((is.null(dimX))||
                                                        (is.null(dimY))))
{
 if(!cotucker)
 {
  windows()
  layout(matrix(c(1,1,2,1),ncol=2),widths=c(4.375,2.625),heights=c(3.5,3.5))
  layout.show(2)
  screeplot(dx)

  # en cada uno de los diagramas representa un esquema con la forma de las matrices de datos correspondientes

  plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,1.25),ylim=c(-1.25,1.5),bty="n",xaxt="n",yaxt="n")
  rect(-1,-1,1,1)
  text(0,0,"X")
  text(-1.25,0,filas)
  text(0,1.25,columnas1)
  windows()
  layout(matrix(c(1,1,2,1),ncol=2),widths=c(4.375,2.625),heights=c(3.5,3.5))
  layout.show(2)
  screeplot(dy)
  plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,1.25),ylim=c(-1.25,1.5),bty="n",xaxt="n",yaxt="n")
  rect(-1,-1,1,1)
```

```
      text(0,0,"Y")
      text(-1.25,0,filas)
      text(0,1.25,columnas2)
    }
    windows()
    layout(matrix(c(1,1,2,1),ncol=2),widths=c(35/9,28/9),heights=c(3.5
        ,3.5))
    layout.show(2)
    screeplot(d)
    plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,3.25),ylim=c(-4.25,1
        .5),bty="n",xaxt="n",yaxt="n")
    rect(-1,-1,1,1)
    rect(1,-1,3,1)
    rect(0,-4,2,-2)
    text(0,0,"X")
    text(2,0,"Y")
    text(1,-3,"Y' Dn X")
    text(-1.25,0,filas)
    text(0,1.25,columnas1)
    text(2,1.25,columnas2)
    text(-0.25,-3,columnas2)
    text(1,-1.75,columnas1)
  }

  # si se ha elegido representar los ejes para la primera matriz,
      comprueba que los ejes a representar en los graficos
  # sean correctos

  if(!((is.null(dimXx))||(is.null(dimYx))))
```

```
   {
    if (compr(dimXx,dimYx,dx))
    {
     dimensionx<-c(dimXx,dimYx)
     FXd<-FX[,dimensionx]
     CXd<-CX[,dimensionx]

     # representa el grafico para las filas de la primera matriz en la
         izquierda con las etiquetas segun los colores de
     # los grupos a los que pertenecen
     # representa las columnas de la primera matriz en la derecha con
         las etiquetas y vectores desde el origen con su
     # color

     plotm(dim=dimensionx,d=dx,M1=FXd,M2=CXd,lim1=FXd,lim2=CXd,names1=
         namesf,names2=namesc1,colores1=coloresf,
         colores2=coloresc1,contf=contributions2(FX,dimensionx),contc=
             contributions2(CX,dimensionx))
    }
   }
  }

  # si se ha elegido representar los ejes para la segunda matriz,
      comprueba que los ejes a representar en los graficos
  # sean correctos

  if(!((is.null(dimXy))||(is.null(dimYy))))
  {
   if(compr(dimXy,dimYy,dy))
```

```
  {
   dimensiony<-c(dimXy,dimYy)
   FYd<-FY[,dimensiony]
   CYd<-CY[,dimensiony]

   # representa el grafico de las filas segun la segunda matriz en la izquierda con las etiquetas segun los colores de
   # los grupos a los que pertenecen
   # representa las columnas de la segunda matriz en la derecha con las etiquetas y vectores desde el origen con su
   # color

   plotm(dim=dimensiony,d=dy,M1=FYd,M2=CYd,lim1=FYd,lim2=CYd,names1=namesf,names2=namesc2,colores1=coloresf,
       colores2=coloresc2,contf=contributions2(FY,dimensiony),contc=contributions2(CY,dimensiony))
  }
 }

# si se ha elegido representar los ejes para la coinercia, comprueba que los ejes a representar en los graficos sean
# correctos

 if(!((is.null(dimX))||(is.null(dimY))))
 {
  if(compr(dimX,dimY,d))
  {
   dimension<-c(dimX,dimY)
   FXcd<-FXc[,dimension]
```

```
CXcd<-CXc[,dimension]
FYcd<-FYc[,dimension]
CYcd<-CYc[,dimension]
F<-rbind(FXcd,FYcd)

# representa el grafico para la coinercia de las filas en la izquierda con las etiquetas y vectores de la primera a
# la segunda matriz segun los colores de los grupos a los que pertenecen
# representa las columnas de la primera matriz en la derecha con las etiquetas y vectores desde el origen con su
# color

plotm(dim=dimension,d=d,M1=FXcd,M2=CXcd,M3=FYcd,lim1=F,lim2=CXcd,names1=namesf,names2=namesc1,
    colores1=coloresf,colores2=coloresc1,contf=contributions2(FXc,dimension),
    contc=contributions2(CXc,dimension),cotucker=cotucker)

# representa el grafico para la coinercia de las filas en la izquierda con las etiquetas y vectores de la segunda a
# la primera matriz segun los colores de los grupos a los que pertenecen
# representa las columnas de la segunda matriz en la derecha con las etiquetas y vectores desde el origen con su
# color

plotm(dim=dimension,d=d,M1=FYcd,M2=CYcd,M3=FXcd,lim1=F,lim2=CYcd,names1=namesf,names2=namesc2,
```

```
        colores1=coloresf , colores2=coloresc2 , contf=contributions2(FYc,
            dimension ) ,
        contc=contributions2(CYc, dimension ) , cotucker=cotucker )
   }
  }
}
```

6.1.5. Programa para realizar un PTA

```
PTA<-function(X,dimX=NULL,dimY=NULL,coloresf=NULL,coloresc=NULL,norm=FALSE,contr=FALSE)
{

 # lee el cubo de datos con las etiquetas de las filas, las columnas y las repeticiones (si no estan incluidas, se
 # nombran por defecto) y suprime las filas que tengan datos faltantes

 l<-read(X)
 X<-l[[1]]
 filas<-l[[2]]
 columnas<-l[[3]]
 repeticiones<-l[[4]]
 namesf<-l[[5]]
 namesc<-l[[6]]
 namesr<-l[[7]]
 conf<-l[[11]]

 # lee las etiquetas de los colores para las filas y columnas y comprueba que hay tantas como filas y columnas
 # (si no se dan, se asignan por defecto de color negro)

 if(!conf)
 {
  l<-colores(filas,columnas,coloresf,coloresc,conf)
  coloresf<-l[[1]]
```

```
  coloresc<-l[[2]]
  conf<-l[[3]]
 }

 # centra el cubo de datos por columnas y si se introdujo TRUE en
    normalizacion, normaliza por columnas

 if(!conf)
 {
  X<-preproc(X,norm,TRUE)

  # crea la matriz con los pesos uniformes para las filas y las
     repeticiones y sus raices cuadradas

  l<-pesos(rep(1,filas))
  Dn<-l[[1]]
  Dn2<-l[[2]]
  l<-pesos(rep(1,repeticiones))
  Dk<-l[[1]]
  Dk2<-l[[2]]

  # calcula la matriz de varianzas-covarianzas vectoriales

  l<-as.matrix(expand.grid(1:repeticiones,1:repeticiones))
  Covv<-matrix(apply(l,1,function(v,X1,D){return(sum(diag(t(X1[,,v[1
     ]]) %*% D %*% X1[,,v[2]]))))},X1=X,D=Dn),
         nrow=repeticiones,ncol=repeticiones)

```

```
# extrae los vectores propios segun el metodo explicado y calcula
    las coordenadas para la interestructura

a<-(eigen(Covv %*% Dk,symmetric=TRUE))$vectors
Vl<-solve(Dk2) %*% (a[,1:2]) %*% diag(1/sqrt(diag(t(a[,1:2]) %*%
    (a[,1:2]))))
Vl[,1]<-abs(Vl[,1])
l<-Covv %*% Dk %*% Vl

# calcula la matriz compromiso

Xc<-apply(X,1:2,function(v,v2){return(sum(v*v2))},v2=Vl[,1]/sum(Vl
    [,1]))

# extrae los vectores singulares por la izquierda y la derecha y
    los valores singulares segun el metodo explicado

c<-svd(Dn2 %*% Xc)
d<-c$d
u<-c$u
v<-c$v
Ur<-(solve(Dn2)) %*% u %*% diag(1/sqrt(diag(t(u) %*% u)))
Vr<-v %*% diag(1/sqrt(diag(t(v) %*% v)))

# calcula las coordenadas para las filas y las columnas de la
    matriz compromiso y de las trayectorias mediante el
# diagrama de dualidad

Fc<-Xc %*% Vr
```

```
Cc<-t(Xc) %*% Dn %*% Ur
Ft<-array(apply(X,3,function(m,m2){return(m %*% m2)},m2=Vr),dim=c(filas,columnas,repeticiones))
Ct<-array(apply(X,3,function(m,m2){return(t(m) %*% m2)},m2=Dn %*% Ur),dim=c(columnas,columnas,repeticiones))
F<-matrix(aperm(Ft,c(2,1,3)),nrow=filas*repeticiones,ncol=columnas,byrow=TRUE)
C<-matrix(aperm(Ct,c(2,1,3)),nrow=columnas*repeticiones,ncol=columnas,byrow=TRUE)

# si se ha elegido, calcula las contribuciones para las filas y las columnas de la matriz compromiso y de las
# trayectorias

if(contr)
{
 contributions(Fc,namesf,"las filas en el compromiso",TRUE)
 contributions(Cc,namesc,"las columnas en el compromiso")
 contributions(F,rep(namesf,repeticiones),"las filas en todas las repeticiones")
 contributions(C,rep(namesc,repeticiones),"las columnas en todas las repeticiones")
}

# si no se ha elegido representar los ejes, representa los valores singulares en un diagrama de sedimentacion

if((is.null(dimX))||(is.null(dimY)))
{
```

```
    windows()
    layout(matrix(c(1,1,2,1),ncol=2),widths=c(31.5/9,24.5/9),heights=c
        (4.5,2.5))
    layout.show(2)
    screeplot(d)

    # representa un esquema con la forma del cubo de datos

    plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,2.25),ylim=c(-4.25,2
        .25),bty="n",xaxt="n",yaxt="n")
    rect(-1,-1,1,1)
    rect(-1,-4,1,-2)
    segments(-1,1,0,2)
    segments(0,2,2,2)
    segments(2,2,2,0)
    segments(2,0,1,-1)
    segments(1,1,2,2)
    text(0,0,"X")
    text(0,-3,"Xc")
    text(-1.25,0,filas)
    text(0,1.25,columnas)
    text(-0.75,1.75,repeticiones,srt=45)
    text(-1.25,-3,filas)
    text(0,-1.75,columnas)

    # si se ha elegido representar los ejes, comprueba que los ejes a
        representar en los graficos sean correctos

  } else {
```

```
 if(compr(dimX,dimY,d))
 {
  dimension<-c(dimX,dimY)
  Fcd<-Fc[,dimension]
  Ccd<-Cc[,dimension]
  Ftd<-Ft[,dimension,]
  Ctd<-Ct[,dimension,]
  Fd<-F[,dimension]
  Cd<-C[,dimension]
  F2<-rbind(Fcd,Fd)
  C2<-rbind(Ccd,Cd)

  # representa el grafico de la interestructura con las etiquetas
      para las repeticiones y vectores desde el origen

  windows()
  plot(I[,1],I[,2],type="n",xlim=c(min(0,I[,1]),max(0,I[,1])),ylim=
      c(min(0,I[,2]),max(0,I[,2])),
      xlab="",ylab="")
  text(I[,1],I[,2],namesr,cex=0.8,pos=2)
  box(lwd=2)
  abline(h=0,lwd=2)
  abline(v=0,lwd=2)
  arrows(0,0,I[,1],I[,2],angle=10,length=0.1)

  # representa el grafico para las filas del compromiso en la
      izquierda con las etiquetas segun los colores de los
  # grupos a los que pertenecen
```

```
      # representa las columnas del compromiso en la derecha con las etiquetas y vectores desde el origen segun los colores
      # de los grupos a los que pertenecen

      plotm(dim=dimension,d=d,M1=Fcd,M2=Ccd,lim1=F2,lim2=C2,names1=namesf,names2=namesc,colores1=coloresf,
         colores2=coloresc,contf=contributions2(Fc,dimension),contc=contributions2(Cc,dimension))

      # representa el grafico para las trayectorias de las filas en la izquierda con las etiquetas y segmentos segun los
      # colores de los grupos a los que pertenecen
      # representa las trayectorias de las columnas en la derecha con las etiquetas, segmentos y vectores desde el origen
      # segun los colores de los grupos a los que pertenecen

      plotm(dim=dimension,d=d,M1=Ftd[,,1],M2=Ctd[,,1],M4=Ftd,M5=Ctd,lim1=F2,lim2=C2,names1=namesf,
         names2=namesc,colores1=coloresf,colores2=coloresc,contf=contributions2(Ft[,,1],dimension),
         contc=contributions2(Ct[,,1],dimension))
    }
   }
  }
}
```

6.1.6. Programa para realizar un BGCOIA

```
BGCOIA<-function(X,Y,dimXx=NULL,dimYx=NULL,dimXy=NULL,dimYy=NULL,dimX
    =NULL,dimY=NULL,coloresf=NULL,coloresc1=NULL,
        coloresc2=NULL,norm=FALSE,contr=FALSE)
{

 # lee los dos cubos de datos con las etiquetas de las filas, las
    columnas y las repeticiones (si no estan incluidas,
 # se nombran por defecto), comprueba que los dos cubos tengan las
    mismas filas y repeticiones y suprime las filas que
 # tengan datos faltantes

 l<-read(X,Y)
 X<-l[[1]]
 filas<-l[[2]]
 columnas1<-l[[3]]
 repeticiones<-l[[4]]
 namesf<-l[[5]]
 namesc1<-l[[6]]
 namesr<-l[[7]]
 Y<-l[[8]]
 columnas2<-l[[9]]
 namesc2<-l[[10]]
 conf<-l[[11]]

 # lee los colores de las filas y de las columnas de los dos cubos y
    comprueba que hay tantos como filas y como
 # columnas (si no se dan, se asignan por defecto en negro)
```

```

if(!conf)
{
 l<-colores(filas,columnas1,coloresf,coloresc1,conf,columnas2,
    coloresc2)
 coloresf<-l[[1]]
 coloresc1<-l[[2]]
 coloresc2<-l[[3]]
 conf<-l[[4]]
}

# centra los cubo de datos por columnas y si se introdujo TRUE en
   normalizacion, normaliza por columnas

if(!conf)
{
 X<-preproc(X,norm,TRUE)
 Y<-preproc(Y,norm,TRUE)

 # crea la matriz con los pesos uniformes para las repeticiones y su
    raiz cuadrada

 l<-pesos(rep(1,repeticiones))
 Dg<-l[[1]]
 Dg2<-l[[2]]

 # calcula las matrices con las medias por repeticiones de ambos
    cubos

```

```
XB<-matrix(unlist(lapply(1:repeticiones,function(x,X1){return(apply(X1[,,x],2,mean))},X1=X)),ncol=columnas1,
    byrow=TRUE)
YB<-matrix(unlist(lapply(1:repeticiones,function(x,X1){return(apply(X1[,,x],2,mean))},X1=Y)),ncol=columnas2,
    byrow=TRUE)

# centra las matrices de las medias por columnas y si se introdujo TRUE en normalizacion normaliza las matrices de
# medias por columnas

XB<-preproc(XB,norm)
YB<-preproc(YB,norm)

# extrae los vectores singulares por la izquierda y la derecha y los valores singulares para las dos matrices y para
# la coinercia segun el metodo explicado

cX<-svd(Dg2 %*% XB)
dx<-cX$d
ux<-cX$u
vx<-cX$v
cY<-svd(Dg2 %*% YB)
dy<-cY$d
uy<-cY$u
vy<-cY$v
c<-svd(t(YB) %*% Dg %*% XB)
d<-c$d
u<-c$u
```

```
v<-c$v
Ur<-u %*% diag(1/sqrt(diag(t(u) %*% u)))
Vr<-v %*% diag(1/sqrt(diag(t(v) %*% v)))

# calcula las coordenadas para las filas, las columnas y las repeticiones para las matrices de las medias, para la
# coinercia y para las trayectorias mediante el diagrama de dualidad

FXB<-XB %*% vx %*% diag(1/sqrt(diag(t(vx) %*% vx)))
CXB<-t(XB) %*% Dg %*% solve(Dg2) %*% ux %*% diag(1/sqrt(diag(t(ux) %*% ux)))
FYB<-YB %*% vy %*% diag(1/sqrt(diag(t(vy) %*% vy)))
CYB<-t(YB) %*% Dg %*% solve(Dg2) %*% uy %*% diag(1/sqrt(diag(t(uy) %*% uy)))
FXBc<-XB %*% Vr
CXBc<-t(XB) %*% Dg %*% YB %*% Ur
FYBc<-YB %*% Ur
CYBc<-t(YB) %*% Dg %*% XB %*% Vr
FX<-array(apply(X,3,function(m,m2){return(m %*% m2)},m2=Vr),dim=c(filas,columnas2,repeticiones))
FY<-array(apply(Y,3,function(m,m2){return(m %*% m2)},m2=Ur),dim=c(filas,columnas2,repeticiones))
PFX<-matrix(aperm(FX,c(2,1,3)),nrow=filas*repeticiones,ncol=columnas2,byrow=TRUE)
PFY<-matrix(aperm(FY,c(2,1,3)),nrow=filas*repeticiones,ncol=columnas2,byrow=TRUE)
```

```
# si se ha elegido, calcula las contribuciones para las filas, las columnas y las repeticiones para las matrices
# de las medias, para la coinercia y para las trayectorias

if(contr)
{
 contributions(FXB,namesr,"las repeticiones segun el primer cubo",TRUE)
 contributions(CXB,namesc1,"las columnas del primer cubo")
 contributions(FYB,namesr,"las repeticiones segun el segundo cubo")
 contributions(CYB,namesc2,"las columnas del segundo cubo")
 contributions(FXBc,namesr,"las repeticiones segun primer cubo en la co-inercia")
 contributions(CXBc,namesc1,"las columnas del primer cubo en la co-inercia")
 contributions(FYBc,namesr,"las repeticiones segun el segundo cubo en la co-inercia")
 contributions(CYBc,namesc2,"las columnas del segundo cubo en la co-inercia")
 contributions(PFX,rep(namesf,repeticiones),"las filas segun el primer cubo en todas las repeticiones")
 contributions(PFY,rep(namesf,repeticiones),"las filas segun el segundo cubo en todas las repeticiones")
}

# si no se ha elegido representar ninguno de los ejes, representa los valores singulares en tres diagramas de
# sedimentacion

```

```
if (((is.null(dimXx))||(is.null(dimYx)))&&((is.null(dimXy))||(is.null(dimYy)))&&((is.null(dimX))||
                                                   (is.null(dimY))))
{
 windows()
 layout(matrix(c(1,1,2,1),ncol=2),widths=c(3.9375,3.0625),heights=c(3.5,3.5))
 layout.show(2)
 screeplot(dx)

 # en cada uno de los diagramas representa un esquema con la forma de los cubos de datos correspondientes

 plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,2.25),ylim=c(-2.25,2.25),bty="n",xaxt="n",yaxt="n")
 rect(-1,-1,1,1)
 rect(-1,-2,1,-1)
 segments(-1,1,0,2)
 segments(0,2,2,2)
 segments(2,2,2,0)
 segments(2,0,1,-1)
 segments(1,1,2,2)
 text(0,0,"X")
 text(0,-1.5,"XB")
 text(-1.25,0,filas)
 text(0,1.25,columnas1)
 text(-0.75,1.75,repeticiones,srt=45)
 text(-1.25,-1.5,repeticiones)
 windows()
```

```
layout(matrix(c(1,1,2,1),ncol=2),widths=c(3.9375,3.0625),heights=c
    (3.5,3.5))
layout.show(2)
screeplot(dy)
plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,2.25),ylim=c(-2.25,2
    .25),bty="n",xaxt="n",yaxt="n")
rect(-1,-1,1,1)
rect(-1,-2,1,-1)
segments(-1,1,0,2)
segments(0,2,2,2)
segments(2,2,2,0)
segments(2,0,1,-1)
segments(1,1,2,2)
text(0,0,"Y")
text(0,-1.5,"YB")
text(-1.25,0,filas)
text(0,1.25,columnas2)
text(-0.75,1.75,repeticiones,srt=45)
text(-1.25,-1.5,repeticiones)
windows()
layout(matrix(c(1,1,2,1),ncol=2),widths=c(2.85,4.15),heights=c(4.5
    ,2.5))
layout.show(2)
screeplot(d)
plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,4.25),ylim=c(-5.25,2
    .25),bty="n",xaxt="n",yaxt="n")
rect(-1,-1,1,1)
rect(1,-1,3,1)
rect(-1,-2,1,-1)
```

```
     rect(1,-2,3,1)
     rect(0,-5,2,-3)
     segments(-1,1,0,2)
     segments(0,2,4,2)
     segments(4,2,4,0)
     segments(4,0,3,-1)
     segments(1,1,2,2)
     segments(3,1,4,2)
     text(0,0,"X")
     text(2,0,"Y")
     text(0,-1.5,"XB")
     text(2,-1.5,"YB")
     text(1,-4,"YB' Dg XB")
     text(-1.25,0,filas)
     text(0,1.25,columnas1)
     text(2,1.25,columnas2)
     text(-0.75,1.75,repeticiones,srt=45)
     text(-1.25,-1.5,repeticiones)
     text(-0.25,-4,columnas2)
     text(1,-2.75,columnas1)
   }

   # si se ha elegido representar los ejes para la primera matriz,
      comprueba que los ejes a representar en los
   # graficos sean correctos

   if(!((is.null(dimXx))||(is.null(dimYx))))
   {
     if(compr(dimXx,dimYx,dx))
```

```
    {
     dimensionx<-c(dimXx,dimYx)
     FXBd<-FXB[,dimensionx]
     CXBd<-CXB[,dimensionx]

     # representa el grafico para las repeticiones del primer cubo en la izquierda con las etiquetas
     # representa las columnas de la primera matriz de medias en la derecha con las etiquetas y vectores desde el
     # origen segun su color

     plotm(dim=dimensionx,d=dx,M1=FXBd,M2=CXBd,lim1=FXBd,lim2=CXBd,names1=namesr,names2=namesc1,
        colores1=rep("black",repeticiones),colores2=coloresc1,contf=contributions2(FXB,dimensionx),
        contc=contributions2(CXB,dimensionx))
    }
  }

  # si se ha elegido representar los ejes para la segunda matriz, comprueba que los ejes a representar en los
  # graficos sean correctos

  if(!((is.null(dimXy))||(is.null(dimYy))))
  {
   if(compr(dimXy,dimYy,dy))
    {
     dimensiony<-c(dimXy,dimYy)
     FYBd<-FYB[,dimensiony]
```

```
     CYBd<-CYB[,dimensiony]

     # representa el grafico para las repeticiones del segundo cubo en la izquierda con las etiquetas
     # representa las columnas de la segunda matriz de medias en la derecha con las etiquetas y vectores desde el
     # origen segun su color

     plotm(dim=dimensiony,d=dy,M1=FYBd,M2=CYBd,lim1=FYBd,lim2=CYBd,names1=namesr,names2=namesc2,
        colores1=rep("black",repeticiones),colores2=colorescc2,contf=contributions2(FYB,dimensiony),
        contc=contributions2(CYB,dimensiony))
   }
 }

 # si se ha elegido representar los ejes para la coinercia y las trayectorias, comprueba que los ejes a representar
 # en los graficos sean correctos

 if(!((is.null(dimX))||(is.null(dimY))))
 {
   if(compr(dimX,dimY,d))
   {
     dimension<-c(dimX,dimY)
     FXBcd<-FXBc[,dimension]
     CXBcd<-CXBc[,dimension]
     FYBcd<-FYBc[,dimension]
     CYBcd<-CYBc[,dimension]
```

```
    FXd<-FX[,dimension,]
    FYd<-FY[,dimension,]
    PFXd<-PFX[,dimension]
    PFYd<-PFY[,dimension]
    Fc<-rbind(FXBcd,FYBcd)

    # representa el grafico para la coinercia de las repeticiones en
        la izquierda con las etiquetas y vectores de la
    # primera a la segunda matriz de medias segun los colores de los
        grupos a los que pertenecen
    # representa las columnas de la primera matriz en la derecha con
        las etiquetas y vectores desde el origen con su
    # color

    plotm(dim=dimension,d=d,M1=FXBcd,M2=CXBcd,M3=FYBcd,lim1=Fc,lim2=
        CXBcd,names1=namesr,names2=namesc1,
        colores1=rep("black",repeticiones),colores2=coloresc1,contf=
            contributions2(FXBc,dimension),
        contc=contributions2(CXBc,dimension))

    # representa el grafico para la coinercia de las repeticiones en
        la izquierda con las etiquetas y vectores de la
    # segunda a la primera matriz de medias segun los colores de los
        grupos a los que pertenecen
    # representa las columnas de la segunda matriz en la derecha con
        las etiquetas y vectores desde el origen con su
    # color

```

```
    plotm(dim=dimension,d=d,M1=FYBcd,M2=CYBcd,M3=FXBcd,lim1=Fc,lim2=CYBcd,names1=namesr,names2=namesc2,
        colores1=rep("black",repeticiones),colores2=coloresc2,contf=contributions2(FYBc,dimension),
        contc=contributions2(CYBc,dimension))

    # representa el grafico para las trayectorias de las filas segun el primer cubo en la izquierda con las etiquetas
    # y segmentos segun los colores de los grupos a los que pertenecen
    # representa las trayectorias de las columnas del primer cubo en la derecha con las etiquetas, segmentos y vectores
    # desde el origen segun su color

    plotm(dim=dimension,d=d,M1=FXd[,,1],M2=CXBcd,M4=FXd,lim1=PFXd,lim2=CXBcd,names1=namesf,names2=namesc1,
        colores1=coloresf,colores2=coloresc1,contf=contributions2(FX[,,1],dimension),
        contc=contributions2(CXBc,dimension))

    # representa el grafico para las trayectorias de las filas segun el segundo cubo en la izquierda con las etiquetas
    # y segmentos segun los colores de los grupos a los que pertenecen
    # representa las trayectorias de las columnas del segundo cubo en la derecha con las etiquetas, segmentos y vectores
    # desde el origen segun su color

```

```
        plotm(dim=dimension,d=d,M1=FYd[,,1],M2=CYBcd,M4=FYd,lim1=PFYd,lim2=CYBcd,names1=namesf,names2=namesc2,
          colores1=coloresf,colores2=coloresc2,contf=contributions2(FY[,,1],dimension),
          contc=contributions2(CYBc,dimension))
      }
    }
  }
}
```

6.1.7. Programa para realizar un STATICO

```
STATICO<-function(X,Y,dimX=NULL,dimY=NULL,coloresf=NULL,coloresc1=NULL,coloresc2=NULL,norm=FALSE,contr=FALSE)
{

 # lee los dos cubo de datos con las etiquetas de las filas, las columnas y las repeticiones (si no estan incluidas,
 # se nombran por defecto), comprueba que los dos cubos tengan las mismas filas y repeticiones y suprime las filas
 # que tengan datos faltantes

 l<-read(X,Y)
 X<-l[[1]]
 filas<-l[[2]]
 columnas1<-l[[3]]
 repeticiones<-l[[4]]
 namesf<-l[[5]]
 namesc1<-l[[6]]
 namesr<-l[[7]]
 Y<-l[[8]]
 columnas2<-l[[9]]
 namesc2<-l[[10]]
 conf<-l[[11]]

 # lee los colores de las filas y de las columnas de los dos cubos y comprueba que hay tantos como filas y como
 # columnas (si no se dan, se asignan por defecto en negro)

```

```
if(!conf)
{
 l<-colores(filas,columnas1,coloresf,coloresc1,conf,columnas2,
    coloresc2)
 coloresf<-l[[1]]
 coloresc1<-l[[2]]
 coloresc2<-l[[3]]
 conf<-l[[4]]
}

# centra los cubos de datos por columnas y si se introdujo TRUE en
   normalizacion, normaliza por columnas

if(!conf)
{
 X<-preproc(X,norm,TRUE)
 Y<-preproc(Y,norm,TRUE)

 # crea la matriz con los pesos uniformes para las filas, las
    repeticiones y la raiz cuadrada de esta

 l<-pesos(rep(1,filas))
 Dn<-l[[1]]
 l<-pesos(rep(1,repeticiones))
 Dk<-l[[1]]
 Dk2<-l[[2]]

 # crea el cubo con las covarianzas entre los dos originales y
    calcula la matriz de varianzas-covarianzas vectoriales
```

```
Z<-array(unlist(lapply(1:repeticiones,function(x,A,B,D){return(t(A[,,x]) %*% D %*% B[,,x])},A=Y,B=X,D=Dn)),
    dim=c(columnas2,columnas1,repeticiones))
I<-as.matrix(expand.grid(1:repeticiones,1:repeticiones))
Covv<-matrix(apply(I,1,function(v,X1){return(sum(diag(t(X1[,,v[1]]) %*% X1[,,v[2]])))},X1=Z),nrow=repeticiones,
    ncol=repeticiones)

# extrae los vectores propios segun el metodo explicado y calcula las coordenadas para la interestructura

a<-(eigen(Covv %*% Dk,symmetric=TRUE))$vectors
VI<-solve(Dk2) %*% (a[,1:2]) %*% diag(1/sqrt(diag(t(a[,1:2]) %*% (a[,1:2]))))
VI[,1]<-abs(VI[,1])
I<-Covv %*% Dk %*% VI

# calcula la matriz compromiso

Zc<-apply(Z,1:2,function(v,v2){return(sum(v*v2))},v2=VI[,1]/sum(VI[,1]))

# extrae los vectores singulares por la izquierda y la derecha y los valores singulares segun el metodo explicado

c<-svd(Zc)
d<-c$d
u<-c$u
```

```
v<-c$v
Ur<-u %*% diag(1/sqrt(diag(t(u) %*% u)))
Vr<-v %*% diag(1/sqrt(diag(t(v) %*% v)))

# calcula las coordenadas para las filas y las columnas del
    compromiso y de las trayectorias mediante el diagrama
# de dualidad

Fc<-Zc %*% Vr
Cc<-t(Zc) %*% Ur
FZ<-array(apply(Z,3,function(m,m2){return(m %*% m2)},m2=Vr),dim=c(
    columnas2,columnas2,repeticiones))
CZ<-array(apply(Z,3,function(m,m2){return(t(m) %*% m2)},m2=Ur),dim
    =c(columnas1,columnas2,repeticiones))
PFZ<-matrix(aperm(FZ,c(2,1,3)),nrow=columnas2*repeticiones,ncol=
    columnas2,byrow=TRUE)
PCZ<-matrix(aperm(CZ,c(2,1,3)),nrow=columnas1*repeticiones,ncol=
    columnas2,byrow=TRUE)
FX<-array(apply(X,3,function(m,m2){return(m %*% m2)},m2=Vr),dim=c(
    filas,columnas2,repeticiones))
FY<-array(apply(Y,3,function(m,m2){return(m %*% m2)},m2=Ur),dim=c(
    filas,columnas2,repeticiones))
PFX<-matrix(aperm(FX,c(2,1,3)),nrow=filas*repeticiones,ncol=
    columnas2,byrow=TRUE)
PFY<-matrix(aperm(FY,c(2,1,3)),nrow=filas*repeticiones,ncol=
    columnas2,byrow=TRUE)
CZ2<-rbind(Cc,PCZ)
FZ2<-rbind(Fc,PFZ)

```

```
# si se ha elegido, calcula las contribuciones para las filas y las columnas de la matriz compromiso y de las
# trayectorias de las filas y las columnas

if(contr)
{
 contributions(Fc,namesc2,"las columnas del segundo cubo en el compromiso",TRUE)
 contributions(Cc,namesc1,"las columnas del primer cubo en el compromiso")
 contributions(PFZ,rep(namesc2,repeticiones),"las columnas del segundo cubo en todas las repeticiones")
 contributions(PCZ,rep(namesc1,repeticiones),"las columnas del primer cubo en todas las repeticiones")
 contributions(PFX,rep(namesf,repeticiones),"las filas segun el primer cubo en todas las repeticiones")
 contributions(PFY,rep(namesf,repeticiones),"las filas segun el segundo cubo en todas las repeticiones")
}

# si no se ha elegido representar los ejes, representa los valores singulares en un diagrama de sedimentacion

if((is.null(dimX))||(is.null(dimY)))
{
 windows()
 layout(matrix(c(1,1,2,1),ncol=2),widths=c(3.375,3.625),heights=c(5.5,1.5))
 layout.show(2)
```

```
screeplot(d)

# representa un esquema con la forma de los cubos de datos

plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,4.25),ylim=c(-8.25,2
    .25),bty="n",xaxt="n",yaxt="n")
rect(-1,-1,1,1)
rect(1,-1,3,1)
rect(0,-5,2,-3)
rect(0,-8,2,-6)
segments(-1,1,0,2)
segments(0,2,4,2)
segments(4,2,4,0)
segments(4,0,3,-1)
segments(1,1,2,2)
segments(3,1,4,2)
segments(0,-3,1,-2)
segments(1,-2,3,-2)
segments(3,-2,3,-4)
segments(3,-4,2,-5)
segments(2,-3,3,-2)
text(0,0,"X")
text(2,0,"Y")
text(1,-4,"Z")
text(1,-7,"Zc")
text(-1.25,0,filas)
text(0,1.25,columnas1)
text(2,1.25,columnas2)
text(-0.75,1.75,repeticiones,srt=45)
```

```
    text(-0.25,-4,columnas2)
    text(1,-2.75,columnas1)
    text(0.25,-2.25,repeticiones,srt=45)
    text(-0.25,-7,columnas2)
    text(1,-5.75,columnas1)

    # si se ha elegido representar los ejes, comprueba que los ejes a
        representar en los graficos sean correctos

  } else {
    if(compr(dimX,dimY,d))
    {
     dimension<-c(dimX,dimY)
     Fcd<-Fc[,dimension]
     Ccd<-Cc[,dimension]
     FZd<-FZ[,dimension,]
     CZd<-CZ[,dimension,]
     FXd<-FX[,dimension,]
     FYd<-FY[,dimension,]
     PFXd<-PFX[,dimension]
     PFYd<-PFY[,dimension]
     FZ2d<-FZ2[,dimension]
     CZ2d<-CZ2[,dimension]

     # representa el grafico de la interestructura con las etiquetas
         para las repeticiones y vectores desde el origen

     windows()
```

```
    plot(I[,1],I[,2],type="n",xlim=c(min(0,I[,1]),max(0,I[,1])),ylim=
        c(min(0,I[,2]),max(0,I[,2])),xlab="",
        ylab="")
    text(I[,1],I[,2],namesr,cex=0.8,pos=2)
    box(lwd=2)
    abline(h=0,lwd=2)
    abline(v=0,lwd=2)
    arrows(0,0,I[,1],I[,2],angle=10,length=0.1)

    # representa el grafico para las columnas del primer cubo en la
        izquierda con las etiquetas y vectores desde el
    # origen segun su color
    # representa las columnas del segundo cubo en la derecha con las
        etiquetas y vectores desde el origen segun su color

    plotm(dim=dimension,d=d,M1=Ccd,M2=Fcd,M6=Ccd,lim1=CZ2d,lim2=FZ2d,
        names1=namesc1,names2=namesc2,
        colores1=coloresc1,colores2=coloresc2,contf=contributions2(Cc,
            dimension),
        contc=contributions2(Fc,dimension))

    # representa el grafico para las trayectorias de las columnas del
        primer cubo en la izquierda con las etiquetas y
    # vectores segun su color
    # representa las trayectorias de las columnas del segundo cubo en
        la derecha con las etiquetas y vectores desde el
    # origen segun su color

```

```
    plotm(dim=dimension,d=d,M1=CZd[,,1],M2=FZd[,,1],M4=CZd,M5=FZd,M6=CZd[,,1],lim1=CZ2d,lim2=FZ2d,
        names1=namesc1,names2=namesc2,colores1=coloresc1,colores2=coloresc2,
        contf=contributions2(CZ[,,1],dimension),contc=contributions2(FZ[,,1],dimension))

    # representa el grafico para las trayectorias de las filas segun el primer cubo en la izquierda con las etiquetas
    # segun los colores de los grupos a los que pertenecen
    # representa las trayectorias de las columnas del primer cubo en la derecha con las etiquetas y vectores desde el
    # origen segun su color

    plotm(dim=dimension,d=d,M1=FXd[,,1],M2=CZd[,,1],M4=FXd,M5=CZd,lim1=PFXd,lim2=CZ2d,names1=namesf,
        names2=namesc1,colores1=coloresf,colores2=coloresc1,contf=contributions2(FX[,,1],dimension),
        contc=contributions2(CZ[,,1],dimension))

    # representa el grafico para las trayectorias de las filas segun el segundo cubo en la izquierda con las etiquetas
    # segun los colores de los grupos a los que pertenecen
    # representa las trayectorias de las columnas del segundo cubo en la derecha con las etiquetas y vectores desde el
    # origen segun su color

    plotm(dim=dimension,d=d,M1=FYd[,,1],M2=FZd[,,1],M4=FYd,M5=FZd,lim1=PFYd,lim2=FZ2d,names1=namesf,
```

```
        names2=namesc2 , colores1=coloresf , colores2=coloresc2 , contf=
            contributions2(FY[ , ,1] ,dimension) ,
        contc=contributions2(FZ[ , ,1] ,dimension))
      }
    }
  }
}
```

6.1.8. Programa para realizar un COSTATIS

```
COSTATIS<-function(X,Y,dimX=NULL,dimY=NULL,coloresf=NULL,coloresc1=
    NULL,coloresc2=NULL,norm=FALSE,contr=FALSE)
{

 # lee los dos cubo de datos con las etiquetas de las filas, las
     columnas y las repeticiones (si no estan incluidas,
 # se nombran por defecto), comprueba que los dos cubos tengan las
     mismas filas y repeticiones y suprime las filas
 # que tengan datos faltantes

 l<-read(X,Y)
 X<-l[[1]]
 filas<-l[[2]]
 columnas1<-l[[3]]
 repeticiones<-l[[4]]
 namesf<-l[[5]]
 namesc1<-l[[6]]
 namesr<-l[[7]]
 Y<-l[[8]]
 columnas2<-l[[9]]
 namesc2<-l[[10]]
 conf<-l[[11]]

 # lee los colores de las filas y de las columnas de los dos cubos y
     comprueba que hay tantos como filas y como
 # columnas (si no se dan, se asignan por defecto en negro)

```

```
 if(!conf)
 {
  l<-colores(filas,columnas1,coloresf,coloresc1,conf,columnas2,
     coloresc2)
  coloresf<-l[[1]]
  coloresc1<-l[[2]]
  coloresc2<-l[[3]]
  conf<-l[[4]]
 }

# centra los cubos de datos por columnas y si se introdujo TRUE en
   normalizacion, normaliza por columnas

 if(!conf)
 {
  X<-preproc(X,norm,TRUE)
  Y<-preproc(Y,norm,TRUE)

  # crea la matriz con los pesos uniformes para las filas, las
     repeticiones y la raiz cuadrada de esta

  l<-pesos(rep(1,filas))
  Dn<-l[[1]]
  l<-pesos(rep(1,repeticiones))
  Dk<-l[[1]]
  Dk2<-l[[2]]

  # calcula las matrices de varianzas-covarianzas vectoriales

```

```
l<-as.matrix(expand.grid(1:repeticiones,1:repeticiones))
CovvX<-matrix(apply(l,1,function(v,X1,D){return(sum(diag(t(X1[,,v[1
    ]]) %*% D %*% X1[,,v[2]])))},X1=X,D=Dn),
        nrow=repeticiones,ncol=repeticiones)
CovvY<-matrix(apply(l,1,function(v,X1,D){return(sum(diag(t(X1[,,v[1
    ]]) %*% D %*% X1[,,v[2]])))},X1=Y,D=Dn),
        nrow=repeticiones,ncol=repeticiones)

# extrae los vectores propios segun el metodo explicado y calcula
    las coordenadas para las interestructuras

aX<-(eigen(CovvX %*% Dk,symmetric=TRUE))$vectors
VIX<-solve(Dk2) %*% (aX[,1:2]) %*% diag(1/sqrt(diag(t(aX[,1:2]) %
    *% (aX[,1:2]))))
VIX[,1]<-abs(VIX[,1])
IX<-CovvX %*% Dk %*% VIX
aY<-(eigen(CovvY %*% Dk,symmetric=TRUE))$vectors
VIY<-solve(Dk2) %*% (aY[,1:2]) %*% diag(1/sqrt(diag(t(aY[,1:2]) %
    *% (aY[,1:2]))))
VIY[,1]<-abs(VIY[,1])
IY<-CovvY %*% Dk %*% VIY

# calcula las matrices compromiso

Xc<-apply(X,1:2,function(v,v2){return(sum(v*v2))},v2=VIX[,1]/sum(
    VIX[,1]))
Yc<-apply(Y,1:2,function(v,v2){return(sum(v*v2))},v2=VIY[,1]/sum(
    VIY[,1]))

```

```
# si se introdujo TRUE en normalizacion, centra y normaliza los compromisos por columnas

Xc<-preproc(Xc,norm)
Yc<-preproc(Yc,norm)

# extrae los vectores singulares por la izquierda y la derecha y los valores singulares segun el metodo explicado

c<-svd(t(Yc) %*% Dn %*% Xc)
d<-c$d
u<-c$u
v<-c$v
Ur<-u %*% diag(1/sqrt(diag(t(u) %*% u)))
Vr<-v %*% diag(1/sqrt(diag(t(v) %*% v)))

# calcula las coordenadas para las filas y las columnas de los compromisos y de las trayectorias mediante el
# diagrama de dualidad

FXc<-Xc %*% Vr
CXc<-t(Xc) %*% Dn %*% Yc %*% Ur
FYc<-Yc %*% Ur
CYc<-t(Yc) %*% Dn %*% Xc %*% Vr
FXt<-array(apply(X,3,function(m,m2){return(m %*% m2)},m2=Vr),dim=c(filas,columnas2,repeticiones))
CXt<-array(apply(X,3,function(m,m2){return(t(m) %*% m2)},m2=Dn %*% Yc %*% Ur),dim=c(columnas1,columnas2,repeticiones))
```

```
FYt<-array(apply(Y,3,function(m,m2){return(m %*% m2)},m2=Ur),dim=c
    (filas,columnas2,repeticiones))
CYt<-array(apply(Y,3,function(m,m2){return(t(m) %*% m2)},m2=Dn %
    *% Xc %*% Vr),dim=c(columnas2,columnas2,repeticiones))
FX<-matrix(aperm(FXt,c(2,1,3)),nrow=filas*repeticiones,ncol=
    columnas2,byrow=TRUE)
CX<-matrix(aperm(CXt,c(2,1,3)),nrow=columnas1*repeticiones,ncol=
    columnas2,byrow=TRUE)
FY<-matrix(aperm(FYt,c(2,1,3)),nrow=filas*repeticiones,ncol=
    columnas2,byrow=TRUE)
CY<-matrix(aperm(CYt,c(2,1,3)),nrow=columnas2*repeticiones,ncol=
    columnas2,byrow=TRUE)
CX2<-rbind(CXc,CX)
CY2<-rbind(CYc,CY)

# si se ha elegido, calcula las contribuciones para las filas y las
    columnas de las matrices compromiso y de las
# trayectorias de las filas y las columnas

if(contr)
{
  contributions(FXc,namesf,"las filas segun el primer cubo en el
    compromiso",TRUE)
  contributions(CXc,namesc1,"las columnas del primer cubo en el
    compromiso")
  contributions(FYc,namesf,"las filas segun el segundo cubo en el
    compromiso")
  contributions(CYc,namesc2,"las columnas del segundo cubo en el
    compromiso")
```

```
    contributions(FX,rep(namesf,repeticiones),"las filas segun el primer cubo en todas las repeticiones")
    contributions(CX,rep(namesc1,repeticiones),"las columnas del primer cubo en todas las repeticiones")
    contributions(FY,rep(namesf,repeticiones),"las filas segun el segundo cubo en todas las repeticiones")
    contributions(CY,rep(namesc2,repeticiones),"las columnas del segundo cubo en todas las repeticiones")
  }

  # si no se ha elegido representar los ejes, representa los valores singulares en un diagrama de sedimentacion

  if((is.null(dimX))||(is.null(dimY)))
  {
    windows()
    layout(matrix(c(1,1,2,1),ncol=2),widths=c(310.5/99,382.5/99),heights=c(5.5,1.5))
    layout.show(2)
    screeplot(d)

    # representa un esquema con la forma de los cubos de datos

    plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,4.25),ylim=c(-7.25,2.25),bty="n",xaxt="n",yaxt="n")
    rect(-1,-1,1,1)
    rect(1,-1,3,1)
    rect(-1,-4,1,-2)
    rect(1,-4,3,-2)
```

```
    rect(0,-7,2,-5)
    segments(-1,1,0,2)
    segments(0,2,4,2)
    segments(4,2,4,0)
    segments(4,0,3,-1)
    segments(1,1,2,2)
    segments(3,1,4,2)
    text(0,0,"X")
    text(2,0,"Y")
    text(0,-3,"Xc")
    text(2,-3,"Yc")
    text(1,-6,"Yc' Dn Xc")
    text(-1.25,0,filas)
    text(0,1.25,columnas1)
    text(2,1.25,columnas2)
    text(-0.75,1.75,repeticiones,srt=45)
    text(-1.25,-3,filas)
    text(0,-1.75,columnas1)
    text(2,-1.75,columnas2)
    text(-0.25,-6,columnas2)
    text(1,-4.75,columnas1)

    # si se ha elegido representar los ejes, comprueba que los ejes a
        representar en los graficos sean correctos

  } else {
    if(compr(dimX,dimY,d))
    {
     dimension<-c(dimX,dimY)
```

```
FXcd<-FXc[,dimension]
CXcd<-CXc[,dimension]
FYcd<-FYc[,dimension]
CYcd<-CYc[,dimension]
FXtd<-FXt[,dimension,]
CXtd<-CXt[,dimension,]
FYtd<-FYt[,dimension,]
CYtd<-CYt[,dimension,]
FXd<-FX[,dimension]
FYd<-FY[,dimension]
CX2d<-CX2[,dimension]
CY2d<-CY2[,dimension]
Fc<-rbind(FXcd,FYcd)

# crea el entorno del grafico de las interestructuras, y se divide en dos

windows(width=14,height=7)
layout(matrix(1:2,nrow=1))
layout.show(2)

# representa el grafico para las repeticiones segun el primer cubo en la izquierda con las etiquetas y vectores
# desde el origen

plot(IX[,1],IX[,2],type="n",xlim=c(min(0,IX[,1]),max(0,IX[,1])),ylim=c(min(0,IX[,2]),max(0,IX[,2])),
    xlab="",ylab="")
text(IX[,1],IX[,2],namesr,cex=0.8,pos=2)
```

```
    box(lwd=2)
    abline(h=0,lwd=2)
    abline(v=0,lwd=2)
    arrows(0,0,IX[,1],IX[,2],angle=10,length=0.1)

    # representa las repeticiones segun el segundo cubo en la derecha con las etiquetas y vectores desde el origen

    plot(IY[,1],IY[,2],type="n",xlim=c(min(0,IY[,1]),max(0,IY[,1])), ylim=c(min(0,IY[,2]),max(0,IY[,2])),
        xlab="",ylab="")
    text(IY[,1],IY[,2],namesr,cex=0.8,pos=2)
    box(lwd=2)
    abline(h=0,lwd=2)
    abline(v=0,lwd=2)
    arrows(0,0,IY[,1],IY[,2],angle=10,length=0.1)

    # representa el grafico para la coinercia de las filas de los compromisos en la izquierda con las etiquetas y
    # vectores de la segunda a la primera matriz segun los colores de los grupos
    # representa las columnas de la primera matriz compromiso en la derecha con las etiquetas y vectores desde el
    # origen segun su color

    plotm(dim=dimension,d=d,M1=FXcd,M2=CXcd,M3=FYcd,lim1=Fc,lim2=CX2d,names1=namesf,names2=namesc1,
        colores1=coloresf,colores2=coloresc1,contf=contributions2(FXc,dimension),
```

```
        contc=contributions2(CXc,dimension))

    # representa el grafico para la coinercia de las filas de los compromisos en la izquierda con las etiquetas y
    # vectores de la primera a la segunda matriz segun los colores de los grupos
    # representa las columnas de la segunda matriz compromiso en la derecha con las etiquetas y vectores desde el
    # origen segun su color

    plotm(dim=dimension,d=d,M1=FYcd,M2=CYcd,M3=FXcd,lim1=Fc,lim2=CY2d,names1=namesf,names2=namesc2,
        colores1=coloresf,colores2=coloresc2,contf=contributions2(FYc,dimension),
        contc=contributions2(CYc,dimension))

    # representa el grafico para las trayectorias de las filas del primer cubo en la izquierda con las etiquetas y
    # segmentos segun los colores de los grupos a los que pertenecen
    # representa las trayectorias de las columnas del primer cubo en la derecha con las etiquetas, segmentos y
    # vectores desde el origen segun su color

    plotm(dim=dimension,d=d,M1=FXtd[,,1],M2=CXtd[,,1],M4=FXtd,M5=CXtd,lim1=FXd,lim2=CX2d,names1=namesf,
        names2=namesc1,colores1=coloresf,colores2=coloresc1,contf=contributions2(FXt[,,1],dimension),
        contc=contributions2(CXt[,,1],dimension))

```

```
    # representa el grafico para las trayectorias de las filas del
        segundo cubo en la izquierda con las etiquetas y
    # segmentos segun los colores de los grupos a los que pertenecen
    # representa las trayectorias de las columnas del segundo cubo en
        la derecha con las etiquetas, segmentos y
    # vectores desde el origen segun su color

    plotm(dim=dimension,d=d,M1=FYtd[,,1],M2=CYtd[,,1],M4=FYtd,M5=CYtd
        ,lim1=FYd,lim2=CY2d,names1=namesf,
        names2=namesc2,colores1=coloresf,colores2=coloresc2,contf=
            contributions2(FYt[,,1],dimension),
        contc=contributions2(CYt[,,1],dimension))
   }
  }
 }
}
```

6.1.9. Programa para realizar un Tucker3

```
TUCKER3<-function(X,p=NULL,q=NULL,r=NULL,P1=NULL,P2=NULL,Q1=NULL,Q2=NULL,R1=NULL,R2=NULL,coloresf=NULL,
        coloresc=NULL,maximo=5,iter=100,tol=10^-8,norm=FALSE,contr=FALSE)
{

 # lee el cubo de datos con las etiquetas de las filas, las columnas y las repeticiones (si no estan incluidas,
 # se nombran por defecto) y suprime las filas que tengan datos faltantes

 l<-read(X)
 X<-l[[1]]
 filas<-l[[2]]
 columnas<-l[[3]]
 repeticiones<-l[[4]]
 namesf<-l[[5]]
 namesc<-l[[6]]
 namesr<-l[[7]]
 conf<-l[[11]]

 # lee las etiquetas de los colores para las filas y columnas y comprueba que hay tantas como filas y columnas
 # (si no se dan, se asignan por defecto de color negro)

 if(!conf)
 {
```

```
  l<-colores(filas,columnas,coloresf,coloresc,conf)
  coloresf<-l[[1]]
  coloresc<-l[[2]]
  conf<-l[[3]]
 }

 # centra el cubo de datos por columnas y si se introdujo TRUE en
    normalizacion, normaliza por capas laterales

 if(!conf)
 {
  X<-preproc(X,norm,TRUE,TRUE)

  # comprueba los valores de maximo y de iteraciones

  if((maximo<=1)||(maximo!=(floor(maximo))))
  {
   conf<-TRUE
   print("te has confundido, numero maximo de componentes incorrecto"
      )
  }
  if(!conf){if((iter<1)||(iter!=(floor(iter))))
  {
   conf<-TRUE
   print("te has confundido, numero maximo de iteraciones incorrecto"
      )
  }
  }
  if(!conf)
```

```
{
 c1<-min(maximo,filas)
 c2<-min(maximo,columnas)
 c3<-min(maximo,repeticiones)

 # si no se ha elegido una combinacion de componentes, realiza el analisis para todas las combinaciones posibles

 if((is.null(p))||(is.null(q))||(is.null(r)))
 {

  # crea una tabla con tantas combinaciones de componentes como sean posibles

  l<-as.matrix(expand.grid(1:c3,1:c2,1:c1))
  l<-l[,3:1]
  names<-apply(l,1,function(v){return(paste(v[1],"x",v[2],"x",v[3],sep=""))})

  # para cada combinacion que cumpla la regla del maximo producto realiza la funcion auxiliar y construye la tabla
  # con los resultados

  a<-function(cont,l,T,iter,tol,c1,c2,c3)
  {
   if(((l[cont,1])<=((l[cont,2])*(l[cont,3])))&&((l[cont,2])<=((l[cont,1])*(l[cont,3])))&&
     ((l[cont,3])<=((l[cont,1])*(l[cont,2]))))
   {
```

```
      return(c(cont,every(l[cont,1],l[cont,2],l[cont,3],T,iter,tol)))
    } else {return(rep(0,5))}
  }
  table1<-matrix(unlist(lapply(1:(dim(l)[1]),a,l=l,T=X,iter=iter,tol=tol,c1=c1,c2=c2,c3=c3)),ncol=5,
         byrow=TRUE)

  # elimina de la tabla las combinaciones que no cumplen la regla del maximo producto

  cual<-apply(table1,1,function(v){return(any(v!=0))})
  table1<-table1[cual,]
  names<-names[cual]
  filas1<-dim(table1)[1]
  ini<-rbind(c("Number","Model_Size","Sum","Best_given_Sum","SS(Res)","Prop._SS(Fit)","Number_of_iterations"),
        cbind(table1[,1],names,table1[,2],rep("",filas1),table1[,3:5]))

  # crea otra tabla con las combinaciones con un mejor ajuste para cada valor de la suma de componentes

  cual<-unlist(lapply(3:(c1+c2+c3),function(i,m){if(any(m[,2]==i)){min(which(m[,4]==max(m[m[,2]==i,4])))}},
          m=table1))
  table2<-table1[cual,]
  names2<-names[cual]
  ini[cual+1,4]<-"*"
  filas2<-dim(table2)[1]
```

```
# guarda las tablas como resultados

results(ini,"todas las combinaciones",first=TRUE)
ini2<-rbind(c("Number","Model_Size","S","SS(Res)","DiffFit","Prop._SS(Fit)","Number_of_iterations"),
      cbind(table2[,1],names2,table2[,2:3],table2[,4]-c(0,table2[-filas2,4]),table2[,4:5]))
results(ini2,"combinaciones con mejor ajuste")

# crea una tabla con las combinaciones que pertenecen a la envolvente convexa de entre todas

table3<-table2[table2[,3]<=(table2[,2]-3)*(table2[filas2,3]-table2[1,3])/(c1+c2+c3-3)+table2[1,3],]
cual<-chull(table3[,2:3])
table3<-table3[cual[order(cual)],]
filas3<-dim(table3)[1]

# elimina todas las combinaciones mas estables menos la mas simple de entre estas

cual<-ifelse(any((table3[,3]-c(table3[-1,3],0))<table3[,3]/100),
        min(which((table3[,3]-c(table3[-1,3],0))<table3[,3]/100)),filas3)
table4<-table3[1:cual,]
filas4<-dim(table4)[1]
if(filas4>=3)
{
```

```
    st<-c(0,((table4[-c(filas4-1,filas4),3]-table4[-c(1,filas4),3])/
           (table4[-c(1,filas4),2]-table4[-c(filas4-1,filas4),2]))/
         ((table4[-c(1,filas4),3]-table4[-c(1,2),3])/(table4[-c(1,2),2]-table4[-c(1,filas4),2])),0)
  } else {st<-0}
  if(any(st!=0)){vertical<-table4[min(which(st==min(st[st!=0])))-1,2]+0.5}

  # representa el grafico scree-plot con todas las combinaciones originales, con vectores para las que pertenecen a
  # la envolvente convexa

  windows(width=14,height=7)
  layout(matrix(c(1,1,2,1),ncol=2),widths=c(9.9375,4.0625),heights=c(3.5,3.5))
  layout.show(2)
  par(mar=c(5,8,4,2)+0.1)
  plot(table1[,2],table1[,3],type="n",xlim=c(min(table1[,2])-1,max(table1[,2])+1),
    ylim=c(min(table1[,3])-1,max(table1[,3])+1),xlab="Suma del numero de componentes",
    ylab="Suma de cuadrados residual")
  text(table1[,2],table1[,3],names,cex=0.8,pos=4)
  box(lwd=2)
  abline(h=0,lwd=2)
  abline(v=0,lwd=2)
  points(table1[,2],table1[,3],pch=3,col=rgb(255,0,0,maxColorValue=255),cex=0.8,lwd=2)
  lapply(1:(filas3-1),function(i,M)
```

```
    {
     arrows(M[i,2],M[i,3],M[i+1,2],M[i+1,3],col=rgb(0,0,255,
        maxColorValue = 255),angle=10,length=0.1,lwd=1.5)
    },M=table3)

    # representa una recta vertical separando las combinaciones
       estables de las no estables, asi la primera a su
    # derecha podria ser la elegida para el resto del analisis

    if(any(st!=0)){abline(v=vertical,col=rgb(255,0,127,maxColorValue=
       255),lwd=2)}

    # representa un esquema con la forma del cubo de datos

    plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,2.25),ylim=c(-1.25,
       2.25),bty="n",xaxt="n",yaxt="n")
    rect(-1,-1,1,1)
    segments(-1,1,0,2)
    segments(0,2,2,2)
    segments(2,2,2,0)
    segments(2,0,1,-1)
    segments(1,1,2,2)
    text(0,0,"X")
    text(-1.25,0,filas)
    text(0,1.25,columnas)
    text(-0.75,1.75,repeticiones,srt=45)

    # si se ha elegido una combinacion de componentes, comprueba que
       sean correctos
```

```

  } else {
   if((p<1)||(p!=(floor(p)))||(p>c1))
   {
    conf<-TRUE
    print("te has confundido, numero de componentes incorrecto")
   }
   if(!conf)
   {
    if((q<1)||(q!=(floor(q)))||(q>c2))
    {
     conf<-TRUE
     print("te has confundido, numero de componentes incorrecto")
    }
   }
   if(!conf)
   {
    if((r<1)||(r!=(floor(r)))||(r>c3))
    {
     conf<-TRUE
     print("te has confundido, numero de componentes incorrecto")
    }
   }
   if(!conf)
   {
    comp<-c(p,q,r)

    # realiza la funcion auxiliar para la combinacion de componentes
        establecida
```

```
l<-every(p,q,r,X,iter,tol,mas=TRUE)
A<-l[[1]]
B<-l[[2]]
C<-l[[3]]
G<-l[[4]]
Aprox<-tensorial(tensorial(tensorial(G,1,t(A)),2,t(B)),3,t(C))

# crea una tabla con los porcentajes de ajuste para cada
   componente de cada dimension, y el ajuste total para
# cada dimension, y la guarda como resultado

sos<-100*(matrix(unlist(lapply(1:3,function(i,T,comp)
{
 return(c(apply(T^2,i,sum),rep(0,max(comp)-comp[i])))
},T=G,comp=comp)),ncol=3))/(sum(X^2))
sos<-rbind(sos,apply(sos,2,sum))
ini3<-rbind(c("Component","Dimension 1","Dimension 2","Dimension
   3"),
      cbind(c(1:(max(comp)),"Total explained variation"),sos))
results(ini3,"porcentajes de ajuste",first=TRUE)

# crea otra tabla con el tensor core y el porcentaje de ajuste
   para cada combinacion de componentes de las tres
# dimensiones, y la guarda como resultado

core<-cbind(matrix(aperm(G,c(2,1,3)),nrow=p*r,ncol=q,byrow=TRUE)
   ,
```

```
          matrix(aperm(100*(G^2)/(sum(X^2)),c(2,1,3)),nrow=p*r,ncol=q,byrow=TRUE))
    ini4<-rbind(c(rep("",3),rep(c("Mode 2 components",rep("",q-1)),2)),
          c(rep("",3),"Residual Sums of Squares",rep("",q-1),"Explained Variation",rep("",q-1)),
          c(rep("",3),rep(1:q,2)),cbind(unlist(lapply(1:r,function(i,p)
          {
           return(c(paste("Mode 3, Component ",i,sep=""),rep("",p-1)))
          },p=p)),rep(c("Mode 1 components",rep("",p-1)),r),rep(1:p,r),core))
    results(ini4,"core array")

    # calcula el ajuste de cada fila, columna y repeticion

    fit1<-apply(Aprox^2,1,sum)
    fit2<-apply(Aprox^2,2,sum)
    fit3<-apply(Aprox^2,3,sum)
    fit1<-cbind(fit1,apply(X^2,1,sum)-fit1)
    fit2<-cbind(fit2,apply(X^2,2,sum)-fit2)
    fit3<-cbind(fit3,apply(X^2,3,sum)-fit3)

    # guarda como resultado las coordenadas de cada fila, columna y repeticion para los graficos

    results(A,"coordenadas de las filas",names=namesf,axis=TRUE)
    results(B,"coordenadas de las columnas",names=namesc,axis=TRUE)
```

```
results(C,"coordenadas de las repeticiones",names=namesr,axis=
    TRUE)

# si se ha elegido, calcula las contribuciones para las filas,
    las columnas y las repeticiones del cubo de datos

if(contr)
{
 contributions(A,namesf,"las filas")
 contributions(B,namesc,"las columnas")
 contributions(C,namesr,"las repeticiones")
}

# representa el grafico con el ajuste para cada fila, columna y
    repeticion con las etiquetas segun los colores de
# los grupos a los que pertenecen
# ademas pinta una recta separando las filas, columnas y
    repeticiones con un ajuste superior o inferior a la media

windows(width=14,height=7)
layout(matrix(1:3,nrow=1))
layout.show(3)
par(mar=c(5,6,4,2)+0.1)
plot(fit1[,1],fit1[,2],type="n",xlim=c(min(fit1[,1]),max(fit1[,1
    ])),ylim=c(min(fit1[,2]),max(fit1[,2]))
    ,xlab="",ylab="")
text(fit1[,1],fit1[,2],namesf,col=coloresf)
box(lwd=2)
abline(h=0,lwd=2)
```

```
        abline(v=0,lwd=2)
        abline(0,sum(fit1[,2])/sum(fit1[,1]),col=rgb(255,0,0,
            maxColorValue = 255),lwd=2)
        plot(fit2[,1],fit2[,2],type="n",xlim=c(min(fit2[,1]),max(fit2[,1
            ])),ylim=c(min(fit2[,2]),max(fit2[,2])),
            xlab="",ylab="")
        text(fit2[,1],fit2[,2],namesc,col=coloresc)
        box(lwd=2)
        abline(h=0,lwd=2)
        abline(v=0,lwd=2)
        abline(0,sum(fit2[,2])/sum(fit2[,1]),col=rgb(255,0,0,
            maxColorValue = 255),lwd=2)
        plot(fit3[,1],fit3[,2],type="n",xlim=c(min(fit3[,1]),max(fit3[,1
            ])),ylim=c(min(fit3[,2]),max(fit3[,2])),
            xlab="",ylab="")
        text(fit3[,1],fit3[,2],namesr)
        box(lwd=2)
        abline(h=0,lwd=2)
        abline(v=0,lwd=2)
        abline(0,sum(fit3[,2])/sum(fit3[,1]),col=rgb(255,0,0,
            maxColorValue = 255),lwd=2)

        # si se ha elegido representar los ejes, comprueba que los ejes
            a representar en los graficos sean correctos

        if((!is.null(P1))&&(!is.null(P2))&&(!is.null(Q1))&&(!is.null(Q2)
            )&&(!is.null(R1))&&(!is.null(R2)))
        {
```

```
        if ((compr(P1,P2,c1,p,TRUE))&&(compr(Q1,Q2,c2,q,TRUE))&&(compr(R
            1,R2,c3,r,TRUE)))
        {
         dimensionP<-c(P1,P2)
         dimensionQ<-c(Q1,Q2)
         dimensionR<-c(R1,R2)
         Ad<-A[,dimensionP]
         Bd<-B[,dimensionQ]
         Cd<-C[,dimensionR]

         # representa las filas en la izquierda con las etiquetas segun
              los colores de los grupos a los que pertenecen
         # representa las columnas en el centro con las etiquetas y
            vectores desde el origen segun los colores de los
         # grupos a los que pertenecen
         # representa las repeticiones en la derecha con las etiquetas
            y vectores desde el origen

         plotmt(dim1=dimensionP,dim2=dimensionQ,dim3=dimensionR,sos=sos
             ,M1=Ad,M2=Bd,M3=Cd,lim1=Ad,lim2=Bd,
              lim3=Cd,names1=namesf,names2=namesc,names3=namesr,colores1
                 =coloresf,colores2=coloresc)

         contf<-contributions2(A,dimensionP)
         contc<-contributions2(B,dimensionQ)
         contre<-contributions2(C,dimensionR)
         contf[is.nan(contf)]<-0
         contc[is.nan(contc)]<-0
         contre[is.nan(contre)]<-0
```

```
            mcontf<-max(contf)/2
            mcontc<-max(contc)/2
            mcontre<-max(contre)/2
            lcoloresf<-coloresf[contf>=mcontf]
            lcoloresc<-coloresc[contc>=mcontc]
            lnamesf<-namesf[contf>=mcontf]
            lnamesc<-namesc[contc>=mcontc]
            lnamesr<-namesr[contre>=mcontre]
            lA<-matrix(Ad[contf>=mcontf,],ncol=2)
            lB<-matrix(Bd[contc>=mcontc,],ncol=2)
            lC<-matrix(Cd[contre>=mcontre,],ncol=2)
            plotmt(dim1=dimensionP,dim2=dimensionQ,dim3=dimensionR,sos=sos
                 ,M1=lA,M2=lB,M3=lC,lim1=Ad,lim2=Bd,
                  lim3=Cd,names1=lnamesf,names2=lnamesc,names3=lnamesr,
                       colores1=lcoloresf,colores2=lcoloresc,
                  titles=TRUE)
          }
        }
      }
    }
  }
 }
}
```

6.2. Programa para realizar un Co-Tucker3

El siguiente programa de este trabajo es aquel para el Co-Tucker3, que es el método nuevo propuesto para analizar simultáneamente una sucesión de pares de tablas.

```
COTUCKER3<-function(X,Y,p=NULL,q=NULL,r=NULL,P1=NULL,P2=NULL,Q1=NULL,
    Q2=NULL,R1=NULL,R2=NULL,
             dimAX=NULL,dimAY=NULL,dimBX=NULL,dimBY=NULL,dimCX=NULL,
                 dimCY=NULL,
             coloresf=NULL,coloresc1=NULL,coloresc2=NULL,maximo=5,iter=1
                 00,tol=10^-8,norm=FALSE,contr=FALSE)
{

 # lee los cubos de datos con las etiquetas de las filas, las
     columnas y las repeticiones
 # (si no estan incluidas, se nombran por defecto) y suprime las
     filas que tengan datos faltantes

 l<-read(X,Y)
 X<-l[[1]]
 filas<-l[[2]]
 columnas1<-l[[3]]
 repeticiones<-l[[4]]
 namesf<-l[[5]]
 namesc1<-l[[6]]
 namesr<-l[[7]]
 Y<-l[[8]]
 columnas2<-l[[9]]
 namesc2<-l[[10]]
 conf<-l[[11]]
```

```

# lee las etiquetas de los colores para las filas y columnas y
    comprueba que hay tantas como filas y columnas
# (si no se dan, se asignan por defecto de color negro)

 if(!conf)
{
  l<-colores(filas,columnas1,coloresf,coloresc1,conf,columnas2,
     coloresc2)
  coloresf<-l[[1]]
  coloresc1<-l[[2]]
  coloresc2<-l[[3]]
  conf<-l[[4]]
}

# centra los cubos de datos por columnas y si se introdujo TRUE en
    normalizacion, normaliza por capas laterales

 if(!conf)
{
  X<-preproc(X,norm,TRUE,TRUE)
  Y<-preproc(Y,norm,TRUE,TRUE)

  # comprueba los valores de maximo y de iteraciones

  if((maximo<=1)||(maximo!=(floor(maximo))))
  {
    conf<-TRUE
```

```
  print("te has confundido, numero maximo de componentes incorrecto"
    )
 }
 if(!conf)
 {
  if((iter<1)||(iter!=(floor(iter))))
  {
   conf<-TRUE
   print("te has confundido, numero maximo de iteraciones incorrecto
     ")
  }
 }
 if(!conf)
 {
  c1<-min(maximo,filas)
  c2<-min(maximo,columnas1,columnas2)
  c3<-min(maximo,repeticiones)

  # si no se ha elegido una combinacion de componentes, realiza el
    analisis para todas las combinaciones posibles

  if((is.null(p))||(is.null(q))||(is.null(r)))
  {

   # crea una tabla con tantas combinaciones de componentes como
     sean posibles

   l<-as.matrix(expand.grid(1:c3,1:c2,1:c1))
   l<-l[,3:1]
```

```
names<-apply(l,1,function(v){return(paste(v[1],"x",v[2],"x",v[3],sep=""))})

# para cada combinacion que cumpla la regla del maximo producto realiza la funcion auxiliar y construye la tabla
# con los resultados

a<-function(cont,l,T,iter,tol,c1,c2,c3)
{
  if(((l[cont,1])<=((l[cont,2])*(l[cont,3])))&&((l[cont,2])<=((l[cont,1])*(l[cont,3])))&&
    ((l[cont,3])<=((l[cont,1])*(l[cont,2]))))
  {
    return(c(cont,every(l[cont,1],l[cont,2],l[cont,3],T,iter,tol)))
  } else {return(rep(0,5))}
}
table1X<-matrix(unlist(lapply(1:(dim(l)[1]),a,l=l,T=X,iter=iter,tol=tol,c1=c1,c2=c2,c3=c3)),ncol=5,byrow=TRUE)
table1Y<-matrix(unlist(lapply(1:(dim(l)[1]),a,l=l,T=Y,iter=iter,tol=tol,c1=c1,c2=c2,c3=c3)),ncol=5,byrow=TRUE)

# elimina de la tabla las combinaciones que no cumplen la regla del maximo producto

cual<-apply(table1X,1,function(v){return(any(v!=0))})
table1X<-table1X[cual,]
table1Y<-table1Y[cual,]
names<-names[cual]
filas1<-dim(table1X)[1]
```

```
    table1<-cbind(table1X[,1],table1X[,2],table1X[,3]+table1Y[,3],
            100-100*(table1X[,3]+table1Y[,3])/(100*(table1X[,3]/(100-
                table1X[,4])+table1Y[,3]/(100-table1Y[,4]))),
            apply(cbind(table1X[,5],table1Y[,5]),1,max))
    ini<-rbind(c("Number","Model_Size","Sum","Best_given_Sum","SS(Res
        )","Prop._SS(Fit)","Number_of_iterations"),
          cbind(table1[,1],names,table1[,2],rep("",filas1),table1[,3:
            5]))

    # crea otra tabla con las combinaciones con un mejor ajuste para
        cada valor de la suma de componentes

    cual<-unlist(lapply(3:(c1+c2+c3),function(i,m){if(any(m[,2]==i)){
        min(which(m[,4]==max(m[m[,2]==i,4])))}},m=table1))
    table2<-table1[cual,]
    names2<-names[cual]
    ini[cual+1,4]<-"*"
    filas2<-dim(table2)[1]

    # guarda las tablas como resultados

    results(ini,"todas las combinaciones",first=TRUE)
    ini2<-rbind(c("Number","Model_Size","S","SS(Res)","DifFit","Prop
        ._SS(Fit)","Number_of_iterations"),
          cbind(table2[,1],names2,table2[,2:3],table2[,4]-c(0,table2
            [-filas2,4]),table2[,4:5]))
    results(ini2,"combinaciones con mejor ajuste")

```

```
# crea una tabla con las combinaciones que pertenecen a la envolvente convexa de entre todas

cual<-table2[,3]<=((table2[,2]-3)*(table2[filas2,3]-table2[1,3])/(c1+c2+c3-3)+table2[1,3])
if((table2[filas2,2])==(c1+c2+c3)){cual[filas2]<-TRUE}
table3<-table2[cual,]
cual<-chull(table3[,2:3])
table3<-table3[cual[order(cual)],]
filas3<-dim(table3)[1]

# elimina todas las combinaciones mas estables menos la mas simple de entre estas

cual<-ifelse(any((table3[,3]-c(table3[-1,3],0))<table3[,3]/100),
        min(which((table3[,3]-c(table3[-1,3],0))<table3[,3]/100)),
            filas3)
table4<-table3[1:cual,]
filas4<-dim(table4)[1]
if(filas4>=3)
{
 st<-c(0,((table4[-c(filas4-1,filas4),3]-table4[-c(1,filas4),3])/
        (table4[-c(1,filas4),2]-table4[-c(filas4-1,filas4),2]))/
      ((table4[-c(1,filas4),3]-table4[-c(1,2),3])/(table4[-c(1,2),2]-table4[-c(1,filas4),2])),0)
} else {st<-0}
if(any(st!=0)){vertical<-table4[min(which(st==min(st[st!=0])))-1,2]+0.5}

```

```
# representa el grafico scree-plot con todas las combinaciones
    originales, con vectores para las que pertenecen
# a la envolvente convexa

windows(width=14,height=7)
layout(matrix(c(1,1,2,1),ncol=2),widths=c(8.85,5.15),heights=c(3
    .5,3.5))
layout.show(2)
par(mar=c(5,8,4,2)+0.1)
plot(table1[,2],table1[,3],type="n",xlim=c(min(table1[,2])-1,max(
    table1[,2])+1),
    ylim=c(min(table1[,3])-1,max(table1[,3])+1),xlab="Suma del
       numero de componentes",
    ylab="Suma de cuadrados residual")
text(table1[,2],table1[,3],names,cex=0.8,pos=4)
box(lwd=2)
abline(h=0,lwd=2)
abline(v=0,lwd=2)
points(table1[,2],table1[,3],pch=3,col=rgb(255,0,0,maxColorValue=
    255),cex=0.8,lwd=2)
lapply(1:(filas3-1),function(i,M){arrows(M[i,2],M[i,3],M[i+1,2],M
    [i+1,3],col=rgb(0,0,255,maxColorValue = 255),
                         angle=10,length=0.1,lwd=1.5)},M=table3)

# representa una recta vertical separando las combinaciones
    estables de las no estables, asi la primera a su derecha
# podria ser la elegida para el resto del analisis

```

```
    if(any(st!=0)){abline(v=vertical,col=rgb(255,0,127,maxColorValue=
        255),lwd=2)}

    # representa un esquema con la forma de los cubos de datos

    plot(0,0,type="n",xlab="",ylab="",xlim=c(-1.5,4.25),ylim=c(-1.25,
        2.25),bty="n",xaxt="n",yaxt="n")
    rect(-1,-1,1,1)
    rect(1,-1,3,1)
    segments(-1,1,0,2)
    segments(0,2,4,2)
    segments(4,2,4,0)
    segments(4,0,3,-1)
    segments(1,1,2,2)
    segments(3,1,4,2)
    text(0,0,"X")
    text(2,0,"Y")
    text(-1.25,0,filas)
    text(0,1.25,columnas1)
    text(2,1.25,columnas2)
    text(-0.75,1.75,repeticiones,srt=45)

    # si se ha elegido una combinacion de componentes, comprueba que
        sean correctos

  } else {
    if((p<1)||(p!=(floor(p)))||(p>c1))
    {
     conf<-TRUE
```

```
    print("te has confundido, numero de componentes incorrecto")
   }
   if(!conf)
   {
    if((q<1)||(q!=(floor(q)))||(q>c2))
    {
     conf<-TRUE
     print("te has confundido, numero de componentes incorrecto")
    }
   }
   if(!conf)
   {
    if((r<1)||(r!=(floor(r)))||(r>c3))
    {
     conf<-TRUE
     print("te has confundido, numero de componentes incorrecto")
    }
   }
   if(!conf)
   {
    comp<-c(p,q,r)

    # realiza la funcion auxiliar para la combinacion de componentes
        establecida

    l<-every(p,q,r,X,iter,tol,mas=TRUE)
    AX<-l[[1]]
    BX<-l[[2]]
    CX<-l[[3]]
```

```
GX<-l[[4]]
AproxX<-tensorial(tensorial(tensorial(GX,1,t(AX)),2,t(BX)),3,t(CX))
l<-every(p,q,r,Y,iter,tol,mas=TRUE)
AY<-l[[1]]
BY<-l[[2]]
CY<-l[[3]]
GY<-l[[4]]
AproxY<-tensorial(tensorial(tensorial(GY,1,t(AY)),2,t(BY)),3,t(CY))

# crea una tabla con los porcentajes de ajuste para cada componente de cada dimension,
# y el ajuste total para cada dimension, una tabla para cada cubo

sosX<-100*(matrix(unlist(lapply(1:3,function(i,T,comp)
{
 return(c(apply(T^2,i,sum),rep(0,max(comp)-comp[i])))
},T=GX,comp=comp)),ncol=3))/(sum(X^2))
sosY<-100*(matrix(unlist(lapply(1:3,function(i,T,comp)
{
 return(c(apply(T^2,i,sum),rep(0,max(comp)-comp[i])))
},T=GY,comp=comp)),ncol=3))/(sum(Y^2))
sosX<-rbind(sosX,apply(sosX,2,sum))
sosY<-rbind(sosY,apply(sosY,2,sum))
ini3X<-rbind(c("Component","Dimension_1","Dimension_2","Dimension_3"),
      cbind(c(1:(max(comp)),"Total_explained_variation"),sosX))
```

```
        ini3Y<-rbind(c("Component","Dimension 1","Dimension 2","Dimension 3"),
              cbind(c(1:(max(comp)),"Total explained variation"),sosY))

        # crea otra tabla con el tensor core y el porcentaje de ajuste para combinacion de componentes de las tres dimensiones,
        # una tabla para cada cubo

        coreX<-cbind(matrix(aperm(GX,c(2,1,3)),nrow=p*r,ncol=q,byrow=TRUE),
              matrix(aperm(100*(GX^2)/(sum(X^2)),c(2,1,3)),nrow=p*r,ncol=q,byrow=TRUE))
        coreY<-cbind(matrix(aperm(GY,c(2,1,3)),nrow=p*r,ncol=q,byrow=TRUE),
              matrix(aperm(100*(GY^2)/(sum(Y^2)),c(2,1,3)),nrow=p*r,ncol=q,byrow=TRUE))
        ini4X<-rbind(c(rep("",3),rep(c("Mode 2 components",rep("",q-1)),2)),
              c(rep("",3),"Residual Sums of Squares",rep("",q-1),"Explained Variation",rep("",q-1)),
              c(rep("",3),rep(1:q,2)),cbind(unlist(lapply(1:r,function(i,p)
              {
               return(c(paste("Mode 3, Component ",i,sep=""),rep("",p-1)))
              },p=p)),rep(c("Mode 1 components",rep("",p-1)),r),rep(1:p,r),coreX))
        ini4Y<-rbind(c(rep("",3),rep(c("Mode 2 components",rep("",q-1)),2)),
```

```
            c(rep("",3),"Residual Sums of Squares",rep("",q-1),"
                Explained Variation",rep("",q-1)),
            c(rep("",3),rep(1:q,2)),cbind(unlist(lapply(1:r,function(
                i,p)
            {
             return(c(paste("Mode 3, Component ",i,sep=""),rep("",p-1
                 )))
            },p=p)),rep(c("Mode 1 components",rep("",p-1)),r),rep(1:p
                ,r),coreY))

    # calcula el ajuste de cada fila, columna de los dos cubos y
        repeticion

    fit1X<-apply(AproxX^2,1,sum)
    fit2X<-apply(AproxX^2,2,sum)
    fit3X<-apply(AproxX^2,3,sum)
    fit1X<-cbind(fit1X,apply(X^2,1,sum)-fit1X)
    fit2X<-cbind(fit2X,apply(X^2,2,sum)-fit2X)
    fit3X<-cbind(fit3X,apply(X^2,3,sum)-fit3X)
    fit1Y<-apply(AproxY^2,1,sum)
    fit2Y<-apply(AproxY^2,2,sum)
    fit3Y<-apply(AproxY^2,3,sum)
    fit1Y<-cbind(fit1Y,apply(Y^2,1,sum)-fit1Y)
    fit2Y<-cbind(fit2Y,apply(Y^2,2,sum)-fit2Y)
    fit3Y<-cbind(fit3Y,apply(Y^2,3,sum)-fit3Y)

    # si no se ha elegido representar los ejes, guarda como
        resultados las tablas con los porcentajes de ajuste
```

```
# para cada componente de cada dimension, y el ajuste total para cada dimension

if(!((!is.null(P1))&&(!is.null(P2))&&(!is.null(Q1))&&(!is.null(Q2))&&(!is.null(R1))&&(!is.null(R2))))
{
 results(ini3X,"porcentajes de ajuste primer cubo",first=TRUE)
 results(ini3Y,"porcentajes de ajuste segundo cubo")

 # guarda como resultados los tensores core y los porcentajes de ajuste para cada combinacion de componentes
 # de las tres dimensiones

 results(ini4X,"core array primer cubo")
 results(ini4Y,"core array segundo cubo")

 # guarda como resultado las coordenadas de cada fila, columna de los dos cubos y repeticion para los graficos

 results(AX,"coordenadas de las filas primer cubo",names=namesf,axis=TRUE)
 results(BX,"coordenadas de las columnas primer cubo",names=namesc1,axis=TRUE)
 results(CX,"coordenadas de las repeticiones primer cubo",names=namesr,axis=TRUE)
 results(AY,"coordenadas de las filas segundo cubo",names=namesf,axis=TRUE)
 results(BY,"coordenadas de las columnas segundo cubo",names=namesc2,axis=TRUE)
```

```
    results(CY,"coordenadas de las repeticiones segundo cubo",names
        =namesr,axis=TRUE)

    # si se ha elegido, calcula las contribuciones para las filas,
        las columnas y las repeticiones de los dos cubo de datos

    if(contr)
    {
     contributions(AX,namesf,"las filas primer cubo")
     contributions(BX,namesc1,"las columnas primer cubo")
     contributions(CX,namesr,"las repeticiones primer cubo")
     contributions(AY,namesf,"las filas segundo cubo")
     contributions(BY,namesc2,"las columnas segundo cubo")
     contributions(CY,namesr,"las repeticiones segundo cubo")
    }

    # representa el grafico con el ajuste para cada fila, columna
        de los dos cubos y repeticion
    # con las etiquetas segun los colores de los grupos a los que
        pertenecen
    # ademas pinta una recta separando las filas, columnas de los
        dos cubos y repeticiones con un ajuste superior
    # o inferior a la media

    if((is.null(dimAX)||is.null(dimAY))&&(is.null(dimBX)||is.null(
        dimBY))&&(is.null(dimCX)||is.null(dimCY)))
    {
     contr=FALSE
     windows(width=14,height=7)
```

```
layout(matrix(1:3,nrow=1))
layout.show(3)
par(mar=c(5,6,4,2)+0.1)
plot(fit1X[,1],fit1X[,2],type="n",xlim=c(min(fit1X[,1]),max(
    fit1X[,1])),
    ylim=c(min(fit1X[,2]),max(fit1X[,2])),xlab="",ylab="")
text(fit1X[,1],fit1X[,2],namesf,col=coloresf)
box(lwd=2)
abline(h=0,lwd=2)
abline(v=0,lwd=2)
abline(0,sum(fit1X[,2])/sum(fit1X[,1]),col=rgb(255,0,0,
    maxColorValue = 255),lwd=2)
plot(fit2X[,1],fit2X[,2],type="n",xlim=c(min(fit2X[,1]),max(
    fit2X[,1])),
    ylim=c(min(fit2X[,2]),max(fit2X[,2])),xlab="",ylab="")
text(fit2X[,1],fit2X[,2],namesc1,col=coloresc1)
box(lwd=2)
abline(h=0,lwd=2)
abline(v=0,lwd=2)
abline(0,sum(fit2X[,2])/sum(fit2X[,1]),col=rgb(255,0,0,
    maxColorValue = 255),lwd=2)
plot(fit3X[,1],fit3X[,2],type="n",xlim=c(min(fit3X[,1]),max(
    fit3X[,1])),
    ylim=c(min(fit3X[,2]),max(fit3X[,2])),xlab="",ylab="")
text(fit3X[,1],fit3X[,2],namesr)
box(lwd=2)
abline(h=0,lwd=2)
abline(v=0,lwd=2)
```

```
    abline(0,sum(fit3X[,2])/sum(fit3X[,1]),col=rgb(255,0,0,
        maxColorValue = 255),lwd=2)
    windows(width=14,height=7)
    layout(matrix(1:3,nrow=1))
    layout.show(3)
    par(mar=c(5,6,4,2)+0.1)
    plot(fit1Y[,1],fit1Y[,2],type="n",xlim=c(min(fit1Y[,1]),max(
        fit1Y[,1])),
        ylim=c(min(fit1Y[,2]),max(fit1Y[,2])),xlab="",ylab="")
    text(fit1Y[,1],fit1Y[,2],namesf,col=coloresf)
    box(lwd=2)
    abline(h=0,lwd=2)
    abline(v=0,lwd=2)
    abline(0,sum(fit1Y[,2])/sum(fit1Y[,1]),col=rgb(255,0,0,
        maxColorValue = 255),lwd=2)
    plot(fit2Y[,1],fit2Y[,2],type="n",xlim=c(min(fit2Y[,1]),max(
        fit2Y[,1])),
        ylim=c(min(fit2Y[,2]),max(fit2Y[,2])),xlab="",ylab="")
    text(fit2Y[,1],fit2Y[,2],namesc2,col=coloresc2)
    box(lwd=2)
    abline(h=0,lwd=2)
    abline(v=0,lwd=2)
    abline(0,sum(fit2Y[,2])/sum(fit2Y[,1]),col=rgb(255,0,0,
        maxColorValue = 255),lwd=2)
    plot(fit3Y[,1],fit3Y[,2],type="n",xlim=c(min(fit3Y[,1]),max(
        fit3Y[,1])),
        ylim=c(min(fit3Y[,2]),max(fit3Y[,2])),xlab="",ylab="")
    text(fit3Y[,1],fit3Y[,2],namesr)
    box(lwd=2)
```

```
    abline(h=0,lwd=2)
    abline(v=0,lwd=2)
    abline(0,sum(fit3Y[,2])/sum(fit3Y[,1]),col=rgb(255,0,0,
        maxColorValue = 255),lwd=2)
  }

  # realiza los tres analisis de co-inercia para las tres
     dimensiones

  dimnames(AX)[[1]]<-namesf
  dimnames(AY)[[1]]<-namesf
  dimnames(BX)[[1]]<-namesc1
  dimnames(BY)[[1]]<-namesc2
  dimnames(CX)[[1]]<-namesr
  dimnames(CY)[[1]]<-namesr
  if((is.null(dimBX)||is.null(dimBY))&&(is.null(dimCX)||is.null(
     dimCY)))
  {
   COIA(AX,AY,dimX=dimAX,dimY=dimAY,coloresf=coloresf,norm=norm,
       contr=contr,cotucker=TRUE)
  }
  if((is.null(dimAX)||is.null(dimAY))&&(is.null(dimCX)||is.null(
     dimCY)))
  {
   COIA(t(BX),t(BY),dimX=dimBX,dimY=dimBY,coloresc1=coloresc1,
       coloresc2=coloresc2,norm=norm,contr=contr,
       cotucker=TRUE,tcotucker=TRUE)
  }
```

```
    if((is.null(dimAX)||is.null(dimAY))&&(is.null(dimBX)||is.null(
       dimBY)))
    {
     COIA(CX,CY,dimX=dimCX,dimY=dimCY,norm=norm,contr=contr,
         cotucker=TRUE)
    }

    # si se ha elegido representar los ejes, comprueba que los ejes
         a representar en los graficos sean correctos

   } else {
    if((compr(P1,P2,c1,p,TRUE))&&(compr(Q1,Q2,c2,q,TRUE))&&(compr(R
       1,R2,c3,r,TRUE)))
    {
     dimensionP<-c(P1,P2)
     dimensionQ<-c(Q1,Q2)
     dimensionR<-c(R1,R2)
     AXd<-AX[,dimensionP]
     BXd<-BX[,dimensionQ]
     CXd<-CX[,dimensionR]
     AYd<-AY[,dimensionP]
     BYd<-BY[,dimensionQ]
     CYd<-CY[,dimensionR]

     # representa las filas segun el primer cubo en la izquierda
         con las etiquetas segun los colores de los grupos
     # a los que pertenecen
     # representa las columnas del primer cubo en el centro con las
          etiquetas y vectores desde el origen
```

```
        # segun los colores de los grupos a los que pertenecen
        # representa las repeticiones segun el primer cubo en la derecha con las etiquetas y vectores desde el origen

        plotmt(dim1=dimensionP,dim2=dimensionQ,dim3=dimensionR,sos=sosX,M1=AXd,M2=BXd,M3=CXd,
              lim1=AXd,lim2=BXd,lim3=CXd,names1=namesf,names2=namesc1,names3=namesr,
              colores1=coloresf,colores2=coloresc1)
        contfX<-contributions2(AX,dimensionP)
        contcX<-contributions2(BX,dimensionQ)
        contreX<-contributions2(CX,dimensionR)
        contfX[is.nan(contfX)]<-0
        contcX[is.nan(contcX)]<-0
        contreX[is.nan(contreX)]<-0
        mcontfX<-max(contfX)/2
        mcontcX<-max(contcX)/2
        mcontreX<-max(contreX)/2
        lcoloresfX<-coloresf[contfX>=mcontfX]
        lcolorescX<-coloresc1[contcX>=mcontcX]
        lnamesfX<-namesf[contfX>=mcontfX]
        lnamescX<-namesc1[contcX>=mcontcX]
        lnamesrX<-namesr[contreX>=mcontreX]
        lAX<-matrix(AXd[contfX>=mcontfX,],ncol=2)
        lBX<-matrix(BXd[contcX>=mcontcX,],ncol=2)
        lCX<-matrix(CXd[contreX>=mcontreX,],ncol=2)
        plotmt(dim1=dimensionP,dim2=dimensionQ,dim3=dimensionR,sos=sosX,M1=lAX,M2=lBX,M3=lCX,
```

```
        lim1=AXd, lim2=BXd, lim3=CXd, names1=lnamesfX, names2=lnamescX
            , names3=lnamesrX,
        colores1=lcoloresfX, colores2=lcolorescX, titles=TRUE)

    # representa las filas segun el segundo cubo en la izquierda
        con las etiquetas segun los colores de los grupos
    # a los que pertenecen
    # representa las columnas del segundo cubo en el centro con
        las etiquetas y vectores desde el origen
    # segun los colores de los grupos a los que pertenecen
    # representa las repeticiones segun el segundo cubo en la
        derecha con las etiquetas y vectores desde el origen

    plotmt(dim1=dimensionP, dim2=dimensionQ, dim3=dimensionR, sos=
        sosY, M1=AYd, M2=BYd, M3=CYd,
        lim1=AYd, lim2=BYd, lim3=CYd, names1=namesf, names2=namesc2,
            names3=namesr,
        colores1=coloresf, colores2=coloresc2)
    contfY<-contributions2(AY, dimensionP)
    contcY<-contributions2(BY, dimensionQ)
    contreY<-contributions2(CY, dimensionR)
    contfY[is.nan(contfY)]<-0
    contcY[is.nan(contcY)]<-0
    contreY[is.nan(contreY)]<-0
    mcontfY<-max(contfY)/2
    mcontcY<-max(contcY)/2
    mcontreY<-max(contreY)/2
    lcoloresfY<-coloresf[contfY>=mcontfY]
    lcolorescY<-coloresc2[contcY>=mcontcY]
```

```
          lnamesfY<-namesf[contfY>=mcontfY]
          lnamescY<-namesc2[contcY>=mcontcY]
          lnamesrY<-namesr[contreY>=mcontreY]
          lAY<-matrix(AYd[contfY>=mcontfY,],ncol=2)
          lBY<-matrix(BYd[contcY>=mcontcY,],ncol=2)
          lCY<-matrix(CYd[contreY>=mcontreY,],ncol=2)
          plotmt(dim1=dimensionP,dim2=dimensionQ,dim3=dimensionR,sos=sosY,M1=lAY,M2=lBY,M3=lCY,
            lim1=AYd,lim2=BYd,lim3=CYd,names1=lnamesfY,names2=lnamescY,names3=lnamesrY,
            colores1=lcoloresfY,colores2=lcolorescY,titles=TRUE)
        }
      }
    }
   }
  }
 }
}
```

Capítulo 7

Resultados

Se presentan los resultados para todos los tipos de análisis explicados divididos en métodos para matrices o pares de matrices de datos, métodos para una sucesión de matrices de datos, esto es, un cubo de datos, y métodos para una sucesión de pares de matrices de datos, o lo que es lo mismo, dos cubos con datos relacionados entre sí.

7.1. Matrices y pares de matrices de datos

Los datos para el Análisis de Componentes Principales y para el Análisis Entre-Grupos están ordenados en una matriz de 151 filas, los países, con 21 columnas, los indicadores del Sustainable Society Index para el año de estudio 2012. Mientras que para el Análisis de Co-Inercia se divide esta matriz en tres: una con 10 columnas, que contiene las variables de tipo social; otra matriz con 6 columnas, las variables relacionadas con el bienestar medioambiental; y la última, con las 5 variables económicas. Estas tres matrices han sido analizadas mediante el Análisis de Co-Inercia consideradas a pares.
El objetivo de estos análisis es una primera toma de contacto, una familiarización con las matrices con los datos de sostenibilidad. Se descubrirá cómo se comportan los distintos países en las distintas áreas de la sostenibilidad durante el año 2012, y qué relaciones existen entre los indicadores de un tipo y los de otro.

La figura 7.1 es una representación gráfica que incluye los países (ver apéndice B) y las variables relativas al Sustainable Society Index (ver apéndice C) analizados mediante un PCA. Todos los países están representados según su nivel de ingresos mediante diferentes colores: los países con un nivel de ingresos bajo con color negro, los que tienen un nivel de ingresos medio-bajo con color azul, los de un nivel de ingresos medio-alto con violeta, y aquellos con un nivel alto de ingresos de color rosa. Los indicadores están representados con vectores de colores según el pilar de la sostenibilidad con el que tengan que ver: los indicadores sociales en azul, los medioambientales en verde, y los económicos de rojo.

En dicha figura, en su parte izquierda, se puede observar cómo los países con el mismo nivel de ingresos se acercan entre ellos, demostrando que forman grupos en el que los países tienen perfiles similares. Además, los grupos por nivel de ingresos se sitúan en el gráfico configurando un gradiente de izquierda a derecha en orden: ingresos bajos, medio-bajos, medio-altos y altos. Esto significa que el motivo principal de la separación de los países es el nivel de ingresos, puesto que es la característica que queda representada en el eje de abscisas, es decir, el primer eje de la descomposición espectral de la matriz correspondiente, que es el eje que retiene la mayor variación por estar asociado al mayor valor propio.

De la parte derecha de la misma figura se puede interpretar cómo se relacionan las variables del SSI, así, por un lado, las variables sociales y las económicas están situadas formando ángulos pequeños entre ellas (a excepción de Public Debt, P.D.), y en el extremo opuesto se encuentran los indicadores medioambientales; esto quiere decir que los indicadores sociales y los económicos están correlacionados positivamente entre sí, y negativamente con los medioambientales, que a su vez también están correlacionados positivamente entre ellos. También se pueden interpretar las longitudes de los vectores que representan a las variables, así los indicadores sociales y económicos, en general, poseen vectores de módulos más cortos que los de las variables medioambientales, lo que significa que los países presentan una mayor variabilidad en los valores que toman en estos últimos, mientras que presentan una mayor similaridad en los valores que toman en aquellos.

Por último, se pueden interpretar las similaridades entre países y variables con las dos partes de la figura conjuntamente, según en qué semiplanos y cuadrantes se sitúen simultáneamente los países y los indicadores. Por lo tanto, los países con nivel de ingresos alto, y los de ingresos medio-altos en menor medida, se inclinan más por temas de tipo social o económico (a excepción de Public Debt, P.D.), porque ambos, países y variables, están situados en el semiplano derecho, en los cuadrantes I y IV. Y en sentido opuesto, los países con nivel de ingresos bajo y la mayoría con medio-bajo, se preocupan más por los temas medioambientales, por estar situados en los semiplanos izquierdos, es decir, en los cuadrantes II y III. Estos resultados son los mismos que los obtenidos en el primer artículo de los publicados por el autor (Apéndice A).

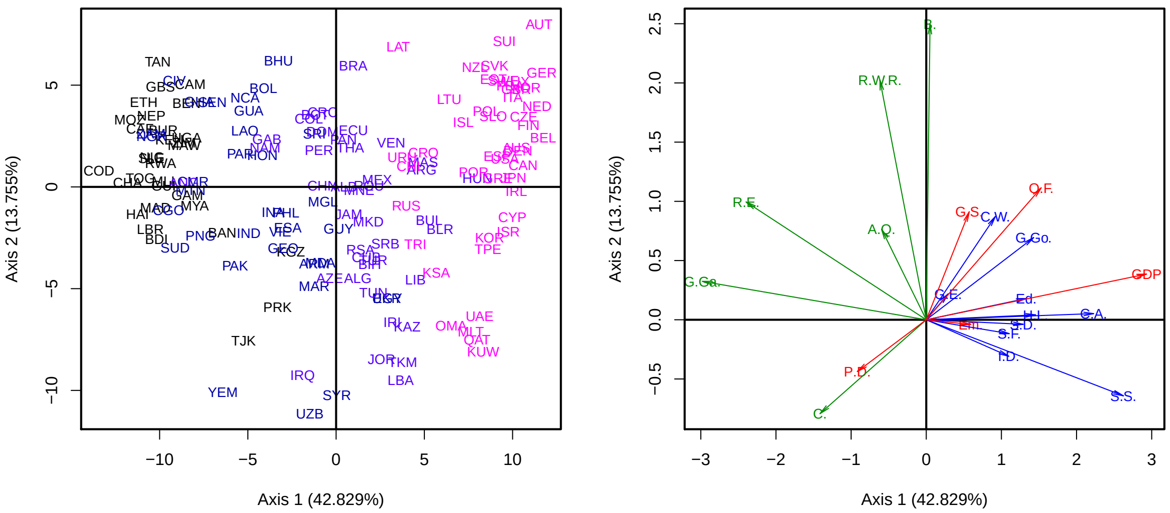


Figura 7.1: *Análisis de Componentes Principales para el año 2012*

La figura 7.2 obtenida tras el análisis BGA, a diferencia de la resultante del PCA, es una representación gráfica que sirve para interpretar, antes que los países, las similaridades y diferencias entre los distintos grupos en que se hayan podido definir estos países, esto es, según su nivel de ingresos, junto con las variables relativas al Sustainable Society Index para los grupos durante el año 2012. Y en un segundo paso, en la representación anterior se pueden proyectar los distintos países, teniendo en cuenta cómo se comportan los grupos a los que pertenecen. Así, la representación de las variables es la misma para la interpretación de los grupos, que para los países.

En este gráfico, para la interpretación de los grupos (parte superior), en su parte izquierda, se puede observar de forma consecuente con lo que se vio tras el análisis PCA, cómo los grupos de países según el nivel de ingresos se sitúan en el gráfico ordenados ascendentemente de izquierda a derecha: ingresos bajos, medio-bajos, medio-altos y altos. Mientras que en la parte derecha, se observa de nuevo que las variables del SSI están relacionadas, por un lado, los indicadores sociales y económicos (excepto Public Debt, P.D.) positivamente, y ambos negativamente con los medioambientales.

De nuevo, se ven las mismas similaridades entre países y variables: los países con nivel de ingresos alto, y medio-altos se relacionan con los temas social y económico (excepto Public Debt, P.D.), y por otro lado los países con nivel de ingresos bajo y medio-bajo están interesados en lo medioambiental.

Por último, se sitúan los países en el mismo subespacio (parte inferior) por si se quisieran interpretar más detalladamente las relaciones individuales entre países e indicadores.

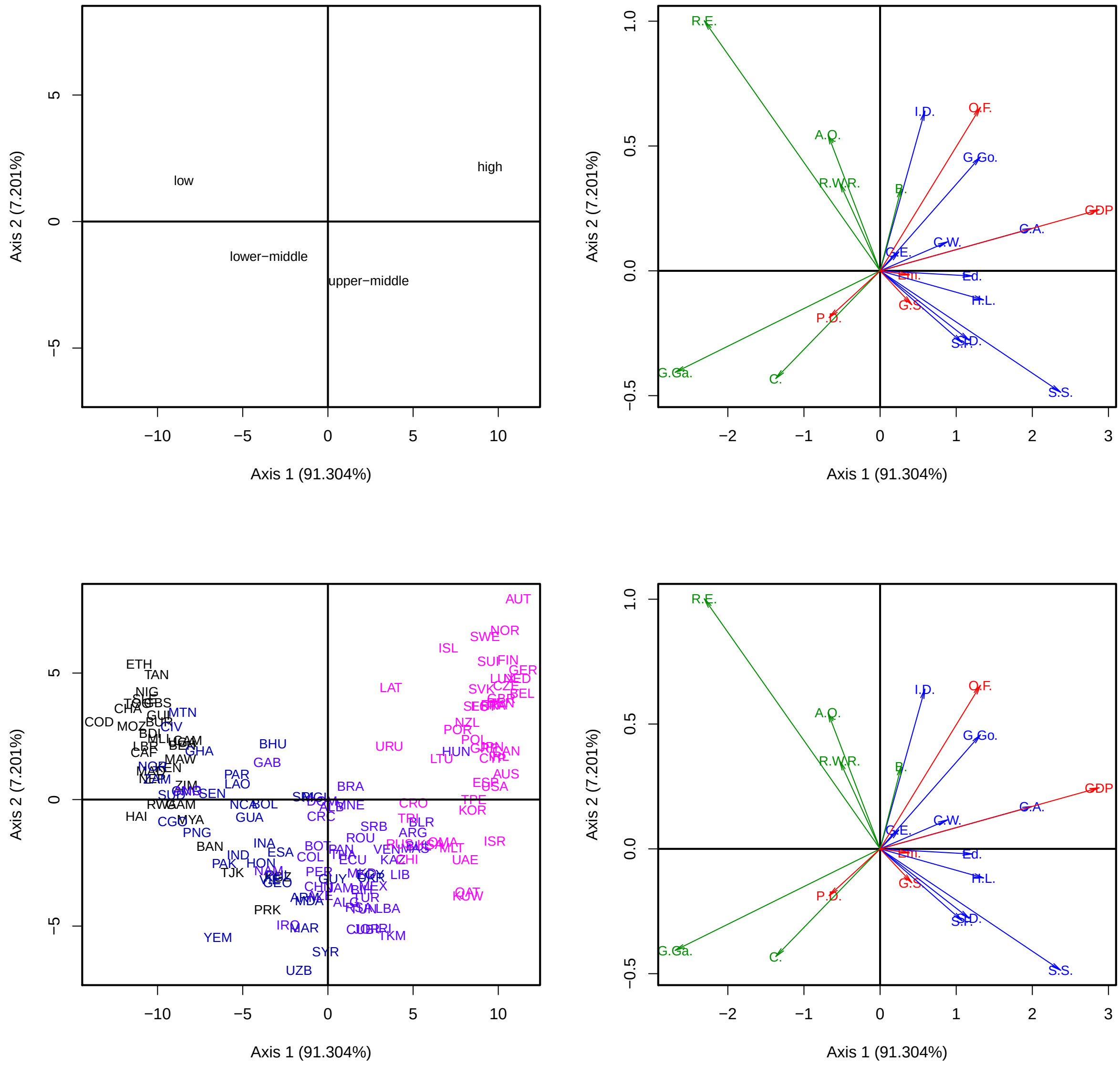


Figura 7.2: *Análisis Entre-Grupos para el año 2012*

Antes de hacer los Análisis de Co-Inercia, se necesita analizar cada tabla separadamente. La figura 7.3 muestra los resultados gráficos tras los Análisis de Co-Inercia para las tres tablas: una para los indicadores sociales, otra para los medioambientales, y la última para los económicos. Las interpretaciones que se pueden hacer de estas figuras son similiares a las llevadas a cabo en los párrafos dedicados al análisis PCA para el año 2012, pero ahora relativas únicamente a cada tipo de variables respectivamente. En este caso, se puede destacar el hecho de que, de nuevo, países con niveles de ingresos similares siguen agrupándose, además de que se sigue formando un gradiente en dirección horizontal para los tres tipos de indicadores, esto viene a significar que los países no solo se separan según su comportamiento económico, si no que también lo hacen según lo medioambiental o lo social.

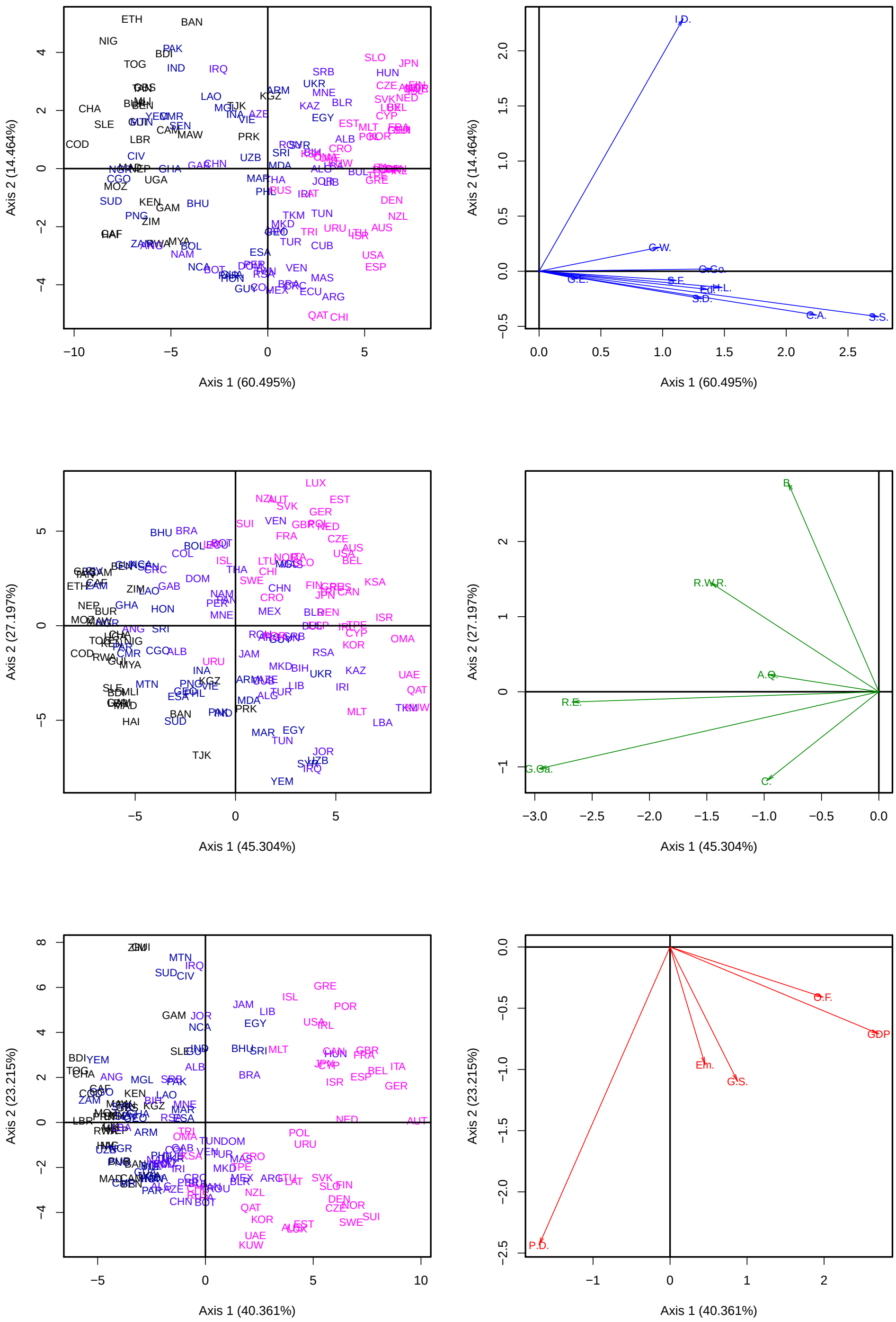


Figura 7.3: *PCAs de los Análisis de Co-Inercia para el año 2012*

Fijándonos en las figuras 7.4, 7.5 y 7.6 se puede explicar qué significan los vectores que parten de las etiquetas de cada país e interpretar también sus longitudes. Así, por ejemplo, hablemos de la figura 7.4: en la parte superior los países se sitúan en el subespacio de Co-Inercia atendiendo a cómo se comportan en relación a los indicadores sociales cuando estos se han estudiado conjuntamente con los medioambientales, y en la parte inferior según los indicadores medioambientales en el mismo estudio. Para que esto quede reflejado en los gráficos, se dibujan vectores partiendo del lugar que ocupa un país según los indicadores, y llegando al lugar que ocuparía según los otros indicadores del estudio. En este caso, en la figura superior izquierda los vectores parten de las coordenadas que le corresponden a los países según las variables sociales, y llegan a las coordenadas donde se situarían los países según las medioambientales; por lo tanto, en el gráfico de abajo a la izquierda, los vectores son los mismos, pero cambiando el origen por el extremo y viceversa.
Interpretemos ahora las longitudes de los vectores. Como un vector representa la distancia entre los lugares que ocupan los países según los dos tipos de indicadores que se estén analizando, un vector corto significa que en ese país los dos tipos de indicadores están fuertemente relacionados y el país se comporta similarmente para ambos; mientras que vectores largos significan que en los países hay diferencias notables entre los dos tipos de variables, y con comportamientos muy distintos. En nuestro caso, se puede observar, en general, en cualquiera de las figuras 7.4, 7.5 y 7.6 que los países con bajo nivel de ingresos (en color negro) poseen vectores cortos, mientras que el resto de países destacan por una mayor longitud.
Si se quieren interpretar estos gráficos en función de relaciones entre países por un lado, indicadores por otro, o países e indicadores conjuntamente, se puede hacer de forma análoga a como se ha explicado esta interpretación para el primer Análisis PCA. Así, en la figura 7.4 se sigue viendo que los indicadores sociales se sitúan opuestamente a los medioambientales y están relacionados con los países de alto y bajo nivel de ingresos respectivamente. En la figura 7.5 se ve que la variable Gross Domestic Product, GDP, es la que más contribuye a la separación de los países en distintos grupos, y de nuevo se puede observar cómo se relaciona positivamente con todos los indicadores sociales. Por último, en la figura 7.6 las variables económicas se sitúan de forma parecida a la figura anterior, mientras que las medioambien-

tales se dividen claramente en dos grupos, uno de ellos en dirección horizontal, que sería el que sirve principalmente para diferenciar a los países según el nivel de ingresos, puesto que países con alto nivel de ingresos se sitúan en el semiplano izquierdo, y los que tienen un bajo nivel en el derecho; por otro lado, existe un segundo grupo de indicadores medioambientales en dirección vertical que explicarían una segunda razón menos importante que el nivel de ingresos por la que se separan los países en los semiplanos superior e inferior.

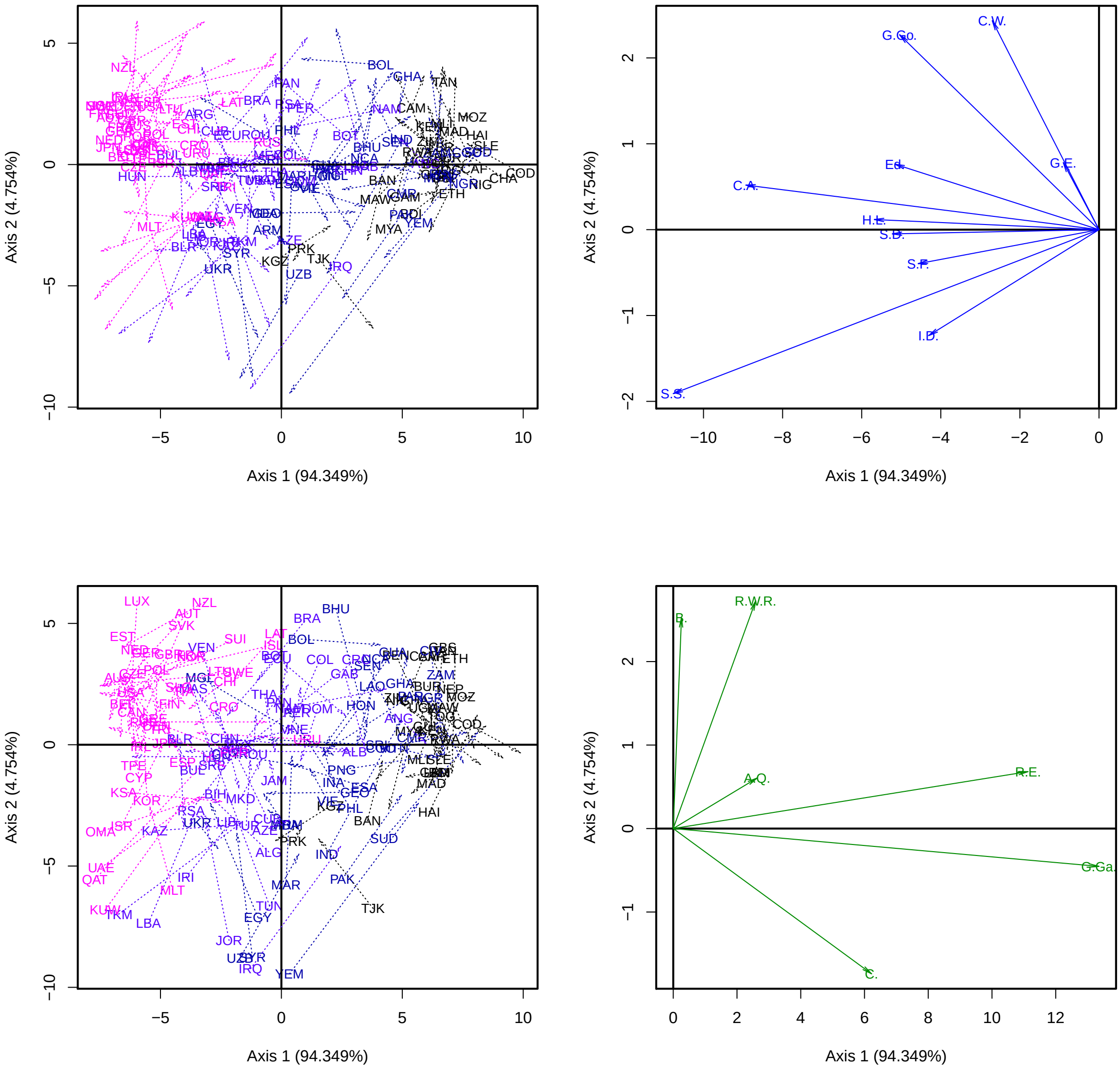


Figura 7.4: *Análisis de Co-Inercia entre variables sociales y medioambientales para el año 2012*

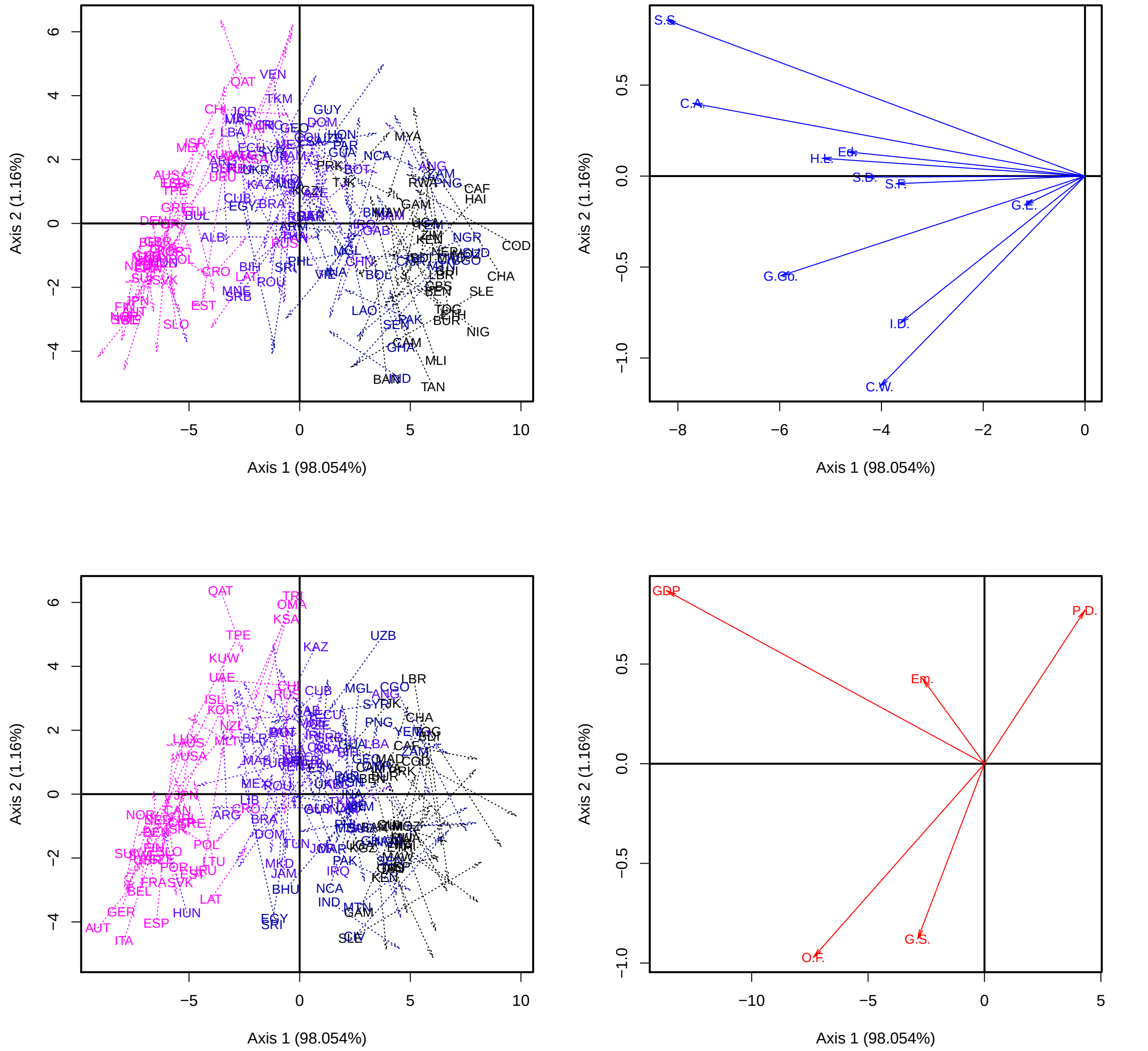


Figura 7.5: *Análisis de Co-Inercia entre variables sociales y económicas para el año 2012*

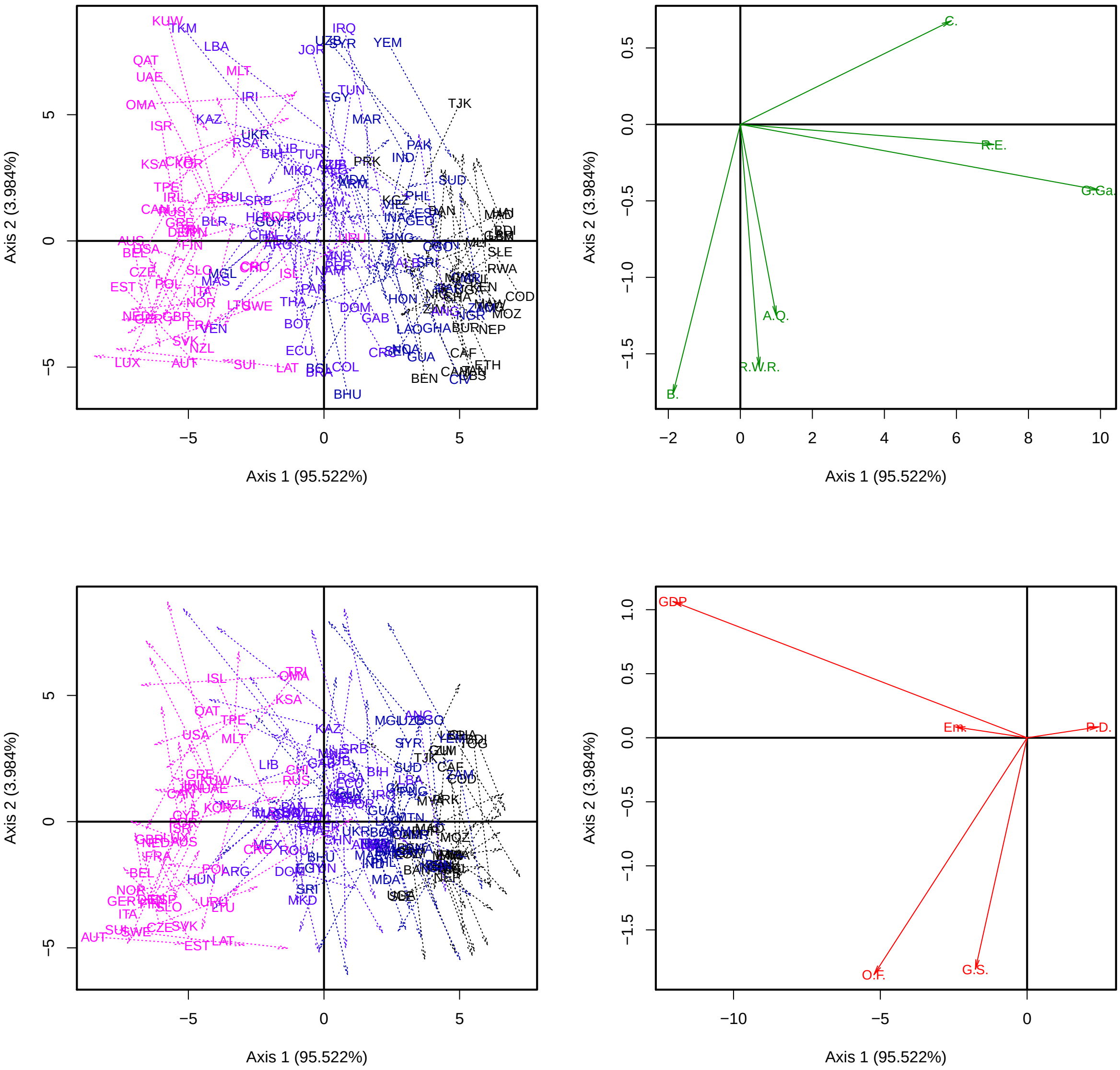


Figura 7.6: *Análisis de Co-Inercia entre variables medioambientales y económicas para el año 2012*

7.2. Sucesiones de matrices de datos

Los datos están ordenados en un cubo constituido por 151 filas, los países, con 21 columnas, los indicadores del Sustainable Society Index, para cuatro repeticiones, los cuatro años de estudio (2006, 2008, 2010 y 2012).

El objetivo del Análisis Parcial Triádico es destacar la estructura estable a lo largo de los cuatro años de los países y los indicadores, es decir, encontrar un "año medio", representar a los países e indicadores en esta estructura estable, y mostrar cómo cada uno de estos se aleja de aquella. Mientras que el objetivo de los métodos Tucker es reducir la dimensionalidad de los datos, en este caso, de 151 países x 21 indicadores x 4 años, mediante tres matrices, una para cada dimensión, y un cubo de datos más pequeño que contenga las interacciones entre filas, columnas y repeticiones. Este método se diferencia del Análisis Parcial Triádico en que ahora se pueden descubrir interacciones más profundas que las estables obtenidas con el Análisis Parcial Triádico.

El primer gráfico resultante que se obtiene después del análisis PTA es la figura 7.7 llamada interestructura. Es una representación gráfica que sirve para interpretar las similaridades y diferencias entre las repeticiones que se hayan estudiado, en nuestro caso, los distintos años, así como evidenciar cuáles de esas repeticiones son las más relevantes a la hora de formar la llamada matriz compromiso, esto es, aquellos años que se parezcan más a un "año medio" que destacará la parte estable de la evolución de los datos a lo largo del tiempo.

Se puede observar que el año más parecido a la configuración más estable, que es el que se encuentre más cerca del eje horizontal, de abscisas, es el 2008, lo que significa que es el que más se parece, en media, a todos los demás años. De la misma forma, se observa también cómo se agrupan los años: por un lado, 2010 y 2012, en el cuadrante IV, se encuentran muy próximos entre sí y alejados de 2006 y 2008, en el cuadrante I, lo cual viene a decir que se han encontrado pruebas significativas para demostrar que entre el año 2008 y el 2010 pudo suceder algo que justifique los diferentes valores de los indicadores del SSI entre los dos pares de años.

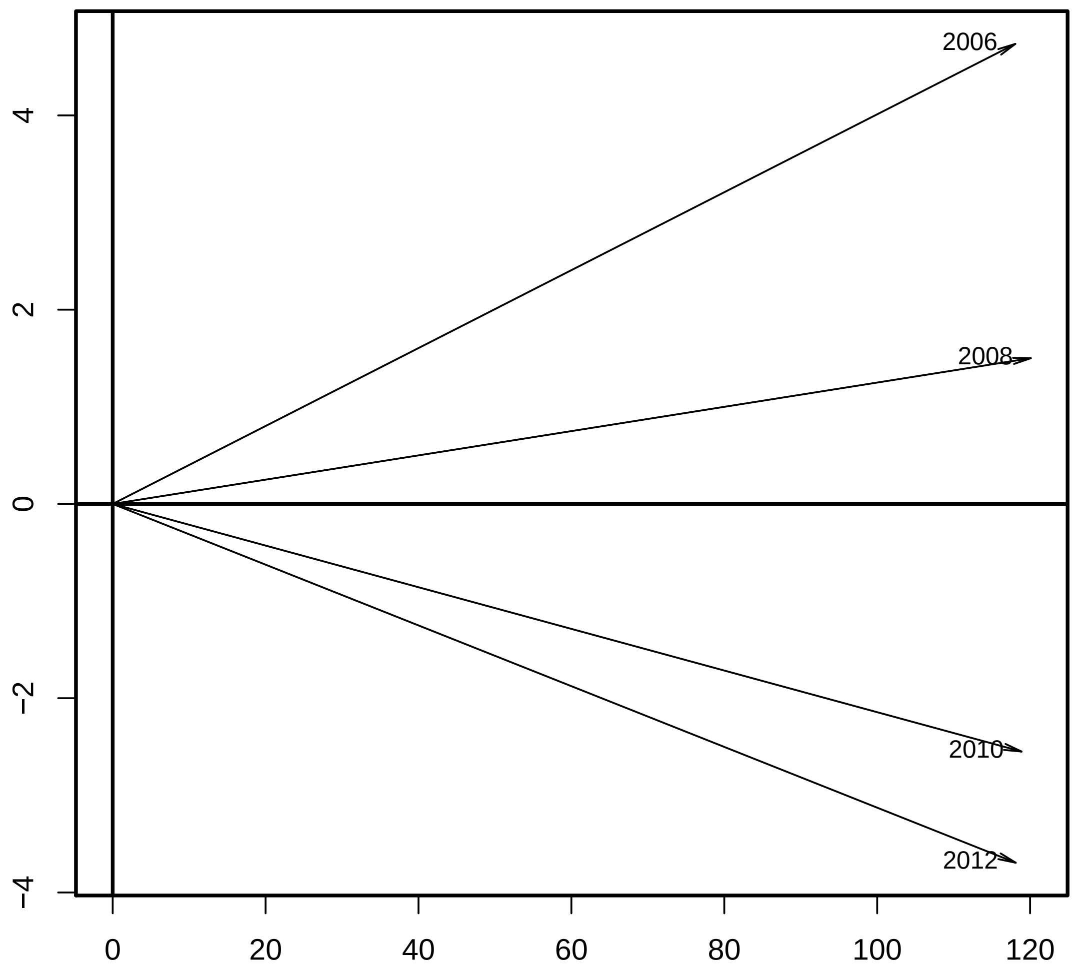


Figura 7.7: *Interestructura del Análisis Parcial Triádico*

En un segundo paso, una vez que se conocen las similaridades y diferencias de las distintas repeticiones con el "año medio", se puede obtener explícitamente como combinación de todas las repeticiones, así se ha calculado la llamada matriz del compromiso, que incluye los países y los valores más estables que toman en los indicadores. Esta matriz se puede analizar mediante el análisis PCA, según el que se ha obtenido la figura 7.8.

Este gráfico se interpreta de la misma forma que se vio en el análisis PCA: de nuevo los grupos de países según el nivel de ingresos se sitúan ordenados; las variables del SSI están relacionadas, por un lado, los indicadores sociales y económicos (excepto Public Debt, P.D.) positivamente, y ambos negativamente con los medioambientales; y las mismas relaciones entre países y variables.
Se puede observar que el gráfico del compromiso guarda cierta semejanza con el gráfico del PCA para el año 2012.

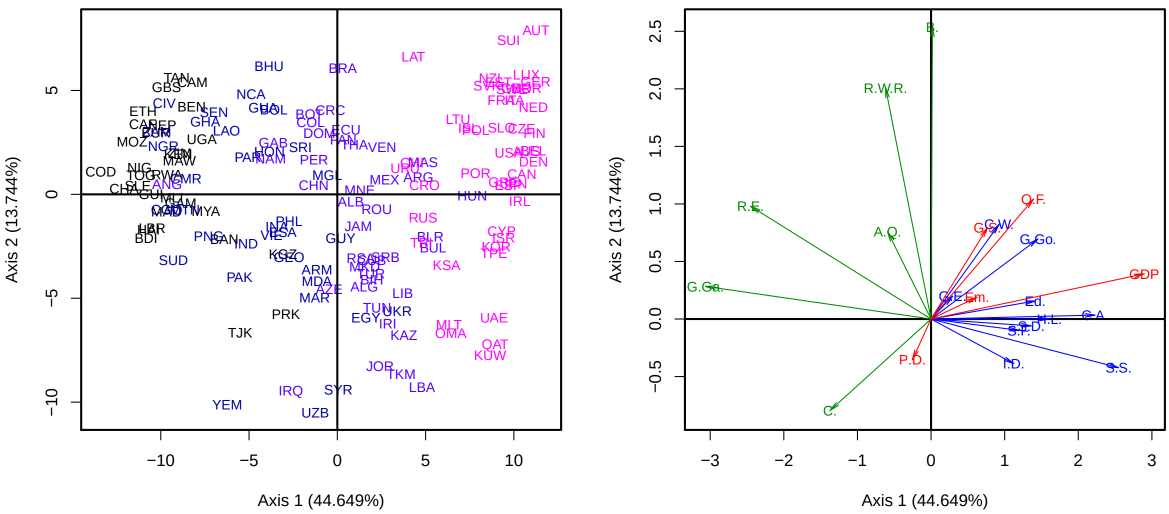


Figura 7.8: *Compromiso del Análisis Parcial Triádico*

Por último, se proyectan los países y las variables de todos las años en el mismo subespacio que la matriz compromiso (figura 7.9), la llamada representación de la intraestructura o de las trayectorias, y con ello se puede interpretar más detalladamente las evoluciones a lo largo del tiempo de los países y los indicadores del SSI. Por ejemplo, los países que tienen las trayectorias más largas, aquellos que han evolucionado más en el tiempo son los que poseen niveles de ingresos bajos o medio-bajos, mientras que los indicadores que más han variado son los de tipo económico, en particular destaca Public Debt, P.D.

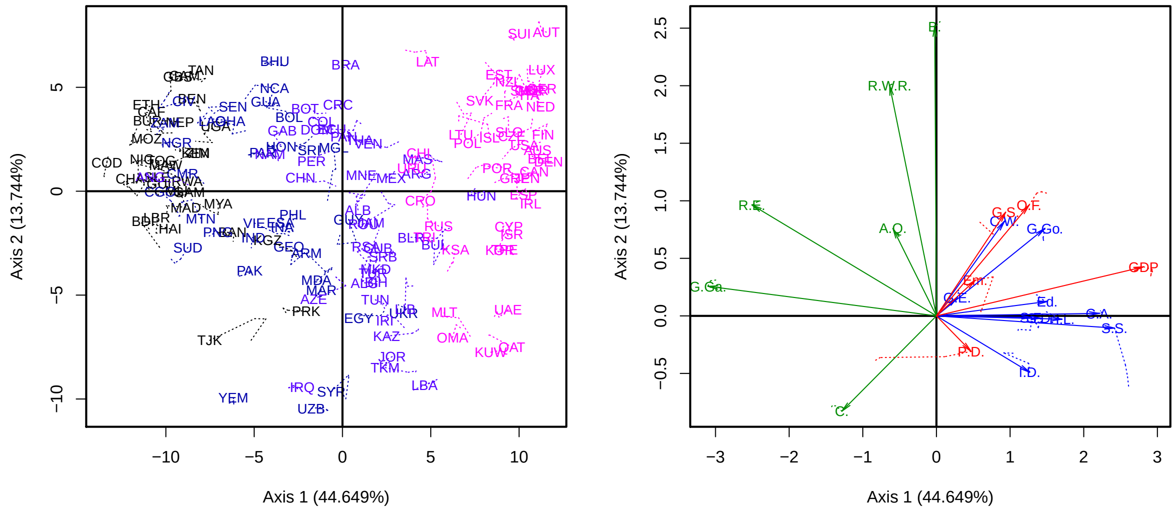


Figura 7.9: *Trayectorias del Análisis Parcial Triádico*

Presentemos ahora los resultados tras el análisis con el método Tucker3. Lo primero es escoger cuántas componentes retener para cada una de las dimensiones, la de los países, la de los indicadores, y la de los años. Para ello nos fijamos en las dos primeras tablas que se obtienen como resultados, la tabla 7.1 que muestra todas las combinaciones posibles de componentes con sus porcentajes de varianza explicada, y la tabla 7.2 que resume la tabla anterior para aquellas combinaciones que tengan una mejor varianza explicada para una suma de componentes fijada.

Tabla 7.1: Todas las combinaciones con el Tucker3

Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones	Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones
1	1x1x1	3	*	42723.454	42.854	4	59	3x5x3	11		26966.856	63.930	15
6	1x2x2	5		42488.928	43.168	5	60	3x5x4	12		26963.461	63.934	18
11	1x3x3	7		42478.898	43.181	12	64	4x1x4	9		42615.310	42.999	3
16	1x4x4	9		42475.130	43.186	8	66	4x2x2	8		32748.470	56.196	14
22	2x1x2	5		42646.805	42.957	8	67	4x2x3	9		32744.952	56.201	15
25	2x2x1	5	*	32861.467	56.045	10	68	4x2x4	10		32743.865	56.203	13
26	2x2x2	6	*	32848.561	56.063	13	70	4x3x2	9		26580.863	64.446	15
27	2x2x3	7		32847.401	56.064	13	71	4x3x3	10		26576.819	64.451	16
28	2x2x4	8		32847.280	56.064	13	72	4x3x4	11		26576.147	64.452	16
30	2x3x2	7		32621.613	56.366	8	**73**	**4x4x1**	**9**	*	**23262.695**	**68.884**	**45**
31	2x3x3	8		32620.148	56.368	8	**74**	**4x4x2**	**10**	*	**23042.134**	**69.179**	**52**
32	2x3x4	9		32620.023	56.368	8	75	4x4x3	11		23031.955	69.193	53
34	2x4x2	8		32587.062	56.412	8	76	4x4x4	12		23029.161	69.197	55
35	2x4x3	9		32582.831	56.418	8	78	4x5x2	11		22966.853	69.280	37
36	2x4x4	10		32582.573	56.418	8	79	4x5x3	12		22955.106	69.296	37
39	2x5x3	10		32573.346	56.431	8	80	4x5x4	13		22948.604	69.305	39
40	2x5x4	11		32570.005	56.435	8	87	5x2x3	10		32723.299	56.230	15
43	3x1x3	7		42625.615	42.985	13	88	5x2x4	11		32722.024	56.232	13
46	3x2x2	7		32777.948	56.157	14	90	5x3x2	10		26520.974	64.526	16
47	3x2x3	8		32776.393	56.159	14	91	5x3x3	11		26435.968	64.640	14
48	3x2x4	9		32775.837	56.160	14	92	5x3x4	12		26435.002	64.641	14
49	3x3x1	7	*	27277.202	63.515	19	94	5x4x2	11		22523.686	69.873	55
50	3x3x2	8	*	27076.362	63.783	20	95	5x4x3	12		22512.436	69.888	55
51	3x3x3	9		27072.916	63.788	21	96	5x4x4	13		22509.026	69.892	56
52	3x3x4	10		27072.302	63.789	21	97	5x5x1	11	*	19744.483	73.590	34
54	3x4x2	9		26996.613	63.890	19	98	5x5x2	12	*	19517.003	73.895	46
55	3x4x3	10		26990.459	63.898	19	99	5x5x3	13	*	19502.269	73.914	47
56	3x4x4	11		26987.795	63.902	19	100	5x5x4	14	*	19494.652	73.924	47
58	3x5x2	10		26974.947	63.919	17							

Tabla 7.2: Combinaciones con mejor ajuste en el Tucker3

Número	Modelo	S	Suma de Cuadrados Residual	Diferencia del Ajuste	Porcentaje del Ajuste	Número de Iteraciones
1	1x1x1	3	42723.454	42.854	42.854	4
25	2x2x1	5	32861.467	13.191	56.045	10
26	2x2x2	6	32848.561	0.017	56.063	13
49	3x3x1	7	27277.202	7.452	63.515	19
50	3x3x2	8	27076.362	0.269	63.783	20
73	**4x4x1**	**9**	**23262.695**	**5.101**	**68.884**	**45**
74	**4x4x2**	**10**	**23042.134**	**0.295**	**69.179**	**52**
97	5x5x1	11	19744.483	4.411	73.590	34
98	5x5x2	12	19517.003	0.304	73.895	46
99	5x5x3	13	19502.269	0.020	73.914	47
100	5x5x4	14	19494.652	0.010	73.924	47

En la segunda se ve que las combinaciones 4x4x1 y 4x4x2 tienen unos porcentajes de ajuste del 68.884 % y 69.179 %, que ya son unos porcentajes suficientemente altos teniendo en cuenta que se ha reducido de un cubo de datos 151x21x4 a uno 4x4x1 o 4x4x2. Además, el incremento en la varianza explicada si se considerase el siguiente modelo más complejo (5x5x1) solo sería de un 4.411 % como se ve en la columna de Diferencia en el Ajuste, lo que ya se considera como insignificante desde el punto de vista estadístico.
También, a la hora de elegir la combinación de componentes, se puede usar un gráfico en el que estén representados todos los modelos según la suma del número de sus componentes frente a la suma de cuadrados residual, tal como se explicó en el apartado de desarrollo. En este caso, el gráfico sería el siguiente (Figura 7.10)

En él se observa que el modelo 4x4x1 es uno de los más simples (tiene la menor suma de cuadrados residual para los modelos que tienen la misma suma del número de componentes) y también es el primero de los más estables (los modelos posteriores tienen una reducción en

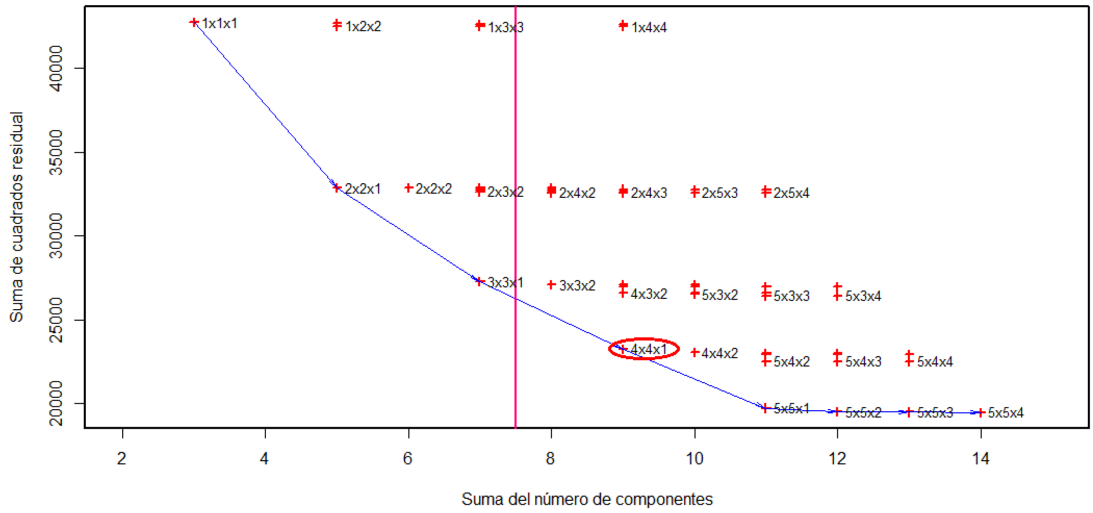


Figura 7.10: *Suma del número de componentes vs. Suma de cuadrados residual en el Tucker3*

la suma de cuadrados residual insignificante desde el punto de vista estadístico).

Sin embargo, se va a elegir el modelo 4x4x2, porque si no se consideran dos componentes para la tercera dimensión no se verán diferencias entre los años, e interesa ver cómo nuestros datos evolucionan en el tiempo.

Los siguientes resultados que se presentan son después de llevar a cabo el mismo análisis Tucker3 pero fijando como componentes el modelo 4x4x2. El primero que se obtiene es la tabla 7.3, una tabla resumen donde se recoge el desglose de la varianza explicada por cada uno de los componentes de cada una de las dimensiones.

Tabla 7.3: Porcentajes de Ajuste con el Tucker3

Componente	Dimensión 1	Dimensión 2	Dimensión 3
1	43.120	42.863	68.875
2	13.202	13.204	0.304
3	7.483	7.722	0
4	5.375	5.391	0
Varianza explicada Total	69.179	69.179	69.179

Pero quizás es más interesante interpretar la siguiente tabla resultado (Tabla 7.4), el llamado core array puesto en forma de matriz, la matriz core. También se refiere a la varianza explicada, pero considerando las diferentes combinaciones de los componentes de las dimensiones. Además, se muestran los signos para interpretar las interacciones entre los componentes de las dimensiones que son estadísticamente significantes, esto es, que no son nulas.

Tabla 7.4: Matriz Core en el Tucker3

			Componentes Modo 2				Componentes Modo 2			
			Suma de Cuadrados Residual				Varianza Explicada			
			1	2	3	4	1	2	3	4
Modo 3, Componente 1	Componentes Modo 1	1	**178.893**	0.111	-0.153	0.058	**42.849**	0.000	0.000	0.000
		2	0.204	**-99.298**	0.991	-0.237	0.000	**13.189**	0.001	0.000
		3	-0.257	-0.734	-74.679	-2.261	0.000	0.001	7.460	0.007
		4	-0.180	-0.213	-1.850	63.327	0.000	0.000	0.005	5.364
Modo 3, Componente 2	Componentes Modo 1	1	-0.409	-2.821	**-13.417**	3.751	0.000	0.011	**0.241**	0.019
		2	-2.132	1.245	1.456	-0.827	0.006	0.002	0.003	0.001
		3	-2.308	-0.744	-2.402	-0.136	0.007	0.001	0.008	0.000
		4	0.154	-0.928	-1.880	0.145	0.000	0.001	0.005	0.000

Por ejemplo, del total de 69.179 %, reteniendo una componente de cada dimensión se obtiene una varianza explicada del 42.849 %.

Se presentan los siguientes gráficos con las tres dimensiones. En cada gráfico se representan dos componentes, una en horizontal y otra en vertical. Estos, junto con los resultados de la anterior tabla, servirán para interpretar las interacciones entre países, indicadores y años.

Se interpretarán dos gráficos: el procedente de las combinaciones de componentes 1x1x1 y 2x2x1 (Figura 7.11); y el procedente de la combinación 1x3x2 (Figura 7.14).

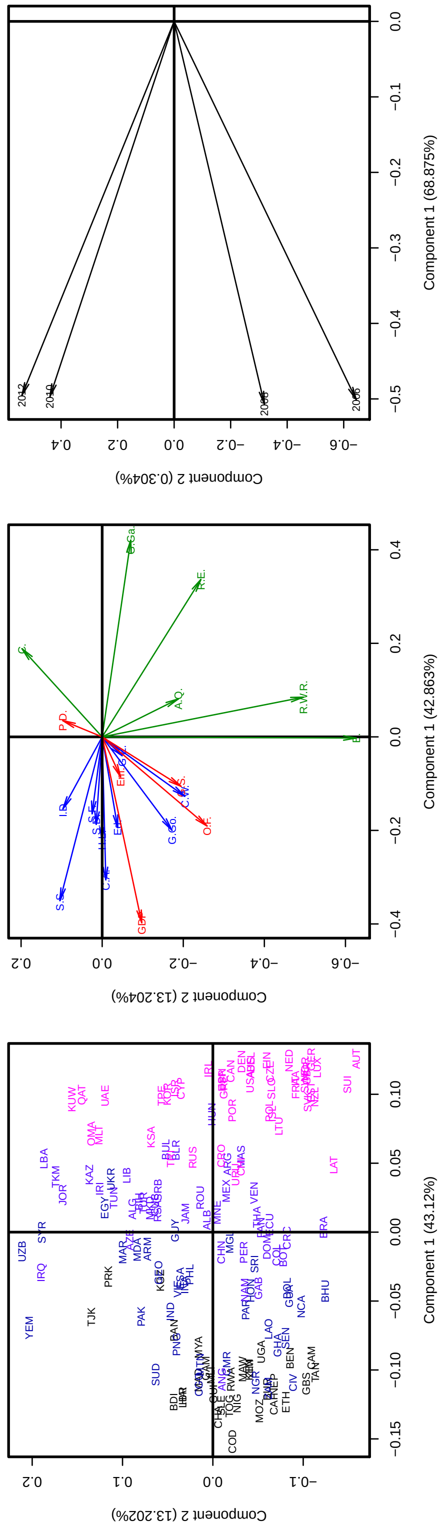


Figura 7.11: *Gráfico para las dos primeras componentes de las tres dimensiones del Tucker3*

Como el elemento 1x1x1 de la matriz core (Tabla 7.4) es positivo (178.983), los países que tengan coordenadas positivas en la primera componente, es decir, los que estén situados en el semiplano derecho (cuadrantes I y IV) de la figura 7.11, tienen una interacción positiva con los indicadores que tienen coordenadas negativas en la componente primera, los situados en el semiplano izquierdo (cuadrantes II y III), en cualquiera de los cuatro años, porque todos se sitúan en el semiplano izquierdo, tienen coordenadas negativas.

$$\text{Países}(+) \times \text{indicadores}(-) \times \text{años}(-) \times \text{matriz core}(+) = \text{interacción}(+)$$

Esto es, en todos los años de estudio, todos los países con ingresos altos y la mayoría con ingresos medio-altos toman valores altos en las variables sociales y económicas (excepto Public Debt, P.D.). De la misma forma, en todos los años de estudio, todos los países con ingresos bajos y la mayoría con ingresos medio-bajos toman valores altos en las variables medioambientales.

$$\text{Países}(-) \times \text{indicadores}(+) \times \text{años}(-) \times \text{matriz core}(+) = \text{interacción}(+)$$

Estas dos conclusiones a partir del producto de los signos se pueden visualizar fácilmente en la figura 7.12, en la que se han representado los países, indicadores y años según los signos explicados. Además, solo han sido representados aquellos con mejor calidad de representación (superior a 500 sobre mil).

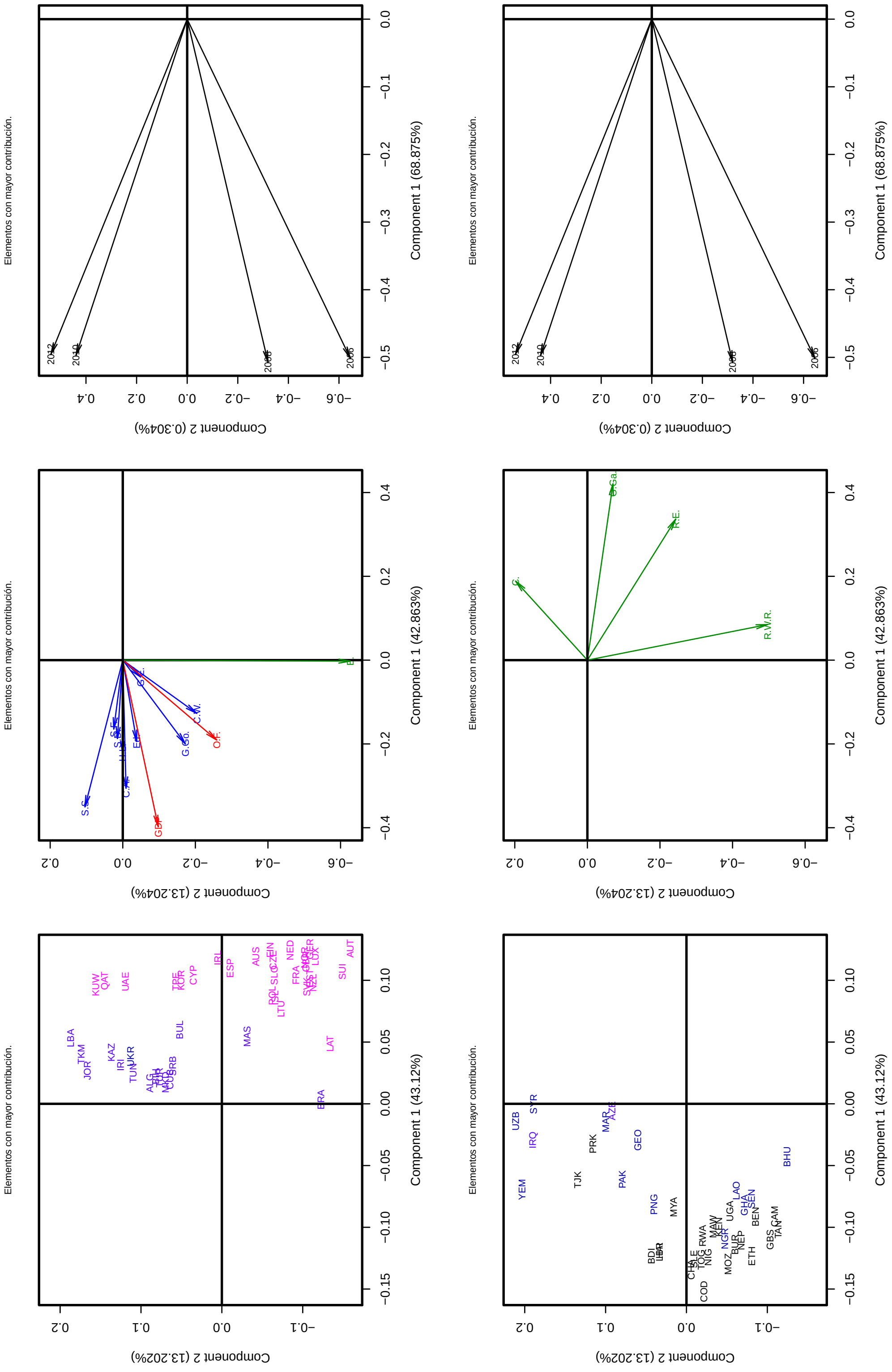


Figura 7.12: *Figura 7.11, con solo los países, indicadores y años correspondientes a los signos*

Con la misma figura y sabiendo que el elemento 2x2x1 de la matriz core es negativo (–99.298), se puede interpretar como en el caso previo, pero considerando los ejes verticales de los subgráficos de los países y los indicadores, que los países pueden ser diferenciados con detalle dependiendo de en qué variables toman valores más altos. Por ejemplo, los países de Rusia, Kuwait, Qatar, Omán, Emiratos Árabes Unidos o Arabia Saudí, que se sitúan en el cuadrante I y por tanto tienen signo positivo en la coordenada para la segunda componente, aunque siguen teniendo valores altos en todas las variables sociales y económicas (excepto Public Debt, P.D.), toman valores ligeramente más altos en Safe Sanitation, S.S., Income Distribution, I.D., Sufficient Food, S.F., y Sufficient to Drink, S.D., porque estos se sitúan en el semiplano superior (cuadrante II).

$$\text{Países(+)} \times \text{indicadores(+)} \times \text{años(–)} \times \text{matriz core(–)} = \text{interacción(+)}$$

Mientras que por otro lado, países como Austria, Suiza, Letonia, Holanda,Croacia o Dinamarca toman valores altos en el resto de indicadores sociales y en todas las variables económicas (excepto Public Debt, P.D.).

$$\text{Países(–)} \times \text{indicadores(–)} \times \text{años(–)} \times \text{matriz core(–)} = \text{interacción(+)}$$

Estas dos últimas diferenciaciones que se han obtenido con los signos de las componentes 2x2x1 tienen una varianza explicada de 13.189 %, así como las obtenidas tras el estudio de los signos para la combinación 1x1x1, de un 42.849 %.
Estas dos conclusiones a partir del producto de los signos, de nuevo, se pueden visualizar fácilmente en la figura 7.13, en la que se han representado los países, indicadores y años según los signos explicados. Además, solo han sido representados aquellos con mejor calidad de representación.

Ahora, como ejemplo de interpretación de la segunda componente para la dimensión de los años, se hablará sobre la combinación 1x3x2, esto es, habrán de observarse la primera componente de los países, la tercera de los indicadores y la segunda de los años. Se usará la figura 7.14, en la que que la componente 1 de los países está representada en horizontal y la tercera de los indicadores en vertical.

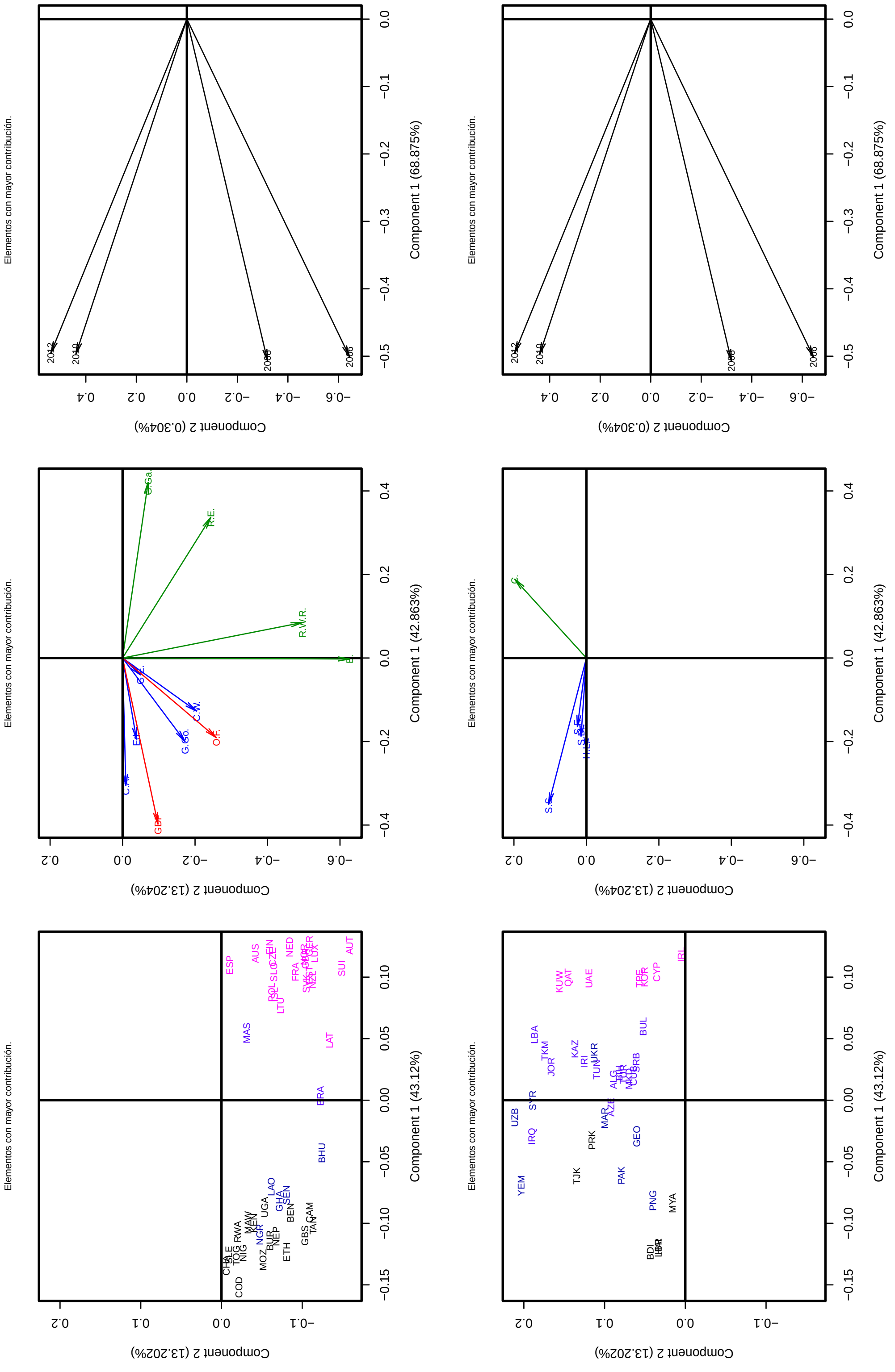


Figura 7.13: *Figura 7.11, con solo los países, indicadores y años correspondientes a los signos*

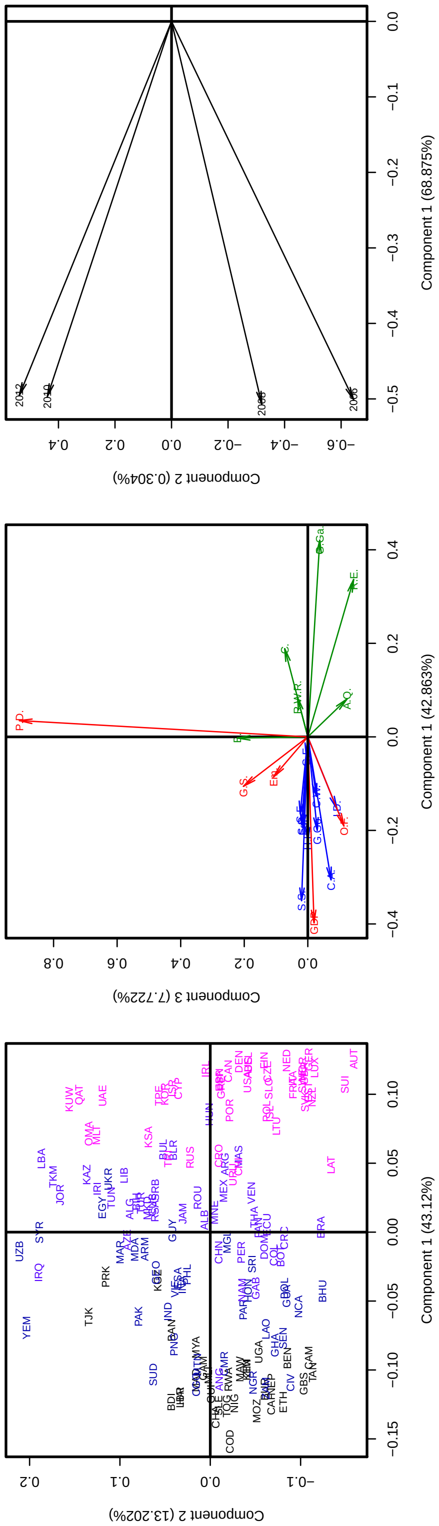


Figura 7.14: Gráfico para las dos primeras componentes en las dimensiones 1 y 3, y para las componentes 1 y 3 en la dimensión segunda del Tucker3

Nos fijamos en que el elmento 1x3x2 de la matriz core es negativo (–13.417).
Durante el 2006 y el 2008, todos los países con ingresos altos y la mayoría con ingresos medio-altos toman valores altos en los indicadores:

- Todos los indicadores sociales excepto Good Governance, G.Go., Clean Water, C.W., Clean Air, C.A., e Income Distribution, I.D.
- Biodiversity, B., Consumption, C., y Renewable Water Resources, R.W.R.
- Public Debt, P.D., Genuine Savings, G.S., y Employment, Em.

$$\text{Países(+)} \times \text{indicadores(+)} \times \text{años(–)} \times \text{matriz core(–)} = \text{interacción(+)}$$

Y por otro lado, durante los mismos años, todos los países con ingresos bajos y la mayoría con ingresos medio-bajos toman valores altos en los indicadores:

- Good Governance, G.Go., Clean Water, C.W., Clean Air, C.A., e Income Distribution, I.D.
- Greenhouse Gases, G.Ga., Air Quality, A.Q., y Renewable Energy, R.E.
- Organic Farming, O.F., y Gross Domestic Product, GDP.

$$\text{Países(–)} \times \text{indicadores(–)} \times \text{años(–)} \times \text{matriz core(–)} = \text{interacción(+)}$$

Mientras que en los años 2010 y 2012 los temas de interés están invertidos con respecto a los años anteriores, porque los años 2010 y 2012 están situados en el semiplano superior y el 2006 y el 2008 en el inferior.
Nótese que estos últimos resultados explican una varianza del 0.241 %, así que solo deberían ser interpretados después de hablar de todos los elementos de la matriz core de mayor varianza explicada y entendiendo que estos resultados sirven para explicar una diferenciación menos manifiesta entre los países, las variables y los años.
Estas dos conclusiones a partir del producto de los signos se pueden visualizar fácilmente en la figura 7.15, en la que se han representado los países, indicadores y años según los signos explicados. Además, solo han sido representados aquellos con mejor calidad de representación.

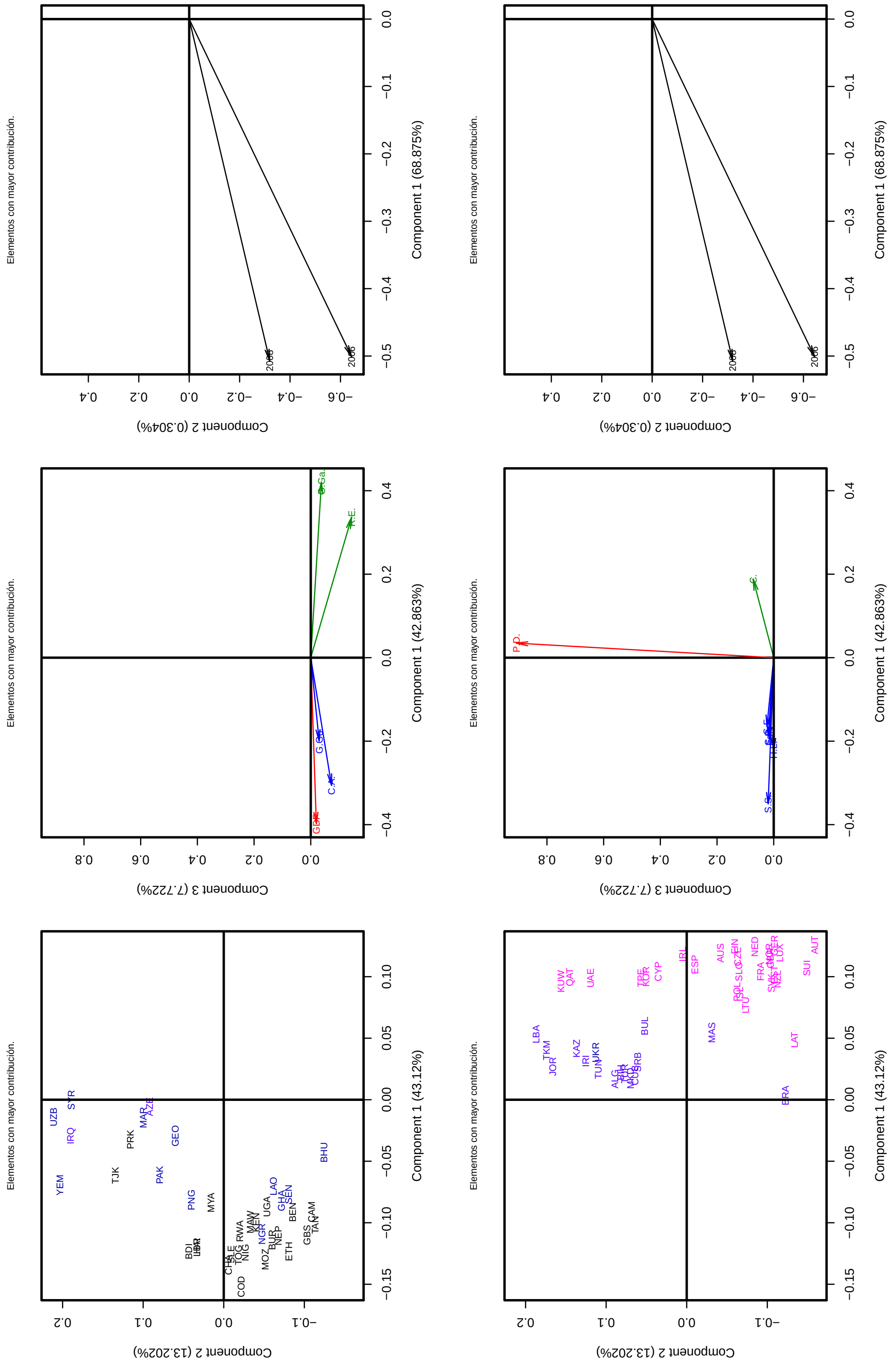


Figura 7.15: *Figura 7.14, con solo los países, indicadores y años correspondientes a los signos*

7.3. Sucesiones de pares de matrices de datos

Los datos para el Análisis de Co-Inercia Entre-Grupos, el STATICO, el COSTATIS y el Co-Tucker3 están ordenados en tres cubos de 151 filas, los países, con 4 repeticiones, los cuatro años de estudio (2006, 2008, 2010 y 2012): un cubo con 10 columnas, que contienen las variables de tipo social; otro cubo con 6 columnas, las variables relacionadas con el bienestar medioambiental; y el último, con las 5 variables económicas.

El objetivo de los análisis de este conjunto de datos es descubrir las relaciones entre países e indicadores de sostenibilidad social, medioambiental y económica. Más precisamente, los análisis ayudarán a descubrir cómo estas relaciones varían a lo largo del espacio (por países de todo el mundo) y del tiempo (los últimos cuatro bienios). Además, se demostrará que las cuatro técnicas deben emplearse de forma complementaria para obtener resultados que se beneficien de las ventajas de cada uno de los métodos.

En una primera parte se hablará de los resultados de los tres primeros métodos, que han sido utilizados de forma complementaria, y en una segunda parte se expondrán los resultados obtenidos tras el análisis Co-Tucker3, y también se demostrará que debe ser usado complementariamente con los otros tres.

7.3.1. BGCOIA, STATICO y COSTATIS

Las figuras 7.16, 7.17 y 7.18 muestran las representaciones de las similaridades existentes entre los distintos años de estudio.

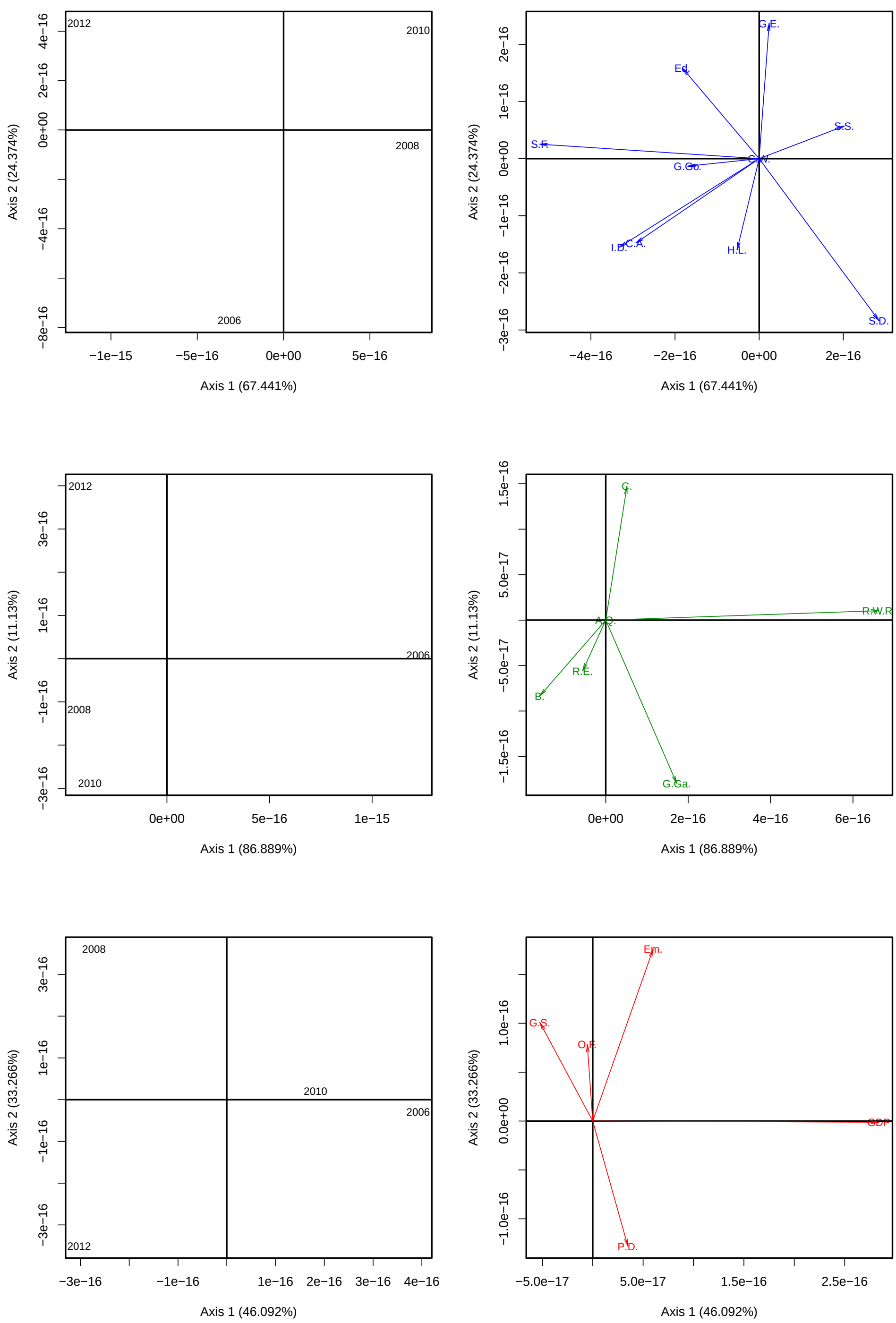


Figura 7.16: *Años según el Análisis BGCOIA*

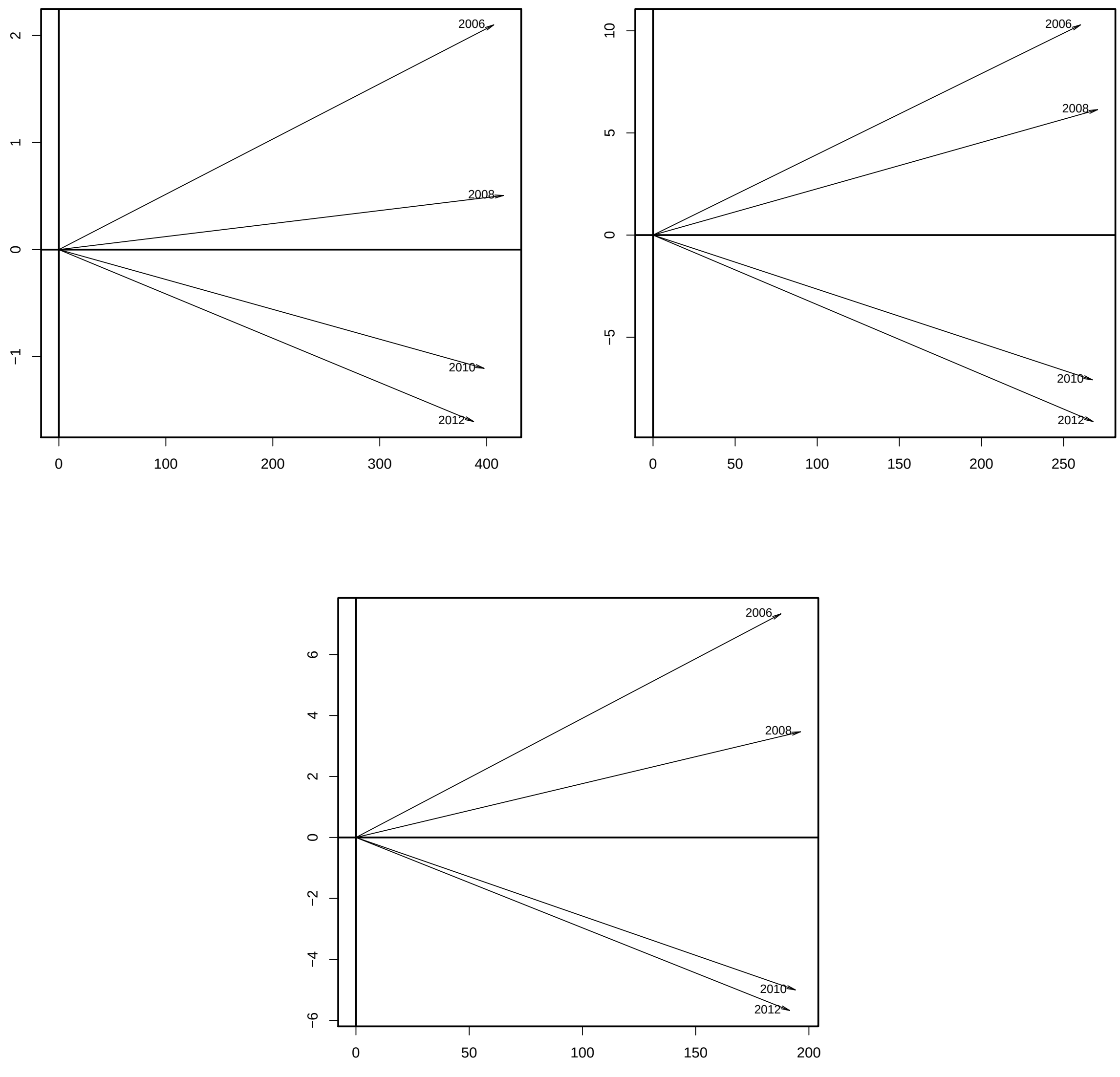


Figura 7.17: *Años según el Análisis STATICO*

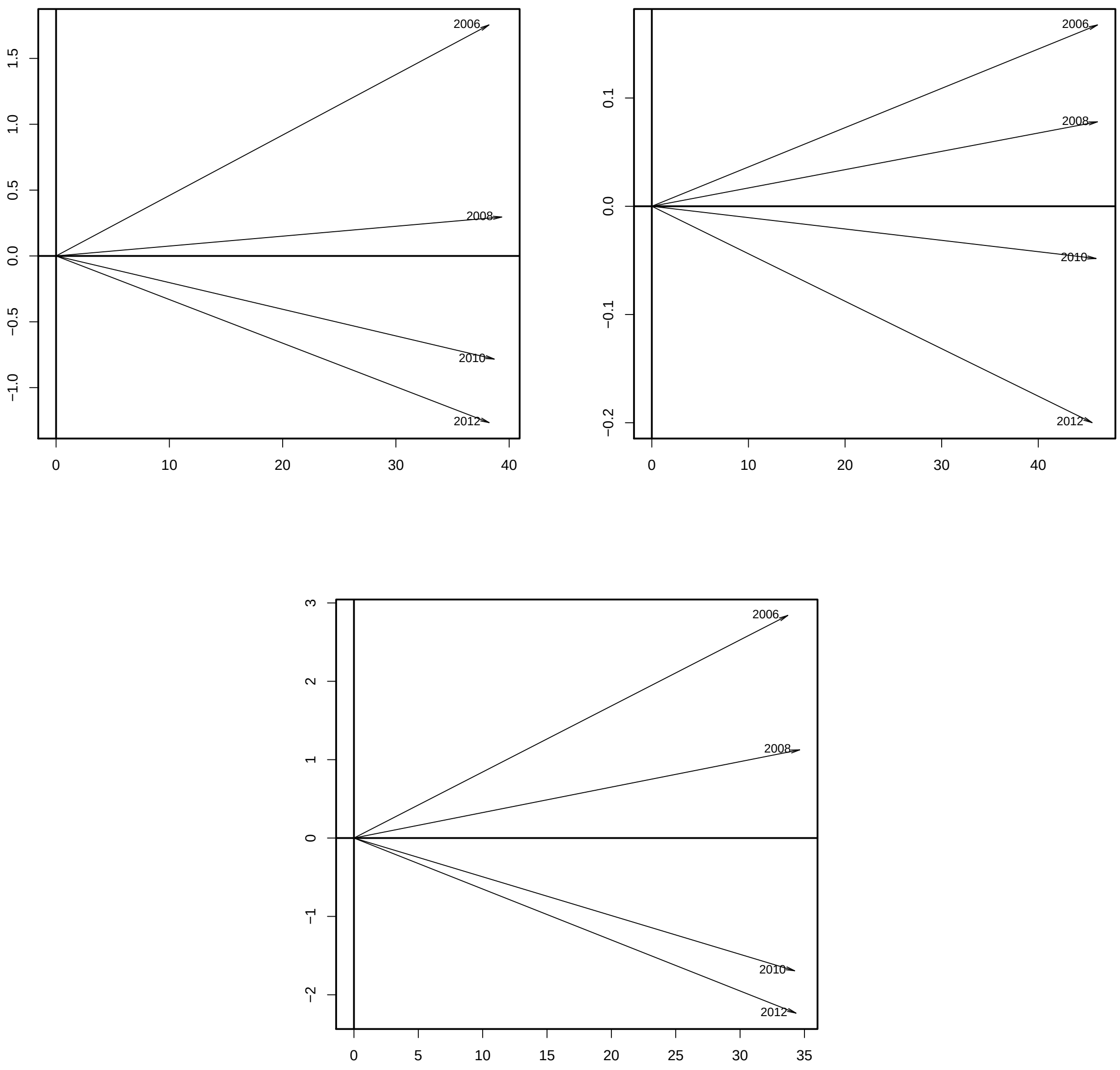


Figura 7.18: *Años según el Análisis COSTATIS*

La primera figura corresponde con los gráficos procedentes del análisis BGCOIA:

- desde el punto de vista Social, el año 2006 da prioridad a los temas de Healthy Life, H.L., y Sufficient to Drink, S.D., los años 2008 y 2010 dan paso a un interés por Safe Sanitation, S.S., y finalmente el 2012 se caracteriza por concentrarse en temas como Education, Ed., y Sufficient Food, S.F.

- desde el punto de vista medioambiental, los cuatro años apenas tocan los temas relacionados con Air Quality, A.Q., y Greenhouse Gases, G.Ga., pero existen diferencias entre los años, en 2008 y 2010 se da prioridad a las cuestiones de Biodiversity, B., y Renewable Energy, R.E. (con mayor énfasis el último año), mientras que en 2006 y 2012 el tema más importante es Renewable Water Resources, R.W.R., en 2006 por interesarse por ello, y en 2012 por ser un tema que pierde mucha importancia. Además, en el año 2012 se da prioridad al tema de Consumption, C.

- desde el punto de vista económico, los años 2006 y 2010 se interesan por Gross Domestic Product, GDP, y el 2008 por Organic Farming, O.F., y Genuine Savings, G.S. Además se observa una evolución del comportamiento respecto a Employment, Em., cuando más importante es, es en el 2006 y el 2008, desciende en el año 2010, y finalmente tiene valores muy bajos en 2012.

Los gráficos del análisis STATICO (segunda figura) muestran las interestructuras del Análisis Parcial Triádico después de realizar el Análisis de Co-Inercia, por lo tanto, muestran las similaridades entre los años teniendo en cuenta los tipos de variables en pares: así, el primero corresponde a variables sociales con medioambientales; el segundo, sociales con económicas; y el tercero, medioambientales con económicas. Además, estos gráficos muestran qué años son los que tendrán mayor peso a la hora de construir el compromiso, así, para el primero, el más importante es el 2008, mientras que para los otros dos gráficos, todos los años son igual de parecidos al compromiso, sin embargo, se observa similaridades entre los años 2006 y 2008, y por otro lado, 2010 y 2012. Estos resultados son los mismos que los obtenidos en el tercer artículo de los publicados por el autor (Apéndice A).

Y los gráficos del análisis COSTATIS (tercera figura) muestran qué años son similares antes del análisis de Co-Inercia, luego corresponden a interestructuras desde el punto de vista de cada tipo de variables por separado: sociales, medioambientales y económicas. De nuevo, se puede observar en estos gráficos qué años son los más relevantes para la construcción del compromiso. En este caso, el primer gráfico muestra que el año 2008 es el que proporciona mayor peso, en el segundo gráfico se observa que los años más importantes son 2008 y 2010 igualmente espaciados de 2006 y 2012, y en el tercero, al igual que mediante el STATICO, todos los años son parecidos al compromiso, pero con similaridades a pares, 2006 con 2008, y 2010 con 2012. Estos resultados también son los mismos que los obtenidos en el tercer artículo de los publicados por el autor (Apéndice A).

Antes del último paso, en el que con los tres análisis se pueden estudiar las trayectorias de los países de estudio a lo largo de los cuatro bienios, se hablará de los resultados gráficos que se obtienen para cada una de las técnicas, puesto que son diferentes, y así quedará manifiesto que se han de usar las tres de forma complementaria.
Con el análisis BGCOIA, lo que se obtiene es un par de gráficos con los que se puede interpretar la Co-Inercia entre los años respecto a los tres tipos de indicadores analizados a pares, que se puede realizar de la misma manera a como se ha explicado anteriormente en el apartado de resultados para pares de matrices de datos, esto es, se puede hablar sobre la longitud de los vectores. Un año con un vector corto se traduce en fuertes relaciones entre los dos tipos de indicadores para ese año y que durante este, los países se comportan de manera similar en las variables de ambos tipos, y viceversa, un año con un vector largo se traduce en diferencias notables entre los dos tipos de indicadores en ese año y que durante este, los países tienen comportamientos muy distintos.
Por ejemplo, en la figura 7.21 se observa que en el año 2008 las variables medioambientales y las económicas están fuertemente relacionadas y los países, por tanto, se comportan de manera similar con respecto a esos indicadores. Mientras que en la figura 7.19 se ve que en el año 2012 hay mucha diferencia entre las variables sociales y las medioambientales, con lo que los países se han comportado de manera muy distinta durante ese año respecto a los dos

tipos de indicadores.

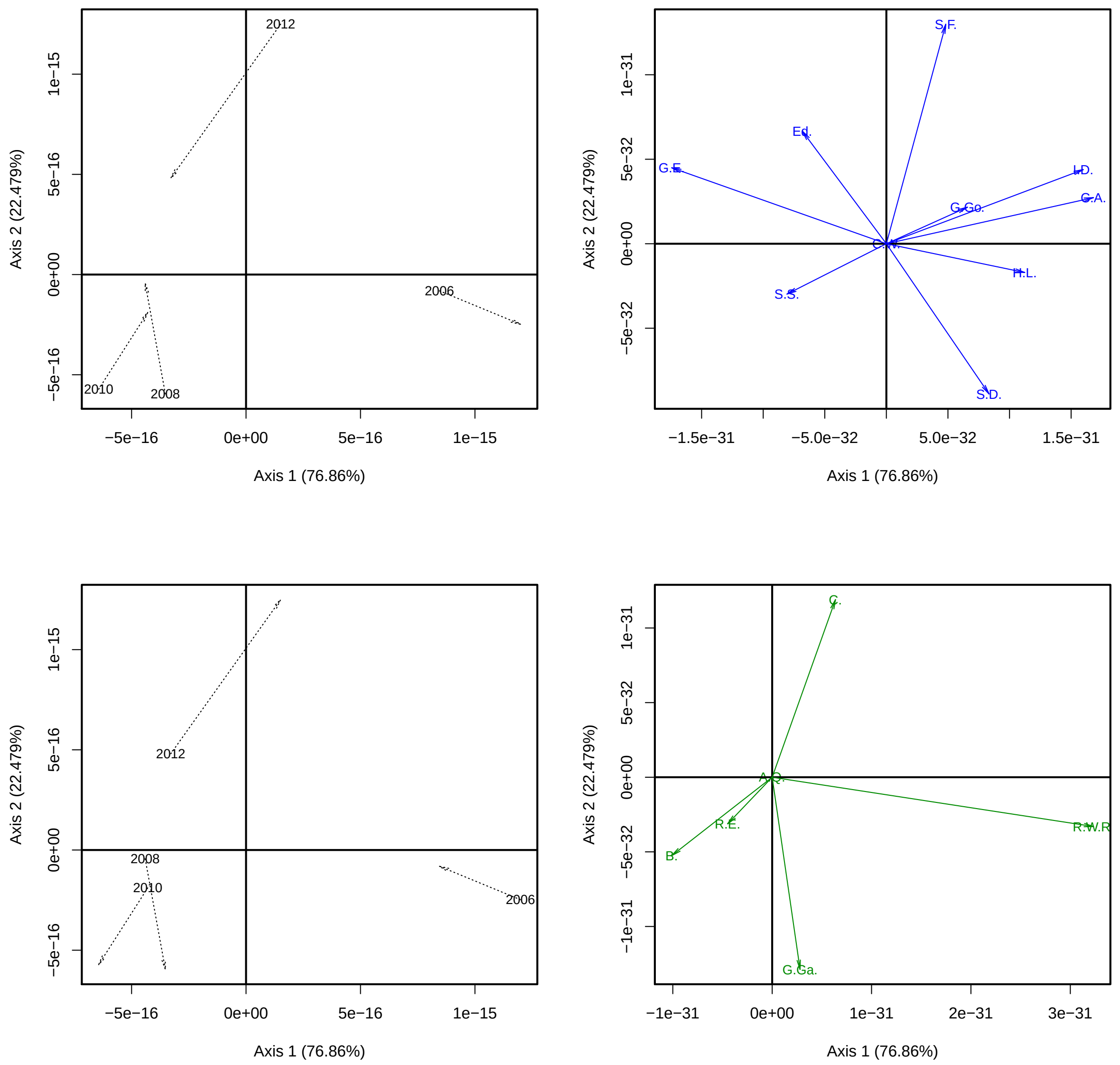


Figura 7.19: *Análisis de Co-Inercia del BGCOIA entre variables sociales y medioambientales para todos los años*

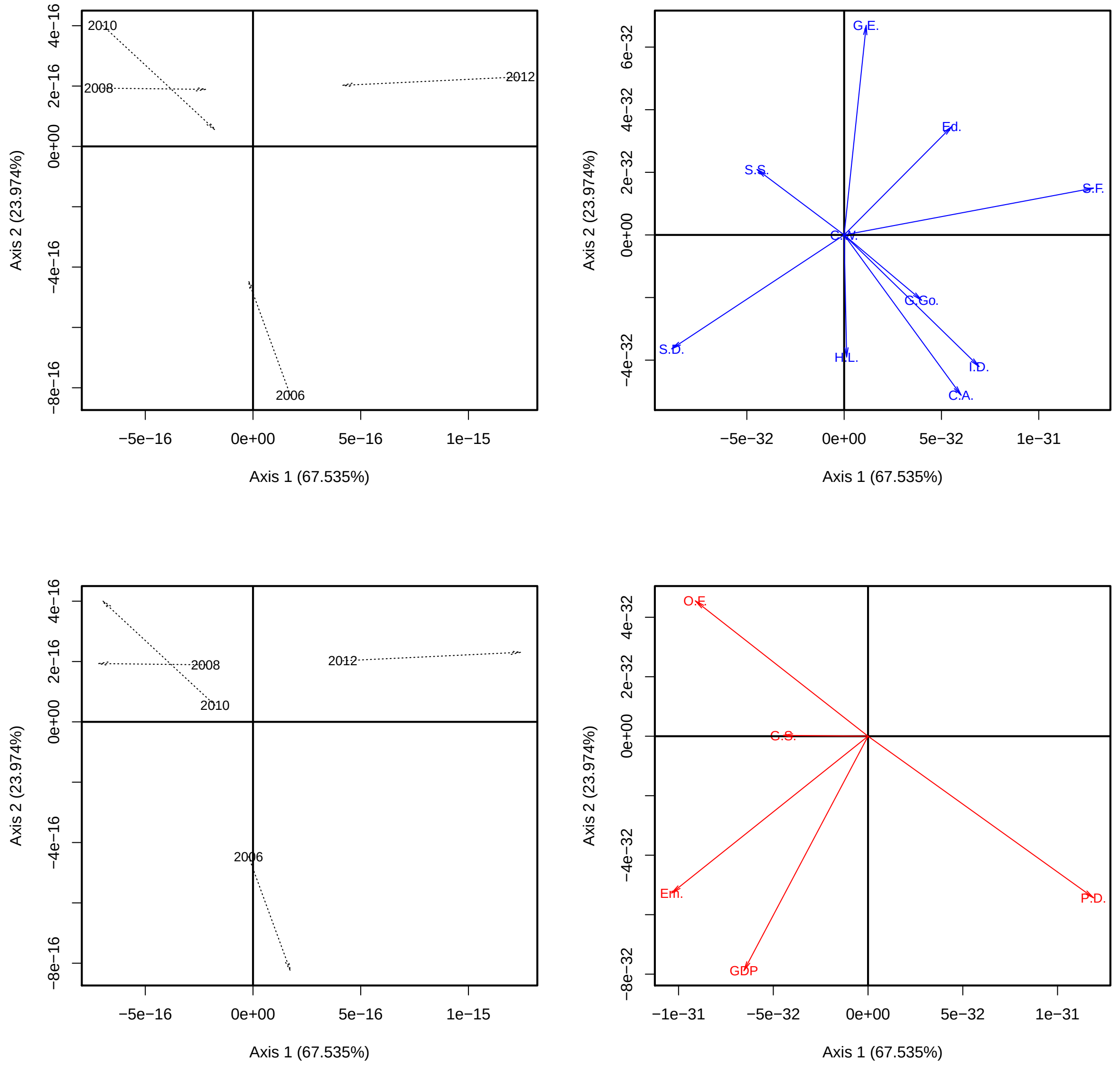


Figura 7.20: *Análisis de Co-Inercia del BGCOIA entre variables sociales y económicas para todos los años*

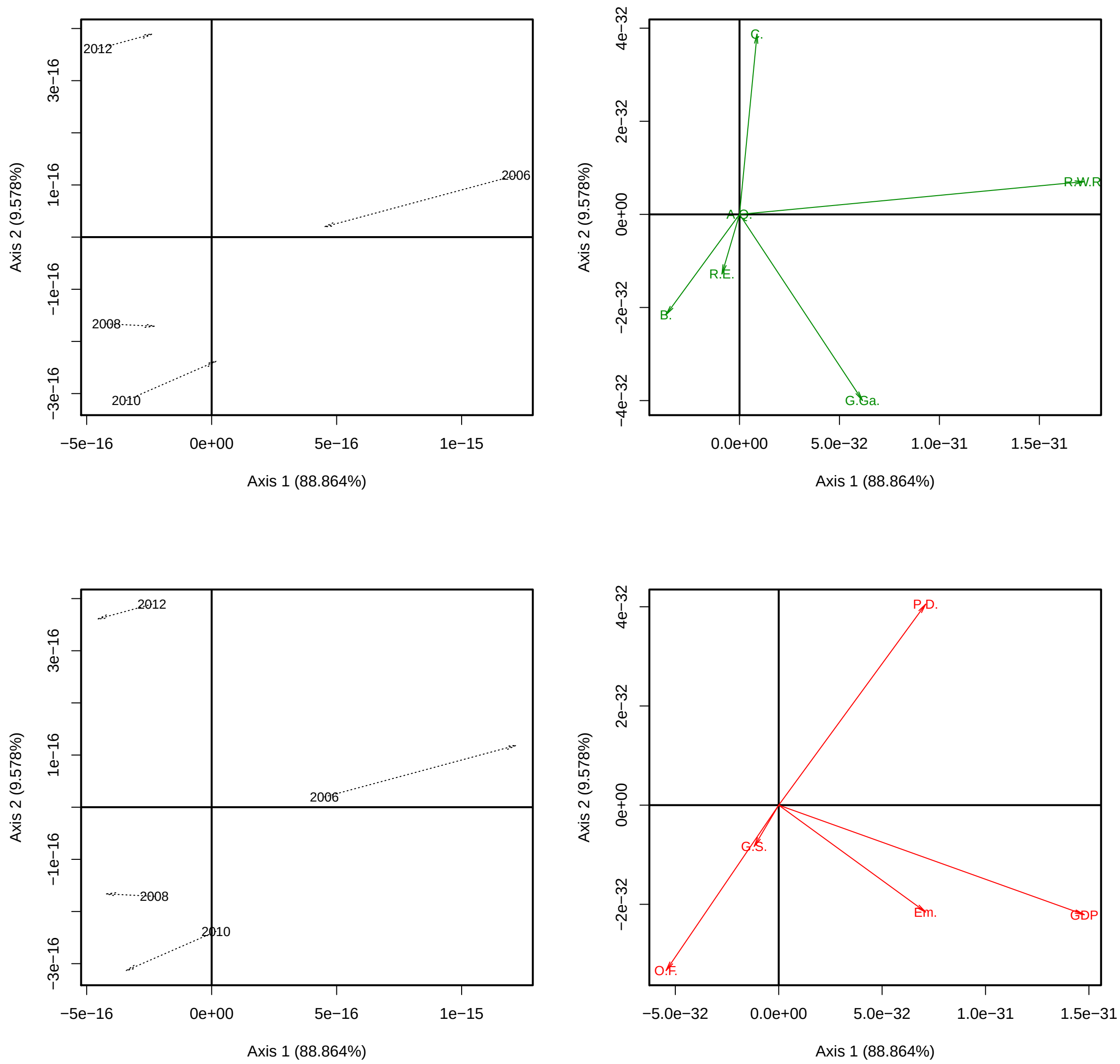


Figura 7.21: *Análisis de Co-Inercia del BGCOIA entre variables medioambientales y económicas para todos los años*

Con el análsis STATICO, se obtienen dos gráficos con los que se pueden interpretar las relaciones entre las variables de dos tipos en función de cómo se comportan en ellas todos los países, y también se puede interpretar cómo varían estas relaciones a lo largo del tiempo.

Lo que se puede destacar, principalmente, es consecuente con lo que ya se había descubierto con el análisis PCA para el año 2012, que las variables de tipo social y las de tipo económico están estrechamente relacionadas entre sí (a excepción de Public Debt, P.D.) e inversamente relacionadas con las de tipo medioambiental; además de que se observa que la variable que más varía a lo largo del tiempo es, de nuevo como se vio tras el análisis PTA para todos los años, la Public Debt, P.D. (figuras 7.23 y 7.24). Con la diferencia de que ahora se puede deducir todo esto para el caso en que se estudien las relaciones entre los tipos de variables a pares, no todas a la vez. De nuevo, estos resultados son los mismos que los obtenidos en el tercer artículo de los publicados por el autor (Apéndice A).

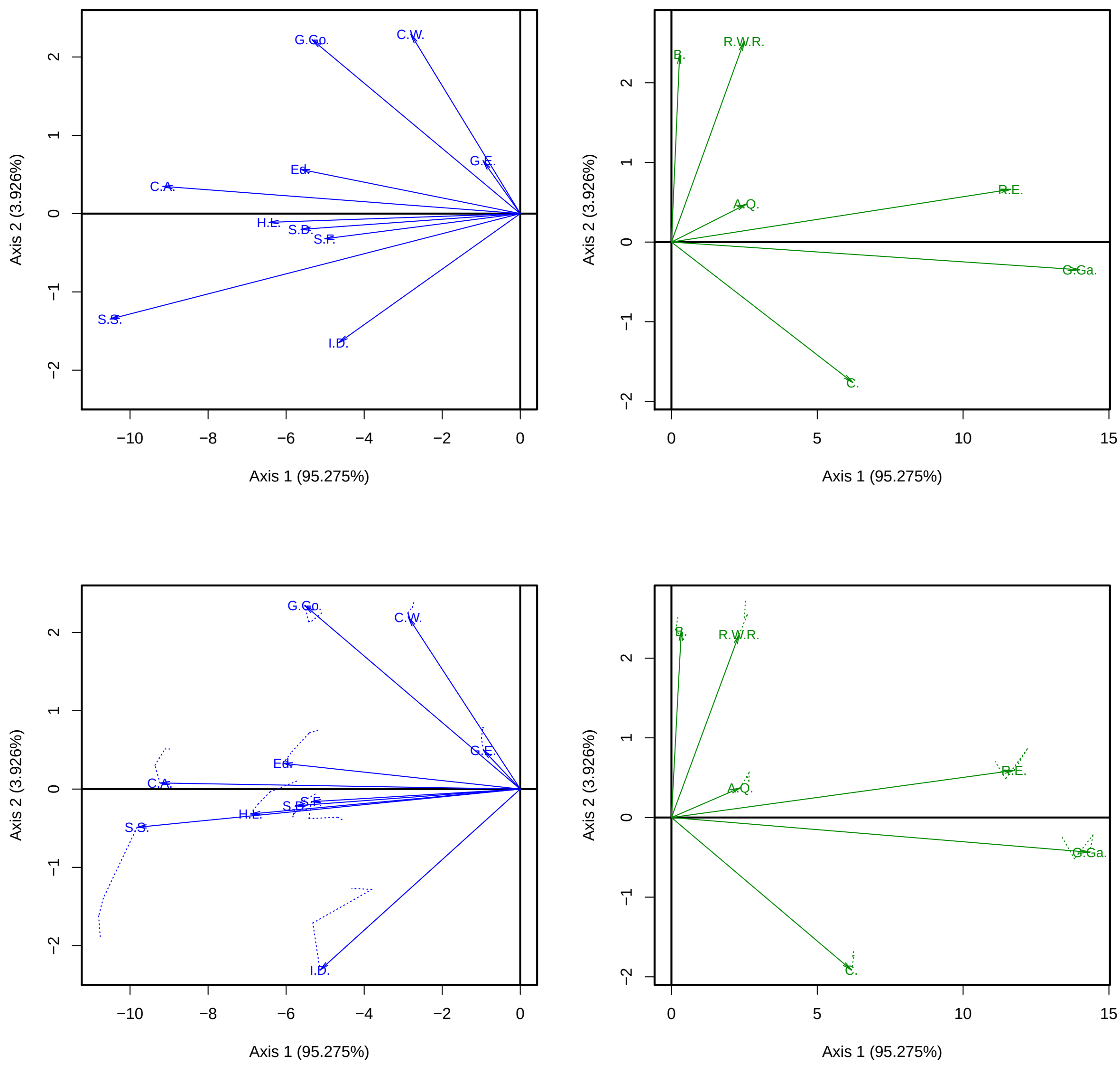


Figura 7.22: *Compromiso y trayectorias del Análisis STATICO entre variables sociales y medioambientales*

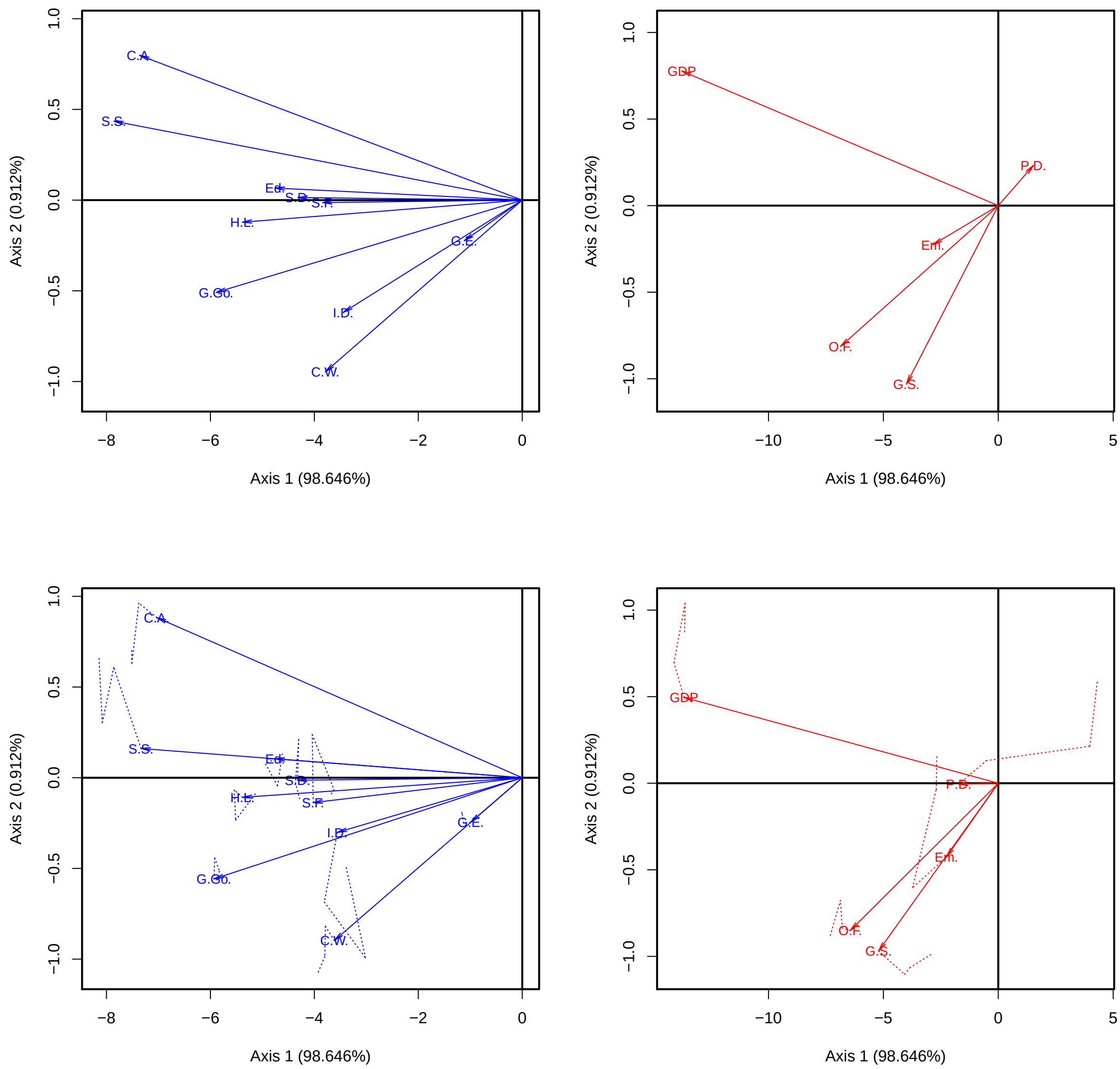


Figura 7.23: *Compromiso y trayectorias del Análisis STATICO entre variables sociales y económicas*

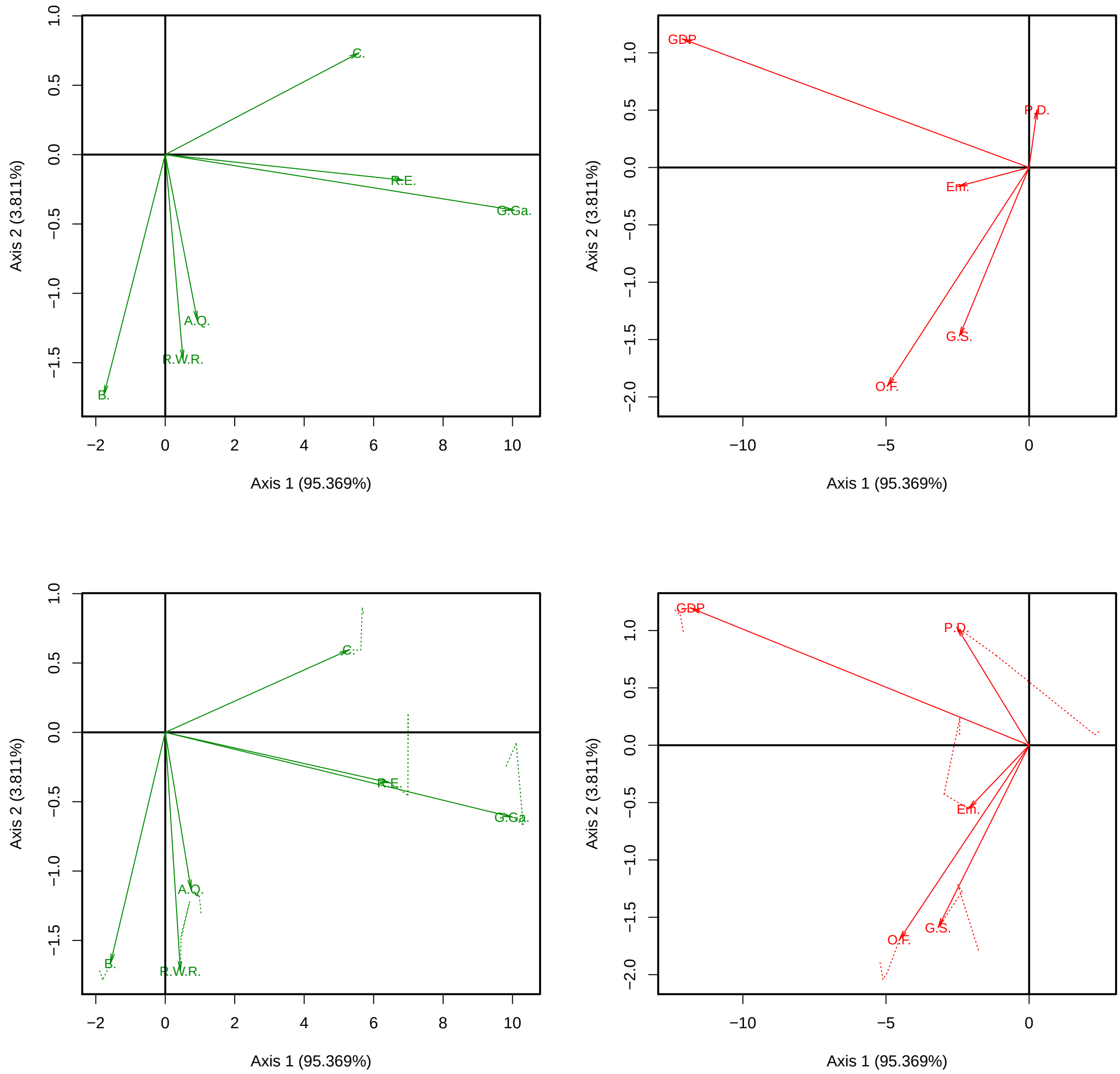


Figura 7.24: *Compromiso y trayectorias del Análisis STATICO entre variables medioambientales y económicas*

Con el análisis COSTATIS, los gráficos que se obtienen hablan sobre la Co-Inercia entre los compromisos después de realizar los dos PTAs, con flechas uniendo cada país según un compromiso con el mismo país del otro compromiso. Así que la interpretación se lleva a cabo de la misma manera que con el BGCOIA pero para paśes en vez de años, puesto que ahora con el COSTATIS solo se puede hablar del comportamiento de los países de media a lo largo del tiempo.

De las figuras 7.25 y 7.27, en las que intervienen los indicadores medioambientales, se puede deducir que los países con un nivel de ingresos bajo tienen vectores mucho más cortos que el resto de países, lo que se traduce en que, de media, las variables sociales o económicas tienen una relación inversa con las medioambientales en la mayoría de los países, y que los países con ingresos bajos se comportan de manera muy similar para las variables medioambientales cuando se estudian con las sociales o las económicas.

Por otro lado, en la figura 7.26, en la que se han analizado las relaciones entre indicadores sociales y económicos, se observa en general que, de media para todos los años de estudio, todos los países, sin importar su nivel de ingresos, tienen vectores cortos, lo que significa que estos indicadores están fuertemente relacionados entre sí independientemente del nivel de ingresos de los países. Pero de todas formas los países se siguen situando en un gradiente por orden de nivel de ingresos, lo que significa que, de nuevo, los países con el mismo nivel de ingresos se siguen comportando de la misma manera.

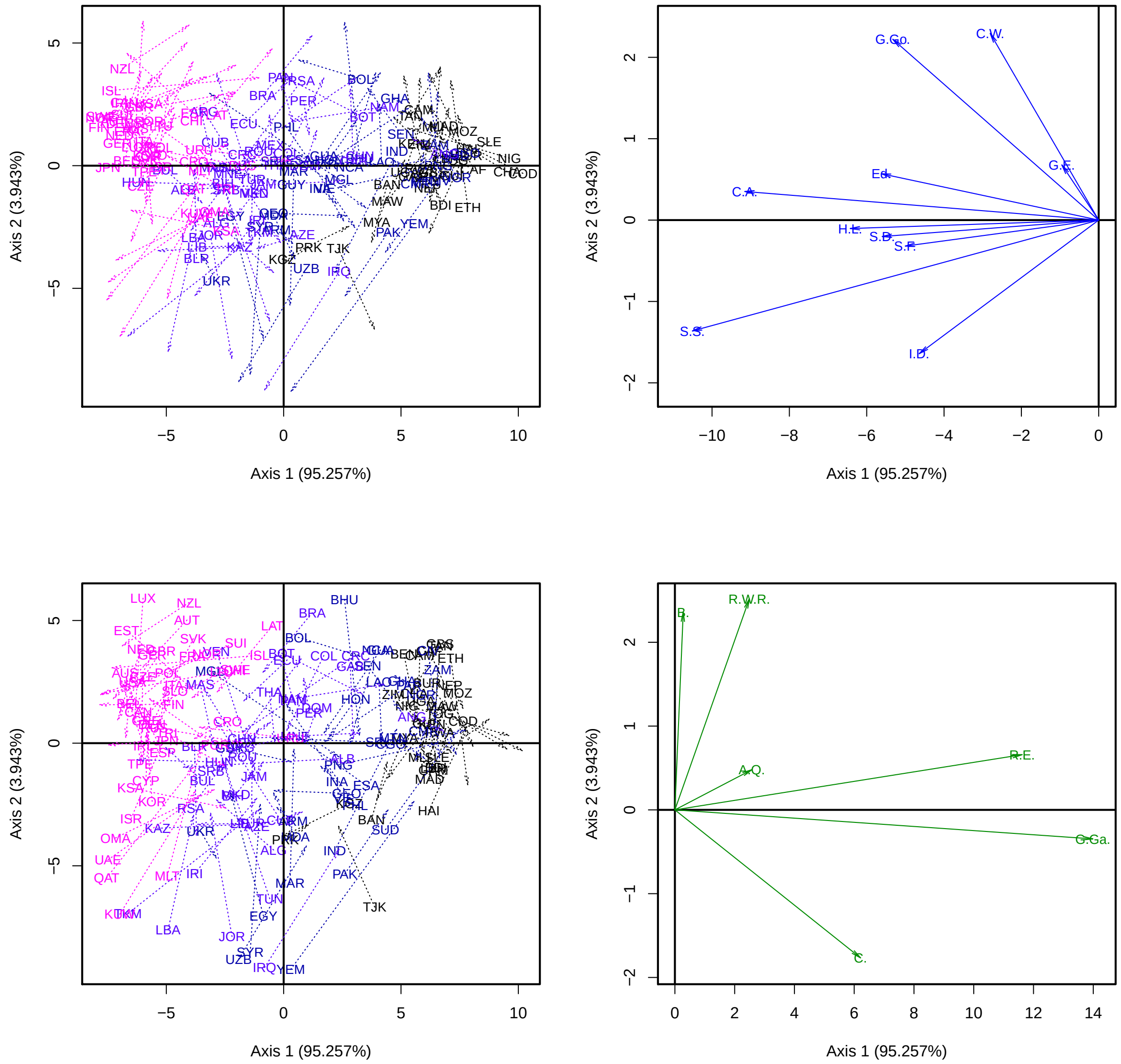


Figura 7.25: *Análisis de Co-Inercia del COSTATIS entre variables sociales y medioambientales para todos los países y años*

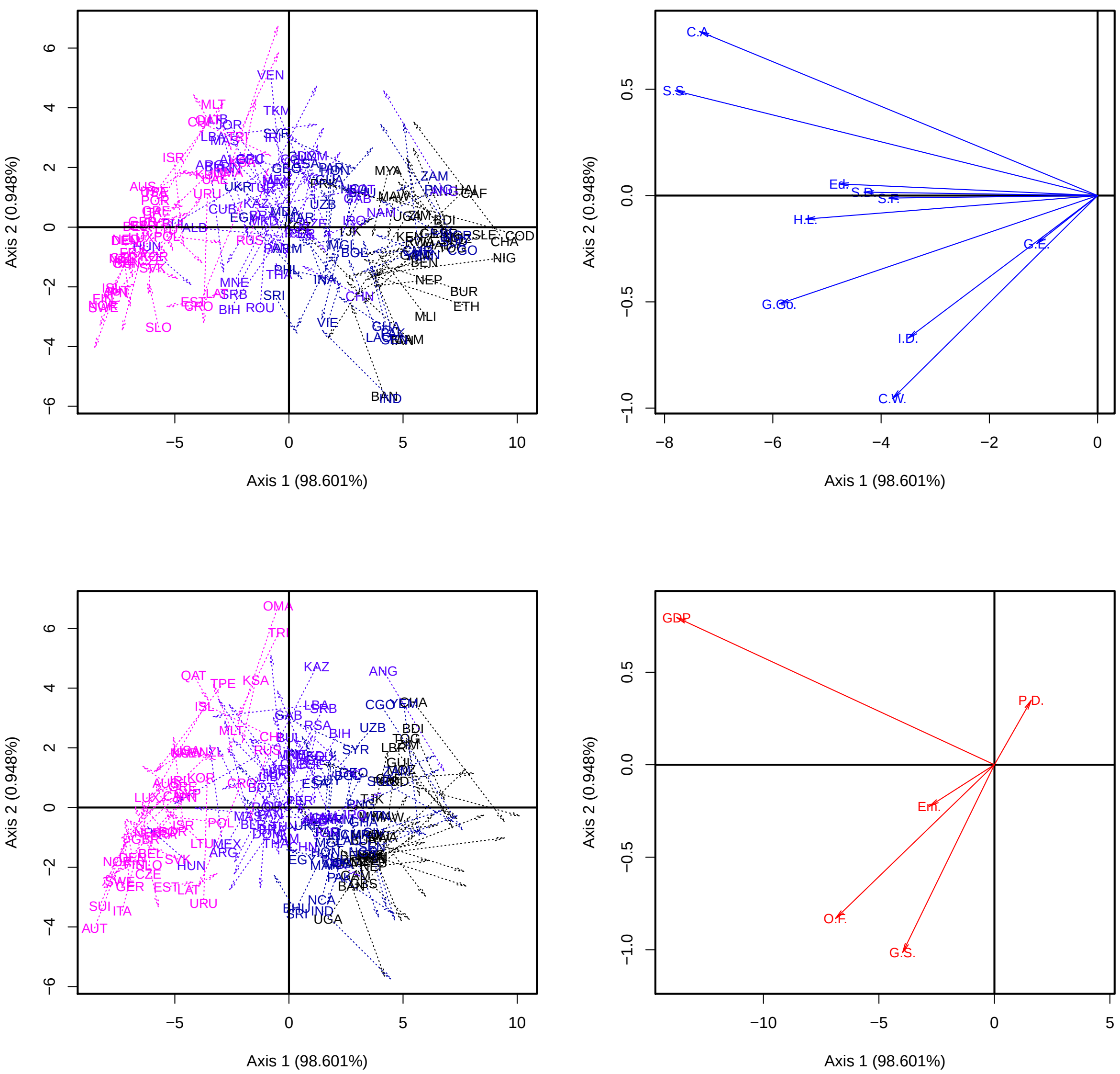


Figura 7.26: *Análisis de Co-Inercia del COSTATIS entre variables sociales y económicas para todos los países y años*

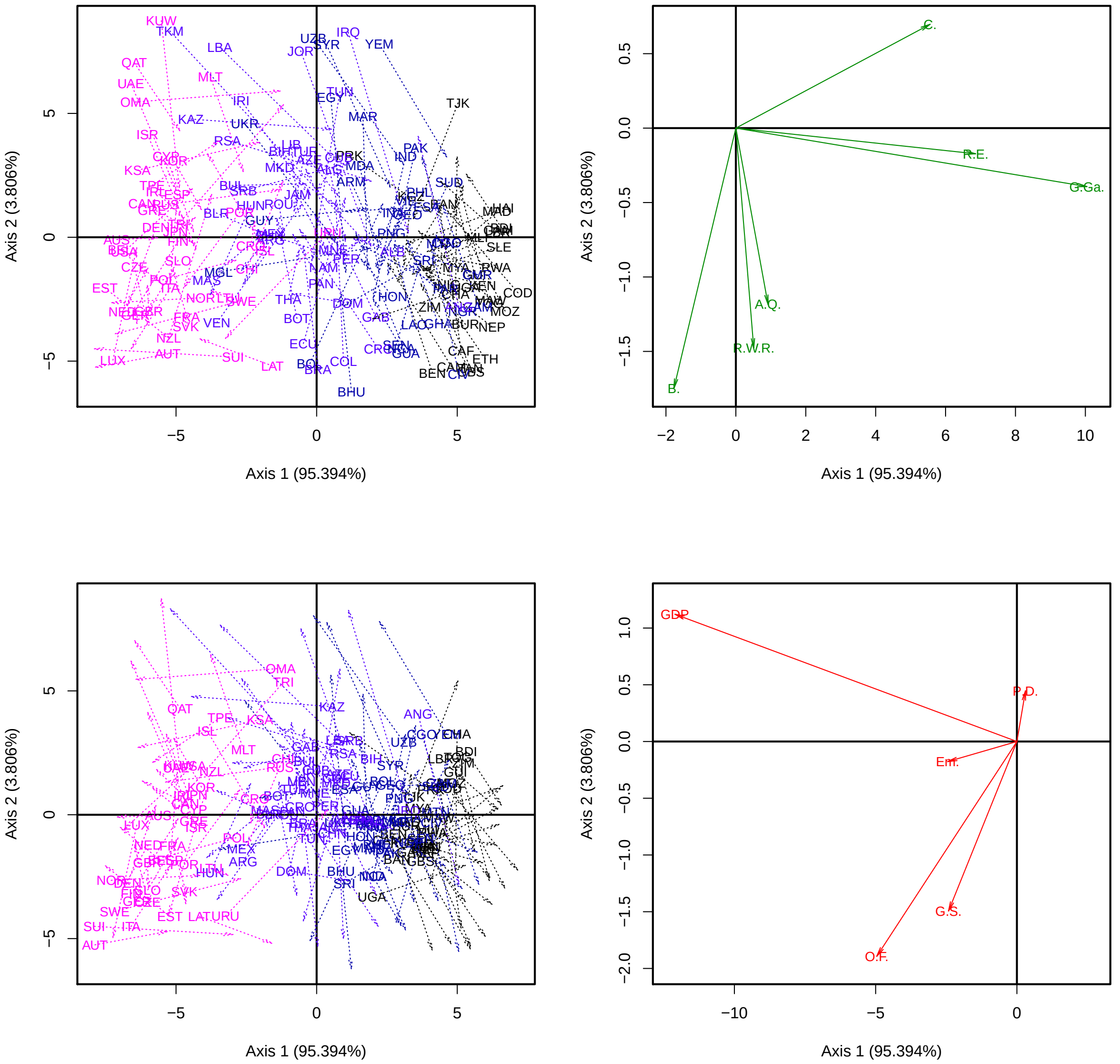


Figura 7.27: *Análisis de Co-Inercia del COSTATIS entre variables medioambientales y económicas para todos los países y años*

En este punto del trabajo se presentan los gráficos de las trayectorias de los países y de las variables, uno de cada análisis (BGCOIA, STATICO Y COSTATIS), agrupados de tres en tres.

A continuación se interpretan algunos de los países individualmente de forma más clara (figuras 7.28, 7.29 y 7.30). Los gráficos con las trayectorias para todos los países y todas las variables a lo largo del tiempo en los subespacios correspondientes a cada tipo de variables (figuras D.1, D.2, D.3, D.4, D.5 y D.6) se muestran sin interpretación en el apéndice D, pero como interpretación general se observa que los gráficos de los análisis STATICO y COSTATIS son muy parecidos y con diferencias mínimas.

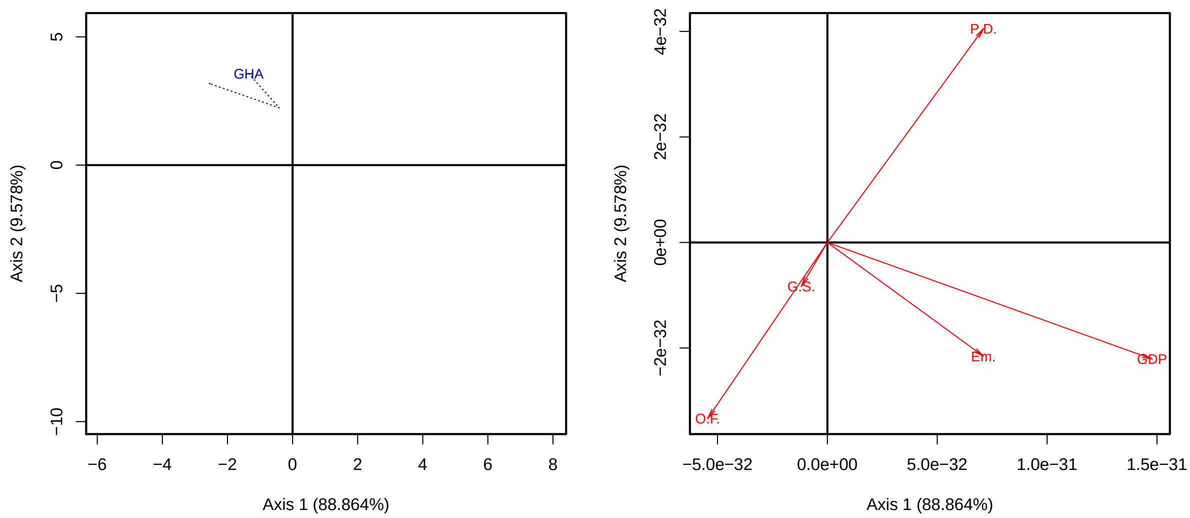


Figura 7.28: *Gráfico de la trayectoria de Ghana con respecto a las variables económicas resultante del Análisis BGCOIA entre variables económicas y medioambientales. Ghana a nivel económico en estos últimos cuatro bienios se caracteriza, en general, por presentar valores medios de Public Debt, P.D., Genuine Savings, G.S., y Organic Farming, O.F., y bajo nivel de Employment, Em., y Gross Domestic Product, GDP. Se observa la siguiente evolución: en 2006 contaba con una baja tasa de Employment, Em., y un bajo Gross Domestic Product, GDP, pero en 2008 sube hasta colocarse casi en la media internacional, para después, en 2010, volver hasta por debajo del punto inicial, que es similar en 2012; pero, además, en estos dos últimos años, Ghana se ha caracterizado por un aumento del nivel de Genuine Savings, G.S., y Organic Farming, O.F.*

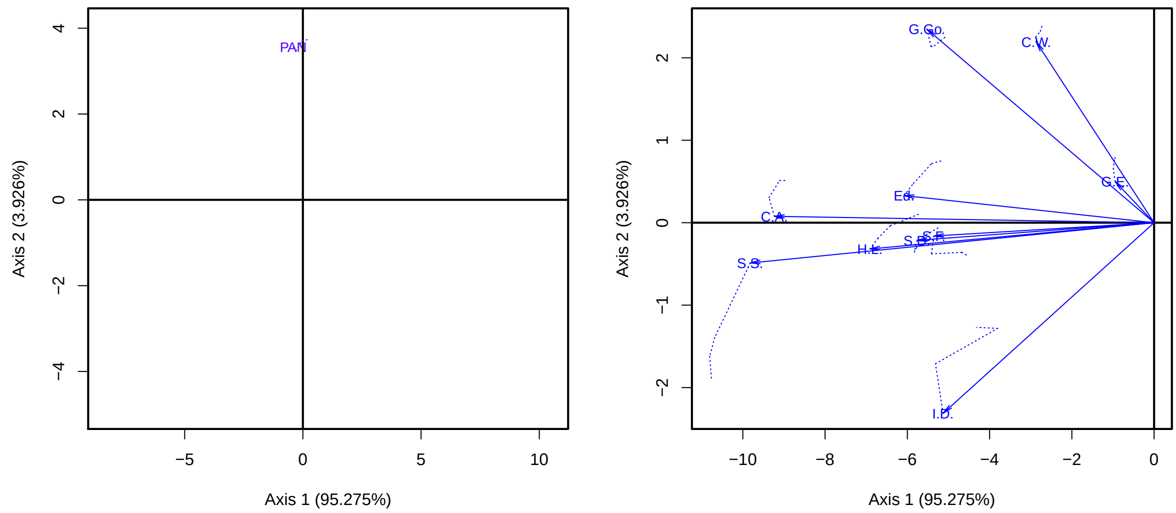


Figura 7.29: *Gráficos de la trayectoria de Panamá con respecto a las trayectorias de las variables sociales resultante del Análisis STATICO entre variables sociales y medioambientales. En este ejemplo, se observa que la trayectoria de Panamá es prácticamente constante, presenta valores altos de todas las variables sociales, entre las que destacan Clean Water, C.W., y Good Governance, G.Go. Este ejemplo sirve para interpretar las trayectorias de las variables: durante todos los años las variables más relacionadas con Panamá fueron Clean Water, C.W., y Good Governance, G.Go., pero existen variables que a lo largo de los años han comenzado a ser de más interés por parte de Panamá como son Clean Air, C.A., Education, Ed., Healthy Life, H.L., e Income Distribution, I.D. (esta última de forma más leve), mientras que el tema de Safe Sanitation, S.S., se aleja cada vez más de Panamá hasta convertirse casi totalmente independiente de este país.*

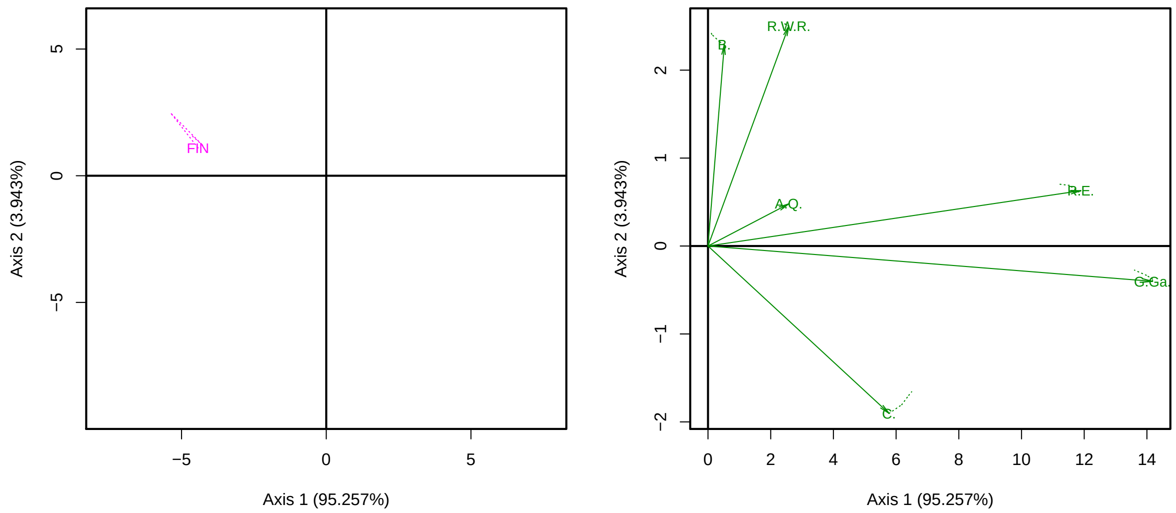


Figura 7.30: *Gráfico de la trayectoria de Finlandia con respecto a las trayectorias de las variables medioambientales resultante del Análisis COSTATIS entre variables medioambientales y sociales. Finlandia durante los últimos cuatro bienios se ha caracterizado por presentar bajos valores en todas las variables medioambientales: Air Quality, A.Q., Consumption, C., Greenhouse Gases, G.Ga., Renewable Energy, R.E, y Renewable Water Resources, R.W.R.; excepto de Biodiversity, B., que presenta valores por encima de la media internacional. Se observa una evolución de tipo cíclica en cuanto a la variable Consumption, C.: en 2008 presenta valores inferiores a 2006 para después, en 2010 y 2012 volver al punto inicial (si se observan las figuras D.2 y D.5 del apéndice D, no solo las resultantes del COSTATIS sino las del BGCOIA y STATICO también, se puede apreciar que, en general, todos los países evolucionan con respecto a la variable Consumption, C., tanto en un sentido como en el opuesto).*

7.3.2. Co-Tucker3

Presentemos ahora los resultados tras los análisis con el método Co-Tucker3. Lo primero es escoger cuántas componentes retener para cada una de las dimensiones, la de los países, la de los indicadores, y la de los años, para cada uno de los tres análisis, para estudiar lo social frente a lo medioambiental, lo social frente a lo económico y lo medioambiental frente a lo económico. Para ello nos fijamos en las dos primeras tablas que se obtienen como resultados en cada análisis, las tablas 7.5, 7.7 y 7.9 que muestran todas las combinaciones posibles de componentes con sus porcentajes de varianza explicada, y las tablas 7.6, 7.8 y 7.10 que resumen las tablas anteriores para aquellas combinaciones que tengan una mejor varianza explicada para una suma de componentes fijada.

Tabla 7.5: Todas las combinaciones con el Co-Tucker3 entre variables sociales y medioambientales

Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones	Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones
1	1x1x1	3	*	24569.000	53.007	5	59	3x5x3	11		8981.313	82.822	13
6	1x2x2	5		24540.368	53.062	6	60	3x5x4	12		8979.166	82.826	13
11	1x3x3	7		24536.168	53.070	10	64	4x1x4	9		24404.445	53.322	5
16	1x4x4	9		24535.671	53.071	9	66	4x2x2	8		13963.652	73.292	17
22	2x1x2	5		24453.295	53.229	8	67	4x2x3	9		13926.144	73.364	28
25	2x2x1	5	*	14372.657	72.510	18	68	4x2x4	10		13921.375	73.373	13
26	2x2x2	6	*	14355.007	72.543	21	70	4x3x2	9		8669.029	83.419	17
27	2x2x3	7		14354.035	72.545	21	71	4x3x3	10		8662.313	83.432	17
28	2x2x4	8		14354.003	72.545	21	72	4x3x4	11		8659.345	83.437	17
30	2x3x2	7		14337.916	72.576	12	73	4x4x1	9	*	5692.008	89.113	10
31	2x3x3	8		14335.295	72.581	11	74	4x4x2	10	*	5629.892	89.232	10
32	2x3x4	9		14334.863	72.582	11	75	4x4x3	11		5623.451	89.244	11
34	2x4x2	8		14336.879	72.578	11	76	4x4x4	12		5618.374	89.254	10
35	2x4x3	9		14333.202	72.585	12	78	4x5x2	11		5593.943	89.301	11
36	2x4x4	10		14332.667	72.586	11	79	4x5x3	12		5585.417	89.317	11
39	2x5x3	10		14332.029	72.587	8	80	4x5x4	13		5579.505	89.328	12
40	2x5x4	11		14330.996	72.589	9	87	5x2x3	10		13838.196	73.532	36
43	3x1x3	7		24418.919	53.294	20	88	5x2x4	11		13829.685	73.548	16
46	3x2x2	7		14042.614	73.141	16	90	5x3x2	10		8588.455	83.573	16
47	3x2x3	8		14036.771	73.152	16	91	5x3x3	11		8542.145	83.662	27
48	3x2x4	9		14036.042	73.154	16	92	5x3x4	12		8535.131	83.675	17
49	**3x3x1**	**7**	*	**9041.929**	**82.706**	**12**	94	5x4x2	11		5305.037	89.853	13
50	**3x3x2**	**8**	*	**9014.115**	**82.759**	**17**	95	5x4x3	12		5296.384	89.870	13
51	3x3x3	9		9010.218	82.766	17	96	5x4x4	13		5289.898	89.882	13
52	3x3x4	10		9008.896	82.769	17	97	5x5x1	11	*	3813.064	92.707	67
54	3x4x2	9		8990.413	82.804	17	98	5x5x2	12	*	3737.423	92.851	75
55	3x4x3	10		8984.687	82.815	17	99	5x5x3	13	*	3726.767	92.872	69
56	3x4x4	11		8982.746	82.819	17	100	5x5x4	14	*	3718.881	92.887	70
58	3x5x2	10		8987.374	82.810	13							

Tabla 7.6: Combinaciones con mejor ajuste en el Co-Tucker3 entre variables sociales y medioambientales

Número	Modelo	S	Suma de Cuadrados Residual	Diferencia del Ajuste	Porcentaje del Ajuste	Número de Iteraciones
1	1x1x1	3	24569.000	53.007	53.007	5
25	2x2x1	5	14372.657	19.502	72.510	18
26	2x2x2	6	14355.007	0.034	72.543	21
49	**3x3x1**	**7**	**9041.929**	**10.162**	**82.706**	**12**
50	**3x3x2**	**8**	**9014.115**	**0.053**	**82.759**	**17**
73	4x4x1	9	5692.008	6.354	89.113	10
74	4x4x2	10	5629.892	0.119	89.232	10
97	5x5x1	11	3813.064	3.475	92.707	67
98	5x5x2	12	3737.423	0.145	92.851	75
99	5x5x3	13	3726.767	0.020	92.872	69
100	5x5x4	14	3718.881	0.015	92.887	70

Tabla 7.7: Todas las combinaciones con el Co-Tucker3 entre variables sociales y económicas

Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones	Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones
1	1x1x1	3	*	23658.352	49.521	13	59	3x5x3	11		10329.416	77.961	13
6	1x2x2	5		23471.747	49.920	7	60	3x5x4	12		10323.415	77.973	13
11	1x3x3	7		23460.698	49.943	8	64	4x1x4	9		23374.795	50.126	8
16	1x4x4	9		23458.522	49.948	7	66	4x2x2	8		14374.903	69.329	17
22	2x1x2	5		23470.854	49.921	17	67	4x2x3	9		14263.381	69.567	28
25	2x2x1	5	*	15598.668	66.718	18	68	4x2x4	10		14258.664	69.577	13
26	2x2x2	6	*	15396.049	67.150	21	70	4x3x2	9		9479.921	79.773	42
27	2x2x3	7		15395.163	67.152	21	71	4x3x3	10		9458.968	79.818	43
28	2x2x4	8		15395.114	67.152	21	72	4x3x4	11		9453.411	79.830	43
30	2x3x2	7		15369.167	67.208	12	73	4x4x1	9	*	6705.658	85.692	13
31	2x3x3	8		15364.299	67.218	16	74	4x4x2	10	*	6438.726	86.262	18
32	2x3x4	9		15363.633	67.219	11	75	4x4x3	11		6410.252	86.323	18
34	2x4x2	8		15367.395	67.211	11	76	4x4x4	12		6399.497	86.346	18
35	2x4x3	9		15355.584	67.237	12	78	4x5x2	11		6402.756	86.339	11
36	2x4x4	10		15353.186	67.242	11	79	4x5x3	12		6371.106	86.406	11
39	2x5x3	10		15353.551	67.241	8	80	4x5x4	13		6359.138	86.432	12
40	2x5x4	11		15350.682	67.247	9	87	5x2x3	10		14104.048	69.907	22
43	3x1x3	7		23404.486	50.063	19	88	5x2x4	11		14093.035	69.930	16
46	3x2x2	7		14523.925	69.011	16	90	5x3x2	10		9189.623	80.393	39
47	3x2x3	8		14517.067	69.026	16	91	5x3x3	11		9124.954	80.531	43
48	3x2x4	9		14516.432	69.027	16	92	5x3x4	12		9115.376	80.551	43
49	**3x3x1**	**7**	*	**10608.009**	**77.366**	**36**	94	5x4x2	11		5559.974	88.137	18
50	**3x3x2**	**8**	*	**10384.786**	**77.843**	**43**	95	5x4x3	12		5527.192	88.207	18
51	3x3x3	9		10367.612	77.879	44	96	5x4x4	13		5514.847	88.233	18
52	3x3x4	10		10363.414	77.888	44	97	5x5x1	11	*	4398.413	90.615	26
54	3x4x2	9		10361.752	77.892	22	98	5x5x2	12	*	4046.941	91.365	27
55	3x4x3	10		10334.040	77.951	18	99	5x5x3	13	*	4009.932	91.444	27
56	3x4x4	11		10328.725	77.962	18	100	5x5x4	14	*	3995.516	91.475	27
58	3x5x2	10		10357.924	77.900	13							

Tabla 7.8: Combinaciones con mejor ajuste en el Co-Tucker3 entre variables sociales y económicas

Número	Modelo	S	Suma de Cuadrados Residual	Diferencia del Ajuste	Porcentaje del Ajuste	Número de Iteraciones
1	1x1x1	3	23658.352	49.521	49.521	13
25	2x2x1	5	15598.668	17.197	66.718	18
26	2x2x2	6	15396.049	0.432	67.150	21
49	**3x3x1**	**7**	**10608.009**	**10.216**	**77.366**	**36**
50	**3x3x2**	**8**	**10384.786**	**0.476**	**77.843**	**43**
73	4x4x1	9	6705.658	7.850	85.692	13
74	4x4x2	10	6438.726	0.570	86.262	18
97	5x5x1	11	4398.413	4.353	90.615	26
98	5x5x2	12	4046.941	0.750	91.365	27
99	5x5x3	13	4009.932	0.079	91.444	27
100	5x5x4	14	3995.516	0.031	91.475	27

Tabla 7.9: Todas las combinaciones con el Co-Tucker3 entre variables medioambientales y económicas

Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones	Número	Modelo	Suma	Mejor Fijada Suma	Suma de cuadrados residual	Porcentaje de Ajuste	Número de Iteraciones
1	1x1x1	3	*	29469.251	41.499	13	59	3x5x3	11		10200.337	79.751	5
6	1x2x2	5		29306.410	41.822	7	60	3x5x4	12		10195.754	79.760	6
11	1x3x3	7		29299.321	41.836	10	64	4x1x4	9		29293.657	41.847	8
16	1x4x4	9		29297.567	41.839	9	66	4x2x2	8		15960.667	68.315	10
22	2x1x2	5		29355.832	41.724	17	67	4x2x3	9		15883.905	68.468	13
25	2x2x1	5	*	16835.529	66.579	10	68	4x2x4	10		15883.613	68.468	13
26	2x2x2	6	*	16647.516	66.952	11	70	4x3x2	9		9619.466	80.904	42
27	2x2x3	7		16647.207	66.953	11	71	4x3x3	10		9603.525	80.935	43
28	2x2x4	8		16647.171	66.953	11	72	4x3x4	11		9600.072	80.942	43
30	2x3x2	7		16633.236	66.980	8	73	4x4x1	9	*	5795.957	88.494	13
31	2x3x3	8		16630.319	66.986	16	74	4x4x2	10	*	5582.846	88.917	18
32	2x3x4	9		16630.015	66.987	9	75	4x4x3	11		5558.500	88.965	18
34	2x4x2	8		16632.397	66.982	6	76	4x4x4	12		5551.430	88.979	18
35	2x4x3	9		16623.479	67.000	9	78	4x5x2	11		5580.091	88.923	7
36	2x4x4	10		16621.498	67.004	9	79	4x5x3	12		5552.090	88.978	7
39	2x5x3	10		16622.552	67.001	5	80	4x5x4	13		5544.403	88.993	7
40	2x5x4	11		16620.508	67.006	3	87	5x2x3	10		15802.306	68.630	36
43	3x1x3	7		29314.610	41.806	20	88	5x2x4	11		15798.838	68.637	12
46	3x2x2	7		16045.920	68.146	11	90	5x3x2	10		9392.970	81.353	39
47	3x2x3	8		16044.057	68.150	11	91	5x3x3	11		9370.492	81.398	43
48	3x2x4	9		16043.890	68.150	11	92	5x3x4	12		9366.007	81.407	43
49	**3x3x1**	**7**	*	**10429.422**	**79.296**	**36**	94	5x4x2	11		4977.559	90.119	18
50	**3x3x2**	**8**	*	**10228.415**	**79.695**	**43**	95	5x4x3	12		4950.601	90.172	18
51	3x3x3	9		10214.145	79.723	44	96	5x4x4	13		4943.092	90.187	18
52	3x3x4	10		10211.084	79.729	44	97	5x5x1	11	*	3020.747	94.003	67
54	3x4x2	9		10225.011	79.702	22	98	5x5x2	12	*	2732.469	94.576	75
55	3x4x3	10		10201.741	79.748	18	99	5x5x3	13	*	2702.409	94.635	69
56	3x4x4	11		10197.816	79.756	18	100	5x5x4	14	*	2694.285	94.651	70
58	3x5x2	10		10224.179	79.703	6							

Tabla 7.10: Combinaciones con mejor ajuste en el Co-Tucker3 entre variables medioambientales y económicas

Número	Modelo	S	Suma de Cuadrados Residual	Diferencia del Ajuste	Porcentaje del Ajuste	Número de Iteraciones
1	1x1x1	3	29469.251	41.499	41.499	13
25	2x2x1	5	16835.529	25.080	66.579	10
26	2x2x2	6	16647.516	0.373	66.952	11
49	**3x3x1**	**7**	**10429.422**	**12.344**	**79.296**	**36**
50	**3x3x2**	**8**	**10228.415**	**0.399**	**79.695**	**43**
73	4x4x1	9	5795.957	8.799	88.494	13
74	4x4x2	10	5582.846	0.423	88.917	18
97	5x5x1	11	3020.747	5.086	94.003	67
98	5x5x2	12	2732.469	0.572	94.576	75
99	5x5x3	13	2702.409	0.060	94.635	69
100	5x5x4	14	2694.285	0.016	94.651	70

Para el análisis entre los indicadores sociales y los medioambientales, en la segunda tabla se ve que las combinaciones 3x3x1 y 3x3x2 tienen unos porcentajes de ajuste del 82.706 % y 82.759 %, que ya son unos porcentajes suficientemente altos. Además, el incremento en la varianza explicada si se considerase el siguiente modelo más complejo (4x4x1) solo sería de un 6.354 % como se ve en la columna de Diferencia en el Ajuste, lo que ya se considera como insignificante desde el punto de vista estadístico.

Para los análisis entre los indicadores sociales y económicos, y entre los medioambientales y los económicos, de forma similar, las mejores combinaciones son también la 3x3x1 y la 3x3x2. También, a la hora de elegir la combinación de componentes, se pueden usar los gráficos en los que están representados todos los modelos según la suma del número de sus componentes frente a la suma de cuadrados residual, tal como se explicó en el apartado de desarrollo y de forma similar a lo explicado en los resultados para el Tucker3. En este caso, los gráficos serían los siguientes (Figuras 7.31, 7.32 y 7.33)

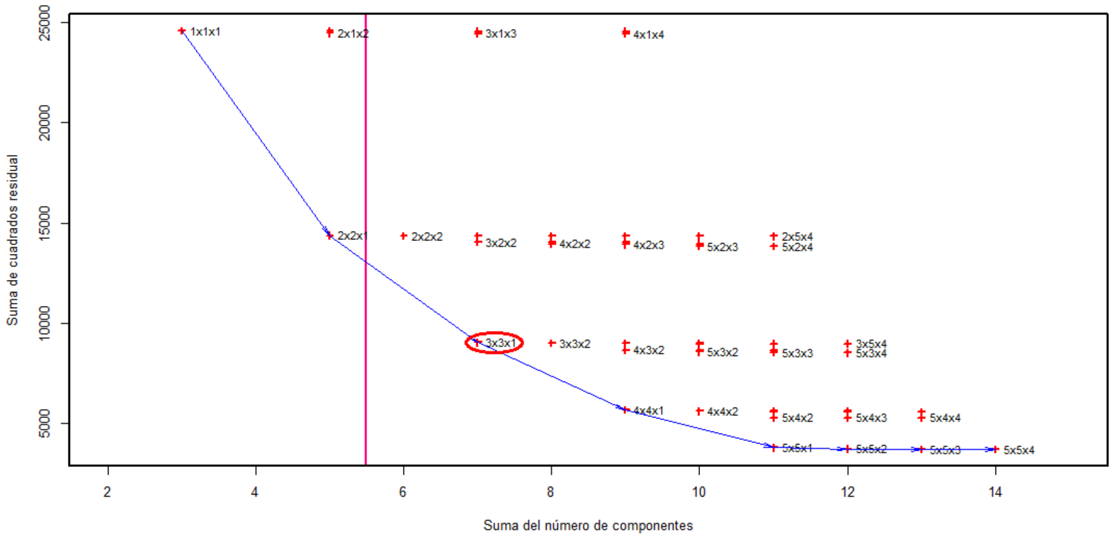


Figura 7.31: *Suma del número de componentes vs. Suma de cuadrados residual del Tucker3 en el análisis Co-Tucker3 entre variables sociales y medioambientales*

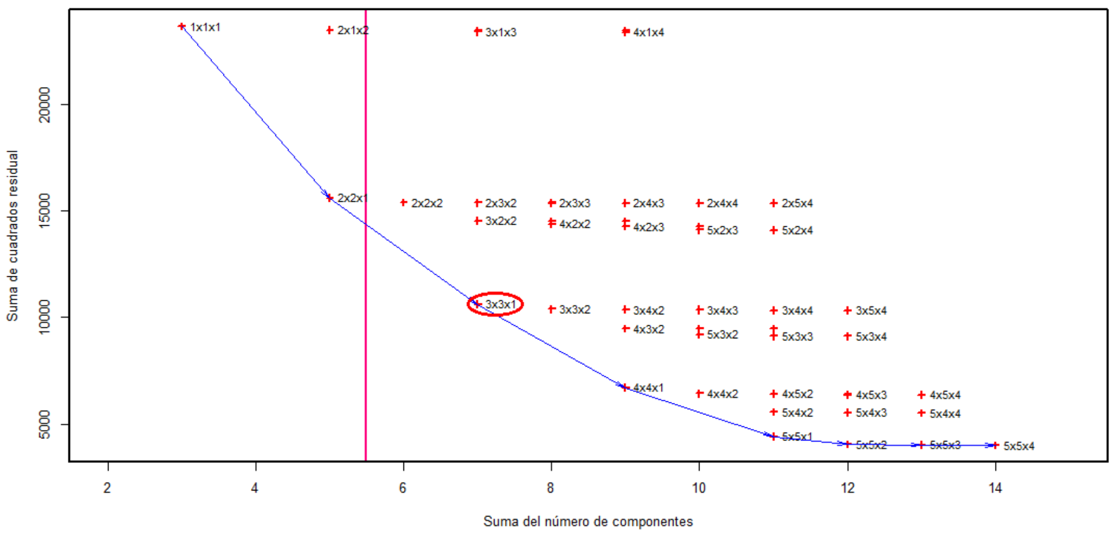


Figura 7.32: *Suma del número de componentes vs. Suma de cuadrados residual del Tucker3 en el análisis Co-Tucker3 entre variables sociales y económicas*

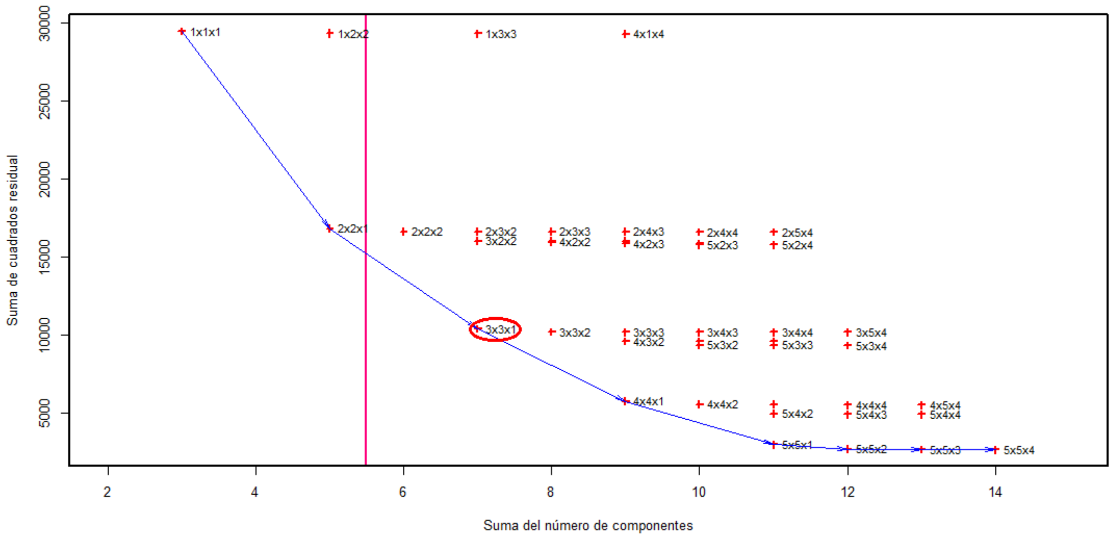


Figura 7.33: *Suma del número de componentes vs. Suma de cuadrados residual del Tucker3 en el análisis Co-Tucker3 entre variables medioambientales y económicas*

En los tres se observa que el modelo 3x3x1 es uno de los más simples (tiene la menor suma de cuadrados residual para los modelos que tienen la misma suma del número de componentes) y también es el primero de los más estables (los modelos posteriores tienen una reducción en la suma de cuadrados residual insignificante desde el punto de vista estadístico). Sin embargo, se va a elegir el modelo 3x3x2, porque, al igual que ocurría en el Tucker3, si no se consideran dos componentes para la tercera dimensión no se verán diferencias entre los años, y ver cómo nuestros datos evolucionan en el tiempo es lo que interesa.
Los siguientes resultados que se presentan son después de llevar a cabo la segunda parte de los análisis Co-Tucker3, esto es, fijando como componentes el modelo 3x3x2. Las primeras tablas que se obtienen son las tablas 7.11, las tablas resumen donde se recogen los desgloses de las varianzas explicadas por cada uno de los componentes de cada una de las dimensiones, para cada uno de los tres tipos de variables.

Tabla 7.11: Porcentajes de Ajuste con el Tucker3 para las variables sociales, medioambientales y económicas

Componente	Dimensión 1	Dimensión 2	Dimensión 3
1	61.593	61.570	81.091
2	11.542	11.593	0.108
3	8.064	8.036	0
Varianza explicada Total	81.199	81.199	81.199
Componente	Dimensión 1	Dimensión 2	Dimensión 3
1	45.546	45.550	84.112
2	26.480	26.479	0.010
3	12.096	12.093	0
Varianza explicada Total	84.123	84.123	84.123
Componente	Dimensión 1	Dimensión 2	Dimensión 3
1	37.188	36.454	73.250
2	23.518	24.200	0.950
3	13.494	13.547	0
Varianza explicada Total	74.201	74.201	74.201

Pero ya se sabe que son más interesantes las siguientes tablas (Tabla 7.12), los llamados core arrays puestos en forma de matrices, las matrices core. Se refieren a las varianzas explicadas, pero considerando las diferentes combinaciones de los componentes de las dimensiones para cada uno de los tres tipos de variables. Además, se muestran los signos para interpretar las interacciones entre los componentes de las dimensiones que son estadísticamente significantes, esto es, que no son nulas.

Tabla 7.12: Matriz Core en el Tucker3 para las variables sociales, medioambientales y económicas

			Componentes Modo 2 Suma de Cuadrados Residual			Componentes Modo 2 Varianza Explicada		
			1	2	3	1	2	3
Modo 3, Componente 1	Componentes Modo 1	1	**122.506**	0.101	-0.068	**61.536**	0.000	0.000
		2	0.096	-53.028	-0.102	0.000	11.530	0.000
		3	-0.083	0.032	-44.241	0.000	0.000	8.025
Modo 3, Componente 2	Componentes Modo 1	1	-1.241	3.446	-0.751	0.006	0.049	0.002
		2	0.480	-1.666	0.124	0.001	0.011	0.000
		3	-2.580	0.843	-1.425	0.027	0.003	0.008

			Componentes Modo 2 Suma de Cuadrados Residual			Componentes Modo 2 Varianza Explicada		
			1	2	3	1	2	3
Modo 3, Componente 1	Componentes Modo 1	1	**-112.712**	0.033	-0.002	**45.543**	0.000	0.000
		2	0.031	85.938	-0.011	0.000	26.476	0.000
		3	-0.005	0.019	58.078	0.000	0.000	12.093
Modo 3, Componente 2	Componentes Modo 1	1	-0.393	-0.539	0.468	0.001	0.001	0.001
		2	-0.808	-0.657	-0.021	0.002	0.002	0.000
		3	-0.952	0.326	0.210	0.003	0.000	0.000

			Componentes Modo 2 Suma de Cuadrados Residual			Componentes Modo 2 Varianza Explicada		
			1	2	3	1	2	3
Modo 3, Componente 1	Componentes Modo 1	1	**-90.381**	-3.018	0.189	**36.338**	0.041	0.000
		2	3.047	-72.428	-0.464	0.041	23.336	0.001
		3	-0.178	0.345	-55.073	0.000	0.001	13.492
Modo 3, Componente 2	Componentes Modo 1	1	3.290	-13.080	0.109	0.048	0.761	0.000
		2	2.421	-3.714	3.438	0.026	0.061	0.053
		3	0.124	0.386	0.310	0.000	0.001	0.000

Para las variables sociales, del total de 81.199 %, reteniendo, por ejemplo, una componente de cada dimensión, se obtiene una varianza explicada del 61.536 %. Para las medioambientales, del total de 84.123 %, se alcanza una varianza explicada del 45.543 %, también reteniendo una componente para cada dimensión; y para las económicas, del 74.201 % se obtiene una varianza del 36.338 % reteniendo una componente de cada dimensión.
A continuación, se presenta uno de los gráficos, con las tres dimensiones, procedente de cada uno de los tres análisis Co-Tucker3 (Figuras 7.34, 7.36 y 7.38). En cada gráfico se representan las mismas dos componentes, la primera en horizontal y la segunda en vertical, que valdrán para estudiar la combinación 1x1x1. Estos gráficos, junto con los resultados de la anterior tabla, sirven para interpretar las interacciones entre países, indicadores y años, cuando se estudian los tres tipos de indicadores.
Para las variables sociales, como el elemento 1x1x1 de la matriz core (Tabla 7.12) es positivo (122.506), los países que tengan coordenadas positivas en la primera componente, es decir, los que estén situados en el semiplano derecho (cuadrantes I y IV) de la figura 7.34, tienen una interacción positiva con todos los indicadores sociales, porque todos tienen coordenadas negativas en la componente primera, todos están situados en el semiplano izquierdo (cuadrantes II y III), y en cualquiera de los cuatro años, porque todos se sitúan en el semiplano izquierdo, todos tienen coordenadas negativas.

$$\text{Países}(+) \times \text{indicadores}(-) \times \text{años}(-) \times \text{matriz core}(+) = \text{interacción}(+)$$

Esto es, en todos los años de estudio, todos los países con ingresos altos y la mayoría con ingresos medio-altos toman valores altos en todas las variables sociales. De la misma forma, en todos los años de estudio, todos los países con ingresos bajos y la mayoría con ingresos medio-bajos toman valores bajos en las variables sociales.

$$\text{Países}(-) \times \text{indicadores}(-) \times \text{años}(-) \times \text{matriz core}(+) = \text{interacción}(-)$$

Estas dos conclusiones a partir del producto de los signos se pueden visualizar fácilmente en la figura 7.35, en la que se han representado los países, indicadores y años según los signos explicados. Además, solo han sido representados aquellos con mejor calidad de representación.

De la misma forma para las variables medioambientales, como el elemento 1x1x1 de la matriz core (Tabla 7.12) ahora es negativo (–112.712), los países que tengan coordenadas positivas en la primera componente de la figura 7.36, tienen una interacción negativa con todos los indicadores medioambientales, porque todos tienen coordenadas negativas en la componente primera, y en cualquiera de los cuatro años, porque todos tienen coordenadas negativas.

$$\text{Países(+)} \times \text{indicadores(–)} \times \text{años(–)} \times \text{matriz core(–)} = \text{interacción(–)}$$

Esto es, en todos los años de estudio, casi todos los países con ingresos altos y la mayoría con ingresos medio-altos toman valores bajos en todas las variables medioambientales. Y por tanto, en todos los años de estudio, todos los países con ingresos bajos y la mayoría con ingresos medio-bajos toman valores altos en las variables medioambientales.

$$\text{Países(–)} \times \text{indicadores(–)} \times \text{años(–)} \times \text{matriz core(–)} = \text{interacción(+)}$$

Estas dos conclusiones a partir del producto de los signos se pueden visualizar fácilmente en la figura 7.37, en la que se han representado los países, indicadores y años según los signos explicados. Además, solo han sido representados aquellos con mejor calidad de representación.

Por último, para las variables económicas, como el elemento 1x1x1 de la matriz core (Tabla 7.12) también es negativo (–90.381), los países que tengan coordenadas positivas en la primera componente de la figura 7.38, tienen una interacción positiva con todos los indicadores económicos excepto Public Debt, P.D., porque estos tienen coordenadas positivas en la componente primera, y en cualquiera de los cuatro años, porque todos tienen coordenadas negativas.

$$\text{Países(+)} \times \text{indicadores(+)} \times \text{años(–)} \times \text{matriz core(–)} = \text{interacción(+)}$$

Lo que significa que en todos los años de estudio, casi todos los países con ingresos altos y la mayoría con ingresos medio-altos toman valores altos en todas las variables económicas excepto en Public Debt, P.D., que toman valores bajos. Mientras que, en todos los años de estudio, todos los países con ingresos bajos y la mayoría con ingresos medio-bajos toman valores bajos en todas las variables económicas excepto en Public Debt, P.D., que toman valores altos.

$$\text{Países}(-) \times \text{indicadores}(+) \times \text{años}(-) \times \text{matriz core}(-) = \text{interacción}(-)$$

Estas dos conclusiones a partir del producto de los signos se pueden visualizar fácilmente en la figura 7.39, en la que se han representado los países, indicadores y años según los signos explicados. Además, solo han sido representados aquellos con mejor calidad de representación.

Se Puede observar que todas estas conclusiones son similares a las obtenidas con cualquiera de los otros métodos.

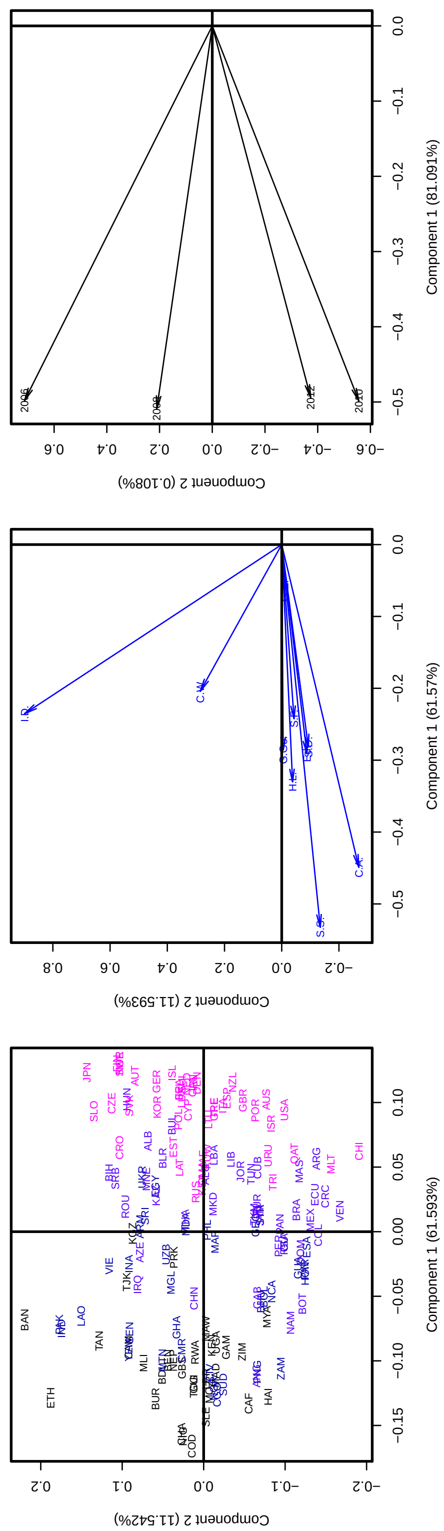


Figura 7.34: Gráfico para las dos primeras componentes de las tres dimensiones del Tucker3 en el análisis Co-Tucker3 para las variables sociales

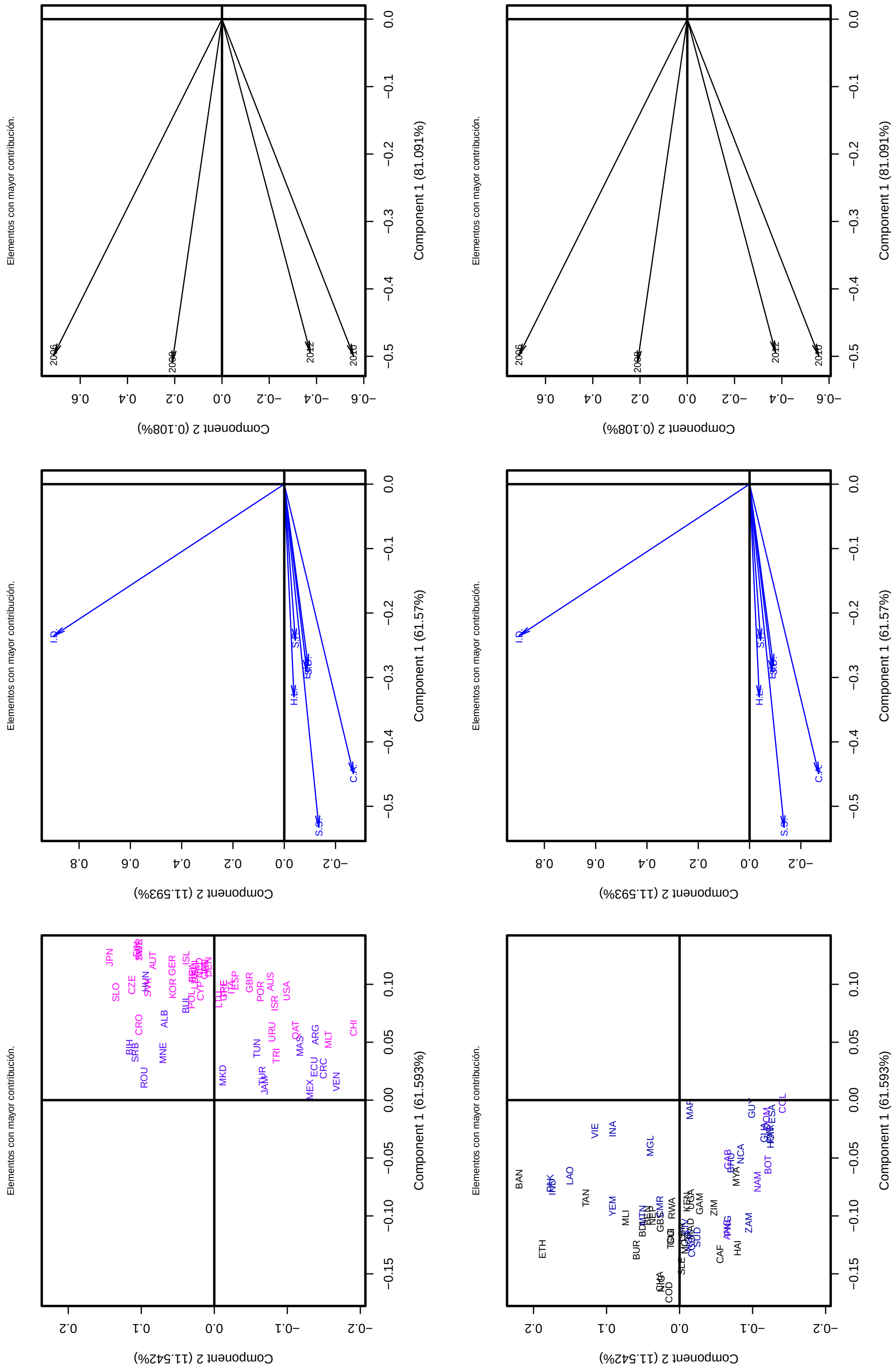


Figura 7.35: Figura 7.34, con solo los países, indicadores y años correspondientes a los signos

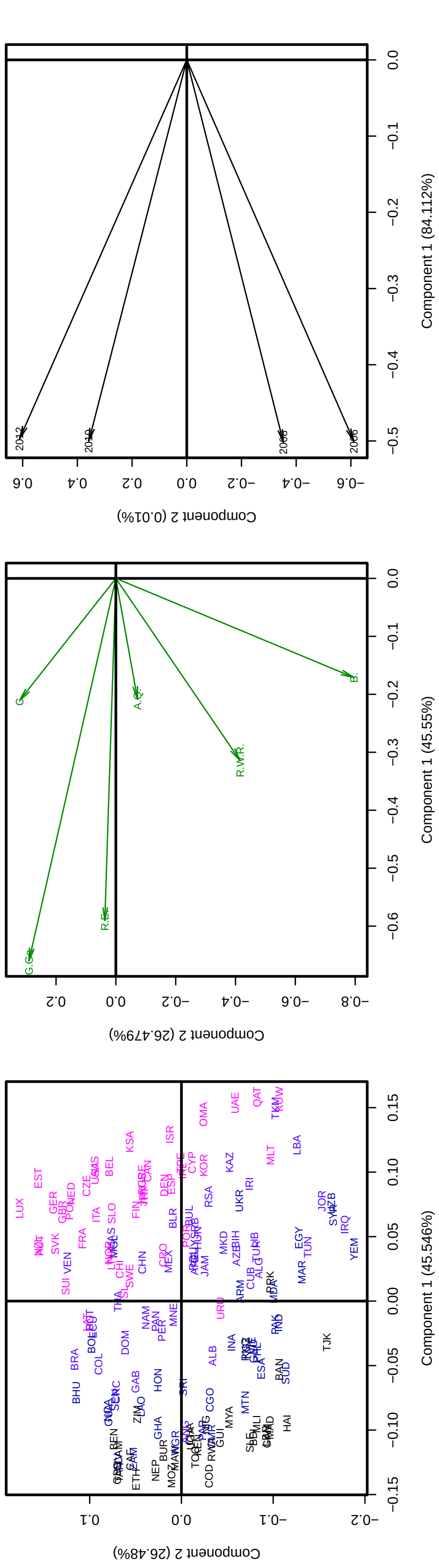


Figura 7.36: *Gráfico para las dos primeras componentes de las tres dimensiones del Tucker3 en el análisis Co-Tucker3 para las variables medioambientales*

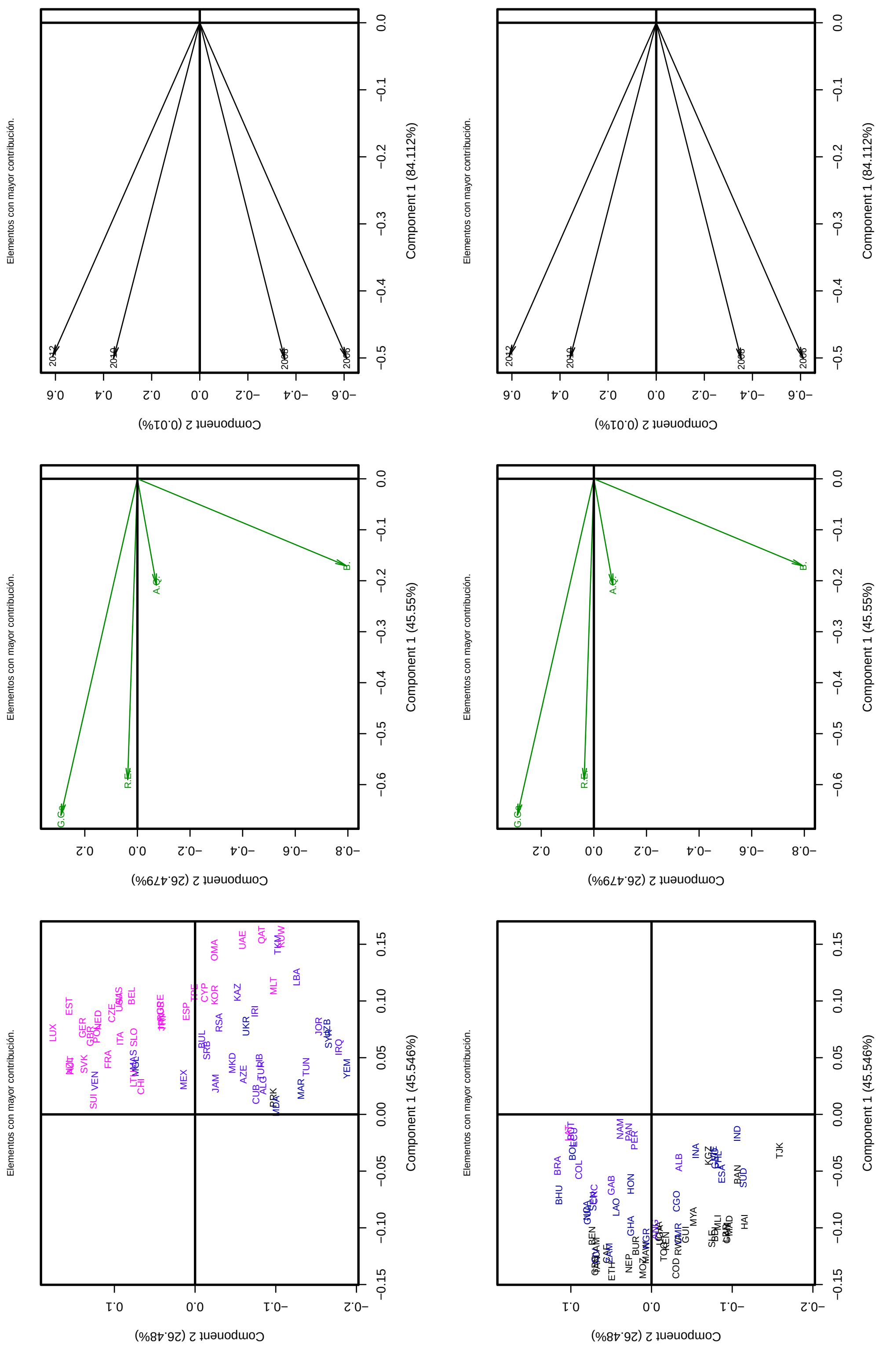


Figura 7.37: *Figura 7.36, con solo los países, indicadores y años correspondientes a los signos*

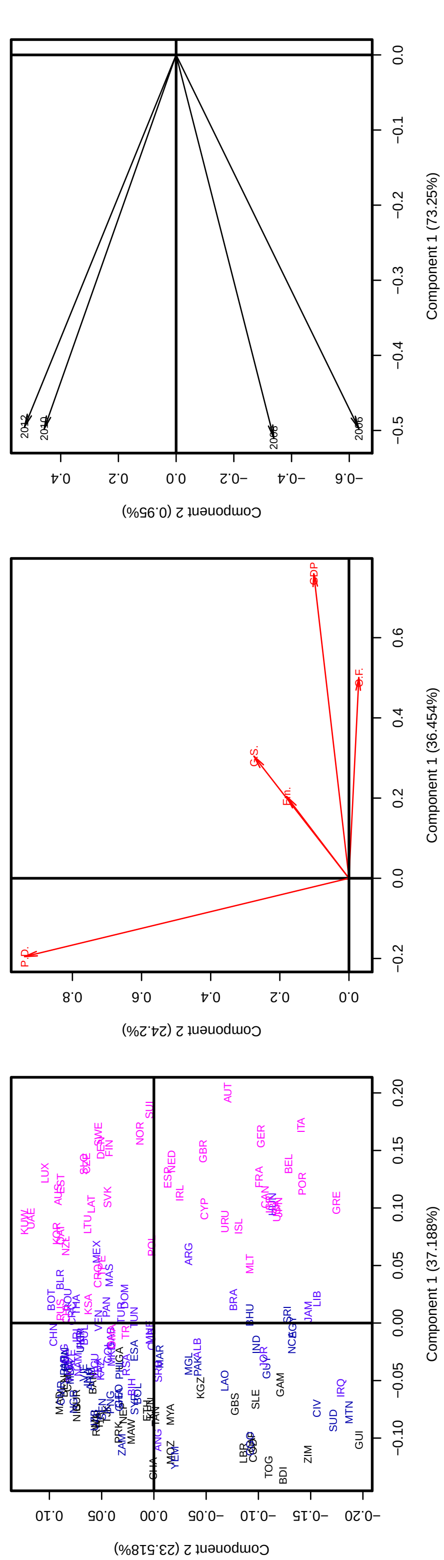


Figura 7.38: Gráfico para las dos primeras componentes de las tres dimensiones del Tucker3 en el análisis Co-Tucker3 para las variables económicas

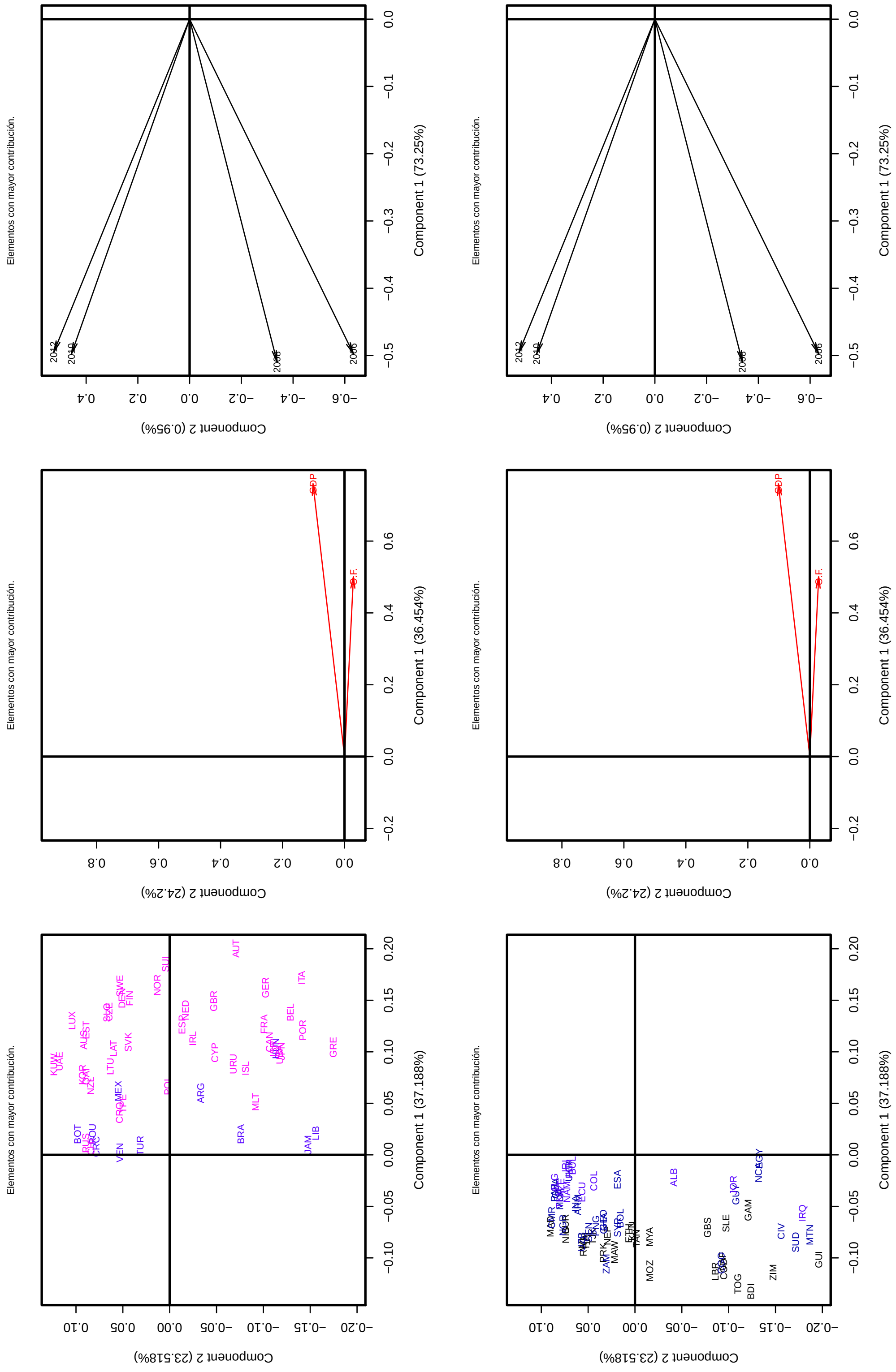


Figura 7.39: Figura 7.38, con solo los países, indicadores y años correspondientes a los signos

Veamos ahora los gráficos resultantes tras la última parte de los análisis con el método Co-Tucker3, los Análisis de la Co-Inercia entre los países, las variables y los años después de haber elegido el modelo 3x3x2 para los tres análisis, uno para cada par de tipos de variables. Así, la figura 7.40 representa los análisis de Co-Inercia cuando se estudian las variables sociales y medioambientales, la figura 7.42 para las variables sociales y económicas, y la figura 7.44 para los indicadores medioambientales y económicos.

Aunque se ha llevado a cabo el análisis para todos los países, para facilitar la visualización y la interpretación a partir de los distintos gráficos, se han representado solamente los países con una alta calidad de representación (superior a 500 sobre mil) y aquellos países con vectores muy largos (vectores más largos que la longitud media, porque todos los vectores eran largos). Así, los indicadores y los años de las figuras 7.41, 7.43 y 7.45 son iguales a los anteriores y estas solo varían en los países.

Con estos gráficos se pueden descubrir las relaciones entre las interacciones más profundas que las obtenidas con los otros tres métodos, de manera que queda demostrado que los cuatro métodos entonces han de usarse de forma complementaria.

De nuevo, la interpretación de estos gráficos se basa en la posición de los países, las variables y los años en los distintos cuadrantes para estudiar agrupaciones o interacciones entre las tres dimensiones; también se puede estudiar la longitud de los vectores que representan a los tres tipos de indicadores y la de los vectores que representan el Análisis de Co-Inercia, es decir, la diferencia entre los países o los años con respecto a un tipo de variables o a otro.

Por ejemplo, al estudiar la relaciones entre las interacciones de las variables sociales con las medioambientales (Figura 7.40), primero se observa que los países se siguen situando en un gradiente horizontal respecto a su nivel de ingresos, y se puede ver también que son los países con niveles de ingresos altos y la mayoría con ingresos medio-altos los que se sitúan en el semiplano izquierdo, los cuadrantes II y III, es decir, están directamente relacionados con todas las variables sociales y con los indicadores medioambientales de Consumption, C., Greenhouse Gases, G.Ga., y Renewable Energy, R.E.; todo esto para los años 2010 y 2012 porque son los situados en el semiplano izquierdo. Mientras que se puede hacer la conclusión opuesta para los países, indicadores y años localizados en los semiplanos derechos.

Hay que notar que, a pesar de que estas conclusiones parecen contradecir lo obtenido con los anteriores métodos (por ejemplo, que algunas variables medioambientales están relacionadas con las sociales), esto es porque ahora se están estudiando las interacciones más profundas, es decir, distintas de las estables que se obtienen con los otros métodos.

Por último, hablemos de las longitudes de los vectores. Aquellas variables que presenten una longitud mayor en su vector son las que servirán para diferenciar a los diferentes países y años por cuadrantes, así, cuando se estudian conjuntamente, por ejemplo, las variables sociales y económicas (figura 7.42), los países con menor nivel de ingresos, los situados en los semiplanos derechos, durante los años 2010 y 2012, que están también localizados en los semiplanos derechos, están fundamentalmente relacionados con los indicadores Income Distribution, I.D., y Public Debt, P.D. Mientras que, si se habla del eje vertical, los países que estén situados en el semiplano superior, durante los años 2006 y 2012 dan mayor importancia a las variables Clean Water, C.W., y Good Governance, G.Go., de entre las sociales, y dan poca importancia a las variables Organic Farming, O.F., y Gross Domestic Product, GDP, de entre las económicas, puesto que estas se sitúan en el semiplano inferior.

En cuanto a las longitudes de los vectores dados por los Análisis de la Co-Inercia, para los países, se puede observar que todos los vectores, en cualquiera de las tres figuras 7.40, 7.42 y 7.44, son largos, lo cual tiene sentido, porque esto significa que los comportamientos de los países respecto a lo social, medioambiental o económico, no se parecen, pero esto solo es porque en este análisis con el Co-Tucker3 se están estudiando las interacciones más profundas que las estables. Ya se ha visto con cualquiera de las otras técnicas que sí que hay, efectivamente, relación entre los tres tipos de indicadores.

Para las longitudes de los vectores de la Co-Inercia de los años, en el análisis entre las variables sociales y las económicas (Figura 7.42), se puede observar que son cortos, lo cual quiere decir que, incluso a un nivel más profundo, durante todos los años hay una relación estrecha entre estos dos tipos de variables. Mientras que para el análisis entre las variables sociales y medioambientales, o entre las medioambientales y las económicas, solo durante los dos últimos bienios, años 2010 y 2012, existe esta relación, puesto que para los otros dos años los vectores son más largos.

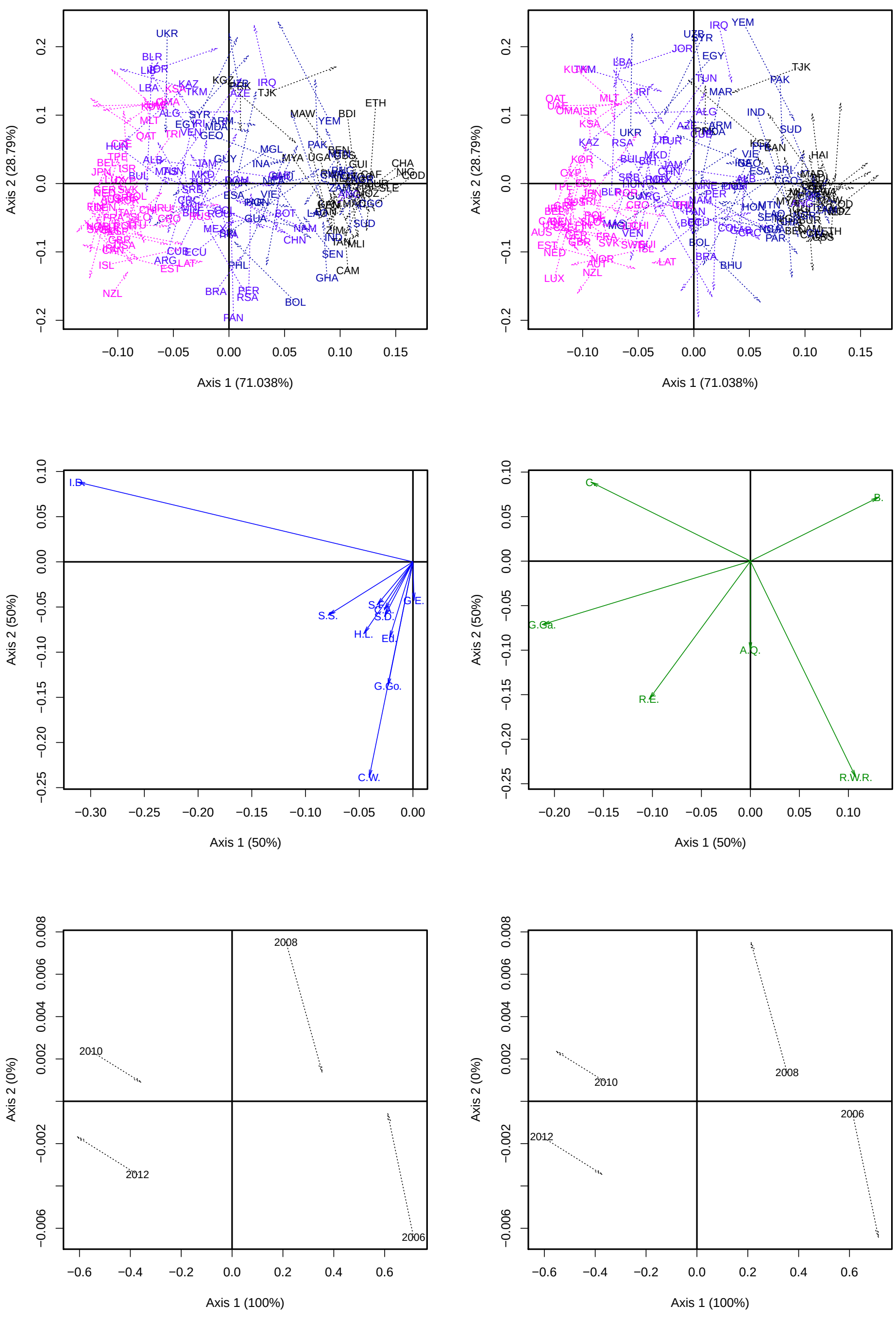


Figura 7.40: *Análisis de Co-Inercia del Co-Tucker3 entre variables sociales y medioambientales*

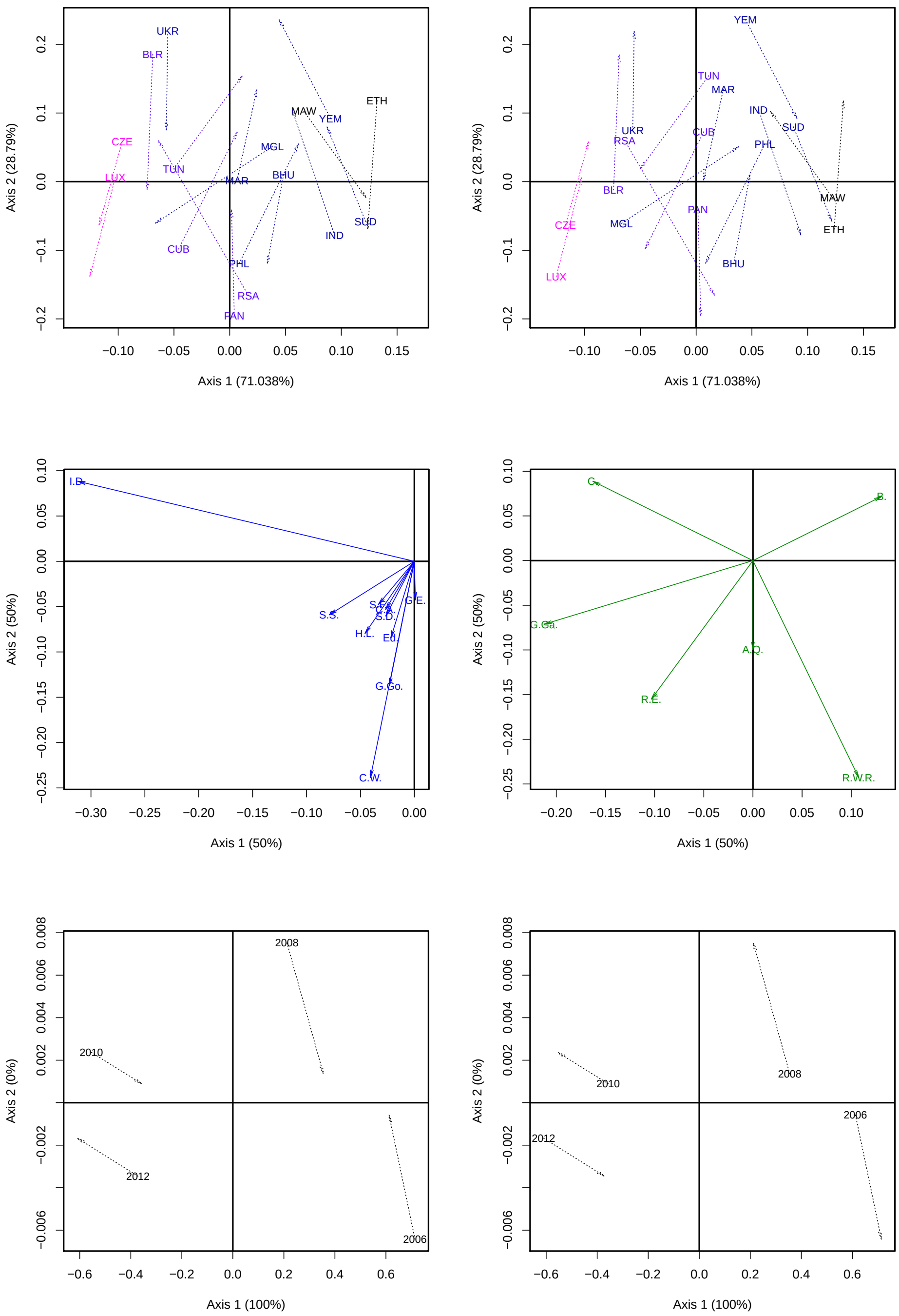


Figura 7.41: *Análisis de Co-Inercia del Co-Tucker3 entre variables sociales y medioambientales, para algunos países*

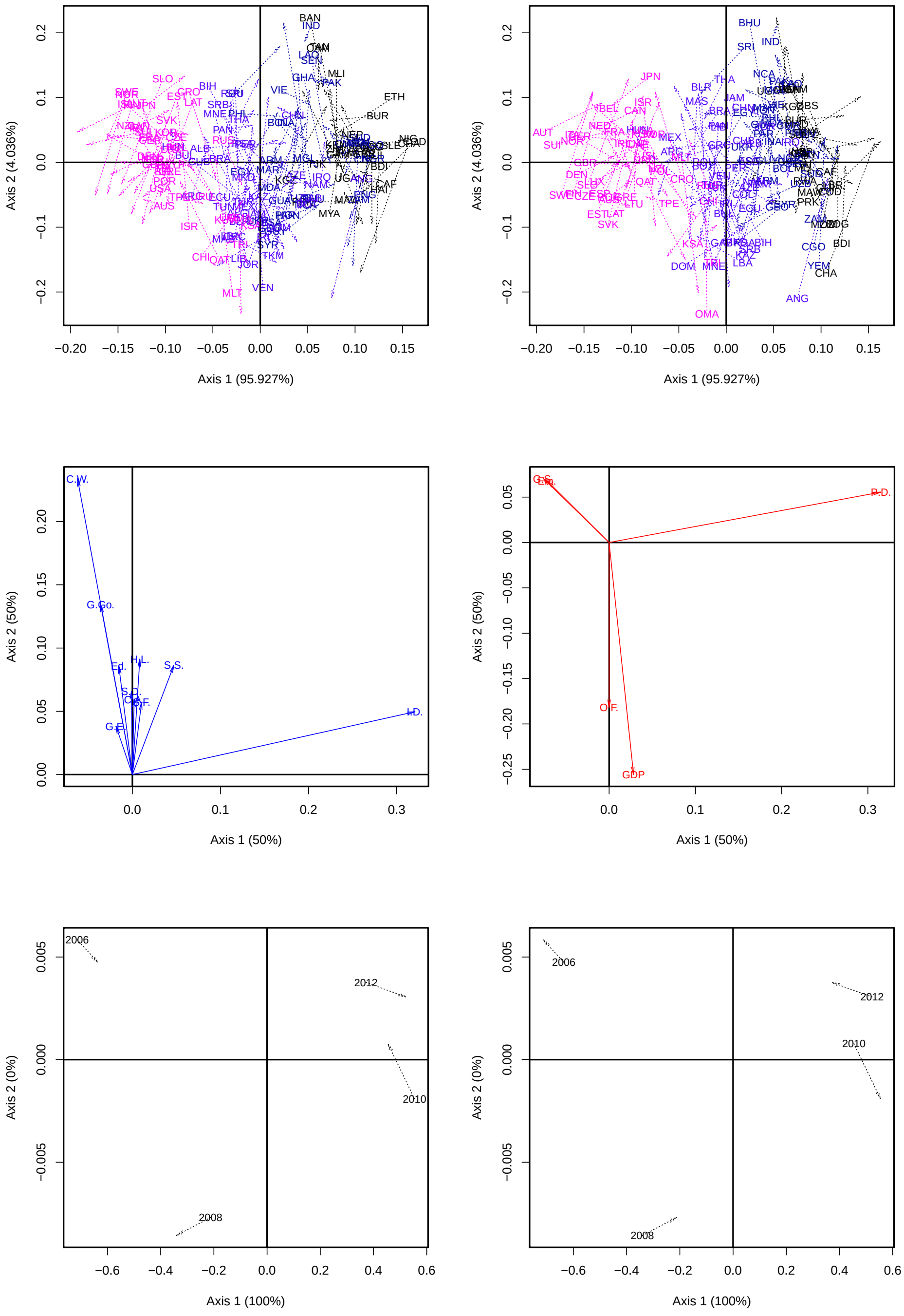


Figura 7.42: *Análisis de Co-Inercia del Co-Tucker3 entre variables sociales y económicas*

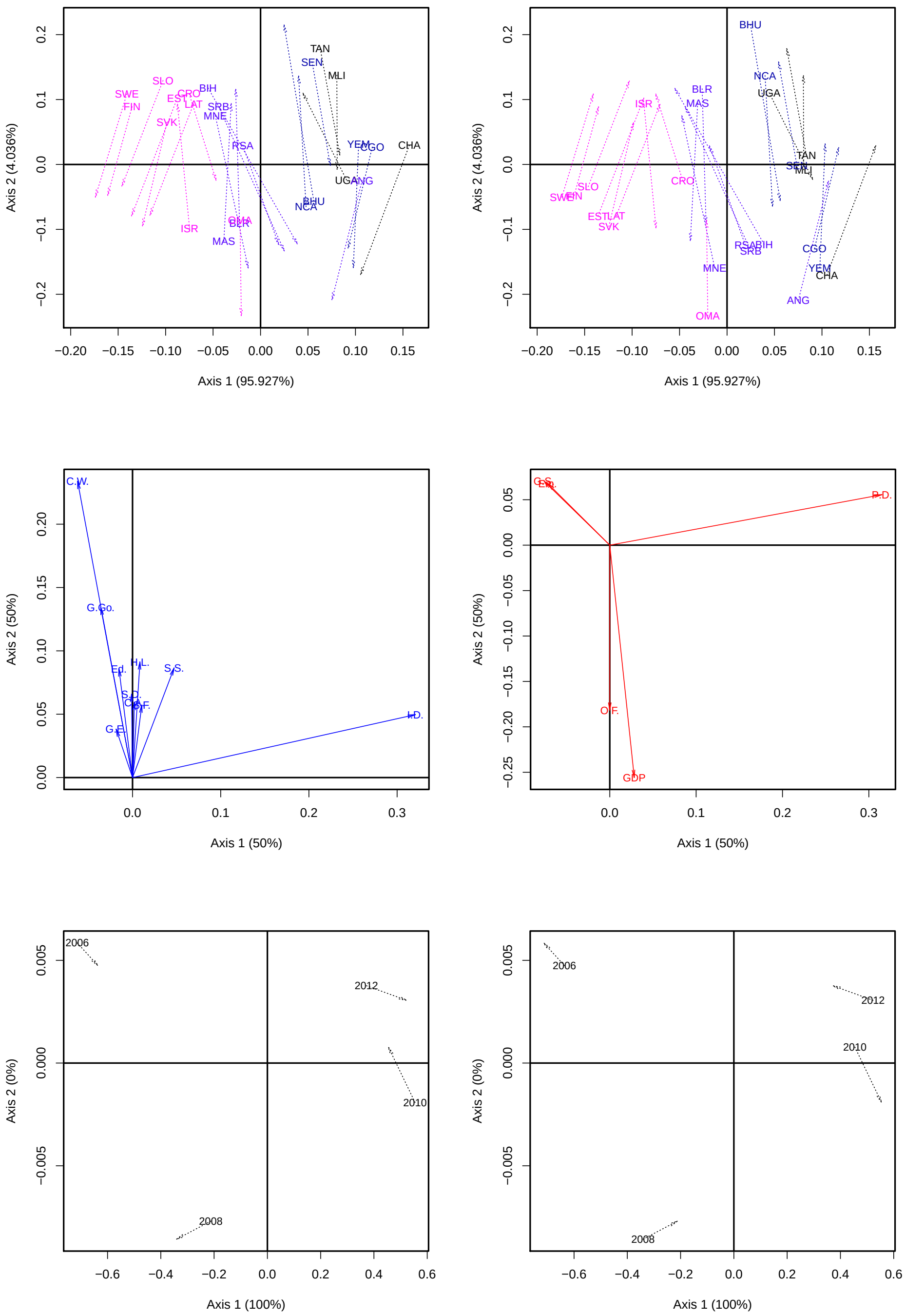


Figura 7.43: *Análisis de Co-Inercia del Co-Tucker3 entre variables sociales y económicas, para algunos países*

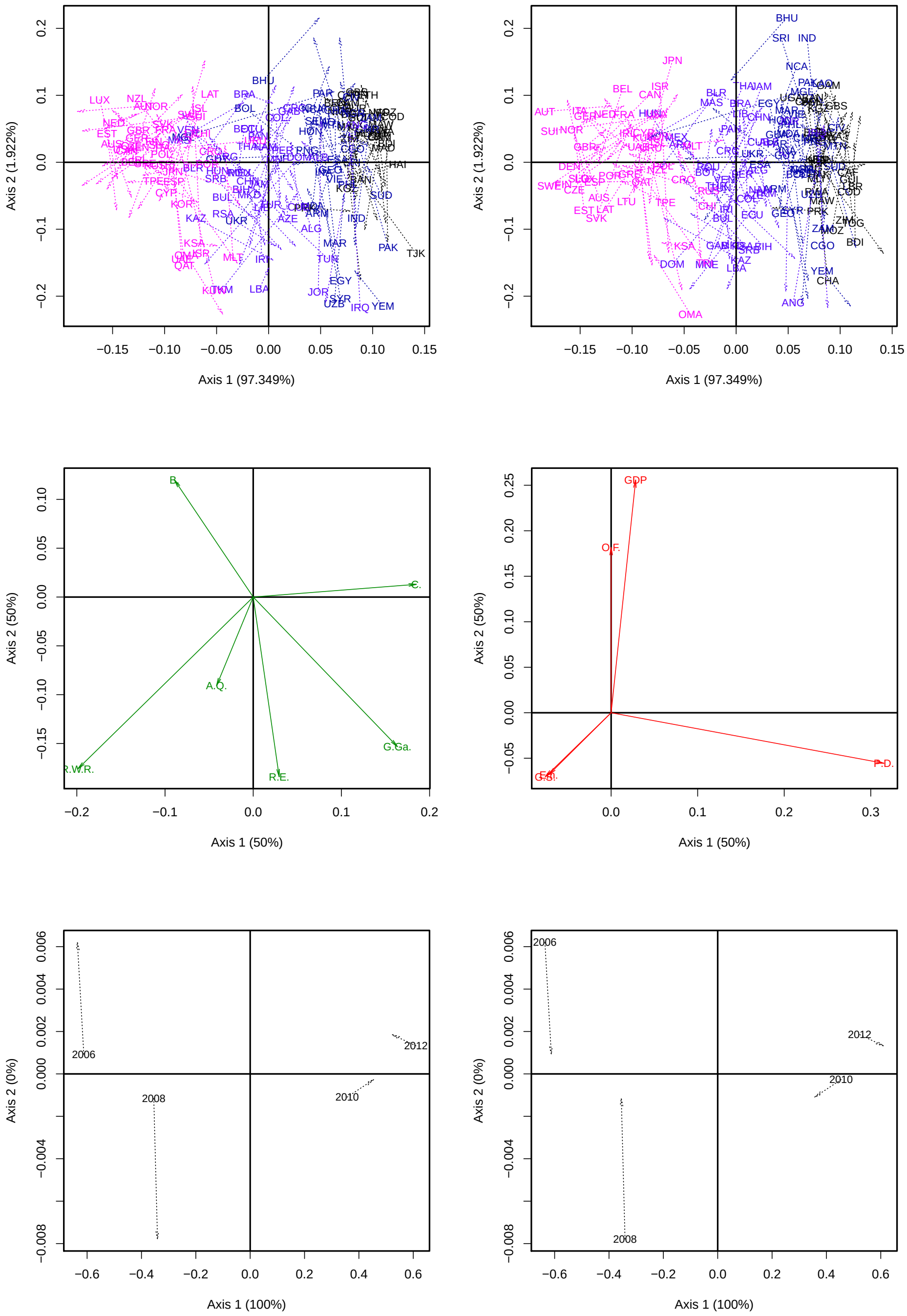


Figura 7.44: *Análisis de Co-Inercia del Co-Tucker3 entre variables medioambientales y económicas*

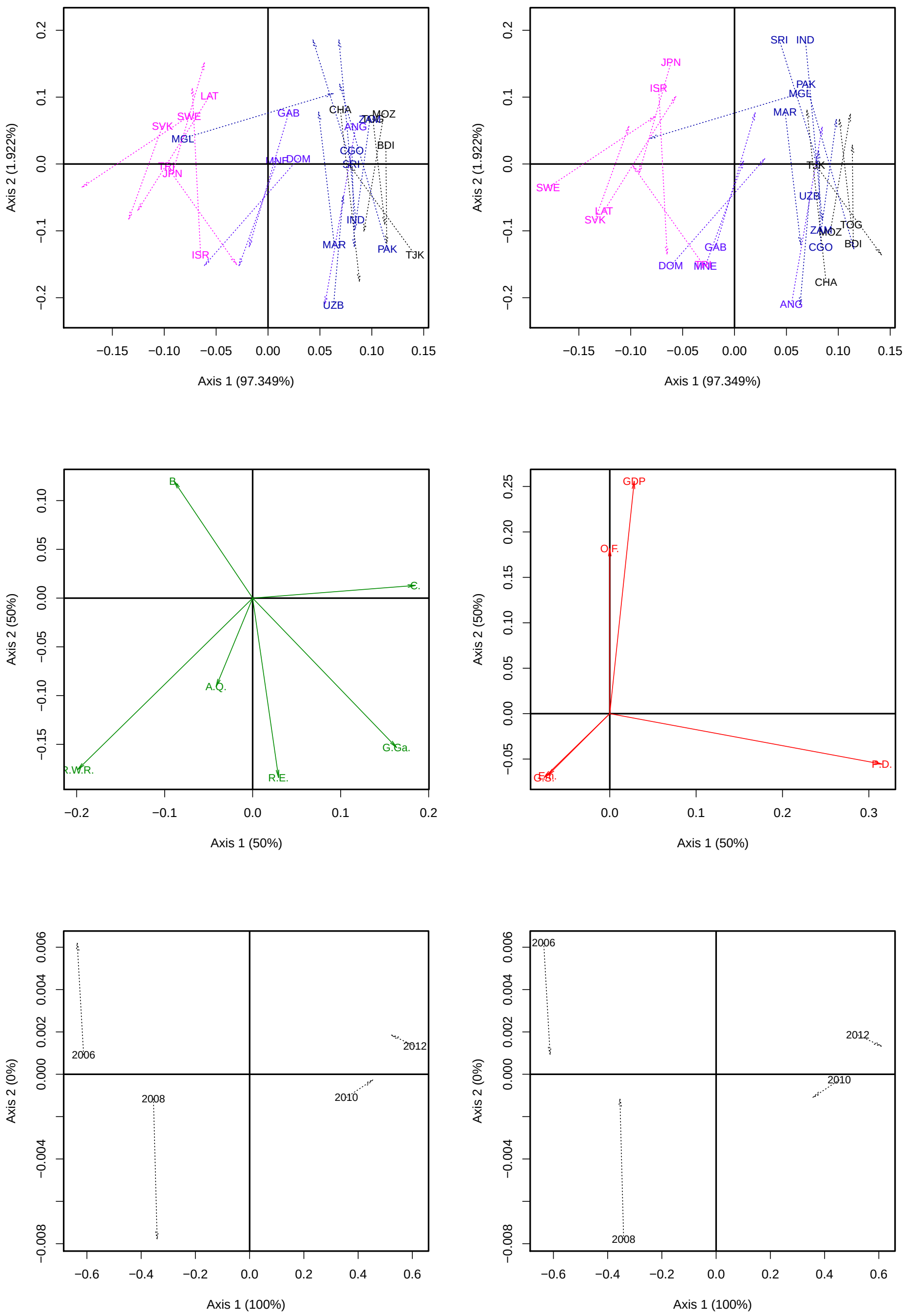


Figura 7.45: *Análisis de Co-Inercia del Co-Tucker3 entre variables medioambientales y económicas, para algunos países*

Conclusiones

1. Tras la exhaustiva revisión bibliográfica, se detecta una clara discordancia entre el crecimiento de las técnicas multivariantes para inspección de tablas de tres vías, y su uso en estudios de sostenibilidad, a pesar de que claramente los datos tiene esa estructura: países, índices de sostenibilidad (sociales, medioambientales y económicos) y años de estudio.

2. Los estudios publicados utilizando el índice de sociedad sostenible (SSI), que es el índice elegido para este trabajo por el hecho de que ha sido auditado por la Comisión Europea, utilizan técnicas estadísticas muy básicas y los estudios multivariantes se limitan, hasta el momento actual, al uso de componentes principales y el HJ-Biplot. Además, son estudios de un solo año y de un único aspecto de sostenibilidad, concretamente la sostenibilidad medioambiental.

3. Existe también un claro desfase entre las propuestas teóricas de técnicas para el estudio de co-estructuras en pares de cubos de datos, y su aplicación en este campo. No existe ningún trabajo publicado que aplique técnicas de tres vías y ninguno que estudie las co-estructuras.

4. El estudio algebraico comparado entre el Análisis de Co-Inercia Entre Grupos (BGCOIA), el STATICO, el COSTATIS y el Co-Tucker3, ha permitido poner de manifiesto las ventajas y las limitaciones de los cuatro métodos, en un lenguaje asequible para los investigadores, en el campo de la sostenibilidad mundial.

5. Consideramos que el uso de cualquiera de las cuatro técnicas daría una visión incom-

pleta del problema y sugerimos la conveniencia de utilizarlas de forma complementaria.

6. El uso de cualquiera de ellas proporcionaría, no obstante, información más completa que los análisis multivariantes individuales, que es, hasta el momento, lo único utilizado.

7. Del análisis de las co-estructuras se deduce que los aspectos más relevantes en los países de los continentes europeo, americano y asiático, son los correspondientes a sostenibilidad social y económica.

8. Las variables asociadas a sostenibilidad medioambiental, emisión de gases, uso de energías renovables y calidad del aire, son temas de mayor interés en los países africanos y, en menor medida, en los asiáticos.

9. El análisis de las trayectorias nos ha permitido, además, evaluar la evolución de los distintos aspectos de la sostenibilidad, en los diferentes continentes, a lo largo de los cuatro bienios en estudio.

Conclusions in English

1. After the exhaustive literature review, a clear mismatch is detected between the growth of the multivariate techniques for the inspection of three-way tables, and their use in studies of sustainability, although the data clearly have that structure: countries, indices of sustainability (social, environmental and economic) and years of study.

2. The published studies using the Sustainable Society Index (SSI), which is the index chosen for this paper by the fact that it has been audited by the European Commission, use very basic statistical techniques and the multivariate studies are limited, up to the present time, to the use of principal components and the HJ-Biplot. Moreover, they are studies of only one year and only one aspect of sustainability, namely environmental sustainability.

3. There is also a clear gap between the theoretical proposals of techniques for the study of co-structures in pairs of data cubes, and its application in this field. There is no published paper that applies three-way techniques and none that studies the co-structures.

4. The study of the algebraic comparison between the Between-Groups Co-Inertia Analysis (BGCOIA), the STATICO, the COSTATIS and the Co-Tucker3, has allowed to highlight the advantages and limitations of the four methods, in an affordable language for the researchers, in the field of global sustainability.

5. We consider that the use of any of the four techniques would give an incomplete view of the problem and we suggest the appropriateness of using them in a complementary way.

6. However, the use of any of them would provide more complete information than the individual multivariate analysis, that is, until the moment, the only thing used.

7. From the analysis of the co-structures it follows that the most relevant aspects in the countries of the European, American and Asian continents, are those corresponding to social and economic sustainability.

8. The variables associated with environmental sustainability, emission of greenhouse gases, use of renewable energies and air quality, are topics of greatest interest in Africa countries and, to a lesser extent, in Asians.

9. The analysis of the trajectories has allowed us, moreover, to evaluate the evolution of the different aspects of sustainability, in the different continents, over the four biennia in the study.

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Apéndice A

Artículos publicados por el autor

Artículos:

1. **Analysis of the Sustainable Society Index Worldwide: A Study from the Biplot Perspective.**
   Gallego-Álvarez, I., Galindo-Villardón, M.P. and Rodríguez-Rosa, M. (2014)
   Social Indicators Research 120 (1), 29-65.

   - Subject categories:
     - Social Sciences, Interdisciplinary.
     - Sociology.
   - Publisher: Springer.
   -

Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
Social Sciences, Interdisciplinary	93	20	Q1
Sociology	142	37	Q2

- Factor de impacto: 1.38; Factor de impacto 5 años: 1.789.
- Citas Google Académico:
  - Ospina, R., Larangeiras, A.M. and Frery, A.C. (2014) *Visualization of Skewed Data: A Tool in R*. Revista Colombiana de Estadística 37(2), 399-417.
  - Nieto, A.B., Galindo, M.P., Leiva, V. and Vicente-Galindo, M.P. (2014) *A Methodology for Biplots Based on Bootstrapping with R*. Revista Colombiana de Estadística 37(2), 367-397.

2. **Environmental Performance in Countries Worldwide: Determinant Factors and Multivariate Analysis.**
   Gallego-Álvarez, I., Vicente-Galindo, M.P., Galindo-Villardón, M.P. and Rodríguez-Rosa, M. (2014) Sustainability 6(11), 7807-7832.

- Subject categories:
  - Environmental Sciences.
  - Environmental Studies.
- Publisher: MDPI AG.
Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
Environmental Sciences	225	146	Q3
Environmental Studies	104	62	Q3
- Factor de impacto: 1.343.

3. **Evolution of sustainability indicator worldwide: A study from the economic perspective based on the X-STATICO method.**
   Gallego-Álvarez, I., Galindo-Villardón, M.P. and Rodríguez-Rosa, M. (2015) Ecological Indicators 58, 139-151.

   - Subject category: Environmental Sciences.
   - Publisher: Elsevier Science B.V.
   -
Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
Environmental Sciences	225	52	Q1

   - Factor de impacto: 3.19; Factor de impacto 5 años: 3.649.

4. **Are Social, Economic and Environmental Well-Being Equally Important in all Countries Around the World? A Study by Income Levels.**
   Rodríguez-Rosa, M., Gallego-Álvarez, I., Vicente-Galindo, M.P. and Galindo-Villardón, M.P. (2016) Social Indicators Research, pp 1-23

   - Subject categories:
     - Social Sciences, Interdisciplinary.
     - Sociology.
   - Publisher: Springer.
   -
Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
Social Sciences, Interdisciplinary	93	20	Q1
Sociology	142	37	Q2

- Factor de impacto: 1.38; Factor de impacto 5 años: 1.789.

5. **Proposal of an algorithm and mathematical modelling to assist policy and decision-makers in the pathway of constructing a sustainable society.**
   Rodríguez-Rosa, M., Gallego-Álvarez, I. and Galindo-Villardón, M.P.

   [This paper has been submitted to European Journal of Operational Research recently].

   - Subject category: Operations Research and Management Science.
   - Publisher: Elsevier Science B.V.
   -

Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
Operations Research and Management Science	82	9	Q1

   - Factor de impacto: 2.679; Factor de impacto 5 años: 3.109.

Artículo 1

Analysis of the Sustainable Society Index Worldwide: A Study from the Biplot Perspective

Isabel Gallego Álvarez, Mª Purificación Galindo Villardón,
Miguel Rodríguez Rosa

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Analysis of the Sustainable Society Index Worldwide: A Study from the Biplot Perspective

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Abstract Since the Brundtland Report defined the concept of sustainability in 1987, several different indices and indicators have been developed in this area, which is becoming an ever greater concern in society, since it will affect future generations. The main objective of this research study is to analyze whether there are differences in the scores obtained by a broad sample of countries in the Sustainable Society Index according to the geographical area in which the country is located. We apply the HJ-biplot method (Galindo in Questíio 10(1):13–23, 1986), a statistical technique that provides a joint graphical representation in a low dimensional Euclidean space (usually a plane), of a multivariate data matrix; in our study, this is formed by the countries grouped by geographical areas and variables relating to sets of economic, social and environmental indicators included in the Sustainable Society Index. Our findings stress that the variables related to Human Wellbeing fall mainly within the proximity of the countries located in the geographical areas of Europe, America and, to a lesser extent, Asia. In contrast, other variables associated with Environmental Wellbeing, such as greenhouse gases, renewable energy, and air quality are mainly located closer to Africa, and more residually to Asia. In order to represent the most relevant variables in each geographical area and corroborate the results obtained using the HJ-biplot methodology, an analysis was carried out of the radial graph that represents the values of each variable along the independent axes in the form of radii that have their starting point in the centre of the plot and end in the outer ring such that each radius corresponds to a variable. The results obtained show the characteristics of each geographical area in relation to the Sustainable Society Index, and confirm the results obtained with the HJ-biplot.

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1 Introduction

One of the most important issues in recent years at the international level is the sustainability of the different countries and geographical areas on our planet. This has become especially crucial since the Brundtland Report, released in 1987, provided a definition of sustainability that has had a deep impact on society at large. The report pointed to the importance of satisfying current needs but in a way that does not compromise the capability of future generations to satisfy their own needs.

From this definition of sustainability it can be deduced that a current and future balance must be sought in three aspects that affect humanity as a whole: the economic aspect, with an optimum combination between economic development and conservation of the natural environment; the social aspect, which involves guaranteeing intergenerational equity in social matters and quality of life; and finally the environmental aspect, which means maintaining the continuity of environmental resources over time, something that can be achieved by limiting the consumption of easily exhaustible resources and products, reducing waste and pollution in all their manifestations, conserving energy and recycling.

All of these aspects are important in attaining a sustainable society in which each individual is able to develop in a healthy way, obtain a decent education, live in a clean environment within a safe and well-balanced society, use non-renewable resources in a responsible way and contribute to a sustainable world (Van de Kerk and Manuel 2008).

To help to understand and manage all these sustainability issues, a number of indicators have been implemented. Some of the most important have been the Human Development Index (HDI), the Millennium Development Indicators, Indicators for the EU Sustainable Development Strategy, and the Index of Sustainable Economic Welfare. In this research, we use the Sustainable Society Index (SSI), employed in previous analyses (e.g. Van de Kerk and Manuel 2008).

This index includes a set of economic, social and environmental wellbeing indicators and has recently been audited by the Joint Research Centre of the European Commission, which considers it an integral and quantitative method for measuring and monitoring the health of human and environmental systems on a world-wide basis. The audit also pointed out that it is a conceptual and statistically solid tool that is widely applicable to the continuous assessment of human and environmental systems and a key point of reference with which to compare future progress and report on the current state of society (Saisana and Philippas 2012).

The technique we have chosen for this research is the HJ-biplot (Galindo 1986), which has been used in other studies (e.g. González-Cabrera et al. 2006; Gardner et al. 2005; Aerni 2009; Basille et al. 2008; Ceschin et al. 2012; Gallego-Álvarez et al. 2013). However, it has not yet been applied to the Sustainable Society Index (SSI), thereby providing some degree of novelty to the current work. This method will allow us to check whether the indicators proposed by the SSI are similar across the different countries, and whether sustainability concerns are similar in different geographical areas.

From a statistical point of view, the eigenvalues, the variance explained, and the relative contribution of factor to the element, ensure the validity of this research. The joint use of the SSI indicators and the HJ-biplot method allow us to depict the geographical zones and the most relevant indicators jointly, showing the proximity of the latter to the former. SSI indicators enable us to extend the analysis beyond a specific country or geographical area, and thus include different contexts in our study. Unlike other techniques, the biplot easily allows us to detect differences between geographical areas in relation to different dimensions (SSI indicators) in a visual way, as well as the proximity of each country to a specific set of indicators. The biplot can also be used to compress the data, by reducing the number of dimensions, without much loss of information. When using HJ-biplot analysis to analyze a data set, it is usually possible to explain a large percentage of the total variance with only a few components, and it allows us to represent the countries and variables in our sample with the maximum quality, at the same time being a technique that is based on simple geometric concepts such as angles, lines and vectors.

Observed in this way, our findings show that the variables related to Human Wellbeing are mainly in closer proximity[1] to the countries located in the geographical areas of Europe, America and, to a lesser extent, to Asian countries. Meanwhile, other variables associated with Environmental Wellbeing, such as greenhouse gas emissions, renewable energy, and air quality are mainly located closer to Africa on the biplot, and more residually to Asia.

The paper is structured as follows: after the introduction, in Sect. 2 we analyze the theoretical framework and the meaning of a sustainable society. Section 3 describes our research methods, including the sample and analysis techniques. In Sect. 4, the results of the empirical analysis are given and then discussed in Sect. 5. Section 6 summarizes the main findings and consequences and presents the conclusions.

2 Sustainable Development and Sustainability Indicators

2.1 Theoretical Framework

Sustainable development and sustainability have become watchwords in recent years owing to the great interest taken in this subject worldwide both at the micro- and macro-economic levels. At the micro-economic level this means sustainability in the business world, as published in sustainability reports presented by companies, a measure that is becoming more and more frequent internationally. Sustainability at the macro-economic level refers to the sustainability of different countries, a research topic that is perhaps less developed than at the business level, but which is unquestionably of great importance.

This research study is focused on sustainability at the country level and the analysis is done according to the geographical area where the country is located, an issue that gained importance starting with the United Nations Conference on Environment and Development held in Rio de Janeiro in 1992. This Conference raised public awareness and placed sustainable development on the world's political agenda, reaffirming the concept introduced in 1987 by the Brundtland Report. That Report was the first to include the concept of

[1] By "closer proximity" we mean the proximity between the countries grouped in geographical areas and the variables that are the sustainability indicators. Put more technically, the countries grouped by geographical area have similar profiles with respect to the variables, since all of them project close to the end of the vector representing the variable (see Fig. 4), e.g. Africa and greenhouse gases. As for the individuals (countries grouped by geographical area), when they are close to a vector, it implies that they take predominant values for that variable, in the sense that the individuals are significant to explain the variable and that the variable is of great value for the individuals.

sustainable development in an official document, defining it as development that satisfies present needs without compromising the capability of future generations to satisfy their own needs (World Commission on Environment and Development, WCED 1987). Thus, general principles were set up to guide relations between the economy and the environment at the global level, with emphasis on the need to find strategies that allow economies to grow while remaining sustainable (Erias Rey 2003).

Attaining sustainable development entails making progress in three fundamental pillars: economic development, social cohesion and protection of the environment. In other words, it involves the integration of three dimensions:

- The environmental (ecological) dimension, through environmental sustainability. This is defined as the need to maintain the continuity of environmental resources over time. This can be achieved by limiting the consumption of easily exhaustible resources and products, reduction of waste and pollution in all their aspects, energy conservation and recycling.
- The social dimension, through social sustainability. This involves guaranteeing intergenerational equity, that is, satisfying the current basic needs of all persons but at the same time guaranteeing that when the time comes, future generations will be able to do the same.
- The economic dimension, through economic sustainability. The means seeking an economic balance by means of an optimum combination between economic development and conservation of natural resources.

Aiming for sustainability implies, first, defining its components in measurable terms and clearly fixing the responsibility to assess progress comprehensively (Hales and Prescott-Allen 2002). Nevertheless, the notion of what is meant by sustainability varies considerably and its definition is still ambiguous (Mori and Christodoulou 2012). It is no wonder that the relevant literature is abundant with studies on sustainability (Hák et al. 2007; Arezki and Van Der Ploeg 2007; Bell and Morse 2008; Betsill and Rabe 2009) and many of them define it in a way similar to that of the Brundtland Report. Thus, Baumgärtner and Quaas (2010) consider sustainability to be a normative notion that indicates the way humans should act towards nature, and how they are responsible towards one another and future generations, and Kates et al. (2001), consider that the essence of sustainable development is to meet fundamental human needs while preserving the life-support systems of planet Earth.

According to Van de Kerk and Manuel (2008), a sustainable society is one in which each human being is capable of developing in a healthy manner and obtaining a proper education; lives in a clean environment; lives in a safe and well-balanced society; uses non-renewable resources responsibly so that future generations will not be left without them, and contributes to a sustainable world.

For Saisana and Philippas (2012), the term sustainability has also been used by politicians and economists to mean that a society is economically viable, environmentally rational and socially responsible, although the great changes taking place in social and economic matters have made the measuring of sustainability very complicated, despite the great progress already achieved in this sense.

Given this situation, more and more new indicators are being developed in an attempt to measure these three aspects of sustainability: environmental, social and economic. Some of these indicators have been established by the OECD and the UN, among others, but the SSI (Sustainable Society Index), developed in 2006, does so in a more complete way, as it covers all three aspects. It can thus be considered innovative (Van de Kerk and Manuel 2012).

Moreover, the SSI has recently been audited by the Joint Research Centre of the European Commission, which considers it to be an integral and quantitative method for measuring and supervising the health of human and environmental systems the world over, in addition to being a conceptually and statistically solid tool that is broadly applicable for a continuous assessment of human and environmental systems and a key reference point with which to compare future progress and report on the current state of society (Saisana and Philippas 2012).

These sustainability indicators can be useful individually to view the state of each country in regard to matters of sustainability, what its deficiencies and most relevant aspects are, and to compare each country's sustainability with that of other countries in its geographical area as well as identify its most effective aspects. For governments, these sustainability indicators serve to show the sustainability situation in each country and geographical area to the general public in a transparent and effective way. They can also help governments make decisions regarding their social, environmental and economic policies, and the projects and strategies to be adopted in this sense. As regards education, sustainability subject matter can be introduced in secondary and higher education to make students aware of the situation of the world around us. As far as the business world is concerned, sustainability indicators for the different countries business is conducted in can help firms to determine whether they will have some kind of competitive advantage and to be innovative.

2.2 Sustainability Indicators: With Special Reference to the Sustainable Society Index

Recently, composite indicators have been used for concerns such as quality of life and the environment, mainly in order to rank performance at country level (Karavanas et al. 2009). Furthermore, they provide information on the status of the environment and assess the economic, social and environmental impact of development.

Generally speaking, indicators have three main functions. First, they reduce the number of measurements necessary to give a description of a situation (OECD 2003). As such, they are indispensable for measuring progress towards policy objectives (Dalal-Clayton and Krikhaar 2007) and for evaluating the effectiveness of policies (European Commission 2005).

Hansen (1996), Jasch (2000) and Perotto et al. (2008) observed that the development of indicators at the national, regional, local or field level had become a commonly used approach to meet the crucial need for assessment tools. Such tools are a prerequisite to the implementation of the concept of sustainability.

With a view to studying sustainability internationally, many current indices and indicators relating to sustainability have been reviewed and it has been found that the good indicators, that is, those that provide a complete picture of all the relevant aspects of sustainability in a transparent and easily understandable way, must fulfil the following criteria (Bell and Morse 2008; Meadows 1998; Guy and Kibert 1998, Van de Kerk and Manuel 2008):

- They must be relevant for one of the issues relating to the above-mentioned definition of sustainability.
- They must cover the complete field of sustainability according to the definition used.
- They must be independent of each other and not overlap.

- They must be measurable.
- They must be easy to access, for the general public as well. This in turn means that the number of indicators should be limited.
- The data used to build the indicators must be publically available.
- The data must be available for all countries, at least for all those except the smallest ones.
- The data must be reliable.
- The data must be recent and regularly updated.
- The complete set of indicators should provide a good picture of the current situation of sustainability and point out the differences between the present situation and the optimum situation of complete sustainability.
- They must permit comparisons among countries.

The general conclusion is that none of the existing indices seems to completely fulfil our needs, since either none is completely suitable or they serve more or less different objectives. Below we list some of the most important indices in the field of sustainability (Van de Kerk and Manuel 2012; Saisana and Philippas 2012):

- *Human Development Index (HDI)* This covers a small part of all the aspects involved in sustainable development and it has sometimes even been considered a redundant indicator that provides little additional information on inter-country development levels, especially in regard to life expectancy, education attained and income per capita. It was drawn up by the United Nations Development Programme (UNDP 2005).
- Environmental Sustainability Index (ESI-2005): This one lacks indicators on gender equality, and good governance does not receive enough attention. The ESI benchmarks the ability of nations to protect the environment over the next several decades. This index was developed by Esty (2005) in the Yale Center for Environmental Law and Policy.
- *Environmental Performance Index (EPI-2006)* This only partially covers sustainable development in its broadest context, particularly environmental items. It was developed by Esty (2005 in the Yale Center for Environmental Law and Policy por Esty (2005).
- *Commitment to Development Index (CDI)* This addresses sustainable development only partly and offers information on no more than 27 countries. It was devised by the Center for Global Development (2007).
- *Index of Sustainable Economic Welfare (ISEW)* This index does not include the main aspects of quality of life and does not offer a clear picture of a country's level of sustainability. Developed by Daly and Cobb (1989a, b), it is available only for a limited number of countries.
- *Genuine Progress Indicator (GPI)* This one has the same deficiencies as the ISEW. It was developed with a view to redefining progress and was first published in 1998. Its importance has increased because of authors such as Talberth et al. (2006).
- *Ecological Footprint* This only partially covers sustainability in the broadest sense. There is quite a bit of debate about the method used in its calculation. It was initially created by Wackernagel and Rees (1996).
- *Wellbeing of Nations* This provides an enormous amount of information, which makes it too complicated. Developed initially by Prescott-Allen (2001), it has only been published once.
- *Millennium Development Indicators* Of limited use for visualizing a country's level of sustainability. It does not cover the whole concept of a sustainable society (United Nations 2005).

- *Indicators for the EU Sustainable Development Strategy* This includes a number of indicators that are not closely related to sustainability, and little or no attention is paid to other topics such as those related to gender equality or access to drinking water. It is limited to member States of the European Union.
- *CSD Indicators* This set comprises many indicators and offers too much information. It does not cover sustainability in its broadest sense (United Nations 2007).

Considering the existing limitations for establishing an index that can be applied generally to all countries, we decided to use the one created by Van de Kerk and Manuel (2012), since their Sustainable Society Index (SSI) was recently audited by the Joint Research Centre of the European Commission, which found that it to be an integral and quantitative method for measuring and monitoring the health of human and environmental systems globally and a conceptually and statistically solid tool that can be broadly applied for the continuous evaluation of these systems. It is also a key reference point for comparing any progress made and for informing about current society (Saisana and Philippas 2012).

According to the recommendations of the Joint Research Centre, the geometric average was used to develop the Sustainable Society Index, aggregating the indicators into categories and then aggregating the categories into the dimensions of sustainability, to finally result in a single index, the Sustainable Society Index (SSI).

According to Van de Kerk and Manuel (2012), owing to the lack of a scientific basis for the attribution of different weights to the indicators, every indicator received the same weight for the aggregation into categories. The same applies for the aggregation of the eight categories into the three wellbeing dimensions and finally into one figure for the overall index.

Our decision to use the Sustainable Society Index instead of other indices such as the Human Development Index or the Environmental Performance Index, which address specific issues (such as life expectancy, education and income in the former and ecosystem vitality and environmental health in the latter), was due to the following aspects: on the one hand it describes societal progress along all three dimensions: human, environmental and economic. Thus, the SSI comprises three wellbeing dimensions (human, environmental and economic) and is calculated for 151 countries accounting for 99 % of the world population. Thus it is much broader in scope than others that refer only to one specific geographical area, such as Europe. Furthermore, the SSI-2012 is based on a definition of sustainability that was provided by the Brundtland Commission (WCED 1987), to make explicitly clear that sustainability includes human wellbeing as well as environmental wellbeing. Another aspect that led us to consider the SSI is that the 21 indicators that populate the SSI-2012 framework come from fifteen sources: the Food and Agriculture Organization (FAO), World Health Organization (WHO)-Unicef Joint Monitoring Programme, WHO, UN Population Division, Yale and Columbia University, UNESCO, World Economic Forum, World Bank, United Nations Environment Programme–World Conservation Monitoring Centre (UNEP-WCMC), FAO global information system on water and agriculture (Aquastat), Global Footprint Network, International Energy Agency (IEA), Research Institute of Organic Agriculture (FiBL), International Monetary Fund (IMF), International Labour Organization (ILO) and Central Intelligence Agency (CIA), World Factbook (Saisana and Philippas 2012, p. 17). Furthermore, it has recently been audited by the Joint Research Centre of the European Commission, in particular its Institute for the Protection and Security of the Citizen, confirming that the SSI is well-structured and guaranteeing a control process to ensure transparency and the credibility of the results. The Joint Research Centre, moreover, is based on the recommendations of the OECD (2008).

It is also important to point out that the SSI is framed within the Pressure-State-Response model proposed by Rapport and Friend (1979) and followed by the OECD. This model has subsequently been used and modified by the UN Commission on Sustainable Development to adapt it to the three dimensions of sustainability (economic, social and environmental aspects), giving rise to the DPSIR, the acronym for the Driving Forces-Pressure-State-Impact-Reponse framework. The model is based on a sequential evolution in which social and economic development give rise to pressures on the environment, which in turn give rise to a series of changes in the state of the environment. As a result of these changes there are impacts on health, availability of resources, natural ecosystems, and so on. These lead to a series of responses on the part of social agents and public authorities addressed to improving economic and social management by eliminating or reducing these pressures, thus restoring and recovering the state of the environment and the alterations that are the result of the impacts. As can be observed, this model adds to the previous model environmentally relevant social and economic trends that are responsible for the situation (driving forces), as well as the adverse effects of the changes in state detected in human health and behaviour, the environment, the economy and society (impacts).

The Sustainable Society Index (SSI) consists of 21 indicators grouped into three dimensions: Human Wellbeing, Environmental Wellbeing, and Economic Wellbeing. The different indicators comprising these dimensions are listed below (Van de Kerk and Manuel 2012).

- Human Wellbeing
  - *Sufficient food* Number of undernourished people in percentage of total population.
  - *Sufficient to drink* Number of people as percentage of the total population, with sustainable access to an improved water source.
  - *Safe sanitation* Number of people in percentage of total population, with sustainable access to improved sanitation.
  - *Healthy life* Life expectancy at birth in number of healthy life years (HALE—Health Adjusted Life Expectancy).
  - *Clean air* Air pollution in its effects on humans.
  - *Clean water* Surface water quality.
  - *Education* Combined gross enrolment ratio for primary, secondary and tertiary schools.
  - *Gender equality* Gender Gap Index.
  - *Income distribution* Ratio of income of the richest 10 % to the poorest 10 % of the people in a country.
  - *Good governance* The average of values of the six Governance Indicators of the World Bank.
- Environmental Wellbeing
  - *Air quality* Air pollution in its effects on nature.
  - *Biodiversity* Size of protected areas (in percentage of land area).
  - *Renewable water resources* Annual water withdrawals (m^2 per capita) as percentage of renewable water resources.
  - *Consumption* Ecological Footprint minus Carbon Footprint.
  - *Renewable energy*: Renewable energy as percentage of total energy consumption.

- *Greenhouse gases* This indicator uses the common measure for greenhouse gas emissions (GHG): CO_2 emissions per capita per year.

- Economic Wellbeing

  - *Organic farming*: Area for organic farming as percentage of total agricultural area of a country.
  - *Genuine savings* Genuine Savings (Adjusted Net Savings) as percentage of Gross National Income (GNI).
  - *Gross domestic product*: GDP, per capita, in Purchasing Power Parity, in current international dollars.
  - *Employment* Unemployment as percentage of total labour force.
  - *Public debt* The level of public debt of a country as percentage of GDP.

In regard to the range of the SSI indicators, if the sustainability value of an indicator in known, the value of the indicator is scored with 10 in the case of 100 % sustainability, and if there is no sustainability at all, the indicator value is 0. If an indicator already has a set of values, the data for this indicator are transformed on a scale of 0–10. This transformation from basic data to indicator values was done by standardization and certain more complex formulas need to be used in the case of some indicators, according to their characteristics.

3 Research Method

3.1 Population and Sample

Taking into account the indicators mentioned above, in this study we selected most countries in the world as our target population. This population was chosen in order to broaden and generalize the results obtained in previous studies, and also in order to overcome two limitations posed previously: the countries being studies and the data analysis techniques used.

Previous studies have usually focused on specific geographical contexts, such as Western industrialised countries (Scruggs 2003; Jahn 1998; Crepaz 1995), 21 OECD countries (Neumayer 2003), 17 industrialised democracies (Scruggs 1999, 2001; 14 OECD countries considering five measures of well-being (Giles and Feng 2005) and 131 countries (Hosseini and Kaneko 2011).

The sample we use comprises the 151 countries selected by Van de Kerk and Manuel (2008) (see Appendix 1) corresponding to the latest information available from 2012, and incorporates the advantages derived from considering different geographical contexts: Europe (Eu), Africa (Afr), America (Am), Asia (As) and Oceania (Oc) (see Appendix 2).

Although the initial population comprised 194 countries, data on these indicators were only available for 151 countries. It was thus possible to calculate the SSI for most large or medium-sized countries. The largest countries that could not be included were Afghanistan, Djibouti, Eritrea, Somalia and Surinam.

3.2 Statistical Analysis

We therefore consider in this research the 151 countries around the world presented in Appendix 1, grouped into 5 geographical areas; the 21 numerical characteristics are the scores obtained by the countries selected concerning the policy categories proposed in the

SSI in the last available year (2012), basically sufficient food, sufficient to drink, safe sanitation, healthy life, clean air, clean water, education, gender equality, income distribution, good governance, air quality, biodiversity, renewable water resources, consumption, renewable energy, greenhouse gases, organic farming, genuine saving, gross domestic product, employment and public debt (see Appendix 2). Hence, in this paper, the data consist of the SSI scores for each country, that is, a $X_{151 \times 21}$ matrix.

The analysis of several sustainability problems at once requires the storage of large volumes of data. In order to explore the data to get a better understanding of several processes, it is important to identify the salient features underlying them. The reduction in the dimensionality of the problem enables us to summarize the information captured in a large number of variables with a smaller number of latent variables. Plots which simultaneously show both the countries and the indices can be of great assistance in this respect. These plots, called biplots, are used in this paper.

A biplot is a graphic display of multivariate data: a joint representation, in a low dimensional Euclidean space (usually a plane), of a matrix $X_{n \times p}$, with markers a_1, …, a_n for its rows and markers b_1, …, b_p for its columns, chosen in such a way that the inner (or scalar) product $a_i^t \cdot b_j$ represents the element x_{ij} of matrix X (Gabriel 1971). First it carries out the approximation of the data matrix by a singular value decomposition (SVD) and then, this matrix is factorized in row and column markers. The biplot is a powerful multivariate data visualization tool, due to its inner product properties. Gabriel proposed several biplots: the JK-biplot (in which only the rows are represented with high quality), and the GH-biplot (only the columns are represented with high quality). Galindo (1986) proposed a new form of representation, the HJ-biplot, in which the coordinates for columns coincide with the column markers in the GH-biplot, and the coordinates for the rows coincide with the row markers in the JK-biplot, but these coordinates may be represented in the same reference system. The HJ-biplot is a joint representation, in a low dimensional vector space (usually two), of the rows and columns of X, using markers (points/vectors), for its rows and for its columns. Like the classic biplots proposed by Gabriel, this alternative allows nearby points and closely angled lines to be interpreted as showing similarity/correlation, but with a very important advantage since it is possible to interpret narrowly between unit-points and variable-points.

All these representations (GH, JK and HJ-biplots) are just exploratory techniques; no parametrical assumptions are considered.

The technique we have chosen for this research is the HJ-biplot, (Galindo 1986) which has been used in other studies (e.g. González-Cabrera et al. 2006; Gardner et al. 2005; Aerni 2009; Basille et al. 2008; Ceschin et al. 2012; Gallego-Álvarez et al. 2013). However, it has not yet been applied to the Sustainable Society Index (SSI), thereby providing some degree of novelty to the current work.

From an analytical point of view, the HJ-biplot is a multivariate graphic display of a matrix $X_{n \times p}$ by means of markers $j_i = (j_{i1}, j_{i2})$ i = 1,…,n for its rows and $h_j = (h_{j1}, h_{j2})$ j = 1,…,p for its columns, such that both markers can be represented in the same reference system, with optimal quality of representation and reaching the best simultaneous representation. The aim of the HJ-biplot is to describe the configuration of the rows and columns and the relationships between them, an aim different from that of the classical biplots in which it is necessary to reproduce each element of matrix X. Usually the row markers are displayed as points on a two-dimensional plot and the column markers as vectors on the same plot.

Let $X = U\sum V^T$ be the usual singular value decomposition (SVD) of X with U and V orthogonal matrices and $\sum = \mathrm{diag}(\lambda_1, \ldots, \lambda_p)$ containing the singular values. Let J and H be the matrices of the first two columns of $U\sum$ and $V\sum$, respectively. This selection provides an HJ-biplot representation in the sense defined. Obviously, if X has rank p $(p > 2)$ the solution is only an approximation, as in the classical biplots.

(a) Rows and columns can be represented in the same reference system.

It is known that V are the eigenvectors of $X'X$ and U are the eigenvectors of XX'. U and V are related

$$U = XV\sum^{-1} \quad V = X'U\sum^{-1} \tag{1}$$

thus

$$\begin{aligned} U\sum &= XV\sum^{-1} \quad \sum = XV \\ V\sum &= X'U\sum^{-1} \quad \sum = X'U \end{aligned} \tag{2}$$

Putting $J = XV$ and $H = X'U$

$$\begin{aligned} H &= X'J\sum^{-1} \\ J &= XH\sum^{-1} \end{aligned} \tag{3}$$

i.e.

$$\begin{aligned} b_{jh} &= \left\{x_{1j}a_{lh}\ldots + x_{nj}\ a_{nh}\right\}(1/\sqrt{\lambda}h) \\ a_{ih} &= \left\{x_{il}b_{lh} + \ldots x_{ip}\ b_{ph}\right\}(1/\sqrt{\lambda}h) \end{aligned}$$

The hth coordinate of the ith row is a function of the hth coordinates of the p variables and each coordinate of the jth variable is a linear combination of the coordinates of the n individuals, where each of these coordinates is weighted by the value that the variable X_j takes on the individuals; likewise, each individual occupies the point of equilibrium of the set of the variables.

The dispersion of both clouds (scatters of rows and columns) is relative to the same eigenvalues

$$\begin{aligned} XX' &= U\sum 2U' \\ X'X &= V\sum 2V' \end{aligned} \tag{4}$$

The relation between the coordinates and the equal dispersion of the two clouds justifies the representation in the same reference system (Greenacre 1984).

Furthermore, HJ-biplot is a symmetrical display. (Galindo 1986).

b. The goodness of fit is identical for rows and columns.

If we take the decompositions of cross-products matrices (4) in the form

$$XX' = \left(U\sum\right)\left(\sum U\right)' \tag{5}$$

$$X'X = \left(V\sum\right)\left(\sum V\right)' \tag{6}$$

it is possible to display the matrices in a biplot. The row and column markers are identical in both displays. The row markers of X are the same as (5) and the column markers of X are the same as (6). It is evident that the goodness of fit for the two approximations of scalar products, (rows and columns) in a k-dimensional display, is the same

$$\left(\lambda_1^4, \ldots, \lambda_k^4\right) \Big/ \left(\lambda_1^4, \ldots, \lambda_p^4\right) \tag{7}$$

Thus, it is possible to interpret the configuration (distances and scalar products) of the row scatter in an optimal representation, the configuration of the column scatter and the relationship between the two representations through the relation shown in (3).

The HJ-biplot is in some ways similar to Correspondence Analysis but is not restricted to categorical data. The markers are obtained from the usual singular value decomposition (SVD) of the data matrix.

The rules for the interpretation of the HJ-biplot are a combination of the rules used in other multidimensional scaling techniques, correspondence analysis, factor analysis and classical biplots: (1) the distances between row markers are interpreted as an inverse function of similarities, in such a way that markers that are closer to each other (countries) are more similar. This property allows for the identification of clusters of countries with similar profiles; (2) the lengths of the column markers (vectors) approximate the standard deviation of the variables; (3) the cosines of the angles between the column vectors approximate the correlations among variables in such a way that small acute angles are associated with variables with high positive correlations; obtuse angles that are almost a straight angle are associated with variables with high negative correlations and right angles are associated with non-correlated variables; (4) the order of the orthogonal projections of the row markers (points) onto a column marker (vector) approximates the order of the row elements (values) in that column. The larger the projection of a country point onto a variable vector, the more this country deviates from the average of that variable.

In short, HJ-biplot is a statistical tool to visualise the data and it is not employed to calculate the indices as such and hence not free from underlying ambiguities and assumptions. It is, however, a technique that allows the dimensionality of the problem to be reduced and allows us to represent the countries and the variables in our sample with the same quality of representation. It is also a technique that is based on simple geometric concepts such as angles, lines and vectors.

In comparison to other, more conventional techniques, such as Principal Component Analysis (PCA) or Correspondence Analysis (CA), the HJ-biplot has important advantages. Thus, according to González-Cabrera et al. (2006, p. 67), when PCA is used the axes are combinations of the variables, but these do not appear on the plots, such that very important information concerning the correlations among them is lost, and Correspondence Analysis tends to use categorical data and it is only possible to work with real positive integers. Moreover, with the HJ-biplot better values are obtained for parameters as relative contributions of the factor to the element and better quality or representation for both rows (where the countries grouped into geographical areas are located and represented as points) and columns (where the variables that are the SSI indictors are represented by vectors), that is, only the points with good quality of representation can be interpreted correctly in the subspace observed.

It is important take into account that the HJ-biplot is just an exploratory, completely nonparametric, technique: any data set can be plugged in and an answer comes out, requiring no parameters to tweak and no regard for how the data were recorded; it is not a statistical method from the viewpoint that there is no probability distribution specified for the observations. Therefore it is important to keep in mind that the HJ-Biplot best serves to represent data in simpler reduced form.

Nonetheless, this method will allow us to check whether the indicators proposed by the SSI are similar across the different countries (for example, whether economic, social or environmental concerns are similar in different geographical areas), to find geographical areas with similar sustainability profiles, to identify the most differentiated ones and to order them according to a sustainability gradient. We will likewise be able to identify the most important components of sustainability in each geographical area.

The software used to implement the HJ-biplot was developed by Vicente-Villardón (2010), and is available free of charge (http://biplot.usal.es/ClassicalBiplot/index.htlm).

4 Results of Empirical Analysis

According to Galindo (1986), several measures are essential for a correct implementation of the HJ biplot; specifically, eigenvalues and explained variance (Table 1) and the relative contribution of the factor to the element (Table 2) through which it is possible to detect the variables responsible for the position of axes and, therefore, the configuration obtained in them.

The first three axes of the HJ-biplot analysis explained 62.33 % of data variability (Fig. 1 and Table 1).

It can be deduced from Table 1 and Fig. 1 that there is a dominant axis (axis 1) that takes 43.73 % of the total inertia of the system. The trend in the eigenvalues is truncated in the third axis, achieving an accumulative inertia of 62.33. In other words, 62 % of the total inertia is absorbed by only the first three factorial axes, indicating that this percentage of the total information is present on these three axes. Factorial plane 1–2 absorbs 56.23 % of the total inertia. This factorial plane is used in the different figures to represent geographical areas and variables (see Figs. 2, 3, 4 where axis 1—horizontal- and axis 2—vertical-are represented). The remaining factors provide a smaller load of information.

Table 2 contains the contribution of each factor to the element, which lets us know the variables responsible for the positions of axes and their configuration.

The variables ‘sufficient food’, ‘sufficient to drink’, ‘safe sanitation’, ‘healthy life’, ‘clean air’, ‘education’, ‘good governance’, ‘GDP’ make a high contribution to Axis 1 and a low contribution to the remaining axes. In contrast, ‘air quality’, ‘biodiversity’ and ‘renewable water resources’ heavily contribute to axis 2 (see Figs. 2, 3, 4 where axis 1—horizontal and axis 2—vertical are represented).

Analysis of the contributions to the different axes shows that the first axis (axis 1 horizontal) is explained by most indicators linked to human wellbeing (see Figs. 3 and 4), such as sufficient food, sufficient to drink, safe sanitation, healthy life, clean air, education, good governance, respectively (621, 703, 739, 840, 694, 757, 684). The second factorial

Table 1 Eigenvalues and explained variance

Axis	Eigenvalue	Expl var	Cummulative
Axis 1	1,377.48	43.73	43.73
Axis 2	393.61	12.50	56.23
Axis 3	192.41	6.11	62.33

Table 2 Relative contribution of the factor to the element

Variables	Axis 1	Axis 2	Axis 3
Sufficient food	621	32	0
Sufficient to drink	703	19	22
Safe sanitation	739	83	5
Healthy life	840	2	3
Clean air	694	5	8
Clean water	285	215	4
Education	757	1	6
Gender equality	330	199	33
Income distribution	152	12	320
Good governance	684	111	4
Air quality	98	334	85
Biodiversity	3	339	38
Renewable water resources	16	650	34
Consumption	449	67	0
Renewable energy	578	195	17
Greenhouse gases	735	38	2
Organic farming	381	162	69
Genuine savings	76	117	51
GDP	885	0	0
Employment	70	1	186
Public debt	85	10	396

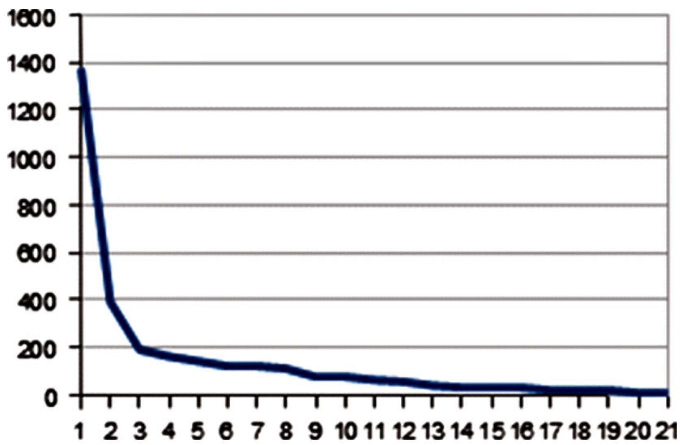

Fig. 1 Eigenvalues

axis (axis 2) is determined by the variables air quality, biodiversity and renewable water resources (334, 339, 650).

The graphic representation of the five geographical areas which include the countries analysed (see Appendix 1; in our biplot, individuals) are presented in Fig. 2.

All the countries grouped in five geographical areas are represented by different forms in four quadrants. The continents are represented as follows: Africa with black five-point stars, America with red inverted triangles, Asia with purple circles, Europe with blue triangles and Oceania with green squares. The countries located in Europe, America and Oceania are mainly represented in quadrants 1 (upper-right) and countries located in Africa

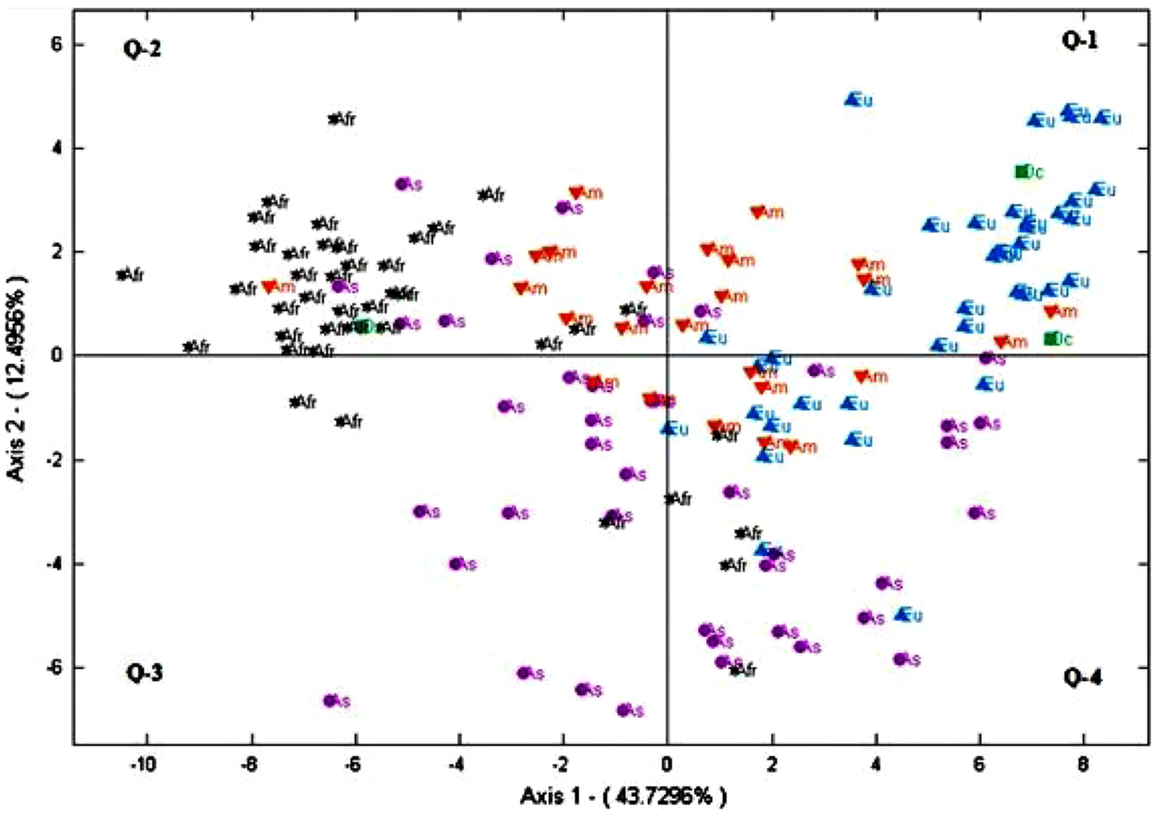

Fig. 2 Factorial plane 1–2 with the geographical areas including the 151 countries

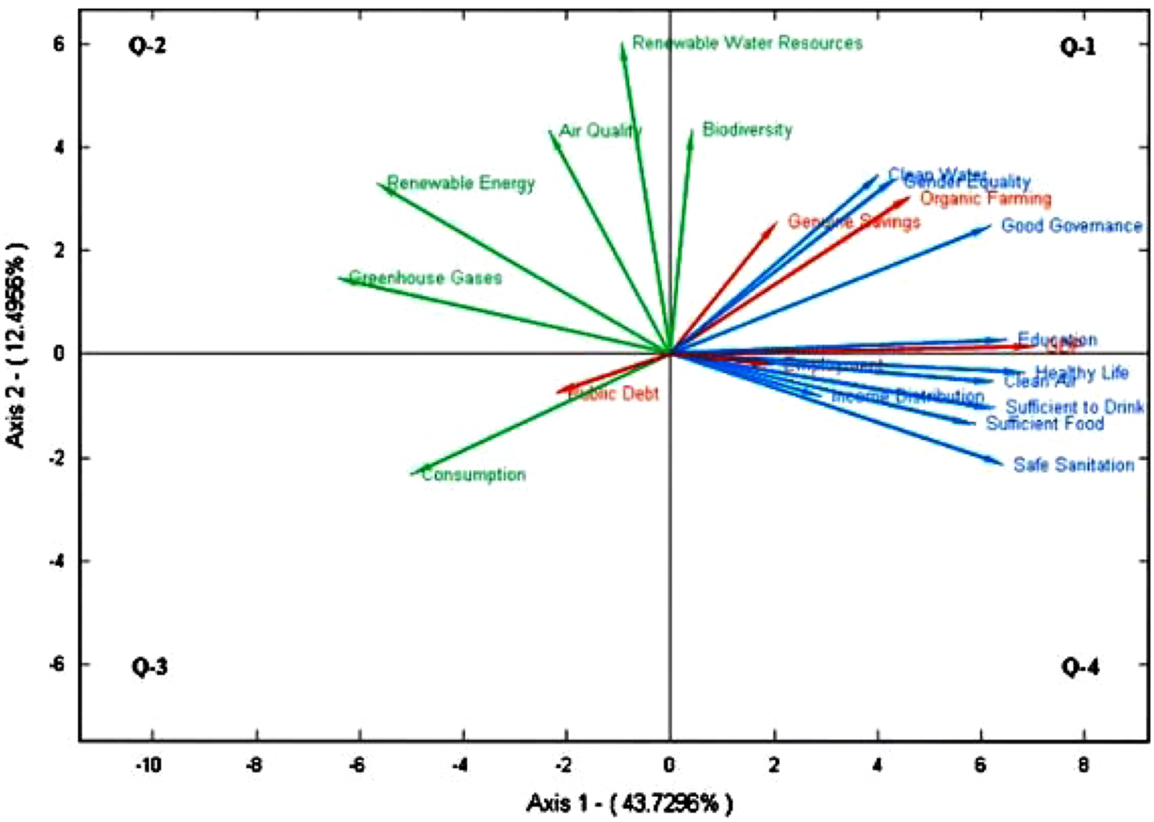

Fig. 3 Factorial plane 1–2 with the Sustainable Society Index indicators

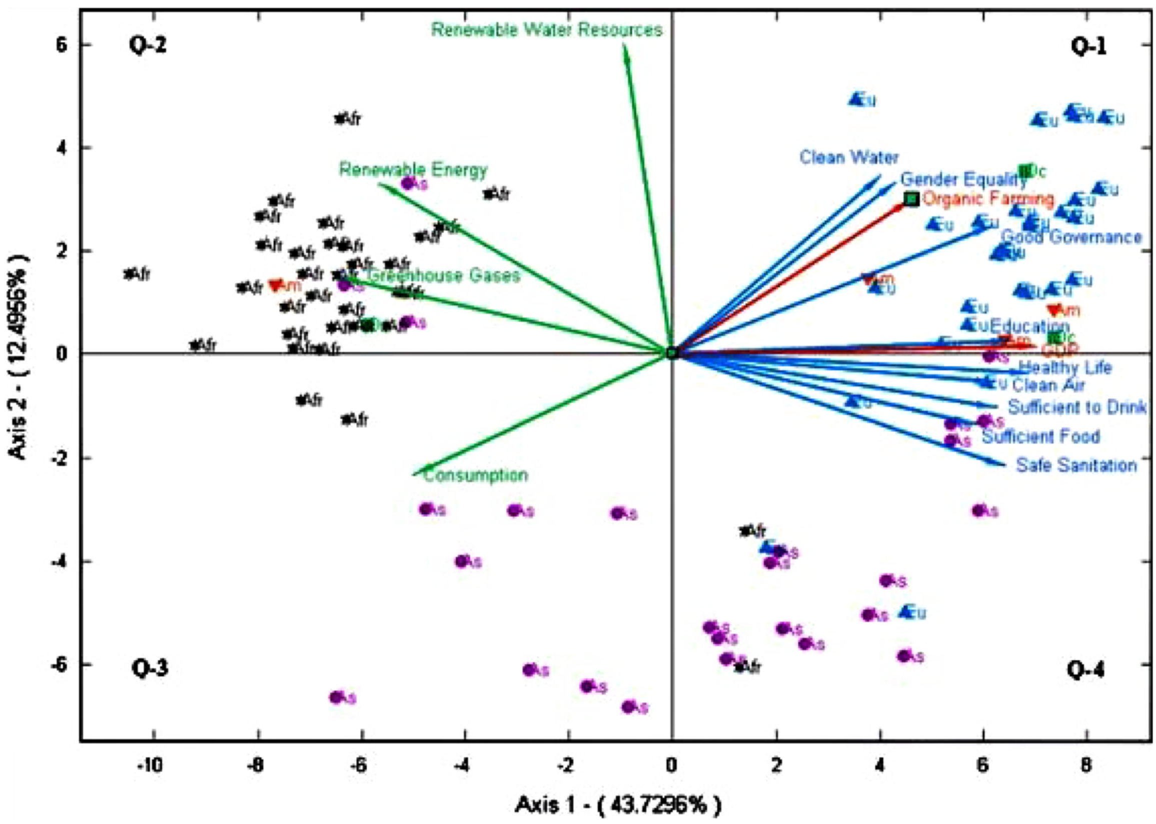

Fig. 4 Factorial plane 1–2 including the countries and variables with high quality of representation

are mainly represented in quadrants 2 (upper-left), whereas quadrants 3 (lower-left) and 4 (lower-right) contain the Asian countries.

In Fig. 3, the following variables are displayed: sufficient food, sufficient to drink, safe sanitation, healthy life, clean air, clean water, education, gender equality, income distribution, good governance, air quality, biodiversity, renewable water resources, consumption, renewable energy, greenhouse gases, organic farming, genuine saving, gross domestic product, employment and public debt. The first ten variables have to do with Human Wellbeing, while the remaining variables are associated with Environmental Wellbeing and Economic Wellbeing, according to the Sustainable Society Index. Environmental wellbeing variables are represented with green arrows, social wellbeing variables with blue arrows, and economic variables with red arrows.

As commented above, interpretation of the variables is based on the angles between the vectors, such that variables with vectors forming small angles are variables with similar behaviours. As can be observed from Fig. 3, the variables linked to Human Wellbeing, such as sufficient food, sufficient to drink, and safe sanitation, show small angles and, therefore, have similar behaviours.

Similarly, for Environmental Wellbeing (variables: air quality, biodiversity, renewable water resources), the variables are quite close, also showing a small angle. Hence, they are highly correlated and behave in a similar way.

In Fig. 4, the geographical areas (different forms) and the variables (vectors) representing human, environmental and economic wellbeing are displayed jointly. Only those countries and variables that obtained a good quality representation or goodness of fit are shown, not the total number of countries and variables.

As for the individuals, that is, countries grouped by geographical areas, when they are close to a vector-variable, it implies that they take predominant values for that variable, in the sense that the individuals are significant to explain the variable and that the variable is of great value for the individuals.

In Fig. 4, it can be observed that the variables related to Human Wellbeing are mainly closer to the countries located in the geographical areas of Europe, America and, to a lesser extent, to Asian countries. Meanwhile, other variables associated with Environmental Wellbeing, such as greenhouse gases, renewable energy, and air quality, are mainly closer to Africa, and more residually to Asia.

The continents can thus be described as follows: Africa is characterized in general by high values in consumption, greenhouse gases, renewable energy and renewable water resources, and by low values in all the social variables.

In contrast to Africa, for America we can see that its countries have high values in the social variables, organic farming and GDP, whereas it shows low values in renewable energy, greenhouse gases and consumption.

Asia, however, does not follow a set pattern for these variables, except for low values in renewable water resources. Some Asian countries also show low values in biodiversity and air quality.

It is clear that European countries all follow the same pattern: high values in all the social variables, organic farming and GDP, and low variables in renewable energy, consumption and greenhouse gases, since all the European countries grouped in the EU geographical area (see Fig. 4) are located in the first quadrant, Q-1, and in the fourth quadrant, Q-4, where the vectors representing social and economic variables such as education, good governance, clean water, clean air, organic farming, and GDP are located.

Finally, as a result of the classification of countries into continents followed, Oceania only has three countries: Australia, New Zealand and Papua New Guinea; a specific pattern therefore cannot be determined.

In order to represent the most relevant variables in each geographical area and to corroborate the results obtained from the HJ-biplot, we carried out an analysis of the radial graphs that represent the values of each variable along the independent axes in the form of radii that have their starting point in the centre of the plot and end in the outer ring, such that each radius corresponds to one variable. Figure 5 shows a geographical area in each star plot.

As can be observed in the star plots, Africa is characterized by high values in the greenhouse gases and renewable water resources indices, as well as by low values in all the social variables. In contrast, the variables related to Human Wellbeing are mainly closer to the countries located in the geographical areas of Europe, America and, to a lesser extent, Asian countries. Asia shows a very bad performance in sustainable use of resources but no specific pattern can be determined for Oceania.

This study has enabled us to verify that this kind of plot helps to corroborate the results obtained in the analyses run using the HJ-biplot methodology.

5 Discussion

Recently, different organizations and authors have developed indicators to measure sustainability issues worldwide. Among these, the Sustainable Society Index (SSI) compiles information from 151 countries.

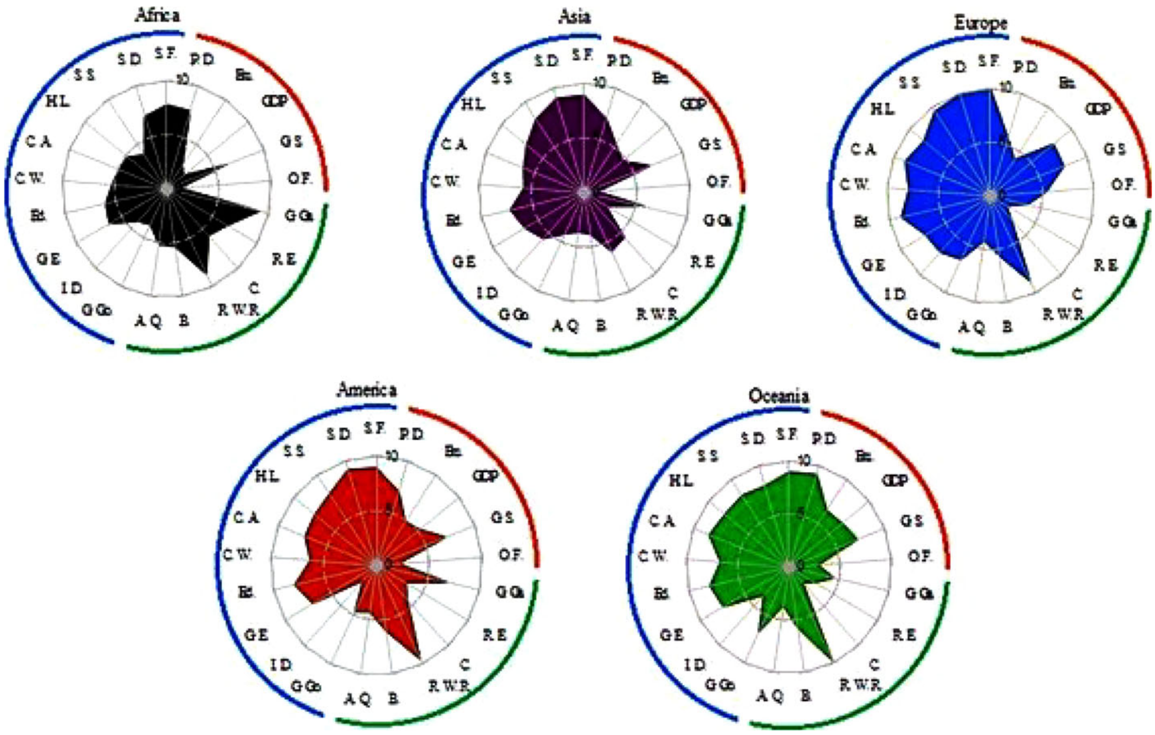

Fig. 5 Radial profiles by geographical area representing the indicators

With the purpose of studying whether these countries grouped into five geographical areas show the same interest concerning economic, social and environmental issues, we employed an HJ-biplot analysis, an exploratory data analysis method that looks for hidden patterns in the data matrix. Unlike other techniques, the biplot can allow us to detect differences easily in the behaviour of geographical areas with regard to different dimensions (SSI indicators) in a visual form, as well as the proximity of each country to a specific set of indicators. This technique enables us to reflect both the indicators and the geographical areas at the same time, showing the proximity of the latter to the former. Also, it permits analysis of different dimensions (sets of SSI indicators) simultaneously.

In the interpretation of the biplots, the different forms reflect individuals (in our study, the countries grouped into five geographical areas) and the vectors represent variables (in our study, the variables related to Basic Needs, Health, Personal and Social Development, Nature and Environment, Natural Resources, Climate and Energy Transition, Economy).

From the results obtained, it can be deduced that the core policy categories for Environmental Wellbeing, such as greenhouse gas emissions, renewable energy, and air quality, are mainly closer to Africa, and more residually to Asia. In contrast, other variables related to Human Wellbeing (education, gender equality, clean water, good governance) are mainly in closer proximity to the countries located in the geographical areas of Europe, America and, to a lesser extent, Asian countries.

Similarly to Van de Kerk and Manuel (2008), we find that the relationship between economic, social and environmental wellbeing is weak, such that countries with high environmental wellbeing do not necessarily attain high scores in social or economic wellbeing. In line with those authors, our findings show that Europe has relatively high values in the category of personal development, in areas such as education, gender equality and good governance, as well as in economic issues, the most salient being organic farming, genuine savings and gross domestic product (GDP).

The relation between GDP and the non-economic dimension of a country was previously analyzed by Cracolici et al. (2010), who suggest that when the GDP of a country rises, there is also a higher level in non-economic aspects such as better health conditions and a higher percentage of educated population. They found that GDP is a basic condition for obtaining a good social performance and a high level of GDP also allows the population to have a longer life expectancy and to achieve a higher level of education (Cracolici et al. 2010, p. 354). Our findings are also in this line, if we consider the economic and social wellbeing indicators in the most developed countries where GDP is higher.

At the same time, and in line with Saisana and Philippas (2012), our findings confirm the inverted shaped relationship between Economic and Environmental Wellbeing (known as Environmental Kuznets Curve) since Environmental Wellbeing has a negative correlation to Human Wellbeing and to Economic Wellbeing, although this is not necessarily the case in all countries (Cole et al. 1997; Daly and Cobb 1989a, b; Ekins 1997; Kuznets 1955).

On the opposite end we have Africa, where issues related to climate and energy take maximum priority, especially renewable energy and greenhouse gas emissions; other environmental issues that also manifest as latent in this geographical area are air quality and renewable water resources. In contrast, a well-balanced society does not appear as a priority in African countries. The results obtained by Hosseini and Kaneko (2011) are also in this line, as they found that Africa has the worst standing relative to other regions for institutional, economic and social pillars; the only positive outcome belongs to the environmental pillar.

This situation may be a result of the fact that many countries are located in arid regions or are in the midst of some type of conflict. This is the case of certain areas of Africa and Asia, and, although it must be said that in these geographical areas improvements have been made in the areas of mother–child health, child education, and infant mortality, there are still many deficiencies.

In relation to climate change, the countries most concerned are the ones with economies based on subsistence agriculture and with low industrial levels owing to economic limitations. Africa, for example, is considered the continent most vulnerable to the adverse effects of climate change (Brown et al. 2007; Hsu et al. 2013; Kotir 2011). For the African Partnership Forum (2007), Africa is particularly vulnerable to climate change because of its dependency on dryland farming, aggravated by factors such as widespread poverty. The main long-term impacts include a change in precipitation patterns that will affect agriculture, a decrease in food safety, a deterioration of water safety, a decrease in fishing resources owing to higher temperatures and an increase in sea level that is affecting low-lying coastal areas with large populations (Wittig et al. 2007).

Looking within Africa, while countries in the northern and Sub-Saharan areas are considered the poorest and worst off, and with a worsening trend, there are nonetheless countries in these regions that are exceptions to the rule and they deserve a more in-depth analysis to find the underlying factors that may have given rise to their better performance in comparison to the other countries in their context. This is the case of Libya and Egypt, which show a certain improvement, and in Sub-Saharan Africa, Angola has shown the best performance of all countries in terms of environmental improvement in the last decade. The southern part of Africa is in a more disadvantageous position in this sense, with Eritrea and Nigeria behind other countries as regards environmental issues.

There are certain measures that could be used to palliate the environmental problems that African countries in general are suffering from. Governments could provide incentives for joint management of natural resources such as water or energy. However, they mainly provide plenty of fine words but little action in the short term, preferring to set up commissions to draft reports and establishing time periods of 10 years or more for solving problems. Another measure could be for African countries to participate in a mechanism derived from the Kyoto Protocol which offers an opportunity to combine measures to mitigate climate change with an outcome that benefits society by means of activities financed through the Clean Development Mechanism (CDM). However, African countries have hardly been given access to participate in this mechanism, and it is therefore considered necessary to foster this financing mechanism. Investing in the design of better and more modern energy systems is another possible measure to take. This could take the form of using biofuels and improving solar technology. Also, taking into account the rising population of large cities such as Lagos, Nairobi, Johannesburg and Accra, new urban designs should be posited, with low carbon emitting technologies.

In Asia, the increase in the use of fossil fuels in China and other parts of South Asia means that there will be an increase in emissions, especially of carbon dioxide. This increase in the use of energy has led to an important increase in air pollution across the whole region. There is also the problem of pollution in the Pacific Ocean, since a high percentage of water has to be treated, at the same time that waste elimination is a huge problem in such densely populated countries, especially in India and China.

The increase in production and consumption in Asia is giving rise to greater energy consumption, more carbon dioxide emissions and other forms of pollution, as well as to a greater concern for environmental issues. In this sense, and according to Roberts and Kanaley (2006), Asia will contribute 56 % of the total growth in global carbon dioxide emissions for the 1990–2025 period, with China alone contributing 34 %. This opinion is shared by Schandl et al. (2009), who refer to the fact that the rapid socio-ecological transition in China is contributing to global high energy consumption and carbon dioxide emissions and now only the United States consumes more energy and emits more carbon dioxide than China.

As regards the social aspect of sustainability, in Asia the population is moving from rural areas to the cities, which offer better quality of life, better access to education, better health care, water supply and sanitation, all of which is reflected in a higher life expectancy and lower infant mortality. Although economic and social conditions have improved in Asia in recent years, there is still great inequality as far as income level, living standards and socio-economic conditions are concerned.

Europe and America include countries with a high and medium Gross National Income (GNI) per capita, the measure used by the World Bank (World Bank 2013) to classify countries according to income level. Thus, countries with a high income, such as Austria, Finland, Australia, the United Kingdom, the United States, Norway, Canada, etc., are characterized as being in close proximity to variables representing human and economic well-being, such as education, good governance, gender equality, clean water, and healthy life, among others. This means that when the wealth of a country increases, there is also an increase in non-economic aspects of well-being, such as better health conditions and a more educated population. The results obtained in our research corroborate those of Cracolici et al. (2010, p. 354), who found that GDP is a basic condition for good social behaviour and a high GDP also provides the population with longer life expectancy and a high level of education.

In order to represent the most significant variables in each geographical area and corroborate the results obtained with the HJ-biplot methodology, an analysis was made of the radial profiles (star plots) that represent the values of each variable along the independent axes in the form of radii that start at the centre of the plot and end in the outer ring; each radius corresponds to one variable. Our findings show the characteristics of each geographical area in relation to the Sustainable Society Index, and confirm the results obtained with the HJ Biplot methodology; furthermore, they are in line with those obtained by Van de Kerk and Manuel (2008).

6 Conclusions

The objective of this paper has been to analyze whether the scores obtained from the indicators proposed in the Sustainable Society Index are similar in different countries or whether there are differences depending on the geographical area in which the country is located. In order to pursue that aim, we analysed a broad sample (151 countries) using a statistical technique—the HJ-biplot—applied to the SSI, in order to depict jointly the geographical areas and the most relevant indicators.

The tables and figures obtained show different objectives concerning Sustainable Society issues. From a statistical point of view, the eigenvalues, the variance explained, and the relative contribution of factor to element ensure the validity of the research. The joint use of the SSI indicators and the HJ-biplot method allow us to depict the geographical zones and the most relevant indicators jointly, showing the proximity of the latter to the former. SSI indicators enable us to extend the analysis beyond a specific country or geographical area, thereby including different contexts in our study. Unlike other techniques, the biplot easily allows us to detect differences in the behaviour of different geographical areas with regard to different dimensions (SSI indicators) in a visual form, as well as the proximity of each country to a specific set of indicators.

From the empirical analyses performed, we were able to draw certain conclusions: the variables linked to Human Wellbeing, such as sufficient food, sufficient to drink, and safe sanitation, show small angles and, therefore, have similar behaviours. Likewise, for Environmental Wellbeing (variables: air quality, biodiversity, renewable water resources), the variables are quite close, also showing a small angle. Hence, they are highly correlated and behave in a similar way.

Additionally, in light of the location of the indicators in different geographical areas, we found that the variables related to Human Wellbeing place more closely to the countries located in the geographical areas of Europe, America and, to a lesser extent, Asia. Meanwhile, other variables associated with Environmental Wellbeing, such as greenhouse gases, renewable energy, and air quality, are mainly closer to Africa, and more residually so to Asia.

After analyzing one of the most significant sets of indicators for sustainability (Sustainable Society Index), the results obtained show that not all geographical areas share the same perspective on economic, social and environmental issues.

In comparison to other related works (e.g. Hosseini and Kaneko 2011), we extend previous literature by analyzing a wider sample of countries, focusing on a broad set of indicators which reflect the main sustainability concerns worldwide. Also, we improve the methodological approach, going beyond the Principal Component Analysis (PCA) and Factor Analysis (FA) used by Srebotnjak et al. (2011) and Zafiriou et al. (2012). Important advantages can be gained from using the HJ-biplot method instead of employing other perhaps more conventional ones. When principal component analysis is used, the axes are combinations of the variables, but these do not appear on the plots, so that very important information concerning the correlations among them is lost, as is the information about the relative situation of the points with respect to the variables, which is interpreted in terms of greater or lesser preponderance in the HJ-biplot method (González-Cabrera et al. 2006). Therefore, the analysis obtained is more representative and better shows the situation of the different geographical areas in regard to sustainable society issues.

As a future line of research we plan to extend this study to include the different years, that is, considering countries, indices and time (three-way-data) in order to gain information about the stability and the differences in the country profiles, according to their level of sustainability throughout the whole time period 2006–2012.

Appendix 1

See Table 3.

Table 3 Countries in the sample

Albania	Cote d'Ivoire	Iraq	Morocco	Spain
Algeria	Croatia	Ireland	Mozambique	Sri Lanka
Angola	Cuba	Israel	Myanmar	Sudan
Argentina	Cyprus	Italy	Namibia	Sweden
Armenia	Czech Republic	Jamaica	Nepal	Switzerland
Australia	Denmark	Japan	Netherlands	Syria
Austria	Dominican Republic	Jordan	New Zealand	Taiwan
Azerbaijan	Ecuador	Kazakhstan	Nicaragua	Tajikistan
Bangladesh	Egypt	Kenya	Niger	Tanzania
Belarus	El Salvador	Korea, North	Nigeria	Thailand
Belgium	Estonia	Korea, South	Norway	Togo
Benin	Ethiopia	Kuwait	Oman	Trinidad and Tobago
Bhutan	Finland	Kyrgyz Republic	Pakistan	Tunisia
Bolivia	France	Laos	Panama	Turkey
Bosnia-Herzegovina	Gabon	Latvia	Papua New Guinea	Turkmenistan
Botswana	Gambia	Lebanon	Paraguay	Uganda
Brazil	Georgia	Liberia	Peru	Ukraine
Bulgaria	Germany	Libya	Philippines	United Arab Emirates
Burkina Faso	Ghana	Lithuania	Poland	United Kingdom
Burundi	Greece	Luxembourg	Portugal	United States
Cambodia	Guatemala	Macedonia	Qatar	Uruguay
Cameroon	Guinea	Madagascar	Romania	Uzbekistan
Canada	Guinea-Bissau	Malawi	Russia	Venezuela
Central African Republic	Guyana	Malaysia	Rwanda	Vietnam
Chad	Haiti	Mali	Saudi Arabia	Yemen
Chile	Honduras	Malta	Senegal	Zambia
China	Hungary	Mauritania	Serbia	Zimbabwe
Colombia	Iceland	Mexico	Sierra Leone	
Congo	India	Moldova	Slovak Republic	
Congo Dem. Rep.	Indonesia	Mongolia	Slovenia	
Costa Rica	Iran	Montenegro	South Africa	

Appendix 2

See Table 4.

Table 4 Geographical areas and variables

	S. F.	S. D.	S. S.	H. L.	C. A.	C. W.	Ed.	G. E.	I. D.	G. Go.	A. Q.
ALB (Eu)	10.00	9.50	9.40	7.38	10.00	8.25	6.79	6.75	6.91	4.67	5.75
ALG (Afr)	10.00	8.30	9.50	7.06	10.00	5.83	7.81	5.99	6.00	3.26	6.17
ANG (Afr)	5.90	5.10	5.80	4.30	4.87	5.18	6.54	6.62	1.00	2.97	7.22
ARG (Am)	10.00	9.70	9.00	7.89	10.00	8.43	9.36	7.24	1.39	4.46	6.17
ARM (As)	7.90	9.80	9.00	6.87	4.60	5.10	7.68	6.65	7.95	4.39	3.41
AUS (Oc)	10.00	10.00	10.00	9.03	10.00	6.17	10.00	7.29	4.40	8.18	1.08
AUT (Eu)	10.00	10.00	10.00	8.68	10.00	9.51	9.13	7.17	9.05	8.13	7.04
AZE (As)	8.90	8.00	8.20	6.58	6.32	4.43	6.49	6.58	7.05	3.36	2.80
BAN (As)	7.40	8.10	5.60	6.08	1.37	8.72	4.87	6.81	8.44	3.28	6.37
BLR (Eu)	10.00	10.00	9.30	7.03	10.00	4.43	9.04	7.10	8.78	3.06	3.91
BEL (Eu)	10.00	10.00	10.00	8.70	10.00	6.63	9.68	7.53	8.66	7.67	5.37
BEN (Afr)	8.80	7.50	1.30	5.12	3.94	3.72	5.78	5.83	5.54	4.42	7.05
BHU (As)	6.70	9.60	4.40	5.93	5.64	4.28	6.56	6.30	3.07	5.20	5.52
BOL (Am)	7.30	8.80	2.70	6.44	5.89	8.34	8.12	6.86	1.00	3.90	5.63
BIH (Eu)	10.00	9.90	9.50	7.88	5.52	9.35	7.60	7.00	5.71	4.23	1.00
BOT (Afr)	7.50	9.60	6.20	4.97	5.59	4.29	6.98	6.83	1.00	6.32	1.75
BRA (Am)	9.40	9.80	7.90	7.44	6.92	8.54	8.75	6.68	1.00	5.29	4.61
BUL (Eu)	9.00	10.00	10.00	7.77	10.00	8.11	7.99	6.99	5.97	5.40	3.18
BUR (Afr)	9.20	7.90	1.70	3.97	3.50	5.05	3.85	6.15	5.33	4.46	6.37
BDI (Afr)	3.80	7.20	4.60	3.97	4.85	3.98	6.43	7.27	7.95	2.67	4.77
CAM (As)	7.50	6.40	3.10	5.64	4.20	9.00	6.17	6.46	4.45	3.30	6.44
CMR (Afr)	7.80	7.70	4.90	4.29	3.36	5.29	6.35	6.07	5.54	3.25	5.77
CAN (Am)	10.00	10.00	10.00	8.82	10.00	9.31	8.95	7.41	6.13	8.24	2.24
CAF (Afr)	6.00	6.70	3.40	3.88	4.84	4.52	4.08	6.00	1.00	2.38	5.40
CHA (Afr)	6.10	5.10	1.30	3.40	3.99	4.47	4.71	5.33	4.82	2.23	5.40
CHI (Am)	10.00	9.60	9.60	8.38	10.00	5.26	8.47	7.03	1.00	7.35	1.00

Table 4 continued

	S. F.	S. D.	S. S.	H. L.	C. A.	C. W.	Ed.	G. E.	I. D.	G. Go.	A. Q.
CHN (As)	9.00	9.10	6.40	7.74	1.97	6.80	6.96	6.87	4.19	3.84	1.82
COL (Am)	9.10	9.20	7.70	7.73	6.47	5.46	8.48	6.71	1.00	4.35	6.11
CGO (Afr)	8.70	7.10	1.80	4.75	2.82	4.90	6.12	6.00	2.64	2.98	2.70
COD (Afr)	3.10	4.50	2.40	4.27	2.91	4.79	5.38	6.00	3.46	1.69	6.78
CRC (Am)	10.00	9.70	9.50	8.21	6.49	4.77	7.41	7.27	1.48	6.22	6.02
CIV (Afr)	8.60	8.00	2.40	4.71	4.53	5.09	3.92	5.77	3.79	2.58	7.11
CRO (Eu)	10.00	9.90	9.90	8.07	6.51	9.25	8.18	7.01	6.15	5.82	3.96
CUB (Am)	10.00	9.40	9.10	8.24	7.80	8.86	9.56	7.39	2.84	3.93	1.93
CYP (As)	10.00	10.00	10.00	8.38	10.00	7.53	8.60	6.57	8.14	7.20	2.82
CZE (Eu)	10.00	10.00	9.80	8.39	10.00	7.45	8.48	6.79	9.22	6.83	3.64
DEN (Eu)	10.00	10.00	10.00	8.73	10.00	7.49	9.91	7.78	5.25	8.64	6.73
DOM (Am)	7.60	8.60	8.30	7.24	6.66	4.65	7.32	6.68	1.76	4.20	4.08
ECU (Am)	8.50	9.40	9.20	7.38	10.00	8.34	8.00	7.04	1.41	3.41	4.72
EGY (Afr)	10.00	9.90	9.50	6.74	8.47	6.24	7.12	5.93	7.63	3.87	4.21
ESA (Am)	9.10	8.80	8.70	6.89	6.09	4.98	7.09	6.57	2.21	4.80	5.19
EST (Eu)	10.00	9.80	9.50	7.97	6.31	9.43	8.95	6.98	6.95	7.16	1.00
ETH (Afr)	5.90	4.40	2.10	5.21	4.20	4.28	5.51	6.14	8.35	3.10	6.85
FIN (Eu)	10.00	10.00	10.00	8.73	10.00	8.76	10.00	8.38	9.31	8.70	4.87
FRA (Eu)	10.00	10.00	10.00	8.88	9.95	8.65	9.44	7.02	7.72	7.54	5.49
GAB (Afr)	10.00	8.70	3.30	5.46	5.95	4.21	7.58	6.00	4.36	3.85	4.51
GAM (Afr)	8.10	8.90	6.80	5.26	1.00	4.86	5.45	6.76	2.37	3.94	6.37
GEO (As)	9.40	9.80	9.50	7.38	5.61	4.02	7.19	6.62	3.26	4.88	5.64
GER (Eu)	10.00	10.00	10.00	8.88	10.00	7.86	9.52	7.59	7.72	7.87	5.89
GHA (Afr)	10.00	8.60	1.40	5.15	4.06	7.78	6.46	6.81	3.13	5.21	5.94
GRE (Eu)	10.00	10.00	9.80	8.79	10.00	7.71	9.86	6.92	5.90	5.82	2.35
GUA (Am)	7.80	9.20	7.80	6.74	5.35	5.99	7.07	6.23	1.00	3.80	4.90

Table 4 continued

	S. F.	S. D.	S. S.	H. L.	C. A.	C. W.	Ed.	G. E.	I. D.	G. Go.	A. Q.
GUI (Afr)	8.40	7.40	1.80	4.65	4.73	4.55	5.23	6.20	5.02	2.45	6.37
GBS (Afr)	7.80	6.40	2.00	3.77	5.02	4.55	6.54	6.20	6.25	2.94	6.37
GUY (Am)	9.20	9.40	8.40	5.62	6.59	4.28	6.92	7.08	1.00	4.27	4.49
HAI (Am)	4.30	6.90	1.70	5.77	5.05	3.97	5.84	7.10	1.00	2.69	6.77
HON (Am)	8.80	8.70	7.70	7.07	5.41	4.97	7.21	6.95	1.00	3.79	4.26
HUN (Eu)	10.00	10.00	10.00	7.77	10.00	7.40	8.94	6.64	9.70	6.49	4.14
ISL (Eu)	10.00	10.00	10.00	9.06	10.00	10.00	9.59	8.53	8.98	7.84	1.10
IND (As)	8.10	9.20	3.40	6.10	1.00	7.89	6.46	6.19	6.64	4.38	3.89
INA (As)	8.70	8.20	5.40	6.78	5.43	6.22	7.56	6.59	6.31	4.04	3.89
IRI (As)	10.00	9.60	10.00	6.92	8.08	4.98	7.77	5.89	5.07	2.65	2.76
IRQ (As)	9.20	7.90	7.30	5.69	3.84	4.27	6.27	6.20	8.03	2.15	2.15
IRL (Eu)	10.00	10.00	9.90	8.91	10.00	9.19	10.00	7.83	6.15	7.91	4.61
ISR (As)	10.00	10.00	10.00	8.98	9.81	5.77	9.21	6.93	4.07	6.04	1.77
ITA (Eu)	10.00	10.00	10.00	9.02	10.00	8.22	9.08	6.80	6.30	6.03	5.81
JAM (Am)	10.00	9.30	8.00	7.42	6.32	4.62	8.27	7.03	3.17	4.88	2.25
JPN (As)	10.00	10.00	10.00	9.39	10.00	8.78	8.92	6.51	10.00	7.38	5.96
JOR (As)	10.00	9.70	9.80	7.21	9.35	3.00	7.52	6.12	5.89	4.83	2.67
KAZ (As)	10.00	9.50	9.70	6.27	6.53	4.34	8.96	7.01	8.03	4.03	3.09
KEN (Afr)	6.70	5.90	3.20	4.87	5.21	5.79	6.70	6.49	2.25	3.66	5.85
PRK (As)	6.50	9.80	8.00	6.51	7.21	4.41	5.95	6.70	6.44	1.80	3.91
KOR (As)	10.00	9.80	10.00	8.65	8.90	8.49	10.00	6.28	7.22	6.44	5.37
KUW (As)	10.00	9.90	10.00	8.20	8.53	4.48	8.31	6.32	6.44	5.36	1.00
KGZ (As)	8.90	9.00	9.30	6.33	4.87	4.13	7.60	7.04	7.87	3.28	3.82
LAO (As)	7.80	6.70	6.30	5.80	3.95	8.51	6.30	6.80	6.31	3.07	4.87
LAT (Eu)	10.00	9.90	7.80	7.65	6.64	9.05	8.24	7.40	4.17	6.33	7.44
LIB (As)	10.00	10.00	9.80	7.04	10.00	4.06	8.14	6.08	5.95	3.78	2.61

Table 4 continued

	S. F.	S. D.	S. S.	H. L.	C. A.	C. W.	Ed.	G. E.	I. D.	G. Go.	A. Q.
LBR (Afr)	6.80	7.30	1.80	4.90	4.60	4.85	6.32	6.20	4.36	3.50	6.37
LBA (Afr)	10.00	9.10	9.70	7.39	10.00	4.94	9.27	6.00	6.44	2.81	4.15
LTU (Eu)	10.00	10.00	9.40	7.46	10.00	8.59	9.07	7.13	3.71	6.45	4.28
LUX (Eu)	10.00	10.00	10.00	8.83	10.00	7.03	7.55	7.22	8.44	8.42	6.25
MKD (Eu)	10.00	10.00	8.80	7.71	5.73	5.97	7.13	6.97	3.33	4.79	2.04
MAD (Afr)	7.50	4.60	1.50	5.44	5.01	4.76	6.75	6.80	3.20	3.51	4.77
MAW (Afr)	7.30	8.30	5.10	4.26	5.01	3.03	6.77	6.85	5.49	4.47	4.77
MAS (As)	10.00	10.00	9.60	7.39	9.73	5.46	7.13	6.53	2.23	5.64	4.15
MLI (Afr)	8.80	6.40	2.20	3.78	3.54	7.86	4.83	5.75	5.12	4.14	6.37
MLT (Eu)	10.00	10.00	10.00	8.74	10.00	2.39	8.19	6.66	8.22	7.42	2.36
MTN (Afr)	9.20	5.00	2.60	5.23	4.57	4.58	5.01	6.16	4.92	3.19	6.37
MEX (Am)	10.00	9.60	8.50	7.91	6.27	6.14	8.09	6.60	1.00	4.60	3.26
MDA (Eu)	10.00	9.60	8.50	6.91	6.38	4.88	6.90	7.08	5.49	4.21	4.51
MGL (As)	7.30	8.20	5.10	6.45	5.19	4.50	8.67	7.14	6.64	4.49	1.30
MNE (Eu)	10.00	9.80	9.00	7.53	5.83	8.36	8.58	7.00	7.95	5.13	3.38
MAR (Afr)	10.00	8.30	7.00	7.09	6.93	6.29	6.04	5.80	4.49	4.36	3.16
MOZ (Afr)	6.20	4.70	1.80	3.77	5.02	4.66	5.89	7.25	2.37	4.49	5.02
MYA (As)	8.40	8.30	7.60	5.11	3.38	4.57	5.74	6.80	1.72	1.51	7.02
NAM (Afr)	8.20	9.30	3.20	5.53	5.41	4.57	6.94	7.18	1.00	5.61	1.00
NEP (As)	8.30	8.90	3.10	5.97	1.80	4.60	5.76	5.89	3.33	3.19	5.52
NED (Eu)	10.00	10.00	10.00	8.89	10.00	7.32	9.94	7.47	8.95	8.30	6.77
NZL (Oc)	10.00	10.00	10.00	8.85	10.00	9.92	10.00	7.81	4.45	8.52	3.92
NCA (Am)	8.10	8.50	5.20	7.42	5.42	4.23	7.01	7.25	1.00	3.66	4.26
NIG (Afr)	8.40	4.90	1.00	4.14	3.64	4.22	3.39	6.20	7.19	3.62	6.37
NGR (Afr)	9.40	5.80	3.10	3.79	2.96	4.48	5.58	6.01	3.07	2.65	6.26
NOR (Eu)	10.00	10.00	10.00	8.89	10.00	9.51	9.80	8.40	9.11	8.39	6.60

Table 4 continued

	S. F.	S. D.	S. S.	H. L.	C. A.	C. W.	Ed.	G. E.	I. D.	G. Go.	A. Q.
OMA (As)	9.20	8.90	9.90	7.49	8.85	4.42	7.54	5.87	6.44	5.59	1.79
PAK (As)	7.50	9.20	4.80	5.90	1.88	6.26	4.37	5.58	7.71	2.74	4.12
PAN (Am)	8.50	9.30	6.90	7.88	6.32	9.22	7.83	7.04	1.00	5.18	4.31
PNG (Oc)	6.90	4.00	4.50	6.12	5.08	3.96	3.70	7.10	1.83	3.62	5.96
PAR (Am)	9.00	8.60	7.10	7.39	5.53	5.18	7.05	6.82	1.00	3.68	6.23
PER (Am)	8.40	8.50	7.10	7.89	5.72	8.34	8.29	6.80	1.20	4.52	1.39
PHI (As)	8.70	9.20	7.40	7.06	5.54	8.93	7.88	7.69	3.83	3.87	3.91
POL (Eu)	10.00	10.00	9.00	7.91	10.00	8.16	8.80	7.04	7.16	6.61	2.54
POR (Eu)	10.00	9.90	10.00	8.56	10.00	7.79	9.56	7.14	6.27	6.91	3.77
QAT (As)	10.00	10.00	10.00	7.88	10.00	4.48	5.74	6.23	1.00	6.32	3.27
ROU (Eu)	10.00	8.80	7.20	7.60	6.06	8.15	8.37	6.81	5.67	5.37	2.14
RUS (Eu)	10.00	9.70	7.00	6.83	6.92	8.24	8.53	7.04	4.23	3.51	1.93
RWA (Afr)	6.80	6.50	5.50	3.97	4.00	4.61	6.84	6.80	1.09	4.50	4.77
KSA (As)	10.00	9.10	8.80	7.06	8.63	4.24	8.43	5.75	6.44	4.46	1.88
SEN (Afr)	8.10	7.20	5.20	5.25	2.77	8.36	4.97	6.57	4.77	4.14	4.86
SRB (Eu)	10.00	9.90	9.20	7.54	5.77	8.36	7.85	7.00	8.69	4.72	3.38
SLE (Afr)	6.50	5.50	1.30	2.65	4.74	4.90	4.67	6.20	4.36	3.67	6.37
SVK (Eu)	10.00	10.00	10.00	7.94	10.00	8.92	8.02	6.80	8.56	6.56	3.95
SLO (Eu)	10.00	9.90	10.00	8.56	6.78	9.30	9.42	7.04	9.58	6.83	3.73
RSA (Afr)	10.00	9.10	7.90	4.76	6.28	8.42	7.99	7.48	1.00	5.47	1.20
ESP (Eu)	10.00	10.00	10.00	9.09	10.00	8.31	10.00	7.58	2.80	6.78	3.38
SRI (As)	8.00	9.10	9.20	7.21	5.18	9.17	6.46	7.21	5.33	4.26	3.65
SUD (Afr)	7.80	5.80	2.60	5.08	3.87	6.52	3.86	6.00	1.72	1.74	6.71
SWE (Eu)	10.00	10.00	10.00	9.08	10.00	9.62	9.28	8.04	9.05	8.53	6.61
SUI (Eu)	10.00	10.00	10.00	9.21	10.00	8.69	8.69	7.63	7.60	8.41	7.26
SYR (As)	10.00	9.00	9.50	7.21	8.08	4.50	6.64	5.90	6.64	3.14	2.74

Table 4 continued

	S. F.	S. D.	S. S.	H. L.	C. A.	C. W.	Ed.	G. E.	I. D.	G. Go.	A. Q.
TPE (As)	10.00	10.00	10.00	8.53	10.00	6.26	9.51	6.70	6.19	6.94	5.09
TJK (As)	7.40	6.40	9.40	6.27	4.17	4.51	7.18	6.53	6.98	2.80	3.80
TAN (Afr)	6.60	5.30	1.00	4.40	5.02	8.50	5.66	6.90	5.49	4.33	6.37
THA (As)	8.40	9.60	9.60	7.04	4.03	8.27	7.19	6.89	4.40	4.32	4.29
TOG (Afr)	7.00	6.10	1.30	5.26	4.12	4.42	6.27	6.20	6.84	3.23	7.00
TRI (Am)	8.90	9.40	9.20	7.06	10.00	4.63	6.41	7.37	3.79	5.22	4.45
TUN (Afr)	10.00	9.40	8.50	7.71	10.00	6.30	7.80	6.26	4.32	4.64	2.72
TUR (As)	10.00	10.00	9.00	7.76	6.48	5.79	7.56	5.95	2.87	4.90	3.06
TKM (As)	9.30	9.10	9.80	5.86	9.76	4.50	6.76	6.90	4.40	2.22	1.68
UGA (Afr)	7.80	7.20	3.40	3.86	4.93	3.95	6.85	7.22	3.36	3.81	4.77
UKR (Eu)	10.00	9.80	9.40	6.81	6.92	2.98	9.20	6.86	9.05	3.88	1.88
UAE (As)	10.00	10.00	9.80	8.06	8.29	4.48	6.72	6.45	6.44	5.82	3.25
GBR (Eu)	10.00	10.00	10.00	8.77	10.00	8.16	9.01	7.46	6.24	7.76	5.13
USA (Am)	10.00	9.90	10.00	8.38	10.00	7.75	9.83	7.41	3.20	7.38	3.04
URU (Am)	10.00	10.00	10.00	7.86	7.31	6.34	9.05	6.91	3.75	6.70	5.14
UZB (As)	8.90	8.70	10.00	6.56	3.97	3.80	7.15	6.90	5.60	2.38	3.06
VEN (Am)	9.30	9.10	8.60	7.71	10.00	4.05	8.92	6.86	2.64	2.41	3.94
VIE (As)	8.90	9.50	7.60	7.39	3.10	7.27	6.58	6.73	6.13	3.92	4.38
YEM (As)	7.00	5.50	5.30	5.79	3.47	4.48	5.47	4.87	5.43	2.56	2.80
ZAM (Afr)	5.60	6.10	4.80	3.56	5.12	4.17	5.90	6.30	1.00	4.28	1.00
ZIM (Afr)	7.00	8.00	4.00	3.55	5.25	7.19	5.71	6.61	1.67	1.84	3.64

	B.	R. W. R.	C.	R. E.	G. Ga.	O. F.	G. S.	GDP	Em.	P. D.
ALB (Eu)	4.21	9.56	6.33	4.16	8.83	1.00	6.57	4.28	3.17	4.86
ALG (Afr)	3.12	4.74	6.56	1.00	7.22	1.00	9.47	4.11	3.68	9.83
ANG (Afr)	6.03	9.96	7.35	5.84	9.13	1.00	1.00	3.48	1.00	9.00

Table 4 continued

	B.	R. W. R.	C.	R. E.	G. Ga.	O. F.	G. S.	GDP	Em.	P. D.
ARG (Am)	2.63	9.60	3.53	1.00	5.79	5.24	8.53	7.17	4.89	7.74
ARM (As)	4.00	6.36	6.27	1.00	8.69	1.00	7.98	3.24	1.50	8.70
AUS (Oc)	6.23	9.54	1.00	1.00	1.00	5.20	8.26	9.50	6.00	9.41
AUT (Eu)	10.00	9.53	2.54	2.68	1.73	9.93	9.01	9.56	6.57	2.29
AZE (As)	3.57	6.53	6.63	1.00	7.27	1.06	7.54	5.20	5.46	9.83
BAN (As)	1.00	9.71	8.31	2.86	9.64	1.00	9.40	1.22	6.07	8.57
BLR (Eu)	3.61	9.25	3.30	1.00	3.12	1.00	9.07	6.61	9.42	6.76
BEL (Eu)	6.58	6.60	1.00	1.00	1.00	5.90	8.95	9.39	4.86	1.00
BEN (Afr)	10.00	9.95	6.49	5.62	9.49	1.00	8.12	1.08	9.32	8.97
BHU (As)	10.00	9.96	1.00	4.76	9.57	1.00	9.67	3.58	6.70	1.27
BOL (Am)	9.25	9.97	2.47	2.69	8.58	1.00	8.09	2.95	5.77	8.87
BIH (Eu)	1.00	9.91	4.75	1.36	4.71	1.00	5.00	4.44	1.00	8.18
BOT (Afr)	10.00	9.84	3.64	2.15	7.71	1.00	9.16	6.84	4.72	9.62
BRA (Am)	10.00	9.93	1.82	4.39	8.01	1.57	7.81	5.71	5.50	3.28
BUL (Eu)	4.43	7.13	3.73	1.00	4.19	1.90	8.57	6.24	2.88	9.63
BUR (Afr)	7.12	9.21	5.11	8.58	9.86	1.00	7.01	1.08	7.19	9.09
BDI (Afr)	2.42	9.77	7.24	8.58	9.86	1.00	1.00	1.00	1.00	8.69
CAM (As)	10.00	9.95	6.94	7.21	9.73	1.00	7.53	1.54	8.44	9.13
CMR (Afr)	4.50	9.97	6.72	6.75	9.74	1.00	7.55	1.56	7.48	9.76
CAN (Am)	3.09	9.84	1.00	1.71	1.00	2.29	7.86	9.51	4.74	1.06
CAF (Afr)	8.87	10.00	5.57	8.58	9.86	1.00	2.84	1.00	4.49	8.15
CHA (Afr)	4.69	9.91	3.73	8.58	9.86	1.00	1.00	1.32	2.42	8.92
CHI (Am)	6.63	9.88	1.65	2.20	5.92	1.00	6.71	7.10	4.90	9.83
CHN (As)	8.02	8.05	6.72	1.16	4.57	1.00	9.56	4.54	6.70	9.28
COL (Am)	10.00	9.94	5.42	2.11	8.69	1.00	7.09	5.22	3.39	8.73
CGO (Afr)	4.84	10.00	6.80	5.42	9.59	1.00	1.00	2.85	2.42	9.44

Table 4 continued

	B.	R. W. R.	C.	R. E.	G. Ga.	O. F.	G. S.	GDP	Em.	P. D.
COD (Afr)	4.99	10.00	7.56	9.63	9.95	1.00	3.51	1.00	2.42	8.93
CRC (Am)	8.82	9.76	4.68	5.24	8.60	1.43	8.48	5.76	4.36	9.01
CIV (Afr)	10.00	9.83	6.68	7.72	9.71	1.00	7.58	1.15	1.64	1.00
CRO (Eu)	4.77	9.94	2.33	1.33	5.70	3.62	8.80	7.30	2.66	7.55
CUB (Am)	2.67	8.02	6.30	1.17	7.33	1.00	5.00	5.10	6.84	8.72
CYP (As)	2.27	8.16	3.68	1.00	1.01	4.58	6.48	8.79	4.60	2.34
CZE (Eu)	7.53	8.71	2.06	1.00	1.00	9.28	8.70	8.60	5.12	8.09
DEN (Eu)	2.04	8.93	1.00	2.03	1.52	7.83	8.90	9.36	5.42	7.42
DOM (Am)	10.00	8.35	7.42	2.32	8.13	8.80	3.50	4.88	2.32	9.10
ECU (Am)	10.00	9.64	4.18	1.15	7.92	1.93	5.11	4.58	5.49	9.60
EGY (Afr)	3.04	1.00	6.32	1.00	7.81	4.27	6.99	3.77	3.54	1.77
ESA (Am)	1.00	9.45	5.27	5.39	9.05	1.03	6.60	4.21	5.58	6.66
EST (Eu)	10.00	8.60	1.00	1.52	1.00	9.56	9.01	7.70	2.87	9.92
ETH (Afr)	9.20	9.54	6.37	9.47	9.94	1.00	8.13	1.00	1.29	8.52
FIN (Eu)	4.24	9.85	3.14	2.53	1.00	8.42	8.49	9.31	4.60	7.07
FRA (Eu)	8.55	8.50	1.10	1.00	4.48	5.37	8.31	9.25	3.80	1.00
GAB (Afr)	7.29	9.99	3.95	5.62	8.24	1.00	5.98	6.88	1.22	9.51
GAM (Afr)	1.00	9.91	5.97	8.58	9.86	1.00	7.95	1.37	1.85	2.82
GEO (As)	1.69	9.71	6.82	3.88	8.89	1.00	5.42	3.29	2.25	8.79
GER (Eu)	10.00	7.90	3.08	1.00	1.00	7.73	8.85	9.40	5.50	1.31
GHA (Afr)	6.98	9.82	4.90	7.03	9.61	1.00	8.33	2.04	1.64	7.85
GRE (Eu)	4.95	8.73	2.02	1.00	2.55	6.08	2.04	8.51	1.77	1.00
GUA (Am)	10.00	9.74	5.63	6.70	9.28	1.00	6.91	3.09	6.64	9.36
GUI (Afr)	3.21	9.93	4.62	8.58	9.86	1.00	1.41	1.00	1.64	2.30
GBS (Afr)	10.00	9.94	6.56	8.58	9.86	1.00	8.83	1.00	1.64	7.60
GUY (Am)	2.38	9.93	3.90	1.28	5.04	1.00	6.04	4.17	3.33	4.21

Table 4 continued

	B.	R. W. R.	C.	R. E.	G. Ga.	O. F.	G. S.	GDP	Em.	P. D.
HAI (Am)	1.00	9.14	8.30	7.05	9.79	1.00	8.98	1.00	1.00	9.82
HON (Am)	6.93	9.88	5.81	4.86	9.04	1.30	8.41	2.72	6.44	9.16
HUN (Eu)	2.57	9.46	3.47	1.00	5.11	5.30	8.82	7.56	3.35	1.39
ISL (Eu)	6.59	9.99	1.00	8.25	3.96	1.00	2.03	9.40	4.76	1.00
IND (As)	2.41	6.61	8.15	2.63	8.61	1.03	9.28	2.38	3.75	2.94
INA (As)	3.21	9.44	6.99	3.45	8.29	1.00	9.17	2.89	5.19	9.32
IRI (As)	3.44	3.23	7.05	1.00	3.12	1.00	7.52	6.09	2.22	9.76
IRQ (As)	1.00	1.27	8.47	1.00	6.77	1.00	5.00	2.48	2.23	1.00
IRL (Eu)	1.00	9.76	1.77	1.00	1.36	2.51	6.93	9.48	2.37	1.00
ISR (As)	7.54	1.00	4.57	1.00	1.07	3.43	8.56	8.96	5.70	2.02
ITA (Eu)	7.93	7.63	2.89	1.06	3.41	8.87	8.12	8.91	4.33	1.00
JAM (Am)	3.67	9.38	6.37	1.52	7.06	1.00	7.68	4.78	2.78	1.00
JPN (As)	5.46	7.91	5.54	1.00	1.03	1.00	8.77	9.22	6.35	1.00
JOR (As)	1.00	1.00	5.35	1.00	6.92	1.00	6.22	3.48	2.75	2.65
KAZ (As)	1.26	7.11	6.01	1.00	1.00	1.00	3.61	6.08	5.83	9.81
KEN (Afr)	5.86	9.11	7.20	8.07	9.73	1.00	8.84	1.25	1.00	7.00
PRK (As)	1.00	8.88	8.15	1.20	7.41	1.00	5.00	1.28	3.36	9.05
KOR (As)	2.50	6.35	4.36	1.00	1.00	1.89	9.29	9.01	7.11	8.77
KUW (As)	1.00	1.00	3.25	1.00	1.00	1.00	8.97	9.56	8.13	9.89
KGZ (As)	3.47	7.94	7.08	3.06	8.70	1.00	8.45	1.63	4.53	6.33
LAO (As)	8.31	9.87	5.92	4.76	9.57	1.00	6.66	1.80	7.79	5.23
LAT (Eu)	8.19	9.88	1.76	3.56	6.40	9.04	8.59	6.76	2.09	8.47
LIB (As)	1.00	8.14	4.94	1.00	5.60	1.00	5.72	6.73	4.54	1.00
LBR (Afr)	1.00	9.99	6.14	8.58	9.86	1.00	1.09	1.00	6.91	9.73
LBA (Afr)	1.00	1.00	5.46	1.00	1.88	1.00	5.00	3.43	1.00	9.95
LTU (Eu)	7.20	9.05	1.00	1.54	5.98	7.42	8.57	7.43	2.12	8.36

Table 4 continued

	B.	R. W. R.	C.	R. E.	G. Ga.	O. F.	G. S.	GDP	Em.	P. D.
LUX (Eu)	10.00	9.81	1.00	1.00	1.00	5.09	8.41	10.00	5.47	9.49
MKD (Eu)	2.44	8.39	5.02	1.48	6.01	5.60	8.71	5.26	1.00	9.16
MAD (Afr)	1.27	9.56	6.36	8.58	9.86	1.00	6.83	1.00	7.71	9.93
MAW (Afr)	7.51	9.44	7.59	8.58	9.86	1.00	8.12	1.00	1.00	7.97
MAS (As)	6.84	9.77	3.73	1.00	3.49	1.00	9.16	6.74	7.26	6.30
MLI (Afr)	1.22	9.35	4.10	8.58	9.86	1.00	8.11	1.00	1.64	9.01
MLT (Eu)	1.00	2.87	3.89	1.00	4.01	1.00	5.25	8.41	5.27	2.47
MTN (Afr)	1.00	8.60	1.45	8.58	9.86	1.00	5.75	1.51	1.00	1.00
MEX (Am)	5.93	8.26	4.66	1.00	6.15	3.21	8.64	6.50	5.93	7.80
MDA (Eu)	1.00	8.36	5.58	1.00	8.28	2.77	8.74	2.20	5.12	9.39
MGL (As)	6.69	9.85	1.00	1.00	5.69	1.00	1.20	2.93	7.41	6.69
MNE (Eu)	5.74	9.75	5.93	3.18	6.69	1.59	4.07	5.64	3.17	7.51
MAR (Afr)	1.00	5.66	6.81	1.00	8.56	1.00	9.35	3.08	4.07	5.91
MOZ (Afr)	7.40	9.97	7.66	9.56	9.89	1.00	7.04	1.00	1.22	8.84
MYA (As)	2.61	9.72	3.80	7.84	9.83	1.00	5.00	1.00	6.69	7.73
NAM (Afr)	7.34	9.83	4.48	1.97	8.54	1.00	9.48	4.13	1.00	9.45
NEP (As)	8.50	9.53	7.69	8.78	9.88	1.00	9.46	1.00	1.00	8.78
NED (Eu)	7.58	8.83	1.00	1.00	1.00	4.51	8.82	9.58	6.38	3.27
NZL (Oc)	10.00	9.85	1.00	3.90	2.96	2.37	8.31	8.66	5.21	8.54
NCA (Am)	10.00	9.93	5.91	5.53	9.23	1.51	7.45	2.11	4.57	2.32
NIG (Afr)	3.53	9.30	1.50	8.58	9.86	1.00	8.99	1.00	1.64	9.57
NGR (Afr)	6.29	9.64	5.70	8.44	9.71	1.00	9.34	1.75	1.00	9.60
NOR (Eu)	5.43	9.92	1.00	3.61	1.99	7.49	9.06	9.86	7.20	6.88
OMA (As)	4.66	1.61	1.95	1.00	1.00	1.00	1.69	8.54	2.23	9.94
PAK (As)	4.91	2.57	8.28	3.73	9.22	1.00	8.45	1.87	5.52	4.58
PAN (Am)	5.75	9.97	3.30	2.23	7.61	1.00	8.57	6.37	6.57	8.46

Table 4 continued

	B.	R. W. R.	C.	R. E.	G. Ga.	O. F.	G. S.	GDP	Em.	P. D.
PNG (Oc)	1.00	10.00	3.86	3.67	9.57	1.00	5.00	1.72	8.27	9.44
PAR (Am)	2.72	9.99	1.63	10.00	9.27	1.00	8.98	3.25	5.71	9.73
PER (Am)	6.54	9.90	4.23	2.52	8.56	2.23	7.77	5.16	4.72	9.46
PHI (As)	2.52	8.30	6.72	3.98	9.18	1.54	9.10	2.58	4.95	8.20
POL (Eu)	10.00	8.06	3.59	1.00	2.01	5.70	8.10	7.69	3.81	5.68
POR (Eu)	3.06	8.77	2.96	2.33	5.47	7.65	3.25	8.15	2.80	1.00
QAT (As)	1.00	1.00	1.00	1.00	1.00	1.00	5.00	10.00	9.61	8.96
ROU (Eu)	3.88	9.68	4.64	1.67	6.48	2.83	9.05	5.92	4.86	8.86
RUS (Eu)	4.60	9.85	3.84	1.00	1.00	1.00	7.32	7.00	5.22	9.84
RWA (Afr)	4.99	9.84	7.80	8.58	9.86	1.00	8.16	1.00	1.00	9.39
KSA (As)	10.00	1.00	4.83	1.00	1.00	1.00	3.01	8.27	3.68	9.89
SEN (Afr)	10.00	9.43	5.60	4.70	9.56	1.00	8.88	1.33	1.00	8.18
SRB (Eu)	2.98	9.75	5.61	1.32	3.69	1.00	4.07	5.35	1.00	7.18
SLE (Afr)	2.15	9.97	6.43	8.58	9.86	3.80	7.63	1.00	1.64	4.62
SVK (Eu)	10.00	9.86	2.06	1.00	3.55	8.95	8.57	8.14	2.62	7.68
SLO (Eu)	6.53	9.70	3.35	1.48	2.52	7.92	8.85	8.75	4.46	7.28
RSA (Afr)	3.44	7.50	6.59	1.07	3.06	1.00	5.97	5.46	1.00	8.37
ESP (Eu)	3.82	7.10	2.16	1.18	4.18	7.68	8.42	8.93	1.15	2.87
SRI (As)	7.48	7.55	6.94	5.61	9.36	3.42	9.03	3.38	6.13	1.57
SUD (Afr)	2.09	4.24	4.86	7.07	9.69	1.00	3.00	1.84	3.00	2.17
SWE (Eu)	5.01	9.85	1.00	3.39	4.93	9.70	9.09	9.51	4.74	8.50
SUI (Eu)	10.00	9.51	4.15	1.90	4.37	9.42	9.37	9.62	7.33	7.05
SYR (As)	1.00	1.36	7.53	1.00	7.18	1.00	2.55	2.95	4.32	8.75
TPE (As)	2.45	7.42	2.37	1.00	1.00	1.00	5.00	9.39	6.45	8.16
TJK (As)	2.07	2.52	7.69	5.90	9.60	1.00	2.75	1.45	8.03	8.68
TAN (Afr)	10.00	9.46	6.30	8.95	9.87	1.00	8.76	1.11	1.00	7.72

Table 4 continued

	B.	R. W. R.	C.	R. E.	G. Ga.	O. F.	G. S.	GDP	Em.	P. D.
THA (As)	8.67	8.69	4.94	1.97	6.41	1.00	9.27	4.92	9.34	8.06
TOG (Afr)	5.52	9.89	6.99	8.30	9.81	1.00	1.31	1.00	1.64	9.00
TRI (Am)	4.80	9.40	5.01	1.00	1.00	1.00	1.00	7.64	5.60	8.90
TUN (Afr)	1.00	3.87	6.31	1.42	7.92	3.61	8.56	4.95	1.51	7.97
TUR (As)	1.00	8.17	5.39	1.11	6.35	3.26	6.60	6.48	3.72	8.31
TKM (As)	1.49	1.00	4.64	1.00	1.00	1.00	9.38	4.33	1.00	9.68
UGA (Afr)	5.13	9.95	4.97	8.58	9.86	3.36	8.44	1.00	6.57	9.10
UKR (Eu)	1.80	7.24	4.96	1.00	4.19	1.51	8.11	4.07	4.41	8.58
UAE (As)	2.35	1.00	1.76	1.00	1.00	1.00	9.49	9.76	6.70	9.63
GBR (Eu)	9.03	9.12	3.12	1.00	2.22	6.62	6.11	9.30	4.49	1.23
USA (Am)	6.83	8.44	2.28	1.00	1.00	1.40	5.26	9.76	4.09	1.00
URU (Am)	1.00	9.74	1.00	4.90	8.08	7.92	7.28	6.63	5.43	5.95
UZB (As)	1.13	1.00	7.58	1.00	6.44	1.00	1.00	2.16	9.80	9.85
VEN (Am)	10.00	9.93	4.32	1.00	3.65	1.00	8.54	5.95	4.45	7.56
VIE (As)	2.29	9.07	6.81	2.88	8.50	1.00	9.02	2.20	6.37	8.45
YEM (As)	1.00	1.00	8.18	1.00	9.10	1.00	1.30	1.59	2.32	7.96
ZAM (Afr)	10.00	9.83	7.52	9.22	9.85	1.00	3.37	1.17	2.47	9.27
ZIM (Afr)	10.00	7.90	6.94	6.88	9.28	1.00	1.41	1.00	1.00	2.57

S. F. sufficient food, *S. D.* sufficient to drink, *S. S.* safe sanitation, *H. L.* healthy life, *C. A.* clean air, *C. W.* clean water, *Ed*. education, *G. E.* gender equality, *I. D*. income distribution, *G. Go*. good governance, *A. Q*. air quality, *B*. biodiversity, *R. W. R.* renewable water resources, *C*. consumption, *R. E.* renewable energy, *G. Ga*. greenhouse gases, *O. F.* organic farming, *G. S.* genuine savings, *GDP* gross domestic product, *Em.* employment, *P. D.* public debt

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Artículo 2

Environmental Performance in Countries Worldwide: Determinant Factors and Multivariate Analysis

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Article

Environmental Performance in Countries Worldwide: Determinant Factors and Multivariate Analysis

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Abstract: The aim of this study is to analyze the environmental performance of countries and the variables that can influence it. At the same time, we performed a multivariate analysis using the HJ-biplot, an exploratory method that looks for hidden patterns in the data, obtained from the usual singular value decomposition (SVD) of the data matrix, to contextualize the countries grouped by geographical areas and the variables relating to environmental indicators included in the environmental performance index. The sample used comprises 149 countries of different geographic areas. The findings obtained from the empirical analysis emphasize that socioeconomic factors, such as economic wealth and education, as well as institutional factors represented by the style of public administration, in particular control of corruption, are determinant factors of environmental performance in the countries analyzed. In contrast, no effect on environmental performance was found for factors relating to the internal characteristics of a country or political factors.

Keywords: environmental sustainability; environmental performance; countries worldwide; socioeconomic factors; institutional factors; biplot

1. Introduction

In recent years, society has shown increased interest in environmental issues, both on the micro- and macro-economic levels.

From the microeconomic point of view, stakeholders are increasingly concerned with the environmental performance of firms and use it to make decisions about their investments. From the macroeconomic point of view, the environmental performance of countries can be defined as a country's ability to produce environmental public goods [1]. Thus, each country will have to be accountable to its citizens for the environmental policies it puts into practice, and political candidates must try to please voters; citizens are therefore expected to want information about the environmental performance [2] of different countries.

Although the relation between environmental performance and different explanatory variables has been analyzed in diverse studies at the firm level [3,4], in this research, we focused on analyzing countries' environmental performance and the variables that can affect it. To measure environmental performance, we use the Environmental Performance Index (EPI), which takes into account objectives, policy categories and indicators corresponding to environmental health and ecosystems. This index was drawn up by Esty *et al.* [5], who form part of a group of environmental experts at Yale University and Columbia University. It focuses on two overarching environmental objectives: reducing environmental stresses to human health and promoting ecosystem vitality and sound natural resource management. Analysis of this index will provide us with a view of the environmental situation worldwide, coinciding with one of the political priorities of environmental authorities around the world and with the international community's intention to adopt Goal 7 of the Millennium Development Goals (MDGs) to ensure environmental sustainability.

In addition, we seek to learn which socioeconomic and institutional factors have an impact on the environmental situation, first jointly and then considering the two most relevant dimensions into which the EPI is divided. Among the socioeconomic factors, we take into account variables of wealth or economic development and education, and among the institutional factors, we have variables relating to the internal characteristics of countries (administration effectiveness), style of public administration (control of corruption) and political factors (political ideology).

Consequently, we carry out multivariate analysis using the biplot methodology to contextualize the countries grouped into geographical areas and the variables relating to environmental indicators included in the EPI. This should give us a view of how the different variables comprising the EPI behave in the different countries of the sample, and we run a regression analysis to test the influence of: (1) socioeconomic factors, such as wealth or economic development and education; and (2) institutional factors, represented by the country's internal characteristics, style of public administration and political factors.

The findings obtained from the empirical analysis point out that socioeconomic factors, such as economic wealth and education, as well as institutional factors represented by the style of public administration and, more specifically, control of corruption, are determining factors of environmental performance in the countries analyzed. In contrast, the factors relating to a country's internal characteristics and political factors do not influence environmental performance. We also found that in regard to the two groups of variables in the EPI, that of environmental health takes higher values in the richest countries of Europe and America, whereas variables related to ecosystem vitality are of more concern in less developed areas, such as Africa, where climate change is a maximum priority.

This study thus extends and improves the previous literature on the topic in the following aspects: (1) we use economic and institutional variables jointly, in contrast to research studies that use only one type of variable, either economic or institutional, and even limit the research to a theoretical perspective; (2) we contextualize the countries and variables (environmental indicators) by means of the HJ-biplot methodology, which provides a graphic representation (plot) that shows that environmental performance indicators can be divided into two categories, one with the variables related to environmental health and the other with those related to ecosystem vitality; this classification neatly coincides with the dimensions into which the EPI is divided. (3) At the same time, this methodology makes it possible to differentiate the environmental concerns of the different countries in the sample, with two types of countries standing out above all: rich countries and poor countries. This classification was corroborated through regression analysis, which showed that economic variables, such as GDP and education, play a fundamental role in environmental performance as measured by the EPI and in its components, especially environmental health. (4) Finally, we use a model in which one of the variables is used in its quadratic form, specifically GDP (GDP^2), which confirms the ecological Kuznets curve theory (EKC), meaning that in the initial stages of development, environmental performance increases along with income level and subsequently decreases in relation to growth in GDP at higher income levels.

The paper is structured as follows: In Section 2, we describe the theoretical framework of environmental performance. Section 3 develops the research hypotheses in relation to the factors that may influence environmental performance worldwide. Section 4 describes the research methods: sample, variables and analysis techniques, including the biplot methodology. In Section 5, the results of the empirical analysis are given, which are then discussed in Section 6. Section 7 summarizes the main findings and consequences and presents the conclusions.

2. Theoretical Framework

Different definitions of environmental performance have been offered in relation to the business sphere. In this regard, Lober [6] considers environmental performance as the commitment of organizations to preserve and protect their natural environment with its multi-dimensional characteristics, such as maintaining the quality of water, air, soil, *etc*. Another definition states that environmental performance refers to the effects of business activities and products on the natural environment, such as resource consumption, waste generation and emissions. For his part, Epstein [7] lists several components of environmental performance, such as the minimization of pollutants, conserving resources, waste reduction, energy conservation, marketing of safe products and reporting potential risks, among others.

However, environmental performance is also a concern in the public sphere, which is the area addressed in our research. The Organization for Economic Co-operation and Development (OECD) considers the following as the primary objectives of environmental performance in the public sphere: helping individual governments assess progress in achieving their environmental goals; promoting continuous policy dialogue and peer learning; and stimulating greater accountability from governments towards each other and towards public opinion.

It is thus necessary to establish a mechanism to examine whether the environmental policies being implemented at the local, regional, national and global levels actually correspond to what was initially planned. This means the adoption of qualitative or quantitative indicators [8] capable of measuring the progress and setbacks that take place in our countries, regions and cities with respect to the environmental objectives initially set [9]. In general, this system of information should be based on batteries of indicators selected by means of a dialogue with all stakeholders.

In this regard, an increase has been observed in the development of indicators by environmental specialists, linked in particular to Local Agenda 21. One of the first initiatives in this regard took place in 1992 at the United Nations Conference on Environment and Development held in Rio de Janeiro, also known as the Earth Summit, specifically in Chapter 40 of Agenda 21, which states the important need for developing sustainability indicators (including environmental indicators) that would be internationally accepted in order to provide a solid foundation for decision making at all levels and contribute to sustainable development.

These indicators are usually classified into areas such as biodiversity, water, energy, transport or agriculture. Many scholars and organizations from around the world also see the need for having a set of common indicators worldwide. This would make it possible to compare environmental action on a global basis. In this context, different organizations, such as the OECD and the United Nations (UN), began to design batteries of indicators to facilitate relevant information for decision making, policy formulation and impact control.

To this effect, the OECD devised a set of indicators following the pressure-state-response model proposed by Rapport and Friend [10], which follows a logic according to which human activity exerts pressure on the environment and on environmental and natural resources, altering their initial state to a greater or lesser extent.

Society as a whole identifies these variations and can decide (policy objectives) to adopt measures (responses) aimed at correcting the negative trends detected. These measures can be directed against the actual pressure mechanisms in a precautionary way or else aimed directly at correcting the environmental factors affected. As a result of these actions, an improvement in the state of the environment is expected.

In short, it is a matter of considering three types of indicators: pressure indicators that quantify the environmental impact of different economic sectors; state indicators that reflect the real situation of the environment; and response indicators that show the action taken to palliate the negative effects of human activity on the environment.

This idea in regard to environmental indicators is shared by Smeets and Weterings [11], who consider that environmental indicators are used for three main objectives: to supply information about environmental problems so that the authorities responsible can assess their severity; to support the development of policies and the setting of priorities through the identification of the factors putting pressure on the environment; and to monitor the effects of the political response.

The pressure-state-response model enables us to propose systems of consistent indicators that consider environmental problems in an integral way, analyzed with all of the connections and interrelations that come between the origin of the problems and their consequences. This view is also shared by Pintér *et al.* [12] when they point out that with a good system of indicators, officials will not have to make decisions blindly and will have objective information available to help them join forces to achieve the objectives proposed, as well as their subsequent assessment.

The OECD has drawn up its proposals for environmental indicators based on this model posited by Rapport and Friend [10]. The UN followed the same model, but adapted to the needs of the concept of sustainable development, which assesses not only environmental aspects, but also economic and social factors.

In addition to the environmental indicators proposed by international organizations such as the OECD and the UN, others, such as the Ecological Footprint, the Environmental Sustainability Index and the Renewability and Energy Sustainability Index, must also be mentioned.

In our research, we use an index derived from the environmental sustainability index, called the environmental performance index (EPI), which can be a useful way of measuring environmental performance. It includes a set of environmental indicators in areas of importance that should be of interest to all politicians and officials in every country and that must be tackled through the use of suitable policies. The environmental indicators proposed by the EPI are focused on two objectives: on the one hand, a reduction in environmental stresses to human health and, on the other, the protection of ecosystems and natural resources [5,13].

The EPI includes a category of environmental health policies to address the effect that the environment has on the quality of life around the world, with a view to reducing environmental pressure on human health. Thus, the EPI uses a set of indicators to reflect environmental health: environmental burden of disease, air pollution (effects on human health) and water (effects on human health). It also includes indicators related to ecosystem vitality aimed at reducing the loss or degradation of ecosystems and natural resources. These indicators are: air pollution effects on ecosystems, water effects on ecosystems, biodiversity and habitat, productive natural resources (forestry, fisheries and agriculture) and climate change [5,13].

Besides these environmental indicators, in the present study, we also consider socioeconomic and institutional variables that represent the institutional environment [14]. The institutional variables we use are: government effectiveness, voice and accountability, political stability and control of corruption [15].

We also incorporate in the model variables that represent wealth or economic development. To do so, we used the variable GDP or gross domestic product per capita [16,17]. GDP is considered to be important, because it reflects a country's ability to offer its citizens good living conditions, taking into account economic, social and environmental aspects. Moreover, a country with a good GDP will improve its health services, access to education and working conditions and will protect its citizens from crime. In short, it will provide a more sustainable habitat [18]. These variables have been used by different authors in research studies that analyze their influence on environmental performance [19–21].

As regards the use of different theories in the determination of the factors that may exert a relevant influence on the environmental performance of countries, economic theory, the ecological Kuznets curve theory and the ecological modernization theory can be mentioned.

Economic theory suggests that control of pollution improves as a country develops, and thus, rich countries not only can, but should, invest in pollution control and other environmental improvements [22]. With the same criteria, Jahn [23] considers that countries with greater economic growth are better able to handle environmental problems, because they have the financial resources to do so. Nonetheless, this same author also found that in wealthy countries, such as Germany, Japan, Canada, the United States and Switzerland, there was no relation between wealth and environmental performance. The reason for this is that wealthy countries may be able to invest money in order to improve their environment in contrast to poorer countries, but they also tend to create environmental problems, since they have a high level of consumption, which can lead to increases in their pollution levels, thereby also generating more waste and using more natural resources.

Other theories related to environmental performance are the so-called ecological Kuznets curve theory and ecological modernization theory, which focus on economic factors to explain social impacts on the environment [1].

With respect to the first theory, Esty and Porter [22] found a significant relation between income and environmental performance, suggesting that the alleviation of poverty should be considered a priority in environmental policy; nonetheless, some authors argue that the ecological Kuznets curve theory is only valid for a small class of environmental impacts and may not be applicable to developing countries [24]. According to Dinda [25], environmental pressure increases more rapidly than income in the initial stage of development and then decreases in relation to growth in GDP at higher income levels.

Ecological modernization theory [26] has its basis in the relation between economic growth and environmental degradation, which can be seen most clearly in advanced industrial countries, and it argues that this created new conditions for environmental protection [1]. Other aspects considered in this theory are: the role of science and technology, the importance of market dynamics, the role of economic agents and the ideology of social movements [27]. Moreover, this theory is necessary for ecological sustainability and needs to show that society modifies their institutions in response to environmental problems, and this modification leads to environmental improvements; at the same time, companies must demonstrate that they are reducing their impacts on the environment and are not contributing to the expansion of negative impacts by other companies.

As mentioned above, besides economic factors, in recent years, there has been an increase in the consideration of other factors that may affect environmental performance (e.g., political factors, structural factors, competitiveness), as manifested in studies, such as the one by Esty and Porter [22]. These authors find significant differences in the environmental performance of countries having similar economic levels, which suggests that environmental results are not merely a function of economic development, but also a consequence of policy choice.

In this vein, authors, such as Fiorino [28], also show that effective, innovative and adaptable governance is a necessary condition for countries seeking a transition to sustainability. Governance aspects include integrating policy, enhancing social capital, improving participation and making and implementing choices more adaptively. In the literature on governance, some researchers have concentrated on regime type, concluding that democratic regimes show higher levels of environmental performance than authoritarian regimes. They attribute these results to the availability of information, opportunities for the public to demonstrate and the independence of scientific researchers. In addition, high levels of democracy are associated with growth in per capita income [29,30]. In contrast, other authors, such as Midlarsky [31], have found a negative relation between democracy and three environmental indicators, deforestation, carbon dioxide emission and soil erosion by water, contrary to prediction.

According to Fiorino [28] and Liefferink *et al.* [32], other institutional factors analyzed that can affect environmental performance are presidential-parliamentary, federalist-unitary, proportional representation and pluralist-corporatist systems; the former ends up affirming that it is difficult to reach clear and consistent conclusions owing to differences in the dependent variables used and the complexity of the interrelation among institutional factors.

3. Factors Affecting Environmental Performance: Research Hypotheses

In light of the above, several factors seem to be involved in the environmental performance of countries, and they can be grouped as socioeconomic factors and institutional factors.

3.1. Socioeconomic Factors

3.1.1. Wealth or Economic Development

According to most of the previous literature, environmental performance overall depends on a country's economic performance. In this regard, [22] argue that the most competitive countries in the world tend to show better environmental performance. Likewise, Scruggs [19,20], using a sample of seventeen industrialized democracies for which he constructs an index of environmental performance to serve as the dependent variable, finds that higher per capita income is positively related to environmental performance.

To represent economic wealth, we used the variable GDP or gross domestic product per capita. This measure of GDP depends on several components, such as private consumption, investment, government consumption, changes in inventories, total exports and total imports [21].

According to Cracolici *et al.* [18], a country's level of GDP can be considered a relevant aspect in its ability to provide citizens with good living conditions from an economic, social and environmental point of view. An increase in GDP per capita is necessary to improve the population's standard of living and to provide better social welfare services, as well as better access to education, better working conditions and a healthier and more sustainable environment. What we wish to demonstrate with the following hypothesis is that the higher the level of a country's GDP, the better its environmental conditions as measured by us using the EPI.

Consequently, on the basis of preceding studies, we posit the following hypothesis:

H1: The economic wealth of a country shows a significant and positive relationship with its environmental performance.

3.1.2. Education

Level of education is also a fundamental factor in a country's environmental performance. A well-educated and trained population will demand a higher volume of information about environmental issues and performance. Thus, Duit [1] considers that a country with a high level of education and culture will be more able to handle environmental problems and initiate environmental cooperation programs.

According to Cracolici *et al.* [18], CO_2 emissions and the literacy rate are variables capable of capturing the differences among countries in regard to social and environmental dimensions, and a similar result regarding literacy rate was obtained by McGillivary [33].

Consequently, the following hypothesis has been established:

> H2: The level of education of a country shows a significant and positive relationship with its environmental performance.

To test this, we use the variable, adult literacy (AL), obtained from the United Nations Human Capital Index.

3.2. Institutional Factors

Among the institutional factors, the following variables can be highlighted: a country's internal characteristics: (administration or government effectiveness), style of public administration (control of corruption) and political factors (political ideology).

3.2.1. Administration Effectiveness

The effectiveness of a country's public administration and government may constitute a relevant driver in the development of environmental performance. This effectiveness encompasses issues such as the quality of the bureaucracy, the competence of civil servants, the independence of the civil service from political pressures and the credibility of the government's commitment to policies [34].

There is a slightly positive relation between government effectiveness and good environmental performance according to the EPI [5]. Particularly, government effectiveness positively correlates with performance on greenhouse gas emissions per capita, health ozone, growing stock and water quality indicators. Government effectiveness shows a slightly negative correlation with performance on the sulfur dioxide indicator.

Consequently, we have formulated the following hypothesis:

> H3: Government effectiveness in a country shows a significant and positive relationship with that country's environmental performance.

Government effectiveness is measured by an index devised by Kaufmann *et al.* [34] for the World Bank. This variable represents the quality of the following factors: a country's bureaucracy, its general infrastructure, budgetary and financial management, institutional effectiveness, public schools, supply of public goods—education and basic health, as well as a quality public administration—among other aspects.

3.2.2. Control of Corruption

Corruption has a negative effect on economic growth. In this regard, the prior literature posits a negative relation between corruption and environmental performance, adducing that corruption reduces a country's income, and this low-income level can lead to high levels of pollution. According to Duit [1], the quality of institutions is argued to be a crucial factor for explaining variation in governance and economy. In this sense, well-functioning institutions, in terms of transparency, rule of law and low levels of corruption, alleviate problems of collective action by providing a structure of rules and sanctions within the institutional realm.

Meyer *et al.* [35] find a correlation between institutional factors and deforestation rates in a study of 117 countries. The negative environmental impact of corruption is corroborated in a study by Welsch [36], cited by Duit [1]. Countries with high levels of corruption tend to have low levels of environmental performance, whereas countries with low levels of corruption perform better on the EPI.

Consequently, the following hypothesis has been formulated:

H4: The control of corruption in a country shows a significant and positive relationship with its environmental performance.

The extent of control of the level of corruption is represented by the Transparency International CPI (Corruption Perceptions Indicator), which indicates the degree of public sector corruption as perceived by business professionals and country analysts and ranges from 10 (highly clean) to 0 (highly corrupt). This variable takes the name CORRUP.

3.2.3. Political Ideology

These factors include both the elements that identify the behavior of parties, such as political stability, understood as a concept of the level of representativeness received in the ballot boxes, and their ideology.

It is of interest to analyze whether the political trend of the ruling party in a country can have any impact (either positive or negative) on environmental performance. Previous research has shown that a more consensual democracy has been beneficial for a higher level of environmental performance [37,38]. In this sense, Scruggs [19,20] considers that in democracies, environmental protection is strengthened via free dissemination of new interests, mobilization of voters (or leaders), *etc*. All of this culminates in better environmental conditions.

More specifically, in a study of 21 OCDE countries, [38] argues that he found a statistically significant effect of left-wing parties on emissions reductions. Likewise, some authors consider that the participation of different agents in the political process tends to favor good environmental performance, a result that is maintained even when certain socio-economic variables are controlled for [19,20,39]. This can be explained by the fact that corporatist accommodation structures tend to favor negotiated solutions, which are a crucial element in the success of an environmental policy [39].

Initially, one would think that left-leaning governments tend to carry out programs or activities addressed to good environmental performance, whereas those that have other types of ideologies more often concentrate on social policies. However, because there is not enough previous evidence to predict which political tendency may be more prone to good environmental performance, we have formulated an open hypothesis:

H5: The political tendency of the ruling party of a country shows a significant and positive relationship with its environmental performance.

To test this hypothesis, we use a dummy variable as an independent variable, CONSERV. This variable takes the value one if the governing party shows a conservative ideology and zero otherwise.

This information is obtained from the World Handbook available on the Central Intelligence Agency (CIA) website.

4. Methodology

4.1. Population and Sample

With our research goals in mind, we selected most countries worldwide as our target population. This population was chosen in the interest of extending and generalizing the results obtained in previous studies and overcoming their limitations, since they focus on specific geographic contexts, such as Western industrialized countries [23,39], 21 OCDE countries [36] and seventeen industrialized democracies [19,20].

The sample used refers to the 149 countries selected by Esty *et al.* [3] (see Appendix Table A1) and incorporates the advantages derived from considering different geographic contexts: in short, countries pertaining to five geographical areas were studied: America, Europe, Africa, Asia and Oceania.

Regarding the data source, for the effects of comparability and consistency, we include international organizations, research institutions, government agencies and academia, all of which are considered objective sources of information. The data are obtained directly at the national level of the countries and are subject to certain requirements regarding information and quality established by the data collection entity. All of the data sources are publicly available and include the following: official statistics calculated at the government level, spatial or satellite data compiled by research or international organizations, observations from monitoring stations and modeled data. Some of the most important data sources are: the World Resources Institute, World Bank, Climate Analysis Indicators Tool, International Energy Agency, United Nations, Department of Economic and Social Affairs, WHO/UNICEF Joint Monitoring Programme for Water Supply and Sanitation, World Database on Protected Areas and United Nations Environment Programme [40].

To make the data comparable across countries, the Environmental Performance Index (EPI) uses a multi-step criterion to devise consistent indicators that will permit comparisons among the different sectors, units of reference and aggregation levels. The first step involves creating standards for transforming raw values according to population, GDP or another denominator to make data comparable across countries. The second step entails applying statistical transformations to the data in order to better differentiate performance amongst countries; the transformed data are then used to calculate performance indicators using a proximity-to-target methodology. This reflects how close a particular country is to an identified policy target. The target, or high performance benchmark, is defined taking into account national or international policy objectives established by scientific thresholds or expert judgment. Scores are converted to a scale of 0–100 by simple arithmetic calculation, with zero for the worst value observed (the furthest from the target) and 100 for the best value (the one closest to the target).

4.2. Dependent Variable

The dependent variable used is the Environmental Performance Index (EPI), devised by Esty *et al.* [5]. This index has also been used by Malul *et al.* [41], who consider that the EPI 2008 offers a composite index of current national environmental protection efforts. This index comprises objectives, policy categories and indicators corresponding to environmental health and ecosystem vitality. The performance indicators are tracked in well-established policy categories, which are then combined to create a final score. Each score is converted to a scale of 0–100 by simple arithmetic calculation, with 0 being the worst observed value and 100 the best observed value.

One of the aggregation methods used to construct the EPI from all of the indicator scores is that of the environmental health and ecosystem vitality subcategories, each representing 50% of the EPI score, *i.e.*, an equal division into human and nature issues. Within this environmental health subcategory, the environmental burden of disease indicator is weighted at 50%, because it is widely considered to be the most comprehensive and accurate measure of environmental health burdens. The water and air pollution (effects on humans) indicators comprise the remaining 50% of this subcategory, each representing a quarter of the total score for environmental health. Within the ecosystem vitality subcategory, the climate change indicator carries 50% of the weight. The air pollution (effects on ecosystems) indicator is weighted at 5% in this subcategory. The remaining indicators—water (effects on ecosystems), biodiversity and habitat and productive natural resources (forestry, fisheries, agriculture)—are each weighted to cover the remaining 45% of this subcategory.

4.3. Independent and Control Variables

Table 1 shows the explanatory variables proposed to test the hypotheses of Section 3.

Table 1. Independent variables.

Variable	Description	Hypothesis
GDP	Economic wealth, measured by gross domestic product per capita	H1
AL	Adult literacy, represented by the index drawn up by the United Nations Human Capital Index	H2
GE	Government effectiveness, represented by the index made by Kaufman *et al*. (2008) for the World Bank	H3
CORRUP	Degree of control of corruption, represented by the indicator from Transparency International	H4
CONSERV	Dummy variable which takes the value 1 if the ruling party is right-wing, and 0 otherwise	H5

In addition, country size, OECD *vs.* non-OECD countries, civil liberties and political stability have been added as control variables representing institutional factors.

Country size is measured by its size of population. Grafton and Knowles [42], in a study using a sample of 53 countries, argue that population growth is an important factor to consider when assessing national environmental performance. In their study, they used the variables proposed in the Environmental Sustainability Index (ESI), an index that preceded the EPI.

Civil liberties measures the perceptions of the extent to which a country's citizens are able to participate in selecting their government, as well as their freedom of expression, freedom of association and a free media. To identify the degree of civil liberties in each country, we used the variable VOICE, the index made by Kaufman *et al.* [34] for the World Bank.

Political stability represents perceptions of the likelihood that the government will be destabilized or overthrown by unconstitutional or violent means, including politically-motivated violence and terrorism. This variable is measured by an index devised by Kaufmann *et al.* [34] for the World Bank.

Each of the four worldwide governance indicators (GE (government effectiveness), CORRUP, VOICE (representing civil liberties) and PE (representing political stability) measures the combined average of the data from several government sources. This is done in three steps as follows: Step 1 is to determine the indices of the individual data sources; Step 2 involves the initial rescaling of data from one source to run from 0 to 1: the individual data sources are rescaled to range from 0 to 1, with higher values corresponding to better results; in Step 3, the 0–1 units are not necessarily comparable across sources, so the unobserved components model (UCM) is used to construct a weighted average of the individual indicators from each source [34]. The UCM assumes that the observed data from any source are a linear function of the unobserved latent variable (level of governance), plus an error term. The linear function for different sources varies and, so, corrects for the non-comparability noted above. The results are a weighted average of the data from each source.

4.4. Multivariate Analysis: The HJ-Biplot Technique

Biplots can be viewed as the multivariate analogue of scatterplots, where countries are plotted as points relative to two latent variables, or in other words, biplot methods are tools for visual inspection of data matrices, allowing the eye to pick up patterns, regularities and outliers. Multivariate data can be biplotted in a manner analogous to principal component analysis. With biplots, the multivariate distribution of a set of variables can be approximated in a low-dimensional space (usually two dimensions), providing a useful visualization of the structure of the samples relative to the variables [43].

Gabriel's biplots focus on the joint plotting of the individuals (countries) and variables of a multivariate data matrix X for descriptive purposes. This graphic representation, in a low-dimensional space, allows interrelations between individuals and variables to be captured visually. According to singular value decomposition (SVD), X is approximated to obtain a Euclidean low dimension map, through row (countries) markers and column (variables) markers that are represented by points/vectors, similar to the case of factorial correspondence analysis, except that in the biplot, interpretation is based on the geometric properties of the scalar product between the row vectors and the column vectors, which allows an approximate representation of the elements of X. Gabriel proposed two different biplots, the JK-biplot and the GH-biplot, JK-biplot represents the interunits distances very well, but not the configuration of the variables. Conversely, the GH-biplot represents the relationships among variables very well, but not the interunits distances.

Galindo [44] demonstrated that, with the appropriate choice of markers, it is possible to represent the rows and columns simultaneously on the same Euclidean space with the highest quality of representation. This was called the HJ-biplot.

Like the classical biplots by Gabriel, this alternative allows closeness between points and vectors to be interpreted. The objective of the HJ-biplot is to describe the rows and columns and the relations between them, a different objective from classical biplots, where the reproduction of each element was necessary. The rules for the interpretation of the HJ-biplot combine the rules for other techniques of multidimensional representation: the distance between rows (points) is an inverse function of the similarity (this allows similar clusters to be identified); the length of a column (vector) approximates its standard deviation; columns with acute angles between them are associated with high positive correlation, an obtuse almost straight angle, with high negative correlation, and a right angle with no correlation; and the order of the orthogonal projections of the rows on a column approaches the order of the row values in that column (so the larger the projection of a point in a vector, the more the row deviates over the average of the column).

The software used to implement the HJ-biplot was developed by Vicente-Villardón [45] and is available for free download [46].

4.5. Explanatory Model Proposed

Based upon the variables selected to test the proposed hypotheses, we defined the following Model (1), in which the environmental performance index is a function of the socioeconomic and institutional factors of a country. The objective of the dependence models is to predict the impact of a set of explanatory variables, considered simultaneously, on a country's environmental performance according to the environmental performance index. This model thus serves to empirically test which variables are the ones that most affect environmental performance.

Environmental Performance Index = f (socioeconomic factors, institutional factors)
Environmental Performance Index = f (economic wealth, adult literacy, government effectiveness, degree of control of corruption, right-wing ruling party, size, OECD and non-OECD countries, level of civil liberties, political stability) (1)

Model (1) can be estimated empirically with Equation (2):

$$EPIINDEX_i = \beta_0 + \beta_1 GDP_i + \beta_2 AL_i + \beta_3 GE_i + \beta_4 CORRUP_i + \beta_5 CONSERV_i + \beta_6 SIZE_i + \beta_7 OECD_i + \beta_8 VOICE_i + \beta_9 PE_i + \varepsilon \quad (2)$$

where *EPIINDEX* is the environmental performance index; GDP_i is economic wealth measured by log gross domestic product per capita; AL_i is adult literacy as represented by the United Nations Human Capital Index; GE_i is government effectiveness, represented by the index made by Kaufman *et al*. [34] for the World Bank; $CORRUP_i$ is the degree of control of corruption represented by the indicator used by Transparency International; $CONSERV_i$ is a dummy variable that takes the value 1 if the ruling party is right-wing, and 0 otherwise; $SIZE_i$ is the size of the public body measured by log of the population of the country; $OECD_i$ is a dummy variable that takes the value 1 if the country belongs to the *OECD*, and 0 otherwise; $VOICE_i$ is the level of civil liberties represented by the index created by Kaufman *et al*. [34] for the World Bank; and PE_i is political stability, represented by the index devised by Kaufman *et al*. [34] for the World Bank.

The above model was checked empirically through a linear regression (OLS), since it was a matter of cross-sectional data for the year 2008. Regression analysis is used to understand which among the independent variables are related to the dependent variable and to explore the forms of these relationships. The regression function is defined in terms of a finite number of unknown parameters that are estimated from the data.

Furthermore, since EKC theory predicts the use of a quadratic form in the model and to empirically test the inverted U [47] that is the result of the relation between income per capita and environmental issues, we developed a new model to be estimated by the following equation:

$$EPIINDEX_i = \beta_0 + \beta_1 GDP_i + \beta_2 GDP^2_i + \beta_3 AL_i + \beta_4 GE_i + \beta_5 CORRUP_i + \beta_6 CONSERV_i + \beta_7 SIZE_i + \beta_8 OECD_i + \beta_9 VOICE_i + \beta_{10} PE_i + \varepsilon$$

5. Results of the Empirical Analysis

5.1. Univariate Analysis: Mean, Standard Deviation and Correlation Matrix

Table 2 displays the descriptive statistics of the numerical dependent, independent and control variables; the minimum, maximum, mean and standard deviation are reported.

Table 2. Descriptive statistics.

Variables	Minimum	Maximum	Mean	Standard Deviation
EPIINDEX	39.10	95.50	71.87	12.74
GDP	2.30	4.86	3.76	0.57
AL	23.60	100.00	81.70	20.43
GE	0.15	0.32	0.18	0.023
CORRUP	1.40	9.40	3.96	2.10
SIZE	5.48	9.12	7.06	0.65
VOICE	0.11	0.21	0.13	0.016
PE *	0.19	0.38	0.21	0.028

* political stability

As can be seen, there is wide dispersion in most of the variables used in the research: in economic wealth, as measured by log gross domestic product per capita (from 2.30 to 4.86), and in adult literacy, as represented by the United Nations Human Capital Index (from 23.60 to 100.00). On average, environmental performance is 71.78, with a minimum value of 39.10 and a maximum value of 95.50. Table 3 shows the correlations matrix, which points to the non-existence of high correlations between the variables analyzed.

Table 3. Correlations matrix.

Variables	1	2	4	5	6	7	8	9	10
1. EPIINDEX	1.00								
2. GDP	0.776 **	1.000							
3. AL	0.784 **	0.727 **	1.000						
4. GE	−0.203 *	−0.094	−0.029	1.000					
5. CORRUP	0.636 **	0.763 **	0.487 **	−0.039	1.000				
6. CONSERV	0.200 *	0.288 **	0.240 **	0.073	0.255 **	1.000			
7. SIZE	−0.038	−0.092	−0.110	−0.530 **	−0.118	0.095	1.000		
8. OECD	0.498 **	0.619 **	0.405 **	−0.039	0.768 **	0.299 **	0.099	1.000	
9. VOICE	0.205 *	0.390 **	0.254 **	0.673 **	0.502 **	0.179 *	−0.550 **	0.430 **	1.000
10. PE	−0.388 **	−0.348 **	−0.228 **	0.876 **	−0.240 **	−0.045	−0.529 **	−0.223 **	0.518 **

** Significant at 0.01; * significant at 0.05.

5.2. Multivariate Technique: Biplot

According to Galindo [44], several measures are essential for a correct implementation of the HJ-biplot; specifically, eigenvalues and explained variance (Table 4) and the relative contribution of the factor to the element (Table 5), through which it is possible to detect the variables responsible for the position of the axes and, therefore, the configuration obtained in them.

Table 4. Eigenvalues and explained variance.

Axis	Eigenvalue	Explained Variance	Cummulative
Axis 1	504.45	37.87%	37.87%
Axis 2	226.29	16.99%	54.86%

It can be deduced from Table 4 that there is a dominant axis (Axis 1) that takes 37.87 percent of the total inertia of the system. The trend in the eigenvalues is truncated in the second axis, achieving an accumulative inertia of 54.86.

In other words, 54.86 percent of the total inertia is absorbed by only the first two factorial axes, indicating that this percentage of the total information is present on these two axes. Factorial Plane 1–2 absorbs 54.86% of the total inertia. This factorial plane is used in the different figures to represent geographical areas and variables (see Figure 1, where Axis 1 (horizontal) and Axis 2 (vertical) are represented). The remaining factors provide a smaller load of information.

Table 5 contains the contribution of each factor to the element, which lets us know the variables responsible for the positions of the axes and their configuration.

Table 5. Relative contribution of the factor to the element.

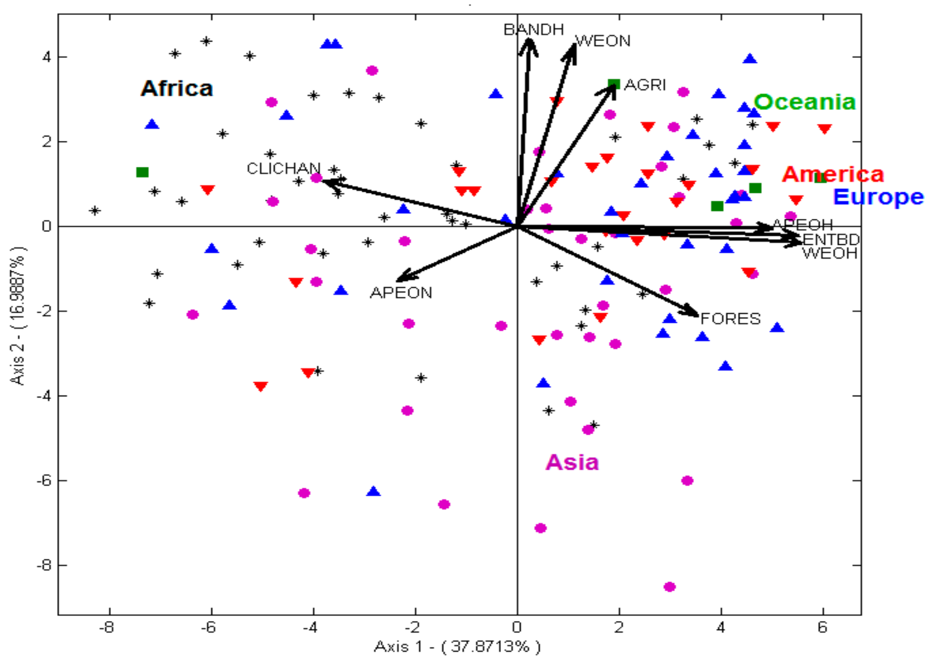
Variables		Axis 1	Axis 2
Environmental burden of disease	ENTBD	842	2
Water (effects on humans)	WEOH	857	4
Air pollution (effects on humans)	APEOH	679	0
Air pollution (effects on nature)	APEON	154	48
Water (effects on nature)	WEON	33	497
Biodiversity and habitat	BANDH	1	524
Forestry	FORES	339	119
Agriculture	AGRI	98	304
Climate change	CLICHAN	406	32

The variables environmental burden of disease (ENTBD), water (effects on humans) (WEOH) and air pollution (effects on humans) (APEOH) make a high contribution (842, 857 and 679, respectively) to Axis 1. In contrast, water (effects on nature) (WEON), biodiversity and habitat (BANDH) and agriculture (AGRI) contribute to Axis 2 (497, 524 and 304, respectively). The graphic representation of the five geographical areas that include the countries analyzed are presented in Figure 1.

All of the countries grouped into five geographical areas are represented by different forms in four quadrants. The continents are represented as follows: Africa with black five-point stars, America with red inverted triangles, Asia with purple circles, Europe with blue triangles and Oceania with green squares. The countries located in Europe, America and Oceania are mainly represented in Quadrants 1 (upper-right), and countries located in Africa are mainly represented in Quadrants 2 (upper-left), whereas Quadrants 3 (lower-left) and 4 (lower-right) contain the Asian countries.

The interpretation of the variables is based on the angles between the vectors, such that variables with vectors forming small angles are variables with similar behaviors. As can be observed from Figure 1, the variables linked to environmental health, such as environmental burden of disease, water (effects on humans) and air pollution (effects on humans), show small angles and, therefore, have similar behaviors. Similarly, for ecosystem vitality (variables: water effects on nature, biodiversity and habitat and agriculture), the variables are quite close, also showing a small angle. Hence, they are highly correlated and behave in a similar way.

Figure 1. Factorial Plane 1–2, including the countries and the indicators in the EPI.



In Figure 1, the geographic areas (points) and the variables (vectors) representing Environmental Health and Ecosystem Vitality are displayed jointly. As for the individuals, when they are close to a vector-variable, it implies that they take predominant values for that variable, in the sense that the individuals are significant in explaining the variable and that the variable is of great value for the individuals. In this same figure, it can be observed that the variables related to Environmental Health are mainly closer to the countries located in the geographic areas of Europe and America and, to a lesser extent, to countries in Oceania. Meanwhile, other variables associated with Ecosystem Vitality are mainly closer to Africa and more residually to Asia.

5.3. Regression Analysis

We analyzed several statistical assumptions of the regression analysis used. Regarding heteroscedasticity, Hair *et al.* [48] recognize that it constitutes a common non-fulfilment of the statistical models in regressions. The usual way to alleviate heteroscedasticity is by transforming variables (e.g., change to an inverse variable or transform the variable into a logarithm); this last measure has been used in the variables SIZE and GDP of our sample.

Despite the issues detected in homoscedasticity, the models fit properly, and their behavior is statistically correct. For example, the value obtained from Durbin–Watson's test (2.102) is close to a value of two. According to Savin and White [49] and David [50], a value close to two reflects the absence of autocorrelation in the residuals from the regression.

To detect the presence or absence of multicollinearity, variance-inflation factors and tolerance were analyzed. No variation-inflation factor is higher than 10, indicating that there is no collinearity between the variables.

With regard to the explanatory power of the model (R^2), Hair *et al.* [48] consider that an R^2 of 0.5 is relatively high, although whether a regression gives a good fit to the model depends on the framework. In our model, the value obtained is 0.745 for a confidence level of 99 percent (p-value < 0.01), and thus, it can be considered a relatively good model. Now that the assumptions of the regression analysis have been analyzed, the results obtained in the estimations of the model proposed are synthesized in Table 6.

Table 6. Linear regression.

Independent and control variables	**Dependent Variable: EPIINDEX**			**Dependent Variable: ENVHEALTH**			**Dependent Variable: ECOSVITALITY**			**Dependent Variable: EPIINDEX ***		
	β	**t**	***p*-value Sig.**	**β**	**T**	***p*-value Sig.**	**β**	**t**	***p*-value Sig.**	**β**	**t**	***p*-value Sig.**
Intercept		2.740	0.007		−2.388	0.018		6.288	0.000		−0.313	0.755
GDP	**0.248**	**2.657**	**0.009**	**0.554**	**7.319**	**0.000**	**−0.690**	**−4.251**	**0.000**	**1.652**	**2.472**	**0.015**
AL	**0.504**	**7.890**	**0.000**	**0.463**	**8.921**	**0.000**	0.127	1.144	0.255	**0.491**	**7.746**	**0.000**
GE	−0.113	−1.037	0.302	−0.011	−0.123	0.902	−0.241	−1.273	0.205	−0.106	−0.988	0.325
CORRUP	**0.228**	**2.577**	**0.011**	0.043	0.604	0.547	**0.443**	**2.871**	**0.005**	**0.301**	**3.202**	**0.002**
CONSERV	−0.029	−0.615	0.540	0.011	0.300	0.765	−0.093	−1.151	0.252	−0.014	−0.305	0.761
SIZE	−0.031	−0.506	0.613	0.023	0.463	0.644	−0.126	−1.184	0.238	−0.039	−0.652	0.515
OECD	−0.010	−0.141	0.888	−0.062	−1.024	0.308	0.118	0.908	0.365	0.028	0.370	0.712
VOICE	−0.052	−0.565	0.573	−0.063	−0.837	0.404	0.021	0.128	0.899	−0.032	−0.349	0.728
PE	−0.026	−0.228	0.820	0.033	0.360	0.719	−0.139	−0.697	0.487	−0.049	−0.429	0.668
GDP^2 *	-	-	-	-	-	-	-	-	-	**-1.499**	**-2.122**	**0.036**
	R^2 = 0.745 F = 45.071 ***			**R^2 = 0.832 F = 76.335 *****			**R^2 = 0.225 F = 4.477 *****			**R^2 = 0.753 F = 42.036 *****		

*** p-value < 0.01; * quadratic form.

The model estimated to determine the explanatory factors of the countries' environmental performance index has an explanatory power of 74.5%, for a confidence level of 99%. Three of the five independent variables proposed are statistically significant.

More specifically, statistically significant positive effects for a confidence level of 99% are detected for GDP, of 99% for AL and of 95% for CORRUP. In contrast, the independent variables, GE and CONSERV, display a non-significant and negative effect. Concerning the control variables, SIZE, OECD, VOICE and PE show a non-significant and negative effect regarding the dependent variable, environmental performance index.

The overall results obtained for the model estimated allow us to accept hypotheses H1, H2 and H4 in regard to positive relationships between the economic wealth of a country and the environmental performance index, positive relationships between the level of education of a country and the environmental performance index and positive relationships between the control of corruption of a country and the environmental performance index. The other hypotheses (H3 and H5) are rejected because of the absence of statistical significance of the variables proposed to test them.

Given that the multivariate analysis (Figure 1, Factorial Plane 1–2 including the countries and the indicators in the EPI index) provided two easily identifiable slopes, the variables close to Axis 1 or the horizontal axis, representing environmental health, and the variables close to Axis 2 or the vertical axis, representing ecosystem vitality, and given that the environmental performance index (EPI) as the overall index is also divided into two dimensions that are exactly the same, we broke down the initial regression model into two models, one in which the dependent variable is environmental health (ENVHEALTH) and the other in which the dependent variable is ecosystem vitality (ECOSVITALITY). This will give us a better understanding of how the different explanatory variables affect the two large components of the environmental performance index.

The results of the regression analysis make it possible to see that when the dependent variable is environmental health, it is a better model, and H1 and H2 are fulfilled as initially posited. That is, economic wealth and education are the variables that most influence the environmental health of the countries. These findings are similar to those obtained by applying the biplot methodology, since it shows that the richest countries (pertaining to America and Europe) are the most concerned with environmental health, as can be observed in Figure 1.

With respect to the model representing ecosystem vitality, that is when the dependent variable only includes aspects related to reducing the loss or degradation of ecosystems and natural resources, the relation is significant, but negative. Thus, H1 is fulfilled, but with the opposite sign to the one posited. That is, if economic growth represented by GDP increases, then ECOSVITALITY decreases (see Table 6, third column).

In regard to EKC theory, as we can see (Table 6, fourth column), when the variable representing economic growth is GDP, a positive and significant relation is obtained (0.015, p-value <0.05) with environmental performance, that is as the level of income increases, the better the environmental performance of the countries, but only up to a point. When this increase in income becomes very high and is represented by the variable GDP [26], a negative relation is obtained with environmental performance ($\beta = -1.499$), meaning that the level of income decreases environmental performance, and that is why it is called an inverted-U. Our results are consistent with those found by Raymond [51], who analyzed this theory considering the components of the ESI, an environmental index that was a precursor to the EPI used here; the results obtained in that study show that EKC theory is consistent with some of the ESI indicators.

6. Discussion of Results

Our findings in this study have increased our knowledge of the environmental performance index, since through the biplot methodology, we were able to obtain a picture of the environmental situation on a global level, which coincides with one of the political priorities of environmental authorities worldwide and with the international community's intention to adopt Goal 7 of the Millennium Development Goals (MDGs) to ensure environmental sustainability. At the same time, we have jointly used economic and institutional variables in order to see which ones have more influence on environmental performance as measured by the EPI.

Regarding the dependence model, our findings allow us to affirm that a higher level of economic wealth represented by the variable GDP per capita is strongly linked to the environmental performance of the countries. This finding is consistent with the statement made by Cracolici *et al*. [18] in the sense that a country's level of GDP is an important aspect if it is to provide its citizens with good living conditions and good social and environmental performance. In the same direction, Esty and Porter [22] also consider that rich countries are the ones with the greatest economic capacity to invest in environmental aspects, such as control of pollution and other aspects that can improve the environment.

These same authors, using an extensive database of 64 countries (many of them developing countries), find a positive and significant relation between GDP and environmental performance. This result shows, as established by Everett *et al*. [51], that economic growth and environmental performance must go hand in hand.

Another variable analyzed was the level of education, and the results obtained in our research corroborate those of Duit [1], who considers that citizens with a higher level of culture and education can be assumed to be in a better position to initiate and implement environmental cooperation schemes of their own. Likewise, Morse [52] considers that education can entail social benefits, such as a greater awareness of environmental issues, leading to greater citizen participation in the social and environmental commitments of a country.

Although Esty *et al.* [5] found a slight positive relationship between government effectiveness and the environmental performance index, our finding is that the relation between government effectiveness and environmental performance is negative and non-significant. In the research carried out by these authors, government effectiveness positively correlates with performance on greenhouse gas emissions per capita, health, ozone, growing stock and water quality indicators, that is, variables that represent ecosystem vitality, for which the target is to reduce the loss or degradation of ecosystems and natural resources. In our research, we also took environmental health into account, and for this reason, the hypothesis posed initially was not fulfilled.

Regarding control of corruption, our results are similar to those found in studies carried out in other contexts, which means that countries with high levels of corruption tend to have low levels of environmental performance, whereas countries with low levels of corruption perform better on the environmental performance index [5,36].

In contrast to the results obtained by Neumayer [38] for ideology, in which he found a positive effect between green/left-wing liberal parties and emissions reductions, the results obtained in our study do not show a statistically significant relation between political ideology and the environmental performance index. This result is in line with Wälti [53], who points out that party politics seem to have no impact on environmental performance. Other authors, such as Midlarsky [31], have found a negative relation between democracy and three environmental indicators, deforestation, carbon dioxide emission and soil erosion by water, contrary to prediction.

Thus, some studies have indeed found a relation between political ideology and other variables, but others did not find any significant relation, as was the case here. One reason for this may be that many studies only considered aspects relating to ecosystem vitality, such as greenhouse gas emissions, deforestation, sulfur dioxide, *etc*., but not aspects concerning environmental health, which we did include in our research. Another possible reason for this discrepancy in results could be the changes in political regimes that have taken place in some of the countries analyzed, such as those in Eastern

Europe. In short, and considering the opinion of Fiorino [28], we may conclude that it is difficult to establish clear and consistent results owing to the differences in the dependent variables used and the complex interaction among institutional factors.

With regard to the control variables and considering that they were empirically tested and found to have a statistically positive relation with environmental performance in studies by Grafton and Knowles [42] for size and by Esty *et al.* [5] for civil liberties, in our research, we did not find a statistically significant relation between them. There may be several reasons for this divergence in results in regard to these two variables. In the case of the size variable, our research employs a broader sample, specifically 149 countries from different geographical areas, America, Europe, Africa, Asia and Oceania, as opposed to the 53 countries used in previous research, and this may have varied the results, significantly owing to the different typologies of the countries and population density employed. Another possible difference may be the index used, since although all of them address measurements of environmental performance, in our study, we employed the Environmental Performance Index as opposed to others, which used the Environmental Sustainability Index. There are some differences between them. For example, the EPI assesses current environmental conditions, whereas the ESI measures the long-term environmental trajectory of countries, focusing on environmental sustainability; the EPI focuses strictly on the area under government control, whereas the ESI considers a wide range of factors that affect sustainability using an adaptation of the pressure-state-response model; the EPI addresses multiple levels with two objectives, six categories and 25 indicators, whereas the EST considers five components, 21 indicators and 76 variables. With respect to the civil liberties variable, the difference between our results and those of other studies may also be due to the way citizens express themselves when they exercise their right to vote or associate, and therefore, the population factor is important in regard to this variable. As mentioned earlier, the fact that in our study a large sample of countries was used may be behind this difference in results with respect to other studies using smaller samples.

The findings obtained from our empirical analysis show that socio-economic factors, such as economic wealth and education, as well as institutional factors represented by control of corruption, are determinant in the environmental performance of the countries analyzed. In contrast, the factors representing the countries' internal characteristics and political factors do not affect environmental performance.

The results obtained are consistent with what is proposed by economic theory in that richer countries not only can, but do invest in pollution control and other environmental improvements; in other words, countries with greater economic growth are better able to combat environmental problems because they have more financial resources to do so.

In addition to the initial model, the findings obtained by applying the biplot methodology add more detail, as two easily identifiable slopes were obtained: the variables close to Axis 1 or the horizontal axis (representing environmental health) and the variables close to Axis 2 or the vertical axis (representing ecosystem vitality). Given that the Environmental Performance Index (EPI) is also divided into two dimensions that coincide exactly with these slopes, we broke down the initial regression model into two models, one in which the dependent variable is environmental health and the other in which the dependent variable is ecosystem vitality. Thus, a better understanding is achieved of how the different explanatory variables affect the two large components of the environmental performance index.

It was observed that when the dependent variable is environmental health (see Table 6), it makes for a better model and confirms our initially posited H1 and H2 (economic wealth and education are the variables that most affect the environmental health of countries). In this sense, rich and developed countries are more likely to have access to the public funds needed to carry out environmental policies. This may be because their citizens demand higher environmental quality when it affects them directly, especially as concerns health-related pollutants; however, when the environmental aspect refers more to ecological or natural resources and factors, such as climate change, biodiversity, and so on, economic wealth as represented by the variable GDP per capita does not behave statistically in the same way, as it is negatively related to ecosystem vitality. That is, when economic wealth increases, ECOSVITALITY decreases, so it could be argued that many wealthy countries do not perform especially well on energy, climate, water stress, biodiversity, *etc*. The result we obtain when environmental performance is separated into its two components, environmental health and ecosystem vitality, are consistent with the findings by Jahn [23], who considers that although wealthy countries may be able to invest money in order to improve their environment in contrast to poorer countries, they also tend to create environmental problems owing to the their high level of consumption, which can lead to an increase in their pollution levels, thereby also generating more waste and using up more natural resources. Thus, Jahn [23] found that in rich countries, such as Germany, Japan, Canada, the United States and Switzerland, there was no relation between a country being wealthier and environmental performance.

With respect to the model used to predict EKC theory, according to Fiorino [28], the research studies derived from EKC recognize the critical role of political and governance factors in explaining environmental performance. This same author postulates that the results of EKC studies should be interpreted with caution, since the dependent variable can consider different indicators; it likewise depends on the type of country, since a developing country need not behave in the same way as other countries, and the economy-environment relationship is not predetermined.

7. Conclusions

The aim of this study was to analyze the following aspects. In the first place, we use economic and institutional variables jointly, in contrast to previous works that only use one type of variable, either economic or institutional, or present only a theoretical perspective. Secondly, we have contextualized the countries and variables (environmental indicators) using the biplot methodology, which provided a graphic representation that differentiates between countries' environmental performance in relation to environmental health, on the one hand, and to ecosystem vitality, on the other. This classification happens to coincide with the division of dimensions in the environmental performance index (EPI). Thirdly, we ran different dependence models to verify which economic and institutional variables have the most impact on environmental performance according to the EPI, finding that economic variables, such as GDP and educational level, play a fundamental role in environmental performance and its components, especially in environmental health. Finally, we tested a model in which one of the variables is used in its quadratic form, specifically GDP (GDP^2), and the results allow us to confirm EKC theory, which means that in the early stages of economic development, environmental performance increases along with income level, but then decreases in relation to growth in GDP at higher income levels.

In regard to the theories discussed—economic theory, ecological Kuznets curve theory and ecological modernization theory—as we have just seen, EKC is confirmed with the data used; economic theory is also confirmed, since as Jahn [23] points out, countries with greater economic growth are better able to handle environmental problems, because they have the financial resources to do so, and in our study, the GDP variable, which represents a country's level of income, is statistically significant. However, we found no statistically significant evidence for the ecological modernization theory, which incorporates in addition to economic growth other variables, such as the role of science and technology, the importance of market dynamics, the role of economic agents and the ideology of social movements. These additional variables were either not used in our study or turned out not to be statistically significant, as in the case of political ideology.

In light of this latter finding, a possible future line of research would be to add these variables to the model. One could also broaden the sample to include more years for a longitudinal study and, in short continue a line of research on this topic in an attempt to ensure environmental sustainability, one of the priorities of environmental authorities around the world.

The results obtained have real-world applications and can be useful for policy makers. Hence, governments in different countries should make a greater effort to control corruption, since it reduces a country's income, and low income levels can lead to higher levels of pollution, with the consequent decrease in environmental performance. Governments should also address educational concerns, because if a country's income is not high enough for good sustainability, then good education may be the next best thing. Population density is also an important factor to consider in attaining good environmental performance, as we can deduce from our research: since there are many different countries, the population density is very high and heterogeneous in comparison to other research studies using fewer countries and in which population density has a positive effect on environmental performance. It can therefore be said that improvements in environmental performance may be best achieved by limiting future increases in population density. Another aspect to consider is that income is not the only explanatory variable for understanding environmental orientation and sustainability across countries; institutional factors must also be taken into account, since they can affect environmental performance indicators. Another important aspect is regime type: democratic regimes show higher levels of environmental performance than authoritarian regimes. These results can be attributed to the availability of information, opportunities to demonstrate and the independence of scientific researchers. High levels of democracy are also associated with growth in per capita income. However, governments should also take into account that being a wealthy country does not always lead to better environmental performance, especially when the environmental health aspect is considered apart from natural or ecological resources. Where the former is concerned, it can be affirmed that there is a positive and significant relation between wealth and environmental performance, but in the latter case, there is either no relation or else it is negative. The reason for this may be that although wealthy countries may be able to invest money in order to improve their environment in contrast to poorer countries, they also tend to create environmental problems owing to their high level of consumption. They have higher pollution levels, thereby also generating more waste and using more natural resources. What is certain is that effective governance leads to better environmental performance; according to Fiorino [28], effective, innovative and adaptable governance is a necessary condition for countries seeking a transition to sustainability. Among these aspects of governance to be considered are integrating policies, enhancing social capital, improving participation and making and implementing choices more adaptively.

Author Contributions

All of the authors have contributed equally in the research design and development, the data analysis and the writing of the paper. All of the authors have read and approved the final manuscript.

Appendix

Table A1. Countries in the sample.

Albania	Chile	Germany	Laos	Norway	Sweden
Algeria	China	Ghana	Latvia	Oman	Switzerland
Angola	Colombia	Greece	Lebanon	Pakistan	Syria
Argentina	Congo	Guatemala	Lithuania	Panama	Taiwan
Armenia	Costa Rica	Guinea	Luxembourg	Papua New Guinea	Tajikistan
Australia	Côte d'Ivoire	Guinea-Bissau	Macedonia	Paraguay	Tanzania
Austria	Croatia	Guyana	Madagascar	Peru	Thailand
Azerbaijan	Cuba	Haiti	Malawi	Philippines	Togo
Bangladesh	Cyprus	Honduras	Malaysia	Poland	Trinidad & Tobago
Belarus	Czech Rep	Hungary	Mali	Portugal	Tunisia
Belgium	Dem. Rep. Congo	Iceland	Mauritania	Romania	Turkey
Belize	Denmark	India	Mauritius	Russia	Turkmenistan
Benin	Djibouti	Indonesia	Mexico	Rwanda	Uganda
Bolivia	Dominican Rep	Iran	Moldova	Saudi Arabia	Ukraine
Bosnia and Herzegovina	Ecuador	Iraq	Mongolia	Senegal	United Arab Emirates
Botswana	Egypt	Ireland	Morocco	Sierra Leone	United Kingdom
Brazil	El Salvador	Israel	Mozambique	Slovakia	United States
Bulgaria	Eritrea	Italy	Myanmar	Slovenia	Uruguay
Burkina Faso	Estonia	Jamaica	Namibia	Solomon Islands	Uzbekistan
Burundi	Ethiopia	Japan	Nepal	South Africa	Venezuela
Cambodia	Fiji	Jordan	Netherlands	South Korea	Viet Nam
Cameroon	Finland	Kazakhstan	New Zealand	Spain	Yemen
Canada	France	Kenya	Nicaragua	Sri Lanka	Zambia
Central African Republic	Gabon	Kuwait	Niger	Sudan	Zimbabwe
Chad	Georgia	Kyrgyzstan	Nigeria	Swaziland	

Conflicts of Interest

The authors declare no conflict of interest.

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Artículo 3

Evolution of sustainability indicator worldwide:
A study from the economic perspective
based on the X-STATICO method

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Review

Evolution of sustainability indicator worldwide: A study from the economic perspective based on the X-STATICO method

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ABSTRACT

Sustainability is becoming an ever greater concern in society as it will affect future generations. Since the Brundtland Report defined the concept of sustainability in 1987, several different indices and indicators have been developed in this area. However, at present, there is no widely accepted indicator of sustainability. Some of the indicators currently used have been established by the OECD and the UN, among others, but the Sustainable Society Index (SSI), developed in 2006, covers three aspects: economic, social and environmental (Van de Kerk and Manuel, 2012). With regard to the SSI, the objective of this study was to analyze the evolution of its indicators over the period 2006–2012. In addition, we consider the evolution of sustainability indicators in different countries within different geographical areas. We thus assessed the spatio-temporal structure of countries worldwide according to the SSI using X-STATIS and X-STATICO in combination with a common tool in statistics, the spider graph. From a methodological point of view, X-STATIS and X-STATICO offer a rigorous theoretical framework for the simultaneous analysis of a three-dimensional set of data. Applied in the field of sustainability, this method constitutes a relevant way of analysing the spatio-temporal organization of countries. Our findings show that by applying these three-way methods, a differentiation can be observed in the sustainability indicators in the period analyzed (2006–2012) towards a greater emphasis on economic indicators, especially public debt, genuine savings and employment.

Contents

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1. Introduction

In recent years, interest in and concern with sustainability has increased worldwide. This concept, which at first seems very general, indicates in the broad sense the way in which humans should act towards nature and how they are responsible towards one another and future generations (Baumgärtner and Quaas, 2010). For authors such as Van de Kerk and Manuel (2008, p. 229), a sustainable society is: "one in which each human being is capable of developing in a healthy manner and obtaining a proper education; lives in a clean environment; lives in a safe and well-balanced society; uses non-renewable resources responsibly so that future generations will not be left without them, and contributes to a sustainable world".

To help to understand and manage all these sustainability issues, a number of indicators have been implemented. Some of the most important are the Human Development Index (HDI), the Millennium Development Indicators, the Indicators for the EU Sustainable Development Strategy and the Index of Sustainable Economic Welfare. In this research, we use the Sustainable Society Index (SSI), employed in previous analyses (e.g. Van de Kerk and Manuel, 2008).

This index comprises a set of economic, social and environmental well-being indicators and in 2012 was audited by the Joint Research Centre of the European Commission, which considers it an integral and quantitative method for measuring and monitoring the health of human and environmental systems on a worldwide basis. The audit also pointed out that it is a conceptual and statistically solid tool that is widely applicable to the continuous assessment of human and environmental systems and a key point of reference with which to compare future progress and report on the current state of society (Saisana and Philippas, 2012).

The aim of this research study was to analyze the behaviour of the Sustainable Society Index (SSI) throughout the period 2006–2012 in an attempt to determine whether the indicators underwent any variation. At the same time, we wanted to examine how these indicators had evolved in countries within different geographical areas. We used the 2012 version of the SSI framework, for which data are available for four years (2006, 2008, 2010 and 2012). In addition to corroborating the results obtained individually we also analyzed the covariation among the different groupings of indicators (social/environmental, economic/social, economic/environmental) in the period under consideration.

The techniques we chose for this research were X-STATIS, also known as partial triadic analysis, to observe the evolution of the individual sustainability indicators by year and country and X-STATICO to observe the evolution in the covariation of the sustainability indicators. These statistical tools had not yet been applied to the SSI, thereby providing some degree of novelty to the current work.

Using the statistical techniques X-STATIS and X-STATICO, we found that a differentiation emerged in the sustainability indicators during the period analyzed (2006–2012), more sharply so in the case of the economic indicators. When the social/economic and environmental/economic groupings were analyzed jointly, we observed a clear difference between the years 2006/2008 and 2010/2012; once again, when economic variables are entered, there is a large difference between the pairs of years 2006/2008 and 2010/2012.

This discovery led us to delve deeper into the analysis of the economic indicators of sustainability to see which of them changed the most when the different geographical areas were considered. To do so, we chose from each geographical area the country in which this change was most noticeable and where the difference between the two periods could best be observed. We found that in low-income countries, such as Liberia and Malaysia, the score for public debt increases, as does the score for genuine savings score although to a lesser extent. In contrast, the countries with the highest income show a reduction in employment, or in other words, an increase in unemployment. This is the situation in Ireland, the United States (US) and New Zealand, with Ireland being the most outstanding in this regard.

The remainder of the paper is structured as follows: in Section 2 we analyze the theoretical framework of sustainability indicators. Section 3 describes our research methods, including the sample and statistical analysis. In Section 4, the results of the empirical analysis are presented and then discussed. Section 5 summarizes the discussion and presents the conclusions.

2. Sustainability indicators: an overview

2.1. General aspects

In recent years there has been an increase in citizens' need for information concerning both private organizations, such as in the business sector, and public institutions, such as the different levels of government (Bovaird and Löffler, 2003). This study focuses on the dissemination of information about the sustainability of different countries as it has become increasingly important to be able to measure sustainability on a global level since the Brundtland Report (WCED, 1987) issued its famous definition of sustainable development as "development that satisfies present needs without compromising the capability of future generations to satisfy their own needs".

After that report was issued, a meeting was held in Bellagio, Italy, in November 1996 with the goal of establishing a series of principles for monitoring progress towards sustainable development (Hak et al., 2007, 2012; Ramos and Caeiro, 2010). As a result, the Bellagio Principles for sustainable development were issued. According to Pintér et al. (2012) a good system of indicators can help authorities to make knowledgeable decisions, providing them with objective information that will allow them to pull together to attain the goals proposed, as well as their subsequent assessment.

Some authors have agreed on certain criteria concerning what indicators to use in the field of sustainability. For example, Bell and Morse (2003) state that: an indicator must be relevant to an issue according to the definition used; the set of indicators must cover the whole field of sustainability in line with the definition used; data must be available for all countries and the data for the indicators must be available from public sources, be they scientific or institutional. Also, Bauler et al. (2007) and Garnåsjordet et al. (2012) argue that indicators should be purposeful, measurable, representative, reliable and communicable.

According to Steurer and Hametner (2013), indicators have three main functions. First of all, the use of indicators reduces the number of measures needed to describe a situation (OECD, 2003); second, they are essential for measuring progress towards political objectives (Dalal-Clayton and Krikhaar, 2007); third, they serve to evaluate the effectiveness of public policies (European Commission, 2005). Hansen (1996), Jasch (2000) and Perotto et al. (2008) observed that the development of indicators at the national, regional, local or field level has become a commonly used approach to meet the crucial need for assessment tools. Such tools are a prerequisite for the implementation of the concept of sustainability.

Baumgärtner and Quaas (2010) consider sustainability indicators to be normative notions that measure the way humans act towards nature and how we humans are responsible towards one another and future generations. Kates et al. (2001) consider that the essence of sustainable development is to meet fundamental human needs while preserving the life-support systems of planet Earth. According to Van de Kerk and Manuel (2008), a sustainable society is one in which each human being is capable of developing in a healthy manner and obtaining a proper education, lives in a clean environment, lives in a safe and well-balanced society, uses non-renewable resources responsibly so that future generations will not be left without them and contributes to a sustainable world. Sustainability indicators should therefore measure these aspects.

2.2. Sustainability indicators: Sustainable Society Index

Focusing more specifically on sustainability indicators, they can be useful individually to observe how each country is performing with respect to sustainability. They help to highlight the most important aspects, make it possible to see what still needs to be done, permit a comparison of the sustainability of different countries in the same geographic area and enable the identification of effective aspects of sustainability. For governments, these sustainability indicators provide a transparent and effective means of showing the general public the situation of a country with regard to sustainability and help in making decisions concerning the social, environmental and economic policies, projects and strategies to be adopted. At the educational level, they enable the inclusion of topics on sustainability in secondary and higher education, making students aware of the situation of the world around them. As far as business is concerned, knowledge of how different countries perform according to sustainability indicators will help companies to see whether they can gain some kind of competitive advantage and carry out business innovations (Rinne et al., 2013).

With a view to studying sustainability internationally, many current indices and indicators relating to sustainability have been reviewed and it has been found that good indices are those that are able to measure all the relevant aspects of sustainability in a transparent and easily understandable way. Authors such as Bell and Morse (2008), Meadows (1998), Guy and Kibert (1998) and Van de Kerk and Manuel (2008) agree that the following criteria must be met by the indicators: they must be relevant for measuring one of the issues relating to the aforementioned definition of sustainability, and all together they must cover the complete field of sustainability according to the definition used. This in turn means that the complete set of indicators should provide a good picture of the current situation with regard to sustainability and point out the differences between the present situation and the optimal situation of complete sustainability; they must also permit comparisons among countries.

In current practice, sustainability indicators are often selected based on historical practices and regulations or expert knowledge and the extent to which they individually comply with a series of criteria (Niemeijer and Groot, 2008); thus, the indicators and the indices they comprise, which are continuously measured and calculated, permit long-term monitoring of the sustainability trend from a retrospective point of view (Ness et al., 2007).

From all of the above, it can be deduced that there is currently no single set of indicators as new indicators are constantly being developed to measure sustainability in its three facets: economic, social and environmental. Some of these sets of indicators have been established by the OECD and the UN, among others, but the SSI has been developed by Van de Kerk and Manuel (2012).

In short, none of the existing indices seems completely to fulfil our needs as no single one is completely suitable or else they do not fit our research needs. Below we list some of the most important sets of indicators that have predominantly been employed in the context of sustainability (Moldan et al., 2012; Saisana and Philippas, 2012; Singh et al., 2012; Van de Kerk and Manuel, 2012) (see Table 1).

Of the current indices, for the present study we decided to use the one created by Van de Kerk and Manuel (2012) as their SSI was recently audited by the Joint Research Centre of the European Commission, which found it to be an integral and quantitative method for measuring and monitoring the health of human and environmental systems globally and a conceptually and statistically solid tool that can be applied broadly for the continuous evaluation of these systems. It is also a key reference point for comparing any progress made and for providing information on the current state of society (Saisana and Philippas, 2012).

In addition to having been audited by the Joint Research Centre of the European Commission, the SSI is based on the definition of sustainability given by the Brundtland Commission (WCED, 1987). To reflect more clearly and explicitly that sustainability – and in particular the SSI – should measure human well-being as well as environmental well-being, the Sustainable Society Foundation has extended the definition of the Brundtland report with a third sentence, as follows:

> A sustainable society is a society that: a) meets the needs of the present generation, b) does not compromise the ability of future generations to meet their own needs, and c) in which each human being has the opportunity to develop itself in freedom within a well-balanced society and in harmony with its surroundings. (Saisana and Philippas, 2012, p. 15).

Although the SSI is not the only way to measure sustainability, it is considered to be a statistically solid conceptual tool that is broadly applicable for the continuous assessment of human and environmental systems and a key point of reference with which to compare future progress and inform global society. The SSI can be used to simulate the consequences of a series of potential actions, making it a powerful tool for informing decisions related to achieving human and environmental growth without compromising environmental well-being. In short, the framework of the SSI is conceptually consistent, fulfils the statistical requirements established by the Joint Research Centre of the European Commission and is suitable for evaluating the development of nations in the wider sense: economic, human and environmental well-being. It is also important to point out that the SSI is framed within the pressure-state-response model proposed by Rapport and Friend (1979) and followed by the OECD.

The SSI consists of 21 indicators grouped into three dimensions: human well-being, environmental well-being, and economic well-being. The different indicators comprising these dimensions are listed below (Van de Kerk and Manuel, 2012) (see Appendix 1, SSI Indicators).

3. Research method

3.1. Population and sample

The sample we use comprises the 151 countries selected by Van de Kerk and Manuel (2008) (see Appendix 2), and incorporates the advantages derived from considering different geographic contexts: Europe (Eu), Africa (Afr), America (Am), Asia (As) and Oceania (Oc) (see Appendix 2). The aim of this study is to add to the countries studied and the techniques used in previous studies, such as those carried out by Neumayer (2003) for 21 OECD countries and by Hosseini and Kaneko (2011) for 131 countries. Although the initial population comprised 194 countries, data on these indicators were only available for 151 countries.

3.2. Statistical analysis

The temporal monitoring of the spatial organization of the countries during the study period required the combined analysis of the data tables issued for each year. For that purpose, we employed the X-STATIS and the X-STATICO multi-way methods, methodologies which despite having been employed previously in other areas

Table 1
Sustainability Indicators.

Sustainability indicators	What the indicators measure	Limitations	Year
Genuine Progress Indicator (GPI)	The major goal of this index is to measure the component of economic activity that leads to the welfare of a society. This index comprises: personal consumption, public non-defensive expenditures, private defensive expenditures, capital formation and costs of environmental degradation.	It does not include the main aspects of quality of life and does not offer a clear insight into the level of sustainability of a country. The GPI is available for a limited number of countries only.	1998
Wellbeing of Nations	This covers the whole field of sustainable development. It consists of the human well-being index and the ecosystem well-being index.	This provides an enormous amount of information, which makes it too complicated. It has only been published once.	2001
Index of Sustainable Economic Welfare (ISEW)	The major goal of this index is to measure the component of economic activity that leads to the welfare of a society.	This index does not include the main aspects of quality of life and does not offer a clear picture of a country's level of sustainability. It is available only for limited number of countries.	2003
Environmental Sustainability Index (ESI)	ESI was developed to measure overall progress towards environmental sustainability. The index comprises: environmental systems, reducing stresses, reducing human vulnerability, social and institutional capacity and global stewardship.	This index lacks indicators on gender equality and good governance receives minor attention. It is not particularly transparent because of the enormous amount of data.	2005
Millennium Development Indicators	These cover eight goals: eradicating extreme poverty and hunger, achieving universal primary education, promoting gender equality and empowering women, reducing child mortality, improving maternal health, combating HIV/AIDS, malaria and other diseases, ensuring environmental sustainability and developing a global partnership for development.	They are of limited use for visualizing a country's level of sustainability. They do not cover the whole concept of a sustainable society.	2005
Environmental Performance Index (EPI)	EPI was developed to measure the environmental burden of disease, water (effects on human health), air pollution (effects on human health), air pollution (ecosystem effects), water resources (ecosystem effects), biodiversity and habitat, forestry, fisheries, agriculture and climate change.	This index only partially covers sustainable development in its broadest context. EPI indicators focus on measurable outcomes, such as emissions or deforestation rates, rather than policy inputs.	2006
Commitment to Development Index (CDI)	The CDI reviews for 21 rich countries the level of support given to poor countries to promote prosperity, good governance and security. It has seven components: aid, trade, investment, migration, environment, security and technology.	The CDI covers sustainable development only partly and offers information concerning no more than 21 countries.	2006
Ecological Footprint	This converts everything a person consumes (house, mobility, energy, food, recreation, etc.) and what is needed to produce all these items into the area required on earth, i.e. the number of hectares per capita.	The Ecological Footprint is a valuable index for providing a quick and inspiring idea of the seriousness of the present lack of sustainability. It encourages people to take action. However, the Footprint is not suited to giving a good idea of sustainability in its broader sense.	1996
Sustainable Society Index	This is a more comprehensive index of sustainability as it covers all three aspects: economic, social and environmental, whereas most other indexes do so only partly.	While this index covers sustainability in its broadest sense, it still has drawbacks. One is subjectivity due to the assumptions made in estimating the measurement error in data, the choice of imputation algorithm, the choice of weights and the choice of aggregation system.	2006–2012
Indicators for the EU Sustainable Development Strategy	The EU indicators comprise a set consisting of three levels, the first two being the most important for policymakers; among other things, the set comprises many macro-economic indicators.	This includes a number of indicators that are not closely related to sustainability and little or no attention is paid to other topics, such as those related to gender equality or access to drinking water. It is limited to member states of the European Union.	2007
CSD Indicators	The set comprises 14 themes, 44 sub-themes, 50 core indicators and 46 other indicators. Some themes are: poverty, education, health, biodiversity, economic development and oceans, seas and coasts.	It does not cover sustainability in its broadest sense. Some indicators are missing, such as the important aspects of gender equality and sufficient food, while others are only partly included (good governance, international cooperation, waste recycling).	2007
European Green City Index	This compares the environmental performance of 30 major cities in 30 European countries playing a leading role in climate protection. It takes into account 30 individual indicators per city, touching on a wide range of environmental areas, from environmental governance and water consumption to waste management and greenhouse gas emissions.	This index does not cover the whole concept of a sustainable society. It is available only for a limited number of European countries.	2010
Human Development Index (HDI)	This index comprises aspects related to a long and healthy life, knowledge and GDP per capita.	This covers a small part of all the aspects involved in sustainable development and it has sometimes even been considered a redundant indicator that provides little additional information on inter-country development levels.	1990
Most Livable Cities	The most livable cities assess living conditions in 140 cities around the world. A rating of relative comfort for 30 indicators is assigned across five broad categories: stability, healthcare, culture and environment, education and infrastructure.	The main limitation of this index is that it only contains information regarding the most livable cities around the world.	2012

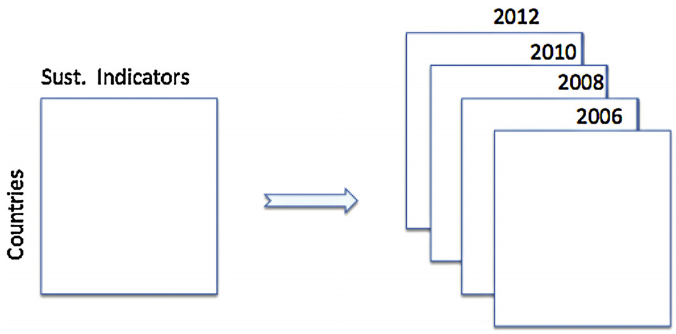

Fig. 1. Three-way scheme ($X_{151\times10\times4}$) for social indicators. Note: There are similar schemes for the environmental and economic indicators.

of knowledge (Adbi et al., 2012; Mendes et al., 2009), have not yet to our knowledge been used in research on the topic of study here. Their use in this study thus provides an original contribution to existing research on sustainability indicators.

We therefore consider in this research the 151 countries around the world presented in Appendix 2, grouped into five geographic areas. The 21 numerical characteristics are the scores obtained by the countries selected concerning the policy categories proposed in the SSI for the latest available years (2006, 2008, 2010 and 2012): sufficient food, sufficient to drink, safe sanitation, healthy life, clean air, clean water, education, gender equality, income distribution, good governance, air quality, biodiversity, renewable water resources, consumption, renewable energy, greenhouse gases, organic farming, genuine saving, gross domestic product, employment and public debt (see Appendix 1). Therefore, in this paper, the data consist of the SSI scores for each country in each time period, that is, three matrices: $X_{151\times10\times4}$ for social indicators, $X_{151\times6\times4}$ for environmental indicators and $X_{151\times5\times4}$ for economic indicators (see scheme in Fig. 1).

The analysis of several sustainability aspects simultaneously requires the storage of large volumes of data. To explore the data and gain a better understanding of the behaviour of several processes, it is important to identify the salient features underlying them. Reducing the dimensionality of the problem enables us to summarize the information captured in a large number of variables with a smaller number of latent variables. Plots which simultaneously show both the countries and the indicators can be of great assistance in this respect.

This method allows us to check whether the indicators proposed by the SSI have similar importance across the different time periods and countries (for example, whether economic, social and environmental concerns are similar in different geographic areas), to find geographical areas with similar sustainability profiles, to identify the most differentiated ones and to order them according to a sustainability gradient. We will likewise be able to identify the most important components of sustainability in each year and their evolution over the time period analyzed.

3.2.1. Partial triadic analysis

The STATIS-ACT method (Lavit, 1988; L'Hermier des Plantes, 1976) is a generalization of principal components analysis (PCA), used to study several data tables measured on the same units or variables simultaneously. The goal of this method is to analyze the relationship between these data tables and to combine them into a compromise matrix corresponding to optimal agreement between the data.

X-STATIS, also called partial triadic analysis (PTA), is a STATIS family method suitable for analysing a three-way table (151 countries, 21 indicators, four years), seen as a sequence of two-way tables. It is an exploratory tool for three-way data analysis and comprises three steps: the interstructure, the analysis of the compromise and the intrastructure (trajectories). PTA requires that all the tables must have the same rows and the same columns.

The objective of PTA is to capture a multivariate structure that is expressed through the different years (2006–2012). The spatio-temporal data are analyzed as follows. The first stage consists of calculating a matrix of scalar products between sustainability indicators for each table (i.e. each year). This step then makes it possible to compare all the tables by calculating a matrix of scalar products between tables (Robert and Escoufier, 1976). Once this has been done, eigenvectors can be obtained, these being a special set of vectors associated with a matrix that allow the transformation of data into information and the answers required.

Given a variance–covariance matrix, for rows, columns or repetitions, the coordinates of the first eigenvector of this matrix are the coefficients of a special linear combination, that which is used to build the row, column or repetition most similar, on average, to all the original rows, columns or repetitions. Thus, in our case, the first eigenvector is able to form the average year in the period 2006–2012. When the first two eigenvectors are calculated, two linear combinations and two average years are built and so on.

The four coefficients of the first eigenvector are used to weight the four tables in the calculation of a "compromise table" (interstructure). This weighting allows the construction of a compromise table which contains the common part of the structures studied. The compromise table is a linear combination of the four initial tables calculated with the aim of constructing a mean table of maximum inertia (compromise analysis). This second stage involves PCA of a fictitious data table constituted by the reorganization of the variable-sample scores (see the flow chart given in Fig. 2).

The compromise analysis allows a multivariate synthesis of the information expressed through the first axis of the data ordination analysis. This step permits a description of sampling sites as a function of the type of variables and the identification of the variables responsible for similar patterns in different years. Our approach therefore focused on the analysis of the countries' spatial pattern and their temporal variability/stability.

The compromise matrix maximizes the similarity with all the initial tables so that the weight of each table is proportional to its inertia and therefore tables that are different from others are weighted by default. The compromise matrix can also be plotted to interpret its structure. The trajectories are obtained by projecting the rows and columns of each table of the sequence in the compromise analysis space.

PTA highlights the stable structure of a sequence of data tables – in our research, the sustainability indicators in the time period 2006–2012. The compromise step represents this stable structure and the trajectories step shows how each table moves away from that structure.

This process for calculating the compromise matrix is equivalent to performing a singular value decomposition of the matrix $Z = U\Lambda Vt$ obtained by placing the concatenated columns of each of the matrices of the sequence in columns as vectors. Then, the first column of ZV will be taken as the compromise, displaying it as a matrix. A scheme of this procedure is shown in Fig. 3.

The compromise analysis using X-STATIS provides two-dimensional representations (graphics of principal axes) to interpret the structure. The intrastructure is obtained by projecting the rows and the columns of each table of the sequence in the compromise analysis. Let V_r be the first r eigenvectors matrix from the compromise analysis. The coordinates of the rows of table X_k are the rows $X_k\ D_p\ V_r$, and the columns are the rows $X_k{}^t\ D_n\ U_r$, U_r being the first r eigenvectors of $X_c\ D_p\ X_c{}^t\ D_n$; however, if D_p and D_n are the Euclidean metrics, the coordinates of the columns are

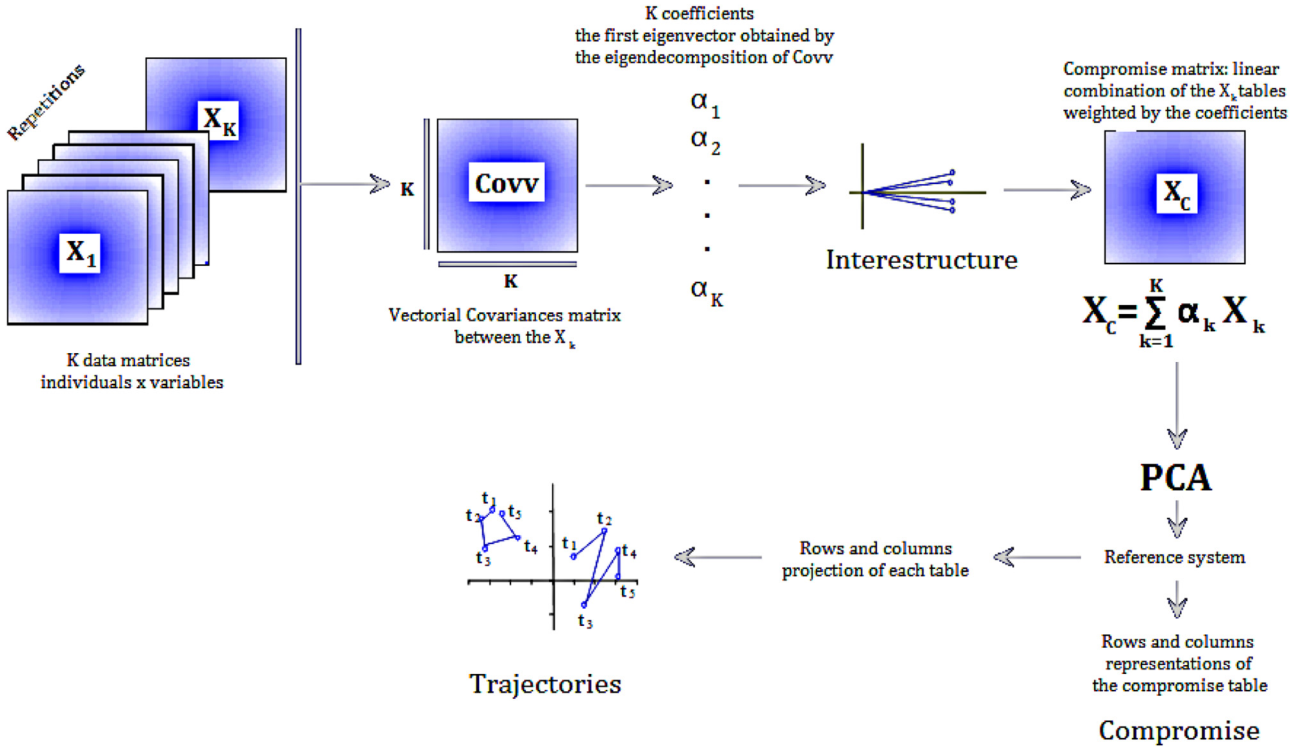

Fig. 2. Compromise table.

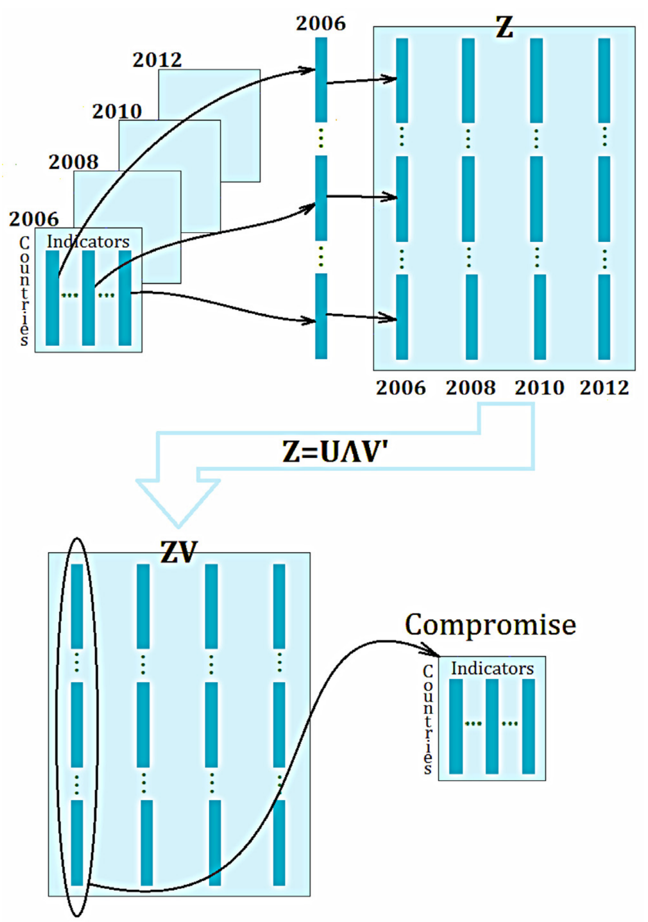

Fig. 3. X-STATIS flow chart.

also the rows $X_k{}^t\, D_n\, X_c\, V_r\, \Lambda_r{}^{-1/2}$, where $\Lambda_r{}^{-1/2}$ is the inverse of the diagonal matrix of the first singular values of the compromise analysis.

All the computations involved in the PTA were directly processed in the STATIS module of the ADE-4 software (Thioulouse et al., 1997). Similarly, graphs were drawn up using the various graphical modules of the ADE-4 software.

3.2.2. X-STATICO

The X-STATICO method is an efficient tool for analysing sequences of paired tables. Its flexibility comes in part from the flexibility of co-inertia analysis, which maximizes the square covariance between the countries' scores according to two different sets of indicators: in our study, economic s environmental indicators, economic versus social indicators and environmental versus social indicators, respectively (see Fig. 4).

This is based on the PTA of a sequence of cross-tables (co-inertia tables). Co-inertia analysis is a multivariate method that explores the covariance between two datasets, that is, it identifies trends or co-relationships in multiple datasets which contain the same samples. Co-inertia simultaneously finds ordinations (dimension reduction diagrams) from the datasets that are most similar. It does this by finding successive axes from the two datasets with maximum covariance; see the flow chart in Fig. 5.

The advantage of this method is that it gives a co-structures compromise, meaning that it first checks how the pairs of variables covary and then it obtains a compromise structure that represents these relations and the trajectories which demonstrate the evolution are drawn over it. Thus, this method is more appropriate if the objective is to find a description of the evolution of the relations between variables rather than a description of the stable part of these relations.

From an analytical point of view, let Z_k be the kth cross-table: $Z_k = Y_k{}^t\, X_k$. The triplet for the co-inertia analysis for the repetition k is (Z_k, D_p, D_q) and the STATIS method is the X-STATIS of the cube made from this sequence of cross-tables.

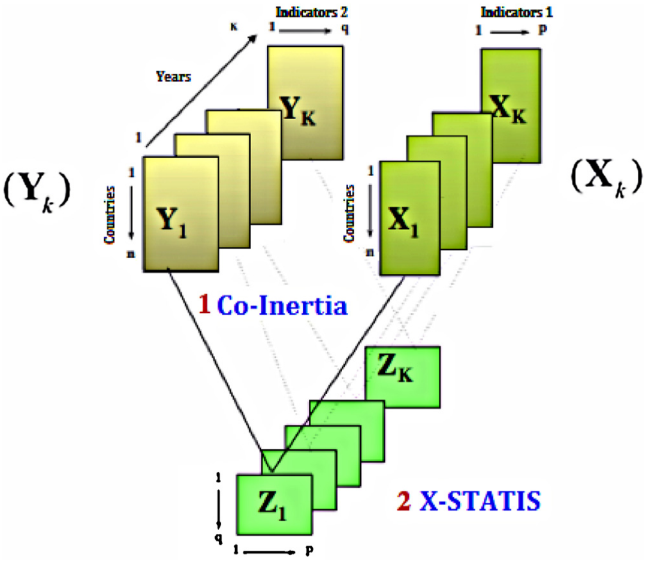

Fig. 4. X-STATICO flow chart. The data structure is a sequence of K paired tables. X_k and Y_k are, respectively, the country and indicator tables in the pair. Z_k is the kth cross-table. Co-inertia analyses allow linkage of the pairs of tables, producing a sequence of cross-tables. PTA is finally used to analyze this sequence.

The compromise of the X-STATICO method (Z) is a weighted average of the cross-tables using the weights α_k: $Z = \sum_k \alpha_k Z_k$.

The intrastructure step projects the rows and the columns of each table of the sequence in the compromise analysis. This gives a representation of the rows of each repetition:

$$X_k D_p V_r \quad \text{and} \quad Y_k D_q Z D_p V_r$$

and the two representations of the variables of each repetition, one from the viewpoint of cross-tables:

$$\text{columns of } X_k : Z_k^t Z V_r \Lambda_r^{-1/2}$$
$$\text{columns of } Y_k : Zk D_p V_r$$

and the other from the viewpoint of the original tables:

$$X_k^t D_n Y_k Z V_r \Lambda_r^{-1/2}$$
$$Y_k^t D_n X_k V_r \Lambda_r^{-1/2}$$

Thioulouse (2011) considered that the main advantage of this method is the optimization of the compromise (maximization of the similarity with all the initial tables). It gives a co-structures compromise, meaning that it represents the stable component of the variations in the relations between the variables of the two cubes. It benefits from the three-step computing scheme of the STATIS method (interstructure, compromise, intrastructure) and the graphical results can be very detailed.

4. Results of empirical analysis

In this section we present the results of the statistical analyses: the plotting of the PTA, X-STATIS and X-STATICO methods for pairs of cubes of data.

The data are ordered in three successions of tables of 151 rows, the countries (with four repetitions), the four years of study (2006, 2008, 2010 and 2012); a cube with 10 columns that contain the social indicators; another cube with six columns containing the environmental indicators; finally, the last cube with the five economic indicators.

The aim of the analysis of this dataset is to discover how the sustainability indicators (social, environmental and economic) evolve over time. More precisely, the analyses help discover how they vary over time (the last four two-year periods) and this variation can be visualized in different countries within different geographical areas. That is, we can observe the evolution of each sustainability indicator, as well as what is responsible for the differences between them. Figs. 6 and 7 portray the representations of the similarities and differences between the different years studied.

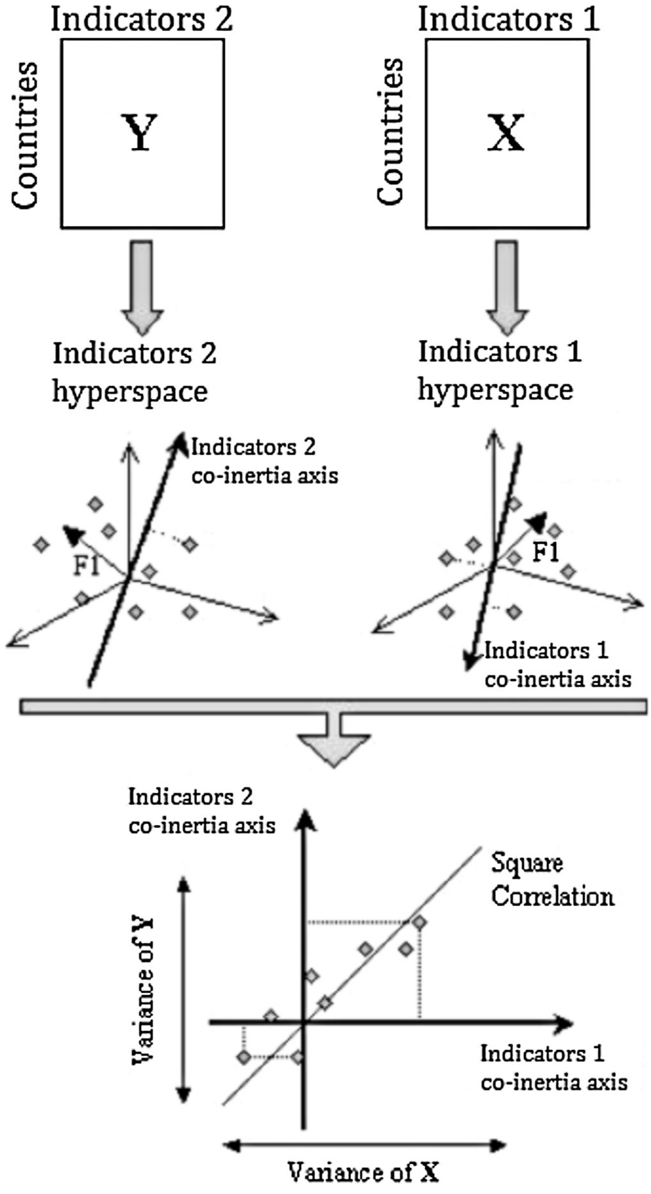

Fig. 5. Principles of co-inertia analysis.

Analysis of the graphs obtained from X-STATIS (Fig. 6) shows that the years are similar from the point of view of each type of sustainability indicator considered separately: social, environmental, and economic, respectively. These graphs show which years are the most relevant for the construction of the compromise, that is, those that are closest to the horizontal axis, or axis 1. In this case, the first graph, corresponding to the social indicators, shows that 2008 is the most important year; the second graph, corresponding to the environmental indicators, shows that the most important years are 2008 and 2010, which are equally spaced from 2006 and 2012.

The most important part of this analysis of the interstructure is that the graph for economic indicators shows differences between pairs of years: on the one hand, 2006 and 2008 are similar to each other, but there is a large difference between them and the years

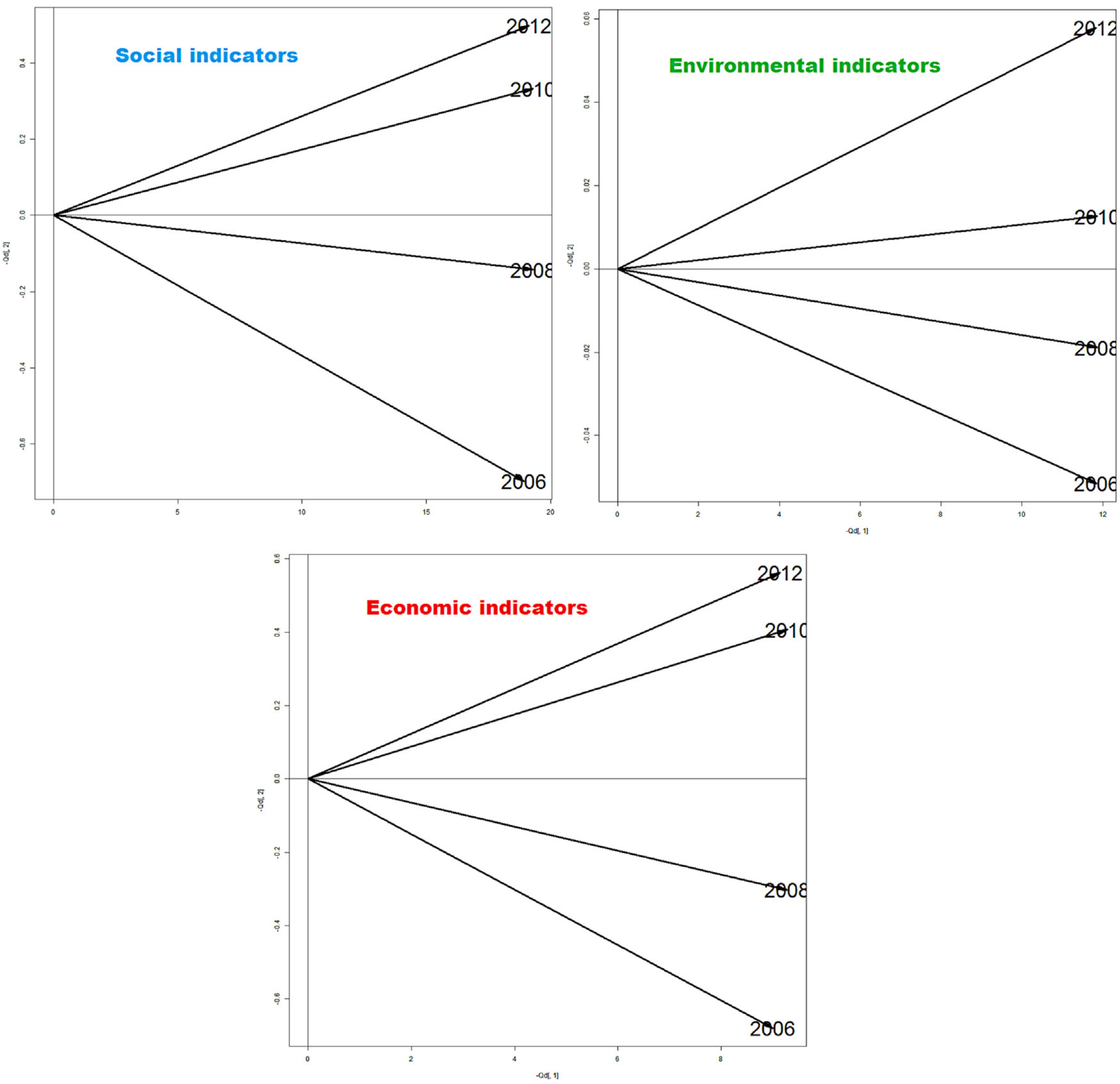

Fig. 6. Ordination of sampling dates (years) on the factorial plan defined by the first two axes of the PCA for the interstructure matrix according to X-STATIS.

2010 and 2012. This difference may be due to economic factors. This means that the economic indicators used within the SSI are those that changed most, which is in line with Van de Ker and Manuel's (2012) proposition that economic well-being is a precondition for the two other dimensions of well-being.

The graphs resulting from X-STATICO (Fig. 7) show the interstructures of PTA after calculating the tables of crossed products. They therefore show the similarities between the years taking into account the types of indicators by pairs: the first one corresponds to the social and environmental variables, the second to social and economic variables and the third to environmental and economic variables. Once again, these graphs show which years have greater weight in constructing the compromise.

Thus, in the first graph the most important year is 2008, whereas the other two graphs, which pair social/economic indicators and environmental/economic indicators, show similarities between 2006 and 2008 on the one hand and between 2010 and 2012 on the other; however, the very large space between the two pairs of years denotes a large difference between them. This corroborates the results obtained through PTA (X-STATIS) in which we analyzed the indicators individually as, once again, when economic variables such as public debt, genuine savings or employment are entered, there is a large difference between the pairs of years 2006/2008 and 2010/2012. This may be because the financial crisis has had much less impact on other aspects of sustainability, such as the environmental and social indicators. In addition, as Van de Kerk and Manuel (2012) point out, economic well-being is a precondition for the other two dimensions of well-being (environmental and social indicators).

Another result obtained from X-STATICO is the analysis of the intrastructure or trajectories, which show the evolution of the indicators over the time period studied. Given that this study aims to analyze the evolution of the indicators over time, more emphasis is placed on the analysis of the trajectories than on that of the compromise as the latter shows the stable part of the relations between the pairs of types of indicators over time.

In our case, these graphic results shows the greater evolution of the economic indicators and how the social and environmental indicators vary less when analyzed in relation to each other by X-STATICO (Fig. 8) than when analyzed by X-STATICO in conjunction with the economic indicators (Figs. 9 and 10).

Another of our objectives was to consider how the different indicators are related to different geographical areas. We made radial profiles for the economic indicators by geographical area and by year to corroborate the results obtained from the X-STATIS and X-STATICO analyses, which showed clear differences in the economic indicators between the two pairs of years 2006/2008 and 2010/2012.

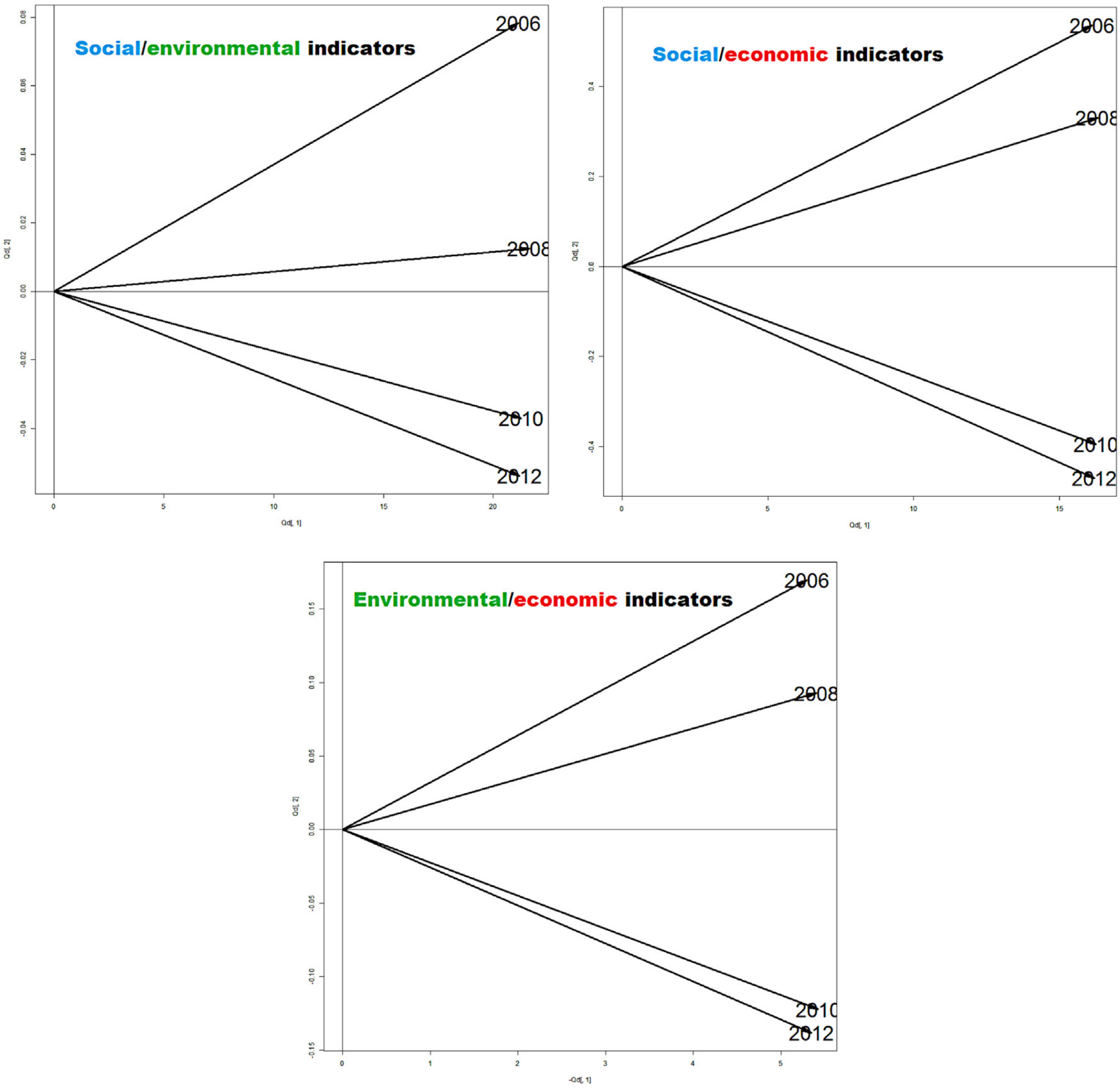

Fig. 7. Ordination of years on the factorial plan defined by the first two axes of the PCA for the interstructure matrix according to X-STATISCO.

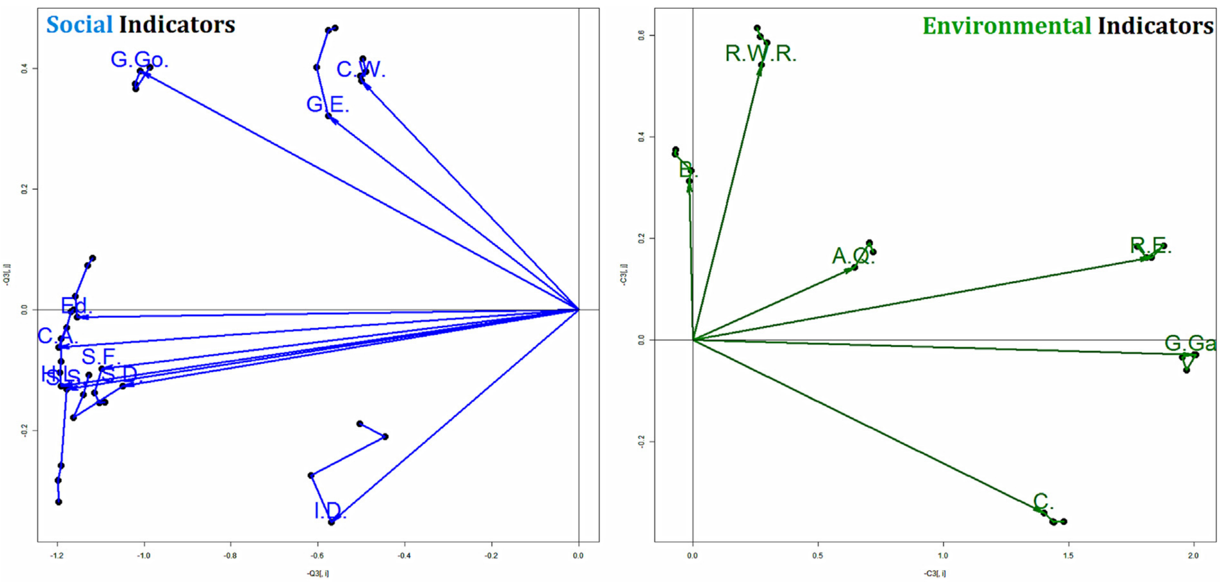

Fig. 8. Analysis of the trajectories between social and environmental indicators with X-STATICO.

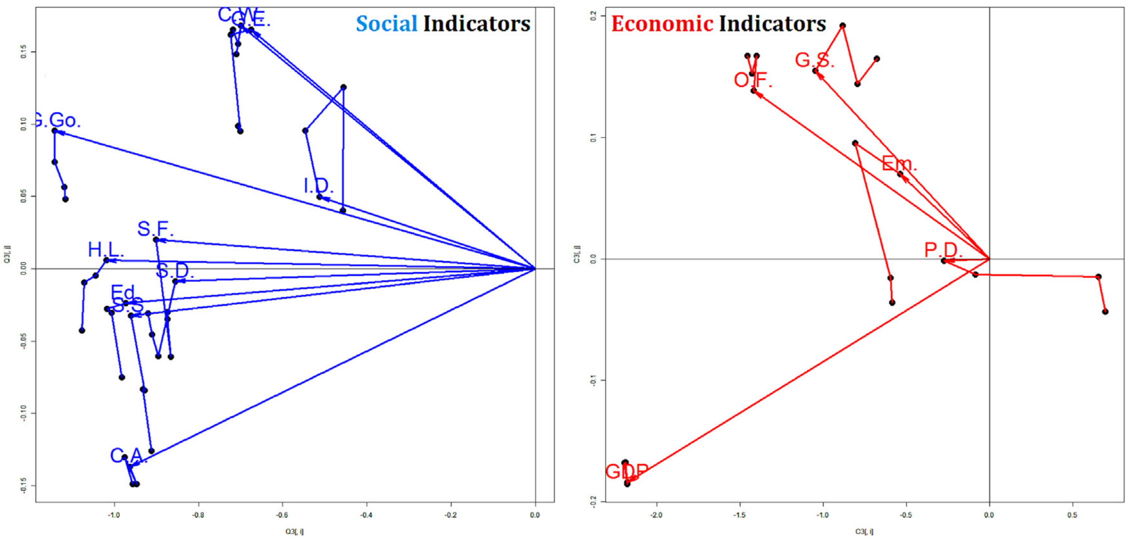

Fig. 9. Analysis of the trajectories between social and economic indicators with X-STATICO.

With the X-STATIS and X-STATICO methods, we have been able to see that the differences found in the years studied are due to the economic factors represented in each radial profile. This means that the economic indicators used within SSI are those that changed the most: as Van de Kerk and Manuel (2012) indicate, economic well-being is a precondition for the two other dimensions of well-being (social and environmental). This led us to delve deeper into the economic indicators of sustainability to see which ones changed the most according to geographical area. To do so, we chose one country from each geographical area where the change was most apparent and where the differences could best be observed between the two pairs of years, 2006/2008 and 2010/2012.

As can be observed in Fig. 11, there are countries within different geographical areas in which the economic indicators show differences over the years studied. This can be observed in the axes of the diagrams or radial profiles in which the variables are represented, in this case, the following economic indicators: OF (organic farming), GS (genuine savings), GDP (gross domestic product), Em (employment) and PD (public debt). With regard to the interpretation of the diagram axes, the closer they are to the value 10.00, the higher the economic indicator of the country in question; the closer they are to the value 0.00, the lower the economic indicator in that country (these scores are those corresponding to the SSI index).

Focusing on Liberia, in Africa, a large difference can be observed in the public debt indicator between 2006/2008 and 2010/2012, with an increase in the later years; this can be seen clearly in the diagram axes as the economic indicator PD (public debt) takes the value of 10.00, meaning that in this country there has been an increase in the economic indicator. Malaysia, in Asia, also shows a difference between these pairs of years, but in this case for the variable GS (genuine savings), which measures the true rate of savings, essential for sustainability. The economic indicator also increases in this case as it is close to 10.00. For authors such as Van de Kerk and Manuel (2012), the comparatively large progress of low-income countries in terms of economic well-being is mainly the result of a sharp increase in the score for public debt and to a lesser extent, of an increase in the genuine savings score. Liberia is precisely the

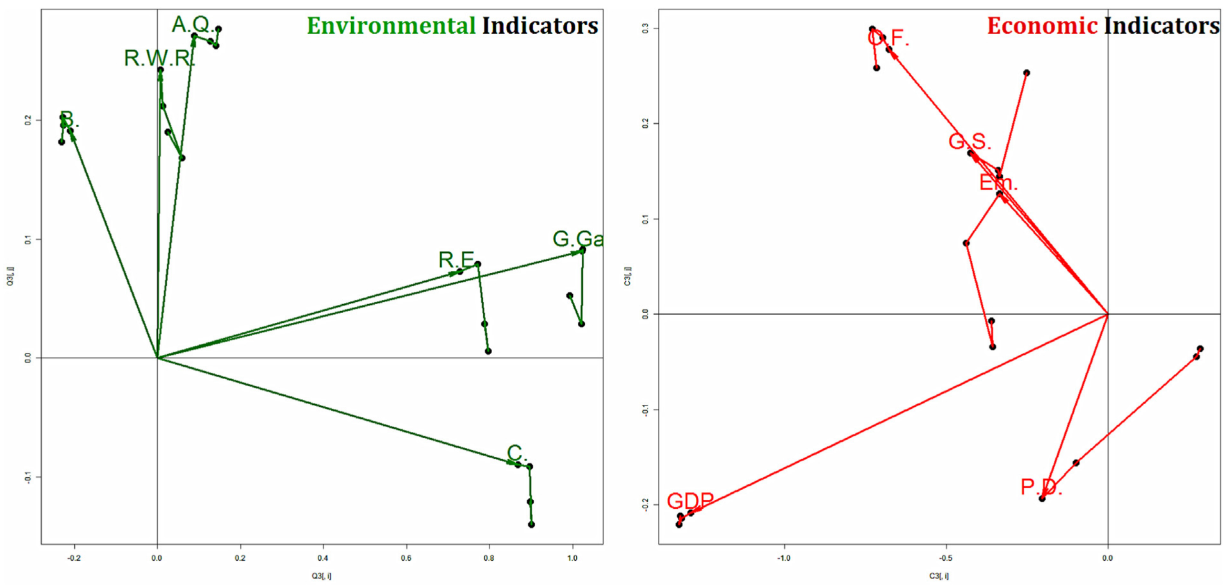

Fig. 10. Analysis of the trajectories between environmental and economic indicators with X-STATICO.

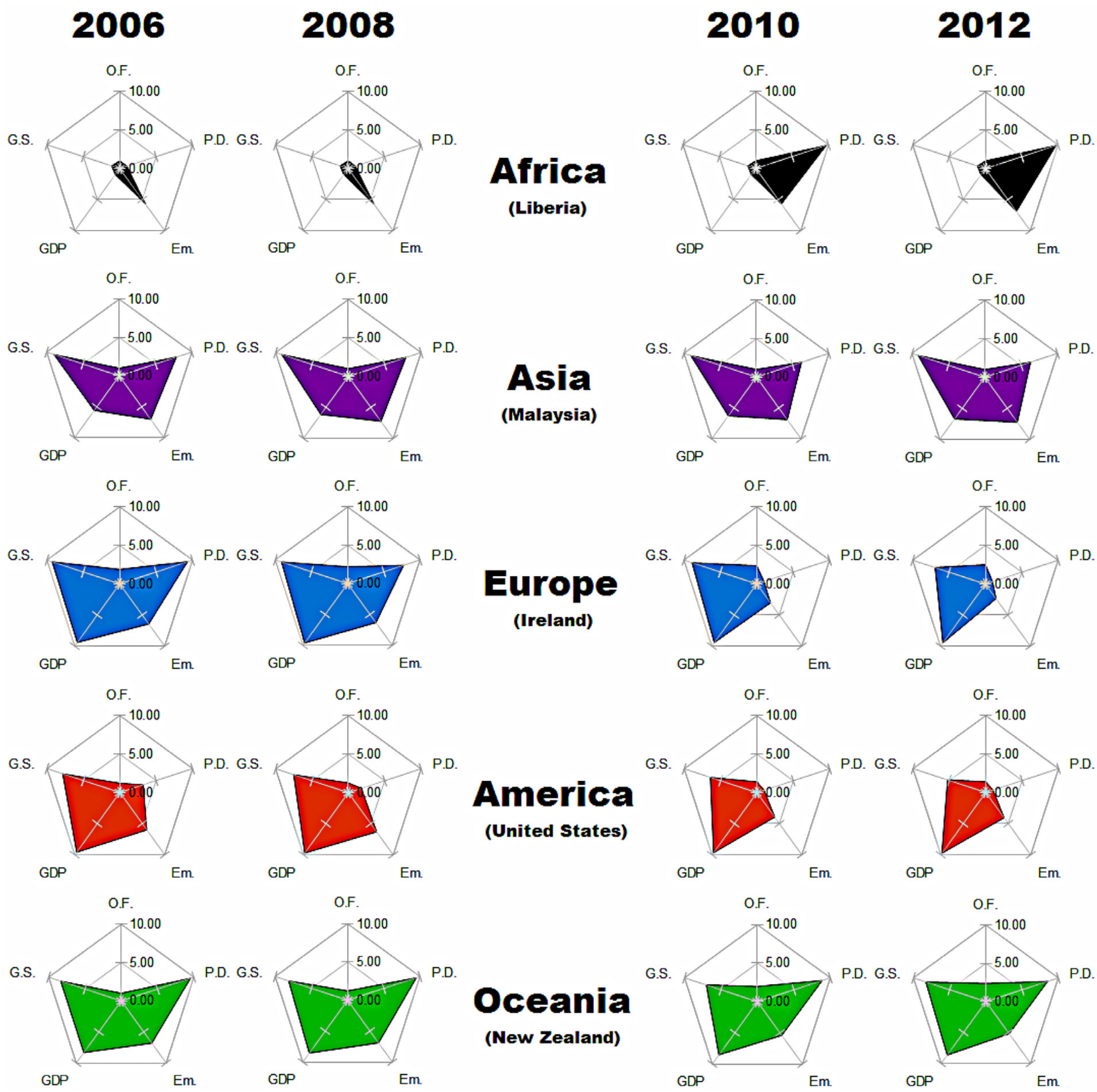

Fig. 11. Radial profiles for economic indicators by year.

one country with the lowest income ($1035 or less) according to the World Bank (2013).

In countries which, according to the World Bank (2013), have a high income ($12,616 or more), such as Ireland in Europe, the US in North America and New Zealand in Oceania, it is a different economic indicator from that of low-income countries which changes between 2006/2008 and 2010/2012. If we look at the radial profiles, in Ireland, the variable Em (employment) moves from approximately 6.00 in 2006/2008 to a value well below 5.00 for the 2010/2012 period. This means that the number of persons employed decreased in that country, as in the US, although with a less severe drop, going from 6.00 to a little under 5.00 in the 2010/2012 period. Thus, employment decreased in the US but to a lesser extent than in Ireland. The same occurs in New Zealand, where the economic indicator Em (Employment) is located between 6.00 and 5.00, meaning that this variable also saw a decrease in the 2010/2012 period.

These analyses confirm what Van de Kerk and Manuel (2012) indicated, namely that the comparatively large progress of low-income countries in terms of economic well-being is mainly the result of a sharp increase in the score for public debt and to a lesser extent, of an increase in the genuine savings score. Liberia and Malaysia can be found among these countries. In contrast, the countries with the highest incomes stand out because of a reduction in employment, or an increase in unemployment, as is the situation of Ireland, the US and New Zealand.

In these countries and for this period the unemployment rate increased rather strongly, as can also be deduced from the information provided by the World Bank (2015): in Ireland the unemployment rate went from 4.4%, 4.6% and 6.0% in the period 2006–2008 to 12.0% in 2009 and 13.9%, 14.0% and 14.7% in the period 2010–2012. In the US the increase in the unemployment rate can also be considered significant, going from 5.9% in 2008 to 9.4% in 2009 and 9.0% on average in the period 2010–2012. There was also an increase in unemployment in New Zealand in 2009, showing a rate of 6.1%, up from a mean for the period 2006–2008 of 3.9%, and again rising in the period 2010–2012 to 6.7%. Among the causes of this significant increase in unemployment during the financial crisis are factors such as labour market reforms, economic freedom, education and part-time employment (Bertola et al., 2007; Choudhry et al., 2012), but undoubtedly the decline in economic activity has been one of the reasons behind the high levels of unemployment during the financial crisis, especially in European countries (Banerji et al., 2015).

5. Discussion and conclusions

The aim of this research study was to analyze the assessment of sustainability indicators throughout the period 2006–2012 in an attempt to establish whether they underwent any variation. At the same time, we wanted to determine how these indicators had evolved in countries within different geographical areas.

In line with what has already been established by different authors, such as Van de Kerk and Manuel (2012), Saisana and Philippas (2012) and Singh et al. (2012), none of the existing indicators seems to fulfil our needs completely as no single one is entirely suitable or else they do not fit our research needs. Taking this into account, we show in Table 1 the indicator measures and limitations of the sustainability indicators that have predominantly been employed in the context of sustainability. The SSI turned out to be the one that best suits our research needs as it covers sustainability in its broadest sense, including economic, social and environmental aspects, whereas most other indices do so only partly.

Analysis of the graphs provided by X-STATIS enabled us to deduce which of the years studied are similar from the perspective of each category of indicator analyzed individually: social, environmental and economic, respectively. Of all the indicators analyzed individually, the economic ones provide the most information concerning the evolution of the period in question (2006–2012). Thus, we can conclude that the economic indicators for the years 2006 and 2008 are similar to each other, but quite different from those corresponding to the years 2010 and 2012, meaning that the economic indicators used within the SSI are those that changed most. As Van de Kerk and Manuel (2012) point out, economic well-being is a precondition for the two other dimensions of well-being.

Likewise, when we used X-STATICO to consider the categories of the sustainability indicators in pairs, it was again the economic indicators such as public debt, genuine savings or employment, combined with social and environmental indicators, which showed large differences in their behaviour and determined the similarities between 2006 and 2008 on the one hand and 2010 and 2012 on the other.

With regard to the evolution of sustainability indicators by geographical area, there are countries within different geographical areas in which the economic indicators are different in 2006/2008 and 2010/2012. Public debt, genuine savings and employment are those marked by large differences for the countries analyzed and within different geographical areas. Radial profile analysis, used previously in studies such as that by Van de Kerk and Manuel (2008), showed that there are economic reasons underlying this large difference in the evolution of the indicators between the two pairs of years, 2006/2008 and 2010/2012, and that this also occurs when the economic indicators are analyzed together with the social and environmental indicators.

With the methodology used in this study, we found that in low-income countries, the score increased for public debt and, to a lesser extent, for genuine savings, Liberia and Malaysia being found in this group of countries. In contrast, the countries with the highest income stand out because of a decrease in employment, or in other words, an increase in unemployment. This is the situation in Ireland, the US and New Zealand, Ireland being salient as a European country (Banerji et al., 2015).

With regard to the results obtained, we are in agreement with Pintér et al. (2012) who affirm that a good system of indicators can help authorities to make knowledgeable decisions because they will have objective information that will allow them to pull together to attain the goals proposed, as well as their subsequent assessment. They can also serve to evaluate the effectiveness of public policies and progress towards political objectives (European Commission, 2005; Dalal-Clayton and Krikhaar, 2007). In the same sense, Rinne et al. (2013) consider that for governments, these sustainability indicators can serve to show the transparency and effectiveness of public sustainability in each country and can help in deciding what policies, projects and social, environmental and economic strategies should be adopted.

The research described here is an attempt to advance our knowledge of the assessment of sustainability indicators; however, the study is not without its limitations. For example, a longer time period could be considered to observe how these sustainability indicators gradually evolve; it would also be advisable to analyze more variables that may influence sustainability indicators, such as technological development, research and innovation, markets or industrial structure.

Acknowledgement

We wish to thank the anonymous reviewers for their comments and suggestions and for the time they spent on improving the initial version of this paper.

Appendix 1.

Table A1
Countries in the sample.

Albania	Ecuador	Latvia	Romania
Algeria	Egypt	Lebanon	Russia
Angola	El Salvador	Liberia	Rwanda
Argentina	Estonia	Libya	Saudi Arabia
Armenia	Ethiopia	Lithuania	Senegal
Australia	Finland	Luxembourg	Serbia
Austria	France	Macedonia	Sierra Leone
Azerbaijan	Gabon	Madagascar	Slovak Republic
Bangladesh	Gambia	Malawi	Slovenia
Belarus	Georgia	Malaysia	South Africa
Belgium	Germany	Mali	Spain
Benin	Ghana	Malta	Sri Lanka
Bhutan	Greece	Mauritania	Sudan
Bolivia	Guatemala	Mexico	Sweden
Bosnia-Herzegovina	Guinea	Moldova	Switzerland
Botswana	Guinea-Bissau	Mongolia	Syria
Brazil	Guyana	Montenegro	Taiwan
Bulgaria	Haiti	Morocco	Tajikistan
Burkina Faso	Honduras	Mozambique	Tanzania
Burundi	Hungary	Myanmar	Thailand
Cambodia	Iceland	Namibia	Togo
Cameroon	India	Nepal	Trinidad and Tobago
Canada	Indonesia	Netherlands	Tunisia
Central African Republic	Iran	New Zealand	Turkey
Chad	Iraq	Nicaragua	Turkmenistan
Chile	Ireland	Niger	Uganda
China	Israel	Nigeria	Ukraine
Colombia	Italy	Norway	United Arab Emirates
Congo	Jamaica	Oman	United Kingdom
Congo. Dem. Rep.	Japan	Pakistan	United States
Costa Rica	Jordan	Panama	Uruguay
Cote d'Ivoire	Kazakhstan	Papua New Guinea	Uzbekistan

Table A1 (*Continued*)

Croatia	Kenya	Paraguay	Venezuela
Cuba	Korea, North	Peru	Vietnam
Cyprus	Korea, South	Philippines	Yemen
Czech Republic	Kuwait	Poland	Zambia
Denmark	Kyrgyz Republic	Portugal	Zimbabwe
Dominican Republic	Laos	Qatar	

Appendix 2.

Table A2
SSI indicators.

Social wellbeing	Environmental wellbeing	Economic wellbeing
S. F. Sufficient food	A. Q. Air quality	O. F. Organic farming
S. D. Sufficient to drink	B. Biodiversity	G. S. Genuine savings
S. S. Safe sanitation	R. W. R. Renewable water resources	GDP Gross domestic product
H. L. Healthy life	C. Consumption	Em. Employment
C. A. Clean air	R. E. Renewable energy	P. D. Public debt
C. W. Clean water	G. Ga. Greenhouse gases	
Ed. Education		
G. E. Gender equality		
I. D. Income distribution		
G. Go. Good governance		

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Artículo 4

Are Social, Economic and Environmental Well-Being Equally Important in all Countries Around the World? A Study by Income Levels

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Are Social, Economic and Environmental Well-Being Equally Important in all Countries Around the World? A Study by Income Levels

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Abstract The main objective of this paper was to see whether different countries around the world show differences in their sustainability levels as captured in the indicators from the Sustainable Society Index (SSI, Van de Kerk and Manuel in Ecol Econ 66:228–242, 2012) according to their level of income. To do so, the X-STATIS and CO-STATIS multivariate techniques were employed. With these methods, our sample of 151 countries and 21 indicators can be jointly represented along four time periods. The results obtained permit us to visualize that the groups of countries by income levels show differences in some of the variables from the SSI, because of the lack of proximities between those variables and the countries. Moreover, with the X-STATIS technique, the possible evolution of the countries or indicators over time can be represented, and with CO-STATIS, the relations between the social, economic and environmental aspects can be shown as well. From our results we were able to deduce that, on the one hand, social and economic indicators, such as Public Debt or Employment, are associated with countries having high and upper-middle incomes, for example, Chile, Israel, Malta, Kuwait, Saudi Arabia, Oman, Spain, Portugal, France, Poland and Czech Republic. On the other hand, countries with low and lower-middle incomes are more associated with environmental issues. Also, after finding that the differences between the countries by income levels are mainly caused by the economic indicators, we carried out two CO-STATIS analyses, one for social and

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economic variables, and the other for economic and environmental variables. These findings led us to deduce that, generally, the social and economic indicators are not related to each other, nor are the economic indicators related to the environmental ones. However, for some of the countries individually both relations may be possible.

Keywords Sustainable society index · Economic index · Social index · Environmental index · Countries worldwide · Multivariate analysis · Income levels · X-STATIS · CO-STATIS

1 Introduction

One of the issues of most concern internationally in recent years has been sustainable development, and hence sustainability. As Waas et al. (2011) affirm, different meetings and events have been taking place since 1972 to address this topic. The latest one was held in Rio in 2012, and focused on attaining different objectives such as a green economy, poverty eradication, and the establishment of an institutional framework for sustainable development. Nevertheless, and despite the effort made, the proposed goals have not been met and according to some scholars, such as Quental et al. (2011), we are still far from reaching global sustainability.

Sustainability is considered by many authors as the best way to deal with huge and complex environmental and social problems, and is considered essential for the benefit of current and future generations. As well, sustainability is not only a solution to environmental and social problems, but also offers a set of principles that involve positive goals, a focus for change and critical thinking and conventional practice.

In addition to the conceptual terms, the need to find a unit of measurement that can be used to represent the aspects of sustainability has also been debated in recent years. Thus, different authors such as (Bell and Morse 2008; Meadows 1998; Guy and Kibert 1998; and Van de Kerk and Manuel 2008) consider that in the last few years there have been different indices or indicators that provide a complete overview of all relevant aspects of sustainability in a transparent and easily understandable manner. Some of the most important have been the Human Development Index, the Millennium Development Indicators, Indicators for the EU Sustainable Development Strategy, and the Index of Sustainable Economic Welfare. In this research, we use the Sustainable Society Index (SSI), employed in previous analyses (e.g. Van de Kerk and Manuel 2008).

Considering that the SSI measures the dimensions of sustainability: environmental, social and economic, the aim of the present study is to analyse how global sustainability evolves depending on level of income, and also analyse the relations among these dimensions of sustainability, using different multivariate techniques. To do so we classify the 151 countries in our sample taking into account Gross National Income (GNI) per capita according to the 2013 World Bank classification. Unlike other research studies made by geographical areas (Scruggs 2003; Jahn 1998; Crepaz 1995; Giles and Feng 2005; Hosseini and Kaneko 2011), in this research we attempt something new: to differentiate according to income level, with a view to obtaining a more accurate assessment of global sustainability in light of the economic capacity of each country. We can then determine whether a country's greater or lesser economic capacity, as measured by GNI per capita, makes a difference in terms of economic, social, and environmental issues. Thus, the countries are classified as follows: countries with the lowest incomes ($1035 or less), countries with low-middle incomes ($1036–4085), countries with upper middle incomes ($4086–12,615) and countries with high incomes ($12,616 or more) (World Bank 2013).

The techniques we have chosen for this research are X-STATIS, to discover how global sustainability evolves depending on the level of income and to be able to analyse how sustainability has evolved over the period 2006–2012, and CO-STATIS, to capture the relationships between environmental and social indicators and economic indicators. This technique also allows us to study the relationships between the respective compromise structures, which means that we have to take into account a weighted average of the indicators along the different years under study. Moreover, these statistical techniques have not yet been applied to the SSI, thereby providing some degree of novelty to the current work.

The paper is structured as follows: after the introduction, in Sect. 2 we analyse the definition and measures of sustainability. Section 3 describes our research methods, including the sample and the statistical analysis. In Sect. 4, the results of the empirical analysis are given and then discussed in Sect. 5. Section 6 summarizes the main findings and consequences and presents the conclusions.

2 Defining and Measuring Sustainability

The term sustainability is often used as a synonym for sustainable development; however, some of the distinctions between the two terms are that when we speak of sustainable development, development and economic growth seem to take precedence, whereas when we talk about sustainability a higher priority is given to the environment. Nonetheless, the two terms have in common environmental considerations, and therefore in line with the determination taken by Waas et al. (2011), in this research the terms sustainable development and sustainability are used interchangeably.

Authors like (Giddings et al. 2002; Sneddon et al. 2006; and Waas et al. 2011) highlight the major stages or key time periods for the development and formalization of this concept: an initial time period until the end of the 1970s, a period of stagnation from 1980 to 1986, a period in which the greatest gains were achieved, from 1987 to 1995, and a fourth period of decline or decay, although undoubtedly the events in each period marked a milestone in the achievement of sustainability.

These events, according to Waas et al. (2011) are: United Nations Conference on the Human Environment (UNCHE, 1972) World Conservation Strategy (WCS, 1980), Our Common Future (1987), United Nations Conference on Environment and Development (UNCED, 1992), United Nations Millennium Summit (2000) Earth Charter (2000), United Nations World Summit on Sustainable Development (WSSD, 2002) and Rio +20 United Nations Conference on Sustainable Development (UNCSD, 2012).

The first event was held in Stockholm in 1972. The most important points to note were the creation of greater environmental awareness worldwide, and the placing on the agenda of an international environmental policy issue (Quental et al. 2011). It resulted in a statement of 36 principles on the preservation and improvement of the environment and 109 recommendations.

Other events that have made a big difference is the report published in 1987, called Our Common Future, which is a milestone in the ongoing development of the thinking for four reasons: first, it establishes the concept of sustainable development; second, it provides a substantial component of international thought; third, it has given rise to a major expansion of publications on the topic; and finally, it represents the breakthrough of the concept of sustainability into the world (Dresner 2008).

Subsequent to this report, in 1992 and in the city of Rio, Agenda 21 was adopted and political support for global sustainability as a new model of sustainable development was achieved, with 27 principles of sustainability also being approved. According to Quental et al. (2011), the conference was considered a success because sustainable development has become a topic of high relevance for governments, companies, academic institutions, both governmental and non-governmental organizations and ultimately for all citizens throughout world.

In 2000, the arrangements set out in Agenda 21 and the Rio Declaration were reaffirmed. Likewise, the so-called Millennium Development Goals were established, to be achieved in 2015. These objectives focus on reducing extreme poverty and meeting the basic needs of the world's poorest people. Also in this year, the Earth Charter was published. It is a declaration of fundamental ethical principles for building a just, sustainable and peaceful global society in the 21st century. That is, a vision of hope for the future.

In 2002, the United Nations World Summit on Sustainable Development met in Johannesburg to launch the necessary mechanisms to actually implement Agenda 21, since there had been little progress so far. In 2012 the Rio +20 United Nations Conference on Sustainable Development was held to focus on securing renewed political commitment for sustainable development, to assess progress and gaps in the implementation of the various principles and statements and to address the difficulties encountered so far. In short, the primary issues focus on a green economy in the context of sustainable development, poverty eradication and the establishment of an institutional framework for sustainability.

As various authors (e.g. Fergus and Rowney 2005; Sneddon et al. 2006; Rees 2010; Quental et al. 2011) point out, despite all efforts at the highest international political level, the objectives set out in each of these statements have not been achieved yet and global sustainability is still far from being reached.

Sustainability is considered by many authors as the best way to deal with huge and complex environmental and social problems, and is considered essential for the benefit of current and future generations; moreover, sustainability is not simply a solution to environmental and social problems, but it also offers a set of principles that involve positive goals, a focus for change and critical thinking and conventional practice. According to Bolis et al. (2014) Sustainability is defined as having several meanings and is still open to several interpretations. In the words of Giannetti et al. (2009) "assuming that sustainability depends on the availability and distribution of natural resources, any proposed measure must include an assessment of the availability of these resources and be able to detect and quantify the balance between the economy, society and environment".

Aiming for sustainability implies, first, defining its components in measurable terms and clearly fixing the responsibility to assess progress comprehensively (Hales and Prescott-Allen 2002). Nevertheless, the notion of what is meant by sustainability varies considerably and its definition is still ambiguous (Mori and Christodoulou 2012). It is no wonder that the relevant literature is abundant with studies on sustainability (Hák et al. 2007; Arezki and Van Der Ploeg 2007; Bell and Morse 2008; Betsill and Rabe 2009) and many of them define it in a way similar to that of the Brundtland Report. Thus, Baumgärtner and Quaas (2010) consider sustainability to be a normative notion that indicates the way humans should act towards nature, and how they are responsible towards one another and future generations, and Kates et al. (2001) consider that the essence of sustainability is to meet fundamental human needs while preserving the life-support systems of planet Earth.

According to Van de Kerk and Manuel (2008), a sustainable society is one in which each human being is capable of developing in a healthy manner and obtaining a proper education; lives in a clean environment; lives in a safe and well-balanced society; uses non-renewable resources responsibly so that future generations will not be left without them, and contributes to a sustainable world.

For Saisana and Philippas (2012), the term sustainability has also been used by politicians and economists to mean that a society is economically viable, environmentally rational and socially responsible, although the great changes taking place in social and economic matters have made the measuring of sustainability very complicated, despite the great progress already achieved in this sense.

Regarding the models used to represent sustainability, Munasinghe (1993) was one of the first authors to introduce a model based on a balance between the economy, society and a sustainable environment, distinguishing between "an economic objective, an ecological objective and a social objective or poverty/equity, and their interaction". Also Robinson and Tinker (1997) consider these dimensions to indicate that sustainability requires the simultaneous reconciliation of three imperatives: the *ecological imperative*, which advocates staying within the biophysical capacity of the planet, *the economic imperative*, which means providing adequate living conditions to all mankind, and the *social imperative*, which comprises the values that everyone would like to have and share with the society in which they live. Among other authors, Lozano (2008) considers these dimensions within the integrational perspective, which focuses on integrating economic, environmental and social aspects and in relation to each other and through time.

Once sustainability has been conceptualized within economic, social and environmental dimensions, the next step is to find a unit of measurement that can be used to represent these dimensions. In the last few years several different indices or indicators have emerged that provide a complete overview of all relevant aspects of sustainability in a transparent and easily understandable manner (Bell and Morse 2008; Meadows 1998; Guy and Kibert 1998, Van de Kerk and Manuel 2008).

Thus, according to Rinne et al. (2013), sustainability indicators serve, individually, to analyze the situation in each country, see what gaps there are and what are considered the most relevant aspects, to compare the sustainability of each country with respect to others in the same geographical area, and to identify the most effective ways to attain it. For governments, these sustainability indicators can serve to show a transparent and effective public sustainability situation in each country and can help in decision-making on what policies, projects and social, environmental and economic strategies to adopt. To achieve this goal the number of indicators should be limited, the data used to construct the indicators should be publicly available for all countries and the set of all indicators should provide a good picture of the current status of sustainability.

In current practice, sustainability indicators are often selected on the basis of historical practices and regulations or expert knowledge, and the degree to which an individual country meets a set of criteria (Niemeijer and Groot 2008). Thus, indicators and indices, which are continuously measured and calculated, allow long-term monitoring of the trend of sustainability from a retrospective point of view (Ness et al. 2007).

From the above it follows that there is currently no single index, an opinion shared by Frugoli et al. (2015) when they point out that, "none of the existing indicators alone can actually measure all significant aspects of economic, social and environmental well-being" since new indicators are increasingly being developed to measure the dimensions of sustainability: economic, social and environmental. Some of these are indices established by the OECD and the UN, among others, but the SSI (hereinafter, SSI), developed in 2006, does so in a more complete way, as it covers economic, social and environmental dimensions; it can thus be considered innovative (Van de Kerk and Manuel 2012).

Furthermore, the SSI has been audited by the Joint Research Centre of the European Commission, and is based on the definition of sustainability given by the Brundtland Commission (WCED 1987). To reflect that it includes sustainability as well as human environmental wellbeing, the Sustainable Society Foundation has extended the Brundtland definition with a third sentence, as follows: “A sustainable society is a society that: (a) meets the needs of the present generation; (b) does not compromise the ability of future generations to meet their own needs, and c) in which each human being has the opportunity to develop itself in freedom within a well-balanced society and in harmony with its surroundings” (Saisana and Philippas 2012, p. 15).

Although the SSI is not the only way to measure sustainability, it is considered a conceptually and statistically robust tool that is widely applicable for the ongoing assessment of human and environmental systems and a key point of reference to compare the progress and inform future global society.

The SSI can be used to simulate the effects of a number of potential actions, providing a powerful tool for informing decisions about how to attain human and economic growth without compromising environmental welfare.

In short, the conceptual framework of the SSI is conceptually coherent, meets the statistical requirements of the Joint Research Centre of the European Commission and is suitable for assessing the development of nations in a broad sense: economic, human and environmental well-being.

The SSI consists of 21 indicators grouped into three dimensions: Human Wellbeing, Environmental Wellbeing, and Economic Wellbeing. The different indicators comprising these dimensions are listed below in Table 1 (Van de Kerk and Manuel 2012).

Regarding the measurement units of the SSI, if the sustainability value of an indicator in known, the value of the indicator is scored with 10 in the case of 100 % sustainability, and if there is no sustainability at all, the indicator value is 0. If an indicator has already a set of values, the data for this indicator are transformed on a scale of 0–10. The transformation from basic data to indicator values has been done by standardization, and for some indicators, more complex calculating formulas have been used considering the characteristics of every indicator.

Table 1 Sustainable Society Index Indicators

Social wellbeing	Environmental wellbeing	Economic wellbeing
S. F. Sufficient Food	A. Q. Air Quality	O. F. Organic Farming
S. D. Sufficient to Drink	B. Biodiversity	G. S. Genuine Savings
S. S. Safe Sanitation	R. W. R. Renewable Water Resources	GDP Gross Domestic Product
H. L. Healthy Life	C. Consumption	Em. Employment
C. A. Clean Air	R. E. Renewable Energy	P. D. Public Debt
C. W. Clean Water	G. Ga. Greenhouse Gases	
Ed. Education		
G. E. Gender Equality		
I. D. Income Distribution		
G. Go. Good Governance		

3 Research Method

3.1 Population and Sample

The sample we use comprises the 151 countries selected by Van de Kerk and Manuel (2008) (see "Appendix") pertaining to different geographical areas around the globe and corresponding to the 2006–2012 period. This sample is different from the one used by Wilkinson and Pickett (2009), who considered the 50 richest countries in world, ruling out countries without reliable data on inequality.

Our sample incorporates the advantages derived from considering different countries grouped according to GNI per capita data taken from the World Bank (2013) (see Table 2). GNI is obtained by the sum of value added by all resident producers plus any product taxes (minus subsidies) not included in the valuation of output, plus net receipts of primary income (compensation of employees and property income) from abroad (World Bank 2013). This measure is calculated based on the currency of each country and is subsequently converted to U.S. dollars at the official exchange rates so that it can be compared across countries.

The classification according to GNI to differentiate countries according to income level is as follows:

Low income $1035 or less
Lower middle income $1036–4085
Upper middle income $4086–12,615
High income $12,616 or more

This classification is quite up to date as it corresponds to July 1, 2013 and will remain effective until July 1, 2014. It will allow us to differentiate countries according to income level, with a view to obtaining a more accurate assessment overall of sustainability in light of the economic capacity of each country.

3.2 Statistical Analysis

We therefore consider in this research the 151 countries around the world presented in Table 2, grouped according to the GNI per capita as reflected in the World Bank (2013) data; the 21 numerical characteristics are the scores obtained by the countries selected concerning the policy categories proposed in the SSI in the latest available years (2006, 2008, 2010, 2012). Therefore, in this paper, the data consist of the SSI scores for each country in each time period, that is three cubes: $X_{151 \times 10 \times 4}$ for social indicators, $X_{151 \times 6 \times 4}$ for environmental indicators and $X_{151 \times 5 \times 4}$ for economical indicators. See diagram below (Fig. 1).

The analysis of many sustainability issues at once requires the storage of a large volume of data. To explore the data to get a better understanding of the behaviour of various processes, it is important to identify the salient features underlying them. The reduction in dimensionality of the problem allows us to summarize the information captured in a large number of variables with a smaller number of latent variables. Plots showing simultaneously both countries and indicators can be helpful in this regard.

These methods will allow us to check what we really wish to with this research. In this study we are attempting something new: to differentiate countries according to income level, with a view to obtaining a more accurate assessment of global sustainability in light

Table 2 Classification of countries by income level

Low incomes	Lower middle incomes	Upper middle incomes	High incomes
$ 1035 or less	$ 1036–4085	$ 4086–12,615	$ 12,616 or more
Bangladesh BAN (As)	Armenia ARM (As)	Albania ALB (Eu)	Australia AUS (Oc)
Benin BEN (Afr)	Bhutan BHU (As)	Algeria ALG (Afr)	Austria AUT (Eu)
Burkina Faso BUR (Afr)	Bolivia BOL (LaAm)	Angola ANG (Afr)	Belgium BEL (Eu)
Burundi BDI (Afr)	Cameroon CMR (Afr)	Argentina ARG (LaAm)	Canada CAN (Am)
Cambodia CAM (As)	Congo CGO (Afr)	Azerbaijan AZE (As)	Chile CHI (LaAm)
Central African Republic CAF (Afr)	Cote d'Ivoire CIV (Afr)	Belarus BLR (Eu)	Croatia CRO (Eu)
Chad CHA (Afr)	Egypt EGY (Afr)	Bosnia-Herzegovina BIH (Eu)	Cyprus CYP (As)
Congo Dem. Rep. COD (Afr)	El Salvador ESA (LaAm)	Botswana BOT (Afr)	Czech Republic CZE (Eu)
Ethiopia ETH (Afr)	Georgia GEO (As)	Brazil BRA (LaAm)	Denmark DEN (Eu)
Gambia GAM (Afr)	Ghana GHA (Afr)	Bulgaria BUL (Eu)	Estonia EST (Eu)
Guinea GUI (Afr)	Guatemala GUA (LaAm)	China CHN (As)	Finland FIN (Eu)
Guinea-Bissau GBS (Afr)	Guyana GUY (LaAm)	Colombia COL (LaAm)	France FRA (Eu)
Haiti HAI (LaAm)	Honduras HON (LaAm)	Costa Rica CRC (LaAm)	Germany GER (Eu)
Kenya KEN (Afr)	India IND (As)	Cuba CUB (LaAm)	Greece GRE (Eu)
Korea, North PRK (As)	Indonesia INA (As)	Dominican Republic DOM (LaAm)	Iceland ISL (Eu)
Kyrgyz Republic KGZ (As)	Laos LAO (As)	Ecuador ECU (LaAm)	Ireland IRL (Eu)
Liberia LBR (Afr)	Mauritania MTN (Afr)	Gabon GAB (Afr)	Israel ISR (As)
Madagascar MAD (Afr)	Moldova MDA (Eu)	Hungary HUN (Eu)	Italy ITA (Eu)
Malawi MAW (Afr)	Mongolia MGL (As)	Iran IRI (As)	Japan JPN (As)
Mali MLI (Afr)	Morocco MAR (Afr)	Iraq IRQ (As)	Korea, South KOR (As)
Mozambique MOZ (Afr)	Nicaragua NCA (LaAm)	Jamaica JAM (LaAm)	Kuwait KUW (As)
Myanmar MYA (As)	Nigeria NGR (Afr)	Jordan JOR (As)	Latvia LAT (Eu)
Nepal NEP (As)	Pakistan PAK (As)	Kazakhstan KAZ (As)	Lithuania LTU (Eu)
Niger NIG (Afr)	Papua New Guinea PNG (Oc)	Lebanon LIB (As)	Luxembourg LUX (Eu)
Rwanda RWA (Afr)	Paraguay PAR (LaAm)	Libya LBA (Afr)	Malta MLT (Eu)
Sierra Leone SLE (Afr)	Philippines PHL (As)	Macedonia MKD (Eu)	Netherlands NED (Eu)
Tajikistan TJK (As)	Senegal SEN (Afr)	Malaysia MAS (As)	New Zealand NZL (Oc)
Tanzania TAN (Afr)	Sri Lanka SRI (As)	Mexico MEX (LaAm)	Norway NOR (Eu)
Togo TOG (Afr)	Sudan SUD (Afr)	Montenegro MNE (Eu)	Oman OMA (As)
Uganda UGA (Afr)	Syria SYR (As)	Namibia NAM (Afr)	Poland POL (Eu)

Table 2 continued

Low incomes	Lower middle incomes	Upper middle incomes	High incomes
Zimbabwe ZIM (Afr)	Ukraine UKR (Eu)	Panama PAN (LaAm)	Portugal POR (Eu)
	Uzbekistan UZB (As)	Peru PER (LaAm)	Qatar QAT (As)
	Vietnam VIE (As)	Romania ROU (Eu)	Russia RUS (Eu)
	Yemen YEM (As)	Serbia SRB (Eu)	Saudi Arabia KSA (As)
	Zambia ZAM (Afr)	South Africa RSA (Afr)	Slovak Republic SVK (Eu)
		Thailand THA (As)	Slovenia SLO (Eu)
		Tunisia TUN (Afr)	Spain ESP (Eu)
		Turkey TUR (As)	Sweden SWE (Eu)
		Turkmenistan TKM (As)	Switzerland SUI (Eu)
		Venezuela VEN (LaAm)	Taiwan TPE (As)
			Trinidad and Tobago TRI (LaAm)
			United Arab Emirates UAE (As)
			United Kingdom GBR (Eu)
			United States USA (Am)
			Uruguay URU (LaAm)

Source: World Bank 2013

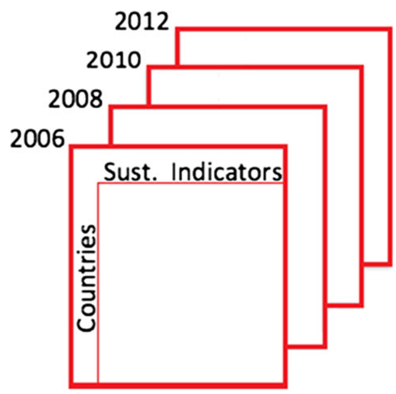

Fig. 1 Representation of the data cube $X_{151 \times 21 \times 4}$, from the data for the sustainability indicators

of the economic capacity of each country. We can then determine whether a country's greater or lesser economic capacity, as measured by GNI per capita, makes a difference in terms of economic, social, and environmental issues. Furthermore, the World Bank classification will also allow us to observe how sustainability has evolved over time.

3.2.1 *Partial Triadic Analysis or X-STATIS*

X-STATIS (Jaffrenou 1978) belongs to the STATIS family of methods (Structuration des Tableaux À Trois Indices de la Statistique) (L'Hermier des Plantes 1976; Lavit et al. 1994). The STATIS method is a generalization of Principal Component Analysis (PCA) used to study several data measured in the same individuals or the same variables in tables.

The aim of the STATIS methods is to analyze the relations between the data tables of a sequence and combine them into one matrix corresponding to an optimal agreement between the data. This means that the same variables can be measured in relation to the same individuals. Every STATIS method follows three steps: interstructure, compromise and intrastructure (or trajectories), see the diagram below (Fig. 2).

The first step is to determine if the data tables have similar structures or not, through the Euclidean representation, in which each point represents a data table. For the case in which they represent different times, similarity indicates that the variables maintain their behaviour over time with regard to the individuals studied.

The compromise is a summary of the different tables expressing the common structure of the variables in these tables, with the analysis of the compromise matrix we can represent the averages for the individuals and variables. So the compromise step represents the stable structure and it can be plotted to interpret that structure.

In the step called trajectories, after the analysis of the compromise matrix, we can represent the variables and individuals in each table. When applying this methodology it is very important to consider the trajectories of the individuals and the variables because this study provides information on the evolution of each of these individuals or variables, and it shows how each table differs from the stable structure. These trajectories are obtained by projecting the rows and columns of each table of the sequence on the compromise graph.

In the last two steps, the correlations of the variables allow us to interpret the positions of the individuals.

From the analytical point of view, the interstructure step provides the coefficients of a particular linear combination of the different data of the sequence, which leads to an optimal average table, the compromise, which is based on the concept of vector variances-covariances between each pair of data tables. For this, a matrix of scalar products is constructed between the tables of the sequence (the matrix of vector variances-covariances). The eigendecomposition of this matrix gives a first eigenvector, whose coordinates are the weights used to calculate the compromise. Furthermore, this interstructure can be plotted to visualize the similarities and differences between each of the data tables of the sequence. The compromise matrix maximizes the similarity with all the matrices of the sequence, so that the weight of each table is proportional to its similarity, and thus the tables that are different from the others will be weighted by default.

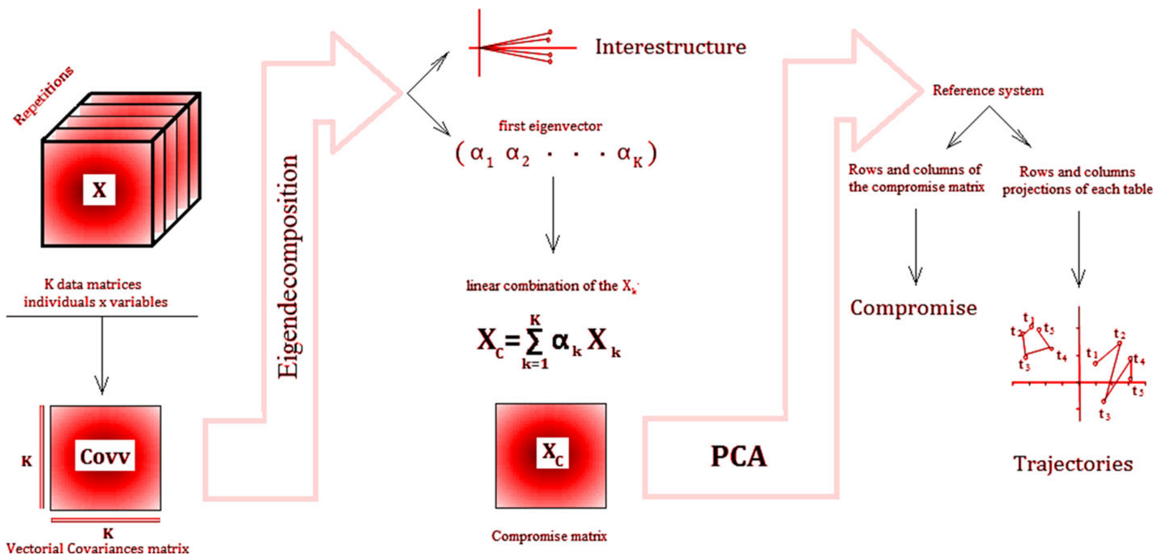

Fig. 2 Interstructure, compromise and trajectories analysis

We have chosen one of the STATIS methods, the X-STATIS, also called Partial Triadic Analysis (PTA): is a STATIS family method able to analyze a three-way Table (151 countries, 21 indicators, four years 2006, 2008, 2010, 2012), seen as a sequence of two-way tables. We have chosen this method because it requires that all tables have the same rows and the same columns.

In our case, the objective of the X-STATIS analysis is to extract a multivariate structure showing the different years 2006–2012. The interstructure analysis allows a multivariate summary of the information, expressed by the first axis of the years' ordination analysis. The compromise step permits a description of the countries in the sample as a function of the types of variables, and an identification of those that are responsible for grouping the countries according to similar patterns over the years. Our results therefore focus on the analysis of the countries and their variability over time.

All calculations included in the X-STATIS analysis were directly studied (Thioulouse et al. 1997) and processed with the STATIS module of ADE-4 software (Thioulouse 2011). Similarly, the plots are drawn using various graphical modules of the ADE-4 software.

In a more analytical way, for the interstructure, a matrix is built of scalar products between tables (the vector variance–covariance matrix), where the element in row k and column l is $Covv(X_k, X_l) = Tr(X_k^t\, D_n\, X_l\, D_p)$, where X_k is the k-th table of the sequence and D_n, D_p are the two metrics for the rows and columns, respectively. Alternatively, a vector correlation matrix that rescales the vector covariance can be used:

$$Rv(X_k, X_l) = Covv(X_k, X_l)/\sqrt{}(Covv(X_k, X_k)\, Covv(X_l, X_l))$$

For ploting the interstructure, we use vectors from the origin that end at the points given by the rows of $V_2\Lambda_2$, V_2 being the first two eigenvectors of the vector covariance matrix and Λ_2 the diagonal matrix with the two associated eigenvalues.

The compromise X_c is a linear combination of the X_k weighted by α_k, the coordinates of the first eigenvector of the interstructure (which are all of the same sign, which we assume are positive, because the vector covariance matrix is symmetric and all its elements are positive): $X_c = \sum_k \alpha_k X_k$. Moreover, the compromise analysis with X-STATIS provides two-dimensional representations (graphics of principal axes) to interpret its structure.

This process for calculating the compromise matrix is equivalent to performing a singular value decomposition of the matrix $Z = U\Lambda V^t$ obtained by placing in columns as vectors, the concatenated columns of each of the matrices of the sequence. Then, the first column of ZV will be taken as the compromise, displaying it as a matrix.

The intrastructure is obtained by projecting the rows and the columns of each table of the sequence on the compromise analysis. Let V_r be the first r eigenvectors matrix from the compromise analysis. The coordinates of the rows of table X_k are the rows of $X_k\, D_p\, V_r$, and the columns are the rows of $X_k^t\, D_n\, U_r$, U_r being the first r eigenvectors of $X_c\, D_p\, X_c^t\, D_n$.

3.2.2 *Costatis*

COSTATIS (or CO-STATIS) (Thioulouse 2011) is a method for simultaneously analysing a couple of data cubes (two sequences of tables) with the same variables in each cube for all repetitions, and with the same individuals in both cubes, although repetitions may differ from one cube to the other.

It is based on the Co-Inertia analysis of two compromise tables. Each compromise is calculated using the two sequences of tables, one for each cube. The first step of the COSTATIS consists in using two X-STATIS for calculating the compromises from both cubes, and then, the second step is Co-Inertia Analysis to analyze the relations between these two compromises.

Co-inertia analysis is a multivariate method that identifies trends or co-relations in multiple datasets which contain the same individuals. Co-Inertia simultaneously finds ordinations (dimension reduction diagrams) from the datasets that are most similar. It does this by finding successive axes from the two datasets with maximum covariance. Co-Inertia can be applied even to datasets where the number of columns far exceeds the number of rows (Fig. 3).

The Co-Inertia Analysis describes the co-structure between two data tables representing as well as possible the squares of the covariances between one table and the other. The fact is that co-inertia analysis is the analysis of a cross-products table and it maximizes the covariances between the coordinates of the rows of the two tables, i.e. the co-inertia is high when the values in both tables are high simultaneously or when they vary inversely, and it is low when they vary independently or do not vary.

The rows and columns of the two original matrices can be plotted in the space obtained after the co-inertia analysis by carrying out the spectral decomposition of the matrix X_c^t D_n Y_c D_q Y_c^t D_n X_c D_p, or if $Z = Y_c^t\ D_n\ X_c$ is the cross-products matrix, it is the decomposition of Z^t D_q ZD_p, where D_n, D_p, D_q are the metrics for the rows and the columns from the two matrices respectively.

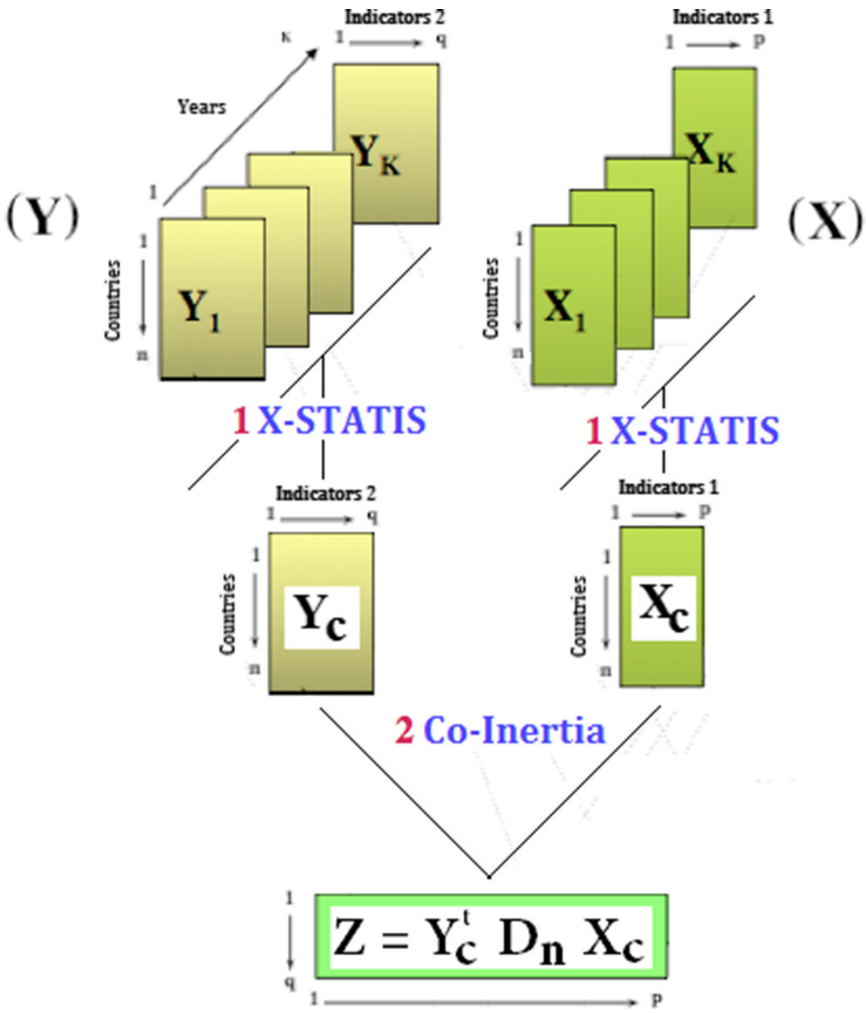

Fig. 3 COSTATIS diagram. The data structure is two sequences of tables. X_k and Y_k are the tables with the countries and the indicators. X-STATIS analysis produces two compromises: X_c and Y_c. Co-Inertia analysis is finally used to analyse these matrices

The compromises after the co-inertia analysis are weighted averages of the tables in the original sequences, with the coordinates of the first eigenvector from the interstructure of both X-STATIS as weights.

An additional step can be implemented, as in the X-STATIS method: it is possible to project the rows and columns from every matrix of the two sequences in this multidimensional space of the Co-Inertia Analysis.

The objective of the COSTATIS analysis is to highlight the relationships between the two compromises, which are the stable structures, and to discard the conflicting variations between the complete sequences. It is therefore very easy to interpret (as in a Co-Inertia Analysis standard), because it retains the optimality properties of the compromises of the two X-STATIS analyses.

COSTATIS is the co-inertia analysis of two compromises, so it looks for relations between two stable structures. This is different from the point of view of the X-STATICO method (Simier et al. 1999), because COSTATIS gives a co-structure of the compromise, which means that it first looks for the stable component of the variables from each cube and then it obtains a co-structure that represents the relations between these compromises. Therefore, this method may be more appropriate if the objective is to find a description of the relations between the stable part of the variables, rather than the evolution of these relations.

4 Results of the Empirical Analysis

4.1 X-STATIS Analysis

Having explained the statistical methodology, in this section the results are presented: the factorial planes, Partial Triadic Analysis methods, STATIS X-and CO-STATIS, for pairs of data cubes.

The data are sorted into three sequences of tables of 151 rows, countries, with 4 repetitions, the four years of study (2006, 2008, 2010 and 2012), a cube with 10 columns, which contain social type indicators, another cube with 6 columns, with environmental indicators, and the last one, with the 5 economic indicators.

We have included a plot with all the variables and indicators (Fig. 4), but for a deeper interpretation, moreover, we present the results in two representations corresponding to the countries with the lower economic levels (Fig. 5) and with the upper income levels (Fig. 6).

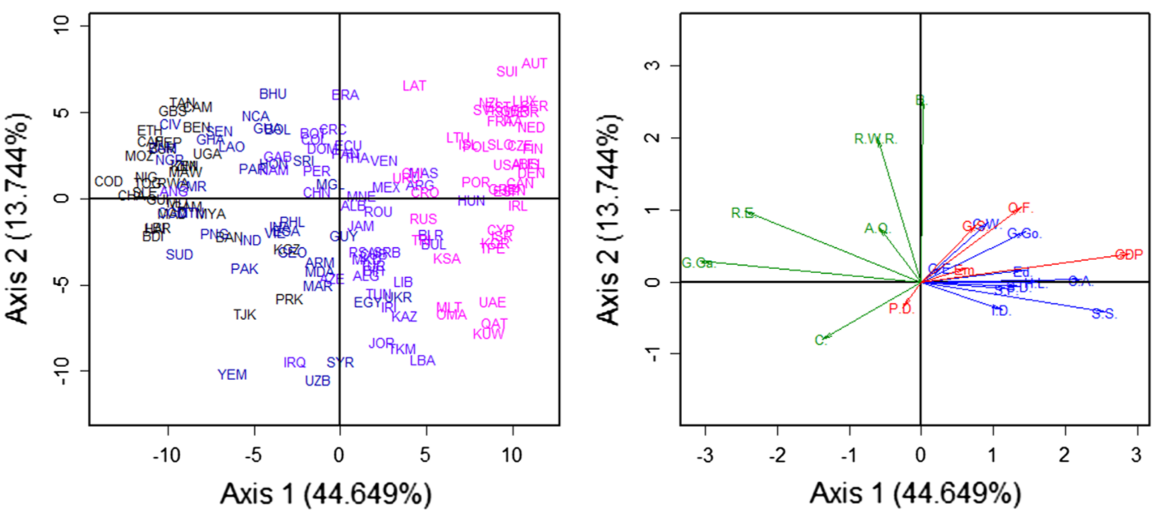

Fig. 4 Compromise analysis representing all the countries and all the indicators, split into two figures

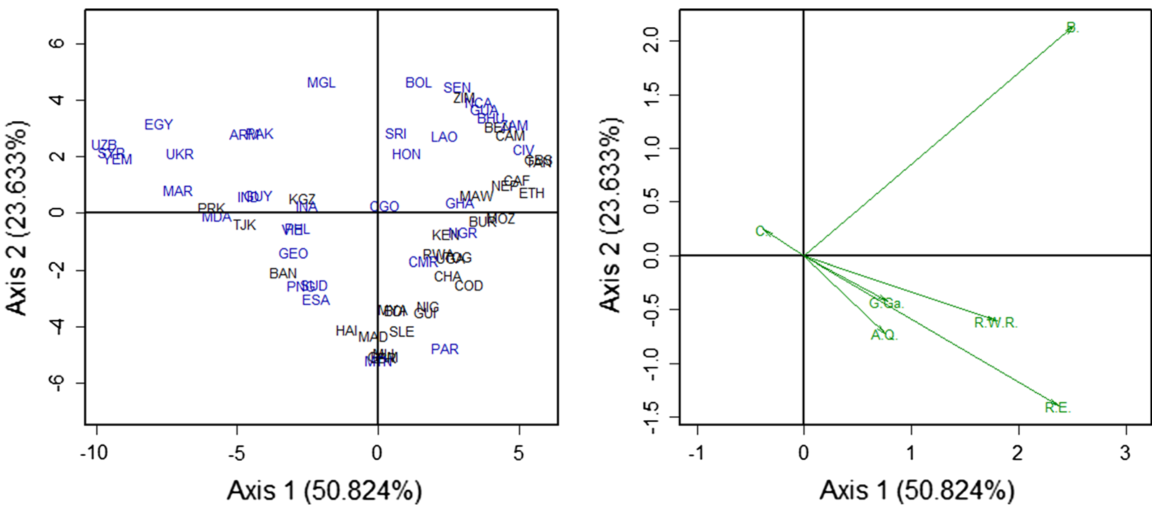

Fig. 5 X-STATIS compromise plot after analysing only the countries with low and lower-middle incomes and only the environmental variables

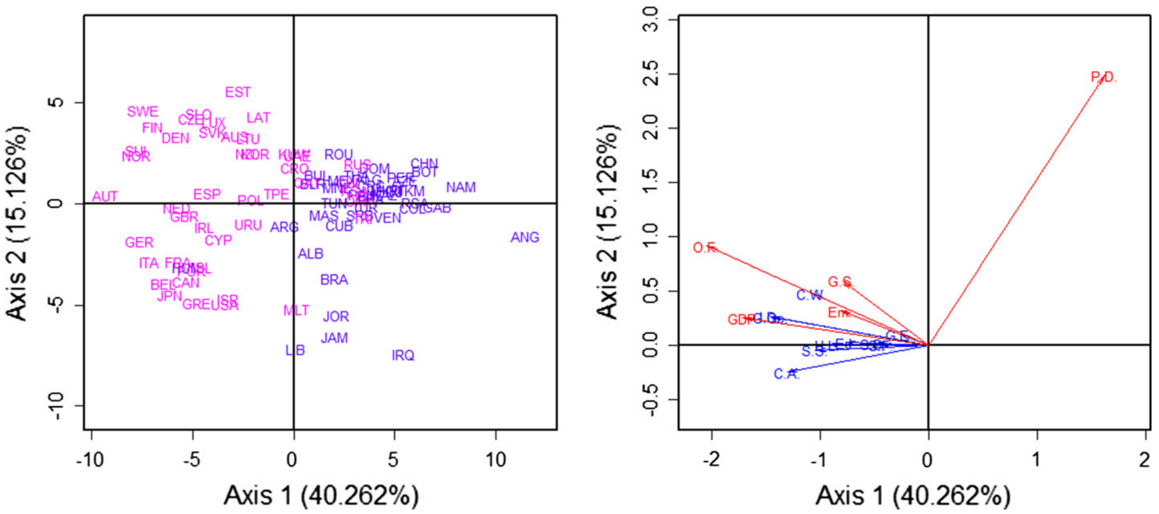

Fig. 6 X-STATIS compromise plot after analysing only the countries with high and upper-middle incomes and only the social and economic variables

The reason for analysing this data set is to discover how global sustainability evolves, depending on the level of income; to do so we have classified the countries as follows: countries with the lowest incomes ($1035 or less) countries with low-middle income ($1036–4085), countries with upper middle income ($4086–12,615) and countries with high income ($12,616 or more) (World Bank 2013) (see Table 2).

In this case the sustainability indicators considered in the study are represented on the right, and all the countries are represented on the left. The first axis (axis 1 horizontal) is explained by most indicators related to human wellbeing, such as sufficient food, sufficient to drink, safe sanitation, healthy life, clean air, education. The second factorial axis (axis 2 vertical) is related to variables such as air quality, biodiversity and renewable water resources.

Moreover, the factorial plane on the left in Fig. 4 shows that low income countries are mainly located in quadrants 2 and three (upper-left and lower-left), and the most of them are African countries, such as Burundi, Chad and Niger. The observation of the position of the indicators in Fig. 4 (right) leads us to affirm that countries with low incomes prioritize environmental concerns over any other topic related to sustainability, that is due to a highly deteriorated environment caused by the adverse effects of climate change, desertification, invasive species and the continuing expansion of human settlements to areas at risk.

As already noted, this factorial plane resulting from X-STATIS analysis was repeated twice, in order to delve deeper into the analysis among countries within the same groups.

In Fig. 5, we have preformed the X-STATIS for the countries with low and lower-middle incomes considering only the environmental indicators. We observed that the biodiversity variable is the most outstanding since some of the countries in this group such as Benin, Zimbabwe and Cambodia conceded greater importance to this environmental variable. Another variable that is emphasized in this group of countries is renewable energy, which has the greatest relevance in countries on the west coast of Africa such as Níger, Mali and Guinea (see Fig. 5).

Now, in Fig. 6, we show the compromise representation for the countries with high and upper-middle incomes and the social and economic indicators in order to preform a more in-depth analysis among these countries.

The high and upper-middle income countries plot confirms the following trend: that in the richest and most developed countries in America and Europe the economic and social issues take on priority.

A deeper analysis among the countries within the same group shows that the most important economic variable is public debt because some of the countries in this group, such as Russia, Oman and Saudi Arabia, show a higher value in the public debt indicator which means that the level of public debt is low, and in contrast, countries such as Italy and Greece show a lower level in this indicator, which points to higher public debt (see Fig. 6).

The Fig. 7 shows the compromise configuration for all the countries (on the left) and the trajectories for all the indicators (on the right), which allow us to study the evolution of countries over the different years of study (2006, 2008, 2010 and 2012) and the development of sustainability indicators.

Analysing the trajectories over time we can state that the variables of sustainability that have changed during the period studied are: Public Debt (Public Debt PD), in particular, which occurs in countries that are represented in the third quadrant, such as Chile, Israel,

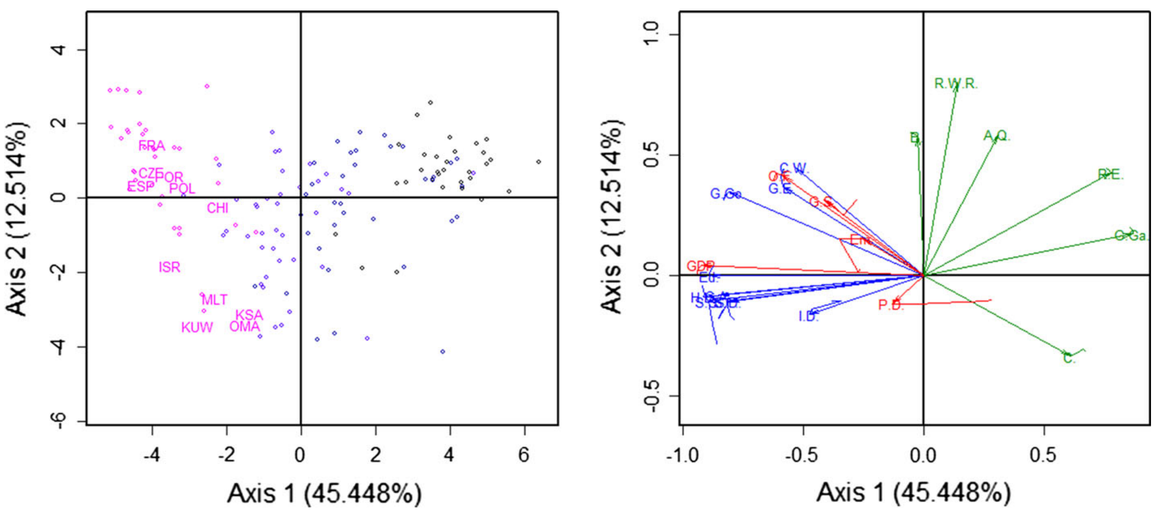

Fig. 7 Compromise analysis for the most representative countries and trajectories analysis for all the indicators

Malta, Kuwait, Saudi Arabia and Oman (see Fig. 7). This has in fact taken place according to data from the Central Intelligence Agency (2014). Another variable is employment (Employment Em.), which decreases in countries, especially in Europe, such as Spain, Portugal, France, Poland, Czech Republic. This is corroborated by consulting the information obtained from the increases in the number of unemployed in developed countries (World Bank 2014), over the study period.

4.2 Co-STATIS Analysis

Going further into the above analysis to capture the relationships between environmental-social indicators and economic indicators, we used the Co-STATIS statistical technique. Two sustainability tables are made from one table containing economic indicators (in columns) and another table containing environmental data (in columns). The rows of these two tables are identical and correspond to the countries where economic variables and environmental data have been measured (similar for economic and social indicators). These data are used to analyse the relationships between the economic and environmental/social aspects of sustainability. Sampling is repeated over time for both tables: one obtains a sequence of pairs of sustainability tables. Analysing this type of data is a way to assess changes in economic-environmental relationships (economic and social), which can be important to explain the global gradient of sustainability.

Co-STATIS is the co-inertia analysis of two compromises, so it looks for the relationships between two stable structures. This is different from the STATICO point of view (co-structure of two compromises versus compromise of a series of co-structures). Co-Inertia Analysis can be seen as the PCA of the table of cross-covariances between the variables of the two tables. In Co-STATIS, a consensus is extracted first, separately and independently. The relationships between these two summaries are then investigated by a co-inertia analysis. We therefore first calculate the compromise structure for the economic indicators, then for the social ones, and in a third step the compromise for the environmental variables. Subsequently, the co-inertia between these compromise matrices will be considered. Co-inertia analysis performs a double inertia analysis of two arrays. Just as inertia is a sum of variances, co-inertia is a sum of squared covariances and Co-Inertia analysis describes the co-structure between two sustainability data tables by summarizing as well as possible the squared covariances between pairs of indicator sets.

A scatter plot with arrows is specific to the coinertia analysis, and represents the countries. The beginning of the arrow is the position of the country described by one set of indicators, say the ones from economic data set; the end of the arrow is the position of the site described by another set of indicators, say the ones from environmental data. There is a similar interpretation for economic and social indicators. Thus, short vectors indicate that the two sets of indicators are related, while long vectors indicate otherwise. In the factor plots of the Co-STATIS analysis, we have represented countries with a high level of income by World Bank (2013) classification, as shown in Fig. 8. Moreover, in Table 3 we can distinguish between countries with short vectors (no colour) and those with long vectors (boxed in black).

According to the Co-STATIS analysis we can see that the position of some countries, such as Malta, Kuwait, Spain, France or Czech Republic, with respect to social indicators is next to their position with respect to economic indicators, short vectors. We can thus state that economic indicators are related to social indicators. However the position of Oman, Chile, Israel, Saudi Arabia, Portugal or Poland indicates that social and economic indicators are not related since their vectors are long.

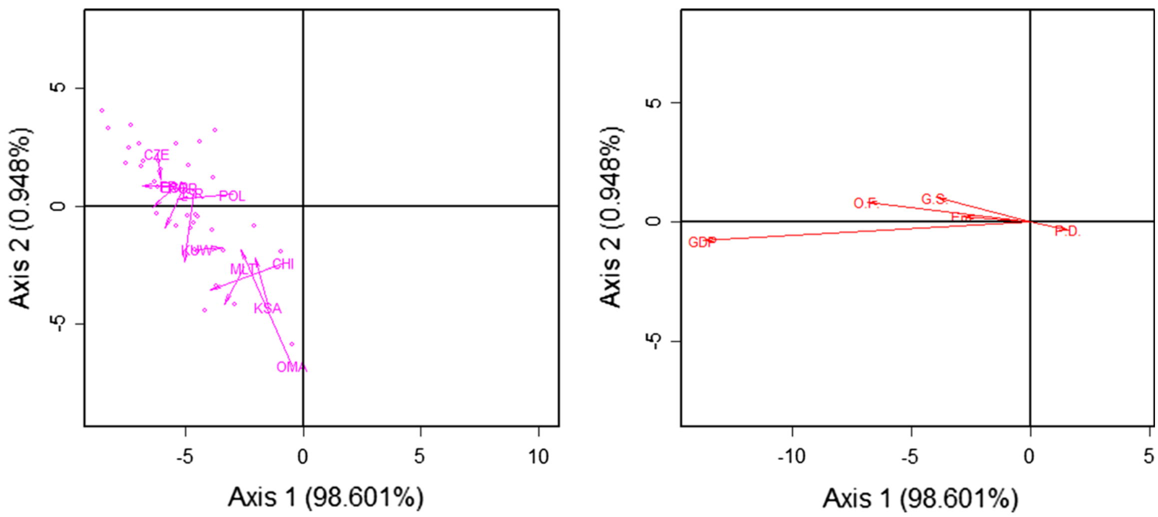

Fig. 8 Factor plots of the Co-STATIS analysis split into two figures: Co-inertia among economic and social indicators

Table 3 Lengths of the vectors in Co-STATIS analysis for high income countries

Country	Social/ economic	Environmental/ economic	Country	Social/ economic	Environmental/ economic
AUS	2.350	0.904	KUW	**3.476**	**2.133**
AUT	**6.16**	**3.61**	LAT	1.638	**1.915**
BEL	**2.793**	0.762	LTU	**2.873**	**1.698**
CAN	1.162	0.66	LUX	1.421	1.404
CHI	2.006	0.959	MLT	**4.131**	**3.098**
CRO	1.180	0.284	NED	1.938	**2.560**
CYP	1.866	1.477	NOR	0.463	0.323
CZE	**4.464**	**1.744**	NZL	**2.455**	**1.860**
DEN	**3.881**	**2.146**	OMA	**2.973**	1.534
ESP	**4.122**	1.135	POL	0.608	1.224
EST	**3.352**	**2.53**	POR	1.500	**1.686**
FIN	2.133	1.436	QAT	**2.873**	1.405
FRA	**3.860**	**1.964**	RUS	**4.128**	**2.232**
GBR	1.596	0.888	SLO	2.207	1.448
GER	**2.516**	0.823	SUI	**4.220**	**3.353**
GRE	1.055	0.514	SVK	**5.994**	**4.414**
IRL	2.326	**2.305**	SWE	0.723	0.495
ISL	**5.491**	**2.457**	TPE	1.584	1.297
ISR	0.906	1.067	TRI	**3.485**	**1.701**
ITA	1.231	0.932	UAE	**3.378**	**1.785**
JPN	**3.263**	**3.176**	URU	0.660	0.283
KOR	**3.396**	**2.514**	USA	**2.940**	**1.888**
KSA	1.669	0.978			

Bold values indicate long vectors

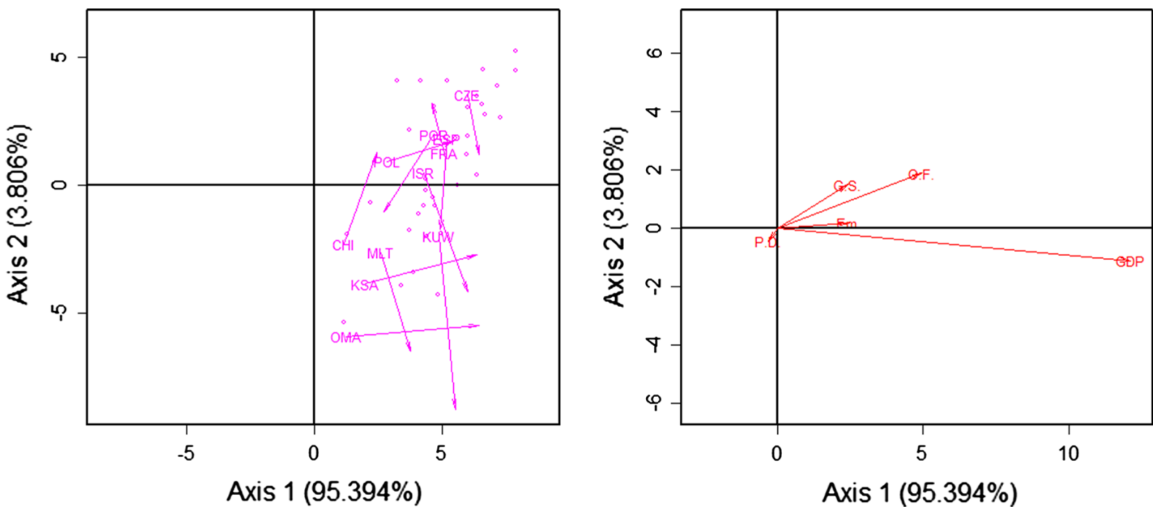

Fig. 9 Factor plots of the Co-STATIS analysis split into two figures: Co-inertia among economic and environmental indicators

Studying both environmental and economic indicators in the same previous countries, we found, for example, that Spain, Poland, Portugal, France and Czech Republic have short vectors. It follows that economic factors influence environmental ones. Hence it follows that richer countries put more effort into protecting the environment. On the other hand, in countries such as Malta, Kuwait, Israel, Saudi Arabia or Oman, the vectors are long, which means that environmental and economic indicators are not directly related (Fig. 9).

In addition to the above representations, we have also analysed these relationships in other countries by income levels, in the upper-middle, lower-middle and low level. In all of these representations long vectors were obtained, which means that the social and environmental aspects do not depend exclusively on economic indicators.

5 Discussion

Increasing concern on a global level for sustainable development and therefore sustainability has led to different conferences and events being held to address this topic (Waas et al. 2011). Since the Rio conference in 1972, different goals have been posited, such as attaining a green economy, eradicating poverty, and establishing an institutional framework for sustainable development. Despite all these efforts, the objectives set are far from being met, and according to authors such as Quental et al. (2011), global sustainability remains a dream for the future.

Thus, in the present study we have focused on sustainability indicators, which according to Rinne et al. (2013) serve to analyze the situation in each country, see what gaps there are and what are considered the most relevant aspects, to compare the sustainability of each country with respect to others in the same geographical area, and to identify the most effective ways to attain it.

Our findings show that low income countries prioritize environmental concerns over any other topic related to sustainability. These are African countries, such as Burundi, Chad, Nepal, and Niger. The results obtained with respect to the relation between low income countries and environmental concerns (air quality, biodiversity, renewable water resources, consumption, renewable energy and greenhouse gases) are in agreement with results obtained previously by Hosseini and Kaneko (2011), who found that "Africa has the worst standing relative to other regions for institutional, economic and social aspects; the only positive outcome belongs to the environmental aspect".

This opinion is shared by authors such as Brown et al. (2009) and Kotir (2011), who take into account that Africa is often cited as the continent most vulnerable to the adverse effects of climate change, and is considered a priority area for raising finds and financing projects aimed at solving the environmental problems resulting from desertification, invasive species and the continuing expansion of human settlements to areas at risk.

On the opposite side we have the high incomes countries, for which economic issues (organic farming, genuine savings, gross domestic product, employment and public debt) and social issues (sufficient food, sufficient to drink, safe sanitation, healthy life, clean air, clean water, education, gender equality, income distribution and good governance) are of maximum importance, especially in the richest and most developed countries located in America and Europe.

Our findings for high income countries are in agreement with Cracolici et al. (2010), who suggest that when the gross domestic product of a country rises, there is also a higher level in non-economic aspects such as better health conditions and a higher percentage of educated population. They found that gross domestic product is a basic condition for obtaining a good social performance and a high level of GDP also allows the population to have a longer life expectancy and to achieve a higher level of education (Cracolici et al. 2010, p. 354).

Regarding the evolution of sustainability during the period under study (2006–2012), we observed that some indicators change more over time, and as Lozano (2008) points out, economic, social and environmental dimensions can also be considered over time; in this sense the most outstanding one is public debt. Thus, in certain countries such as Chile, Israel, Malta, Kuwait, Saudi Arabia and Oman, the variation observed in the level of public debt is higher than in other countries around the world. Another variable with a negative evolution is employment, which decreases in some countries, especially in Europe, such as Spain, Portugal, France, Poland, and the Czech Republic with an ever greater number of unemployed (World Bank 2014).

As pointed out by Rinne et al. (2013), for governments, these sustainability indicators can serve to show a transparent and effective public sustainability situation in each country and can help in decision-making as to what policies, projects and social, environmental and economic strategies should be adopted.

Thus, from our point of view and considering some of the variables analysed, the following are important: education has to be a major concern for governments, because in cases in which income levels are low, a good educational system is needed if the environment is to be protected. Countries with a high educational and cultural level will be better able to understand environmental and social problems and initiate cooperation programs; implementation of national programs addressed to the most vulnerable people by promoting policies of gender equality, human rights and equal access to basic resources; social networks should be created with the idea of participating in the planning for climate change and in the process of implementing the adaptation strategies; better management of natural resources is also considered a necessity in the least developed countries.

Governments should also put greater emphasis on policies to improve the two aspects that showed the greatest changes over the period 2006–2012: public debt and employment, the latter having decreased especially in Europe, in countries such as Spain, Portugal, France, Poland, and the Czech Republic.

6 Conclusions

The main objective of this article was to determine whether different countries around the world showed differences in levels of the sustainability indicators collected in the SSI (SSI Van de Kerk and Manuel 2012), in relation to their income level. Two multivariate statistical techniques, X-STATIS and CO-STATIS were used. With these techniques it was possible to represent the 151 countries and the 21 sustainability indicators of the SSI, in four periods, 2006, 2008, 2010 and 2012.

Multivariate graphics enable us to visualize the differences between groups of countries, according to their income, for some of the indicators of SSI, through observation of the proximity or distance between markers of countries and variables. In addition, the X-STATIS technique takes into account the possible evolution of the countries, or the indicators, over time (trajectories) and the CO-STATIS technique can depict relationships and influences between types of indicators (environmental, social and economic).

Using the X-STATIS analysis we demonstrated that social and economic aspects are directly related to countries with high-incomes, while environmental indicators are closer to countries with a low-income level. Thus, as a first conclusion we can say that it is possible to define a multivariate gradient related to income level: countries with high incomes, or upper-middle incomes, place more importance on social or economic aspects and countries with low incomes, or lower-middle incomes, give more importance to environmental issues. Examples of this cluster of countries with high incomes are Chile, Israel, Malta, Kuwait, Saudi Arabia and Oman, with high levels of public debt, and Spain, Portugal, France, Poland or Czech Republic for their level of unemployment.

Therefore, supported by our results we find that sustainability is not equally important in all countries in the study.

Once the importance of economic indicators in the ranking of countries was demonstrated according to a gradient of sustainability, we analysed the relationship between economic and social indicators and between economic and environmental indicators, applying Co-Statis. From the analysis of these two series of paired tables, we found that in France, Spain and Czech Republic, social and environmental indicators are both related to economic indicators, while in Poland there is no significant relationship between social and economic indicators, but economic and environmental indicators are related. In Malta and Kuwait there is a relationship between social and economic variables, but not between economic and environmental ones. We have obtained similar results in countries with upper-middle, lower-middle and low levels of income.

Appendix

See Table 4.

Table 4 Countries in the sample

Albania	Ecuador	Latvia	Romania
Algeria	Egypt	Lebanon	Russia
Angola	El Salvador	Liberia	Rwanda
Argentina	Estonia	Libya	Saudi Arabia
Armenia	Ethiopia	Lithuania	Senegal

Table 4 continued

Australia	Finland	Luxembourg	Serbia
Austria	France	Macedonia	Sierra Leone
Azerbaijan	Gabon	Madagascar	Slovak Republic
Bangladesh	Gambia	Malawi	Slovenia
Belarus	Georgia	Malaysia	South Africa
Belgium	Germany	Mali	Spain
Benin	Ghana	Malta	Sri Lanka
Bhutan	Greece	Mauritania	Sudan
Bolivia	Guatemala	Mexico	Sweden
Bosnia-Herzegovina	Guinea	Moldova	Switzerland
Botswana	Guinea-Bissau	Mongolia	Syria
Brazil	Guyana	Montenegro	Taiwan
Bulgaria	Haiti	Morocco	Tajikistan
Burkina Faso	Honduras	Mozambique	Tanzania
Burundi	Hungary	Myanmar	Thailand
Cambodia	Iceland	Namibia	Togo
Cameroon	India	Nepal	Trinidad and Tobago
Canada	Indonesia	Netherlands	Tunisia
Central African Republic	Iran	New Zealand	Turkey
Chad	Iraq	Nicaragua	Turkmenistan
Chile	Ireland	Niger	Uganda
China	Israel	Nigeria	Ukraine
Colombia	Italy	Norway	United Arab Emirates
Congo	Jamaica	Oman	United Kingdom
Congo. Dem. Rep.	Japan	Pakistan	United States
Costa Rica	Jordan	Panama	Uruguay
Cote d'Ivoire	Kazakhstan	Papua New Guinea	Uzbekistan
Croatia	Kenya	Paraguay	Venezuela
Cuba	Korea, North	Peru	Vietnam
Cyprus	Korea, South	Philippines	Yemen
Czech Republic	Kuwait	Poland	Zambia
Denmark	Kyrgyz Republic	Portugal	Zimbabwe
Dominican Republic	Laos	Qatar	

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Artículo 5

Proposal of an algorithm and mathematical modelling to assist policy and decision-makers in the pathway of constructing a sustainable society

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Proposal of an algorithm and mathematical modelling to assist policy and decision-makers in the pathway of constructing a sustainable society

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Abstract

For governments, the sustainability indicators provide a transparent and effective way of showing the general public the situation of a country with regard to sustainability and help in making decisions concerning the social, environmental and economic policies, projects and strategies to be adopted. Because the sustainability has become in one of the main topics nowadays, there are institutions that must already face the challenge of the necessity of building new mathematical models and of developing algorithms that confirm the progress towards a more sustainable society. Due to the absence of this kind of methods, we have thought appropriate the creation of a new proposal, a new system without which studying the sustainability would not have been possible, because it helps to explain the effects of each country, of each sustainability indicator, or even of each year, and it describes the behaviors thereof in an objective way. The Co-Tucker3 method can be used for describing the interactions between the different rows, columns and repetitions of a couple of sequences of matrices, by turning them into an amount of data with less elements, by reducing the dimensionality. Plots will be used to graphically visualize differences or similarities in the data, like if there are groups of countries that behave differently in some indicators during some years, if there are relations between the indicators, or if a trend can be found over the time.

Keywords: Decision analysis, Mathematical modelling, Algorithm, Co-Tucker3, Co-Inertia Analysis, Sustainability indicators, Countries worldwide

1. Introduction

In recent years there has been an increase in the needs of information concerning the sustainability of different countries by public institutions, such as the different levels of government. It has become increasingly important to be able to measure sustainability on a global level since the Brundtland Report (WCED, 1987) issued its famous definition of sustainable development as "development that satisfies present needs without compromising the capability of future generations to satisfy their own needs". For this, a good system of indicators can help authorities to make knowledgeable decisions, providing them with objective information that will allow attain the goals proposed, as well as their subsequent assessment.

Indicators must have two indispensable functions: they have to measuring progress towards policy objectives; and they have to serve to evaluate the effectiveness of public policies. Hence, they have to become a commonly used approach to meet the crucial need for assess the concept of sustainability at the national level. With regard to the indicators, McCool and Stankey (2004) argue the functions the indicators serve, and they suggest appropriate roles of science, policy and the public in the indicator selection process.

More specifically, sustainability indicators can help decision-makers to highlight the most important aspects, to see what still needs to be done, to enable the identification of effective aspects of sustainability. For governments, these sustainability indicators provide a transparent and effective means of showing the general public the situation of a country with regard to sustainability and help in making decisions concerning the social, environmental and economic policies, projects and strategies to be adopted. At the business level, knowledge of how different countries perform according to sustainability indicators will help companies to see whether they can gain some kind of competitive advantage and carry out business innovations. An investigation into the implications of various policies and how business can integrate sustainability is needed, since many changes can exist whether no practice is prepared (Linton, Klassen and Jayaraman, 2007). These indicators can be proposed for influencing the marketplace on various levels of sustainable decision-making and policy formation, such as business level (Grundey, 2008; Tang and Zhou, 2012). Therefore, it could be useful to run through all levels in detail.

Any social decision problem is characterized by conflict between competing values and interests and the different groups and communities that represent them. In the framework of sustainability policies, the need to deal with conflicts among various social demanders is unavoidable. Any decision affects the welfare of individuals, regions or groups in different ways; consequently, public support for any decision very much depends on the results of such a decision. In empirical evaluations of projects, decisions based in multivariate analysis seem to be an appropriate tool, since they make possible to take into account a wide range of assessment criteria. This is the main reason why it is desirable to combine matrices with real data with the proposed model based in a multivariate analysis.

Mathematical models can help to provide more systematic information and they are very useful in helping at finding potential social compromises by making a complex situation more transparent to decision-makers and ordinary people. We can use a scientific approach to aid decision-makers in complex systems, through applications, methodologies and analytical techniques like simulation, optimization and probability and statistics (Altay and Walter, 2006).

This is the main objective of the procedure of the model presented in this research: propose an algorithm and mathematical modelling to assist policy and decision-makers in the pathway of constructing a sustainable society, for that, they will use a series of plots with which they will be able to graphically visualize the differences and similarities among the elements of each one of the three dimensions, in our case countries, two types of sustainability indicators and years of study, moreover, they will be able to deduce if differences or similarities also exist among the countries according to if they behave in a similar or opposite way in each one of the types of sustainability indicators, and the same can be deduced for the years of the studied period of time. For this aim, we have developed a mathematical model with which to reduce the dimensionality of our amount of data, that is, to turn the three sequences of matrices with as many rows as countries, as many columns as sustainability indicators and as many matrices in the sequences as years of study, into sequences of matrices with less elements. An algorithm has been also developed that allows that reduction of the dimensionality and the necessary analysis so that the results that are obtained will be the above-mentioned plots.

A real data application is presented too. A real world example is used to emphasize the main operational consequences of the proposed mathematical procedure. The results presented here proceed from the data of sustainability for countries worldwide over four biennia (2006-2012 period).

The remainder of this paper is organized as follows. We describe general aspects of sustainability in section 2. The reduction of dimensionality necessary for formulate the mathematical model is presented in section 3 and a new method for simultaneously analyzing a sequence of pairs of tables is introduced in section 4. Section 5 presents the results of the empirical analysis and an applicability to real sustainability data. The conclusions and the differences and similarities with other existing methods and data are discussed in section 6.

2. Sustainability – an overview: general aspects

According to Van de Kerk and Manuel (2008), a sustainable society is one in which each human being is capable of developing in a healthy manner and obtaining a proper education, lives in a clean environment, lives in a safe and well-balanced society, uses non-renewable resources responsibly so that future generations will not be left without them and contributes to a sustainable world. Espinosa, Harnden and Walker (2008, p. 637) consider a "combination of social needs, ecological limits and quality of life, treating the natural and human social systems as co-evolving in a recurrent dance of interaction, each dependent on outputs from the other and providing inputs to it".

Recently, composite indicators have been used in order to provide information and assess the economic, social and environmental impact of development. Indicators for assessing and benchmarking sustainability impacts of different systems have been developed (Ness, Urbel-Piirsalu, Anderberg and Olsson, 2007).

With a view to studying sustainability internationally, it has been found that the good indicators, that is, those that provide a complete picture of all the relevant aspects of sustainability in a transparent and easily understandable way, must fulfil the following criteria:

- They must be relevant for one of the issues relating to the above-mentioned definition of sustainability.
- They must cover the complete field of sustainability according to the definition used.
- They must be independent of each other and not overlap.
- They must be measurable.
- They must be easy to access, for the general public as well. This in turn means that the number of indicators should be limited.
- The data used to build the indicators must be publically available.
- The data must be available for all countries, at least for all those except the smallest ones.
- The data must be reliable.
- The data must be recent and regularly updated.
- The complete set of indicators should provide a good picture of the current situation of sustainability and point out the differences between the present situation and the optimum situation of complete sustainability.
- They must permit comparisons among countries.

We decided to use the index created by Van de Kerk and Manuel (2012), since their Sustainable Society Index (SSI) was recently audited by the Joint Research Centre of the European Commission

(Saisana and Philippas, 2012), which found that it to be an integral and quantitative method for measuring and monitoring the health of human and environmental systems globally and a conceptually and statistically solid tool that can be broadly applied for the continuous evaluation of these systems.

The decision to use the Sustainable Society Index instead of other indices was due to the following aspects: on the one hand it describes societal progress along all three dimensions: human, environmental and economic. Thus, the SSI comprises three wellbeing dimensions (human, environmental and economic). Furthermore, the SSI is based on a definition of sustainability that was provided by the Brundtland Commission (WCED 1987), to make explicitly clear that sustainability includes human wellbeing as well as environmental wellbeing. Furthermore, it has recently been audited by the Joint Research Centre of the European Commission, in particular its Institute for the Protection and Security of the Citizen, confirming that the SSI is well structured and guaranteeing a control process to ensure transparency and the credibility of Analysis of the Sustainable Society Index (Saisana and Philippas, 2012; Van de Kerk and Manuel, 2012).

The Sustainable Society Index (SSI) consists of 21 indicators grouped into three dimensions: Human Wellbeing, Environmental Wellbeing, and Economic Wellbeing. The different indicators comprising these dimensions are listed below (Van de Kerk and Manuel 2012).

• *Human Wellbeing:*

• Sufficient food, S.F. Number of undernourished people in percentage of total population.

• Sufficient to drink, S.D. Number of people as percentage of the total population, with sustainable access to an improved water source.

• Safe sanitation, S.S. Number of people in percentage of total population, with sustainable access to improved sanitation.

• Healthy life, H.L. Life expectancy at birth in number of healthy life years (HALE—Health Adjusted Life Expectancy).

• Clean air, C.A. Air pollution in its effects on humans.

• Clean water, C.W. Surface water quality.

• Education, Ed. Combined gross enrolment ratio for primary, secondary and tertiary schools.

• Gender equality, G.E. Gender Gap Index.

• Income distribution, I.D. Ratio of income of the richest 10 % to the poorest 10 % of the people in a country.

• Good governance, G.Go. The average of values of the six Governance Indicators of the World Bank.

• *Environmental Wellbeing*

• Air quality, A.Q. Air pollution in its effects on nature.

• Biodiversity, B. Size of protected areas (in percentage of land area).

• Renewable water resources, R.W.R. Annual water withdrawals (m^2 per capita) as percentage of renewable water resources.

• Consumption, C. Ecological Footprint minus Carbon Footprint.

• Renewable energy, R.E. Renewable energy as percentage of total energy consumption.

• Greenhouse gases, G.Ga. This indicator uses the common measure for greenhouse gas emissions (GHG): CO_2 emissions per capita per year.

• *Economic Wellbeing*

• Organic farming, O.F. Area for organic farming as percentage of total agricultural area of a country.

• Genuine savings, G.S. Genuine Savings (Adjusted Net Savings) as percentage of Gross National Income (GNI).

• Gross domestic product, GDP. GDP per capita, in Purchasing Power Parity, in current international dollars.

• Employment, Em. Unemployment as percentage of total labour force.

• Public debt, P.D. The level of public debt of a country as percentage of GDP.

Taking into account the indicators mentioned above, in this study we selected most countries in the world as our target population. This population was chosen in order to extend and generalize the results obtained in previous studies.

The sample we use comprises the 151 countries selected by Van de Kerk and Manuel (2008), and incorporates the advantages derived from considering different geographical contexts. Although the initial population comprised 194 countries, data on these indicators were only available for 151 countries. It was thus possible to calculate the indicators from the SSI for most large or medium-sized countries. We therefore consider in this research the 151 countries around the world (see Appendix A); the 21 numerical characteristics are the scores obtained by the countries selected concerning the policy categories proposed in the Analysis of the Sustainable Society Index Worldwide – SSI – in the last years (period 2006-2012). Hence, in this paper, the data consist of the SSI scores for each country, that is, a X_151x21x4 matrix.

Although the SSI is not the only way to measure sustainability, it is considered to be a statistically solid conceptual tool that is amply applicable for the continuous assessment of human and environmental systems and a key point of reference with which to compare future progress and inform global society. The SSI can be used to simulate the consequences of a series of potential actions, making it a powerful tool for informing decisions related to achieving human and environmental growth without compromising environmental well-being.

3. Reduction of the dimensionality

Before we present the mathematical model for applying to the sustainability data, we should define mathematically what we mean with reduction of the dimensionality, because this is one of the previous steps in order to develop the algorithm that will allow us to obtain the formulation necessary for graphically representing the data under study.

Given a matrix X(nxp), a symmetrical metric $D_p = (d_{ij})_{pxp}$ in the space of dimension p, and a diagonal matrix D_n (nxn) with the weights for the rows of X, that is, n coefficients called ω whose sum is 1, we want find a subspace V(pxr) whose dimension is r, or what is the same, an orthonormal base, so that the differences between the rows of X and their orthogonal projections over V are minimum, that is, we want to find a matrix V and then a matrix $A=XD_pV$ with only r columns and whose n rows are the orthogonal projections of X's, that approximates the best possible to X, that is what is called to reduce the dimensionality of X.

We understand for difference between a row of X and its projection over V, the square of the norm of every row vector of $X-XD_pVV^t$ (because the projection of X over V is XD_pVV^t), that is, the elements in the diagonal of $(X-XD_pVV^t)(X-XD_pVV^t)^t$.

We state our problem using the method of Lagrange multipliers: we want to minimize the weighted sum of the differences between the rows of X and their projections over V with the constraint

$$F\left(v_{ij},\lambda_{ef}\right)=\sum_{a=1}^{n}\omega_a\left[\left(X-XD_pVV^t\right)\left(X-XD_pVV^t\right)^t\right]_{aa}-\sum_{e=1}^{r}\sum_{f=1}^{r}\lambda_{ef}\left(\sum_{g=1}^{p}\sum_{h=1}^{p}v_{ge}d_{eh}v_{hf}-\delta_{ef}\right)$$

that V is an orthonormal base.

Where δ_{ef} is the Kronecker delta, and Λ is the matrix with the Lagrange multipliers, that, because of the simplicity, from now on we suppose diagonal.

Expanding the term between brackets and the first summation we obtain that the minimizing problem turns into the next maximizing problem:

$$F\left(v_{ij},\lambda_{ef}\right)=Tr\left[D_nXD_pVV^tD_pX^t\right]-\sum_{e=1}^{r}\sum_{f=1}^{r}\lambda_{ef}\left(\sum_{g=1}^{p}\sum_{h=1}^{p}v_{ge}d_{eh}v_{hf}-\delta_{ef}\right)$$

Where Tr is the Trace operator of a matrix. That equation holds because there are terms in the sum that do not depend on either v_{ij} or λ_{ef} and they can be removed, and because all the other terms have negative sign and they can be turned into a positive sign (that is why we turn the minimum problem into a maximum problem).

Expanding the first summation to terms with factors x_{ij}, d_{ij}, v_{ij}, deriving F respect v_{ij} and λ_{ef}, matching these derivatives with 0, and regrouping again the sums in a matrix form we obtain the following system of equations where V and Λ are unknown:

$$\begin{cases} X^t D_n X D_p V = V\Lambda \\ V^t D_p V = Id_r \end{cases}$$

If r=1, the first equation means that v_1 is an eigenvector of $X^tD_nXD_p$ with λ_{11} its eigenvalue, and the second one means that v_1 is normalized according to the metric D_p. But, what eigenvector do we choose between all of them with different corresponding eigenvalues?

We put in F that v_1 is an eigenvector with λ_{11} its eigenvalue and we see for what is the highest value:

$$Tr\left[D_n X D_p V V^t D_p X^t\right] = Tr\left[\underbrace{X^t D_n X D_p V}_{V\Lambda} V^t D_p\right] = Tr\left[V\Lambda V^t D_p\right] = Tr\left[\Lambda \underbrace{V^t D_p V}_{Id_r}\right] = Tr\left[\Lambda\right] = \lambda_{11}$$

Then, the maximum is reached if v_1 is the eigenvector of $X^tD_nXD_p$ normalized corresponding to the highest eigenvalue.

By recurrence, if r>1 we obtain $V=(v_1|...|v_r)$ where v_1, ..., v_r are the eigenvectors of $X^tD_nXD_p$ normalized according to D_p whose corresponding eigenvalues are sorted downward.

Because of the matrix D_n is symmetric and positive defined, it defines a symmetric metric and we can reduce the dimensionality of X along the rows using the same method as we did for the columns: $B=X^tD_nU$, where $U=(u_1|...|u_r)$ are the eigenvectors of $XD_pX^tD_n$ normalized using D_n whose corresponding eigenvalues are sorted downward.

If we want to reduce the dimensionality of X along the rows and along the columns too, then we should perform the eigendecompositions of two matrices:

$$XD_p X^t D_n = U\Lambda_{n\times n}U^{-1}$$
$$X^t D_n X D_p = V\Lambda_{p\times p}V^{-1}$$

Where U(nxn) and V(pxp), and then we calculate

$$A = XD_p V$$
$$B = X^t D_n U$$

and retain r columns from A and B.

But we can prove that U and V can be obtained from other matrices obtained after performing the singular value decomposition of only one matrix, the matrix

$$\overline{X} = \overline{U}\,\Lambda^{1/2}_{n\times p}\,\overline{V}^t$$

where $U^tU=Id_n$ and $V^tV=Id_p$
if we define

$$\overline{X} = D_n^{1/2} X D_p^{1/2}$$
$$U = D_n^{-1/2}\overline{U}$$
$$V = D_p^{-1/2}\overline{V}$$
$$\Lambda^{1/2} = \overline{\Lambda}^{1/2}$$

In fact:

$$U^t D_n U = U^t D_n^{1/2} D_n^{1/2} U = \overline{U}^t\,\overline{U} = Id_n$$
$$V^t D_p V = V^t D_p^{1/2} D_p^{1/2} V = \overline{V}^t\,\overline{V} = Id_p$$

moreover

$$XD_p X^t D_n = D_n^{-1/2}\,\overline{X}\,D_p^{-1/2} D_p D_p^{-1/2}\,\overline{X}^t D_n^{-1/2} D_n = D_n^{-1/2}\,\overline{U}\,\overline{\Lambda}^{1/2}\,\overline{V}^t\,\overline{V}\,\overline{\Lambda}^{1/2}\,\overline{U}^t D_n^{-1/2} D_n = U\,\Lambda\,U^t D_n = U\,\Lambda\,U^{-1}$$
$$X^t D_n XD_p = D_p^{-1/2}\,\overline{X}^t D_n^{-1/2} D_n D_n^{-1/2}\,\overline{X}\,D_p^{-1/2} D_p = D_p^{-1/2}\,\overline{V}\,\overline{\Lambda}^{1/2}\,\overline{U}^t\,\overline{U}\,\overline{\Lambda}^{1/2}\,\overline{V}^t D_p^{-1/2} D_p = V\,\Lambda\,V^t D_p = V\,\Lambda\,V^{-1}$$

All this mathematical formulation leads us to a way to reduce the dimensionality of a data matrix, and we will look for the way of using this by adapting it to an algorithm that can be used in order to apply this reduction of the dimensionality of a sequence of data matrices, in our investigation sustainability society indicators.

Thus, with this mathematical model, we consider appropriate to establish a new method with its corresponding algorithm that allows us to give a way of interpreting more logically the sustainability data.

4. Co-Tucker3: A new method for simultaneously analyzing a sequence of pairs of tables

A new proposal is presented for analyzing two sequences of data matrices, the Co-Tucker3 method (Co-Inertia Analysis + Tucker3 method), so it means that, first of all, the Tucker3 is used and then the Co-Inertia Analysis.

It is based on the Tucker3 method and the Co-Inertia Analysis, and it has the benefits of them. One of the advantages of the application of an analysis with the Tucker3 method is that it solves the problem of describing not only the stable part of a data structure, but also the possibility of extract the latent structure, as well as the interactions between the three dimensions, the countries, the indicators and the years. However, it only takes into account only one amount of data, that is, one sequence of data matrices. So, when one pair of sequences of data matrices must be studied, the Tucker3 method can be combined with the Co-Inertia Analysis. The first step of the Co-Tucker3 is to perform two simple Tucker3 analysis: one for each sequence of data matrices. And the second step is three simple Co-Inertia Analysis for the retained components for each one of the dimensions from both sequences of matrices, (see Figure 1). This means that the number of retained components (P, Q and R) for each dimension has to be the same for both data sequences, but the number of rows, columns and repetitions of both sequences of matrices can be different.

The objective of the Co-Tucker3 method is, firstly, to decompose both sequences of data matrices into orthogonal matrices and two data cubes (the core arrays) with dimension PxQxR simpler than the original $X_{I_1 \times J_1 \times K_1}$ and $Y_{I_2 \times J_2 \times K_2}$; and then, to find out the relations between these reduced rows, columns and repetitions to a smaller number of components.

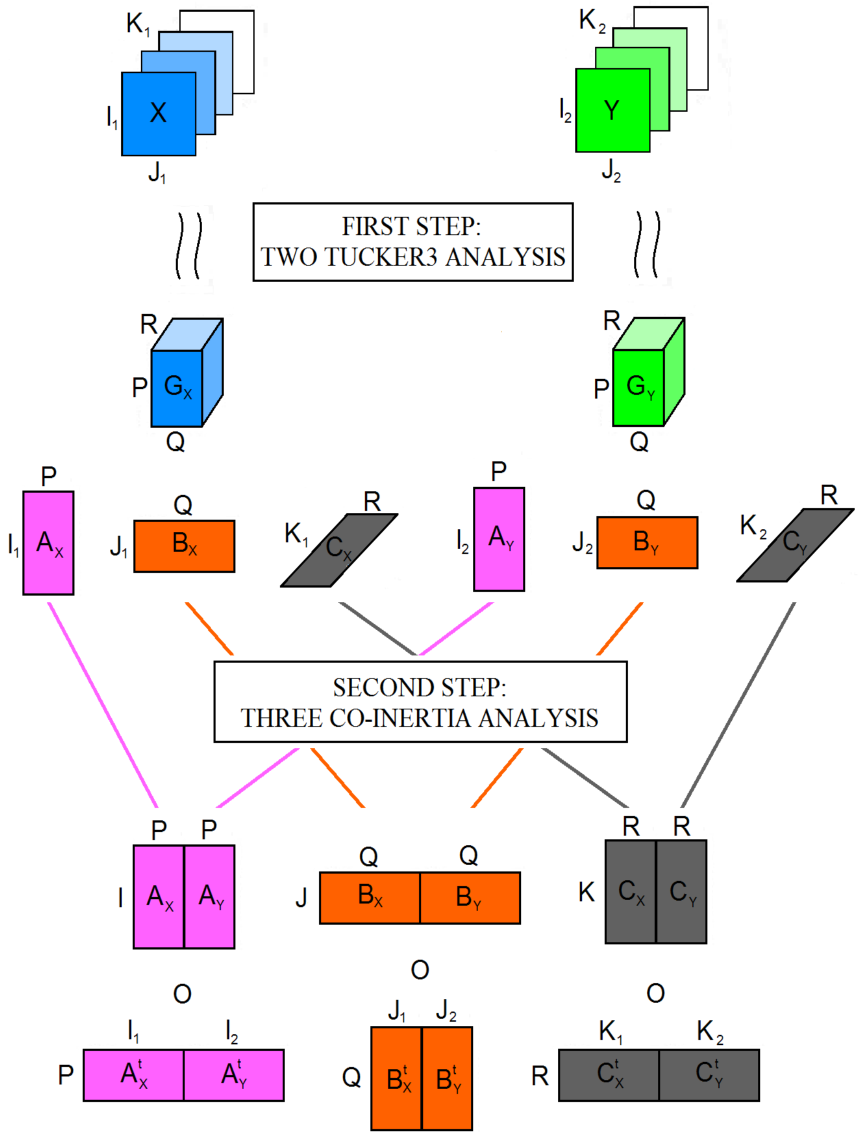


Figure 1: Co-Tucker3 analysis scheme

4.1. First step: two Tucker3 analysis

With this proposal, first of all, each one of the sequences of data matrices $X_{I_1 \times J_1 \times K_1}$ and $Y_{I_2 \times J_2 \times K_2}$ will be analyzed performing the Tucker3 method. Its objective is to reduce the dimensionality of the problem, $I_1xJ_1xK_1$ and $I_2xJ_2xK_2$, in order to summarize the information, building a simplified model for each one, PxQxR, in order to facilitate the data description. Moreover, plots that simultaneously show the three dimensions can be very useful for this.

This first part of the Co-Tucker3 method allows to answer questions like: Which groups of individuals (first dimension of both sequences of matrices) behave in a different way in which variables of each one of the sequences of matrices (second dimension of each sequence of matrices) during which repetitions (third dimension of both sequences of matrices)?, what are the relations between the variables individually within each sequence of matrices?, what trend can be found out over the time?, are there different types of individuals?, and more complex questions, like if the relations between the variables individually within each sequence of matrices vary over the time, or if the structure of the variables changes over the time in a different way for different groups of individuals (the countries in our research).

From the algebraic point of view, the objective of the two Tucker3 methods (one for each sequence of matrices) is to find one decomposition of the sequences of data matrices X and Y into orthogonal matrices and other data cubes: three pairs of orthogonal matrices $\mathbf{A}_X(I_1xP)$ and $\mathbf{A}_Y(I_2xP)$, $\mathbf{B}_X(J_1xQ)$ and $\mathbf{B}_Y(J_2xQ)$, and $\mathbf{C}_X(K_1xR)$ and $\mathbf{C}_Y(K_2xR)$, and two other data cubes $\mathbf{G}_X(PxQxR)$ and $\mathbf{G}_Y(PxQxR)$, the core arrays, with PxQxR simpler than $I_1xJ_1xK_1$ and $I_2xJ_2xK_2$, such as the tensor product of $\mathbf{G}_X$, $\mathbf{A}_X^t$, $\mathbf{B}_X^t$ and $\mathbf{C}_X^t$, and $\mathbf{G}_Y$, $\mathbf{A}_Y^t$, $\mathbf{B}_Y^t$ and $\mathbf{C}_Y^t$ are the best approximations of X and Y (see Figure 2).

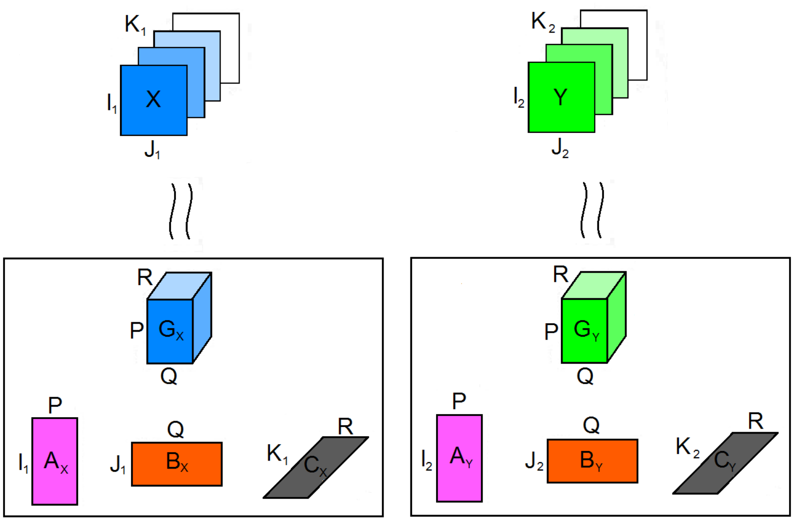


Figure 2: Scheme of the Tucker3 Analysis from the Co-Tucker3

The process for finding the best approximation of X and Y with PxQxR components is the same iterative algorithm as for the Tucker3 method (Krooneberg and De Leeuw, 1980). The way of choosing which model PxQxR will be considered is to calculate the decomposition given by the algorithm for every combination PxQxR with $P \leq min(I_1, I_2)$, $Q \leq min(J_1, J_2)$ and $R \leq min(K_1, K_2)$; and we must take into account the maximum product rule too.

In this algorithm, the term $\mathbf{A}_{X_n}$ represents the matrix for the first dimension of the first sequence of matrices obtained after the n-th iteration, in particular, $\mathbf{A}_{X_1}$ is the matrix calculated in the initial iteration; $\mathbf{B}_{X_n}$,
$\mathbf{C}_{X_n}$, $\mathbf{G}_{X_n}$ and $\mathbf{A}_{Y_n}$, $\mathbf{B}_{Y_n}$, $\mathbf{C}_{Y_n}$, $\mathbf{G}_{Y_n}$ are similarly defined:

1. $\mathbf{A}_{X_1}$ is defined as the first P left-singular vectors of $X_{(1)}$. Similarly, $\mathbf{B}_{X_1}$ and $\mathbf{C}_{X_1}$ are defined as the first Q and R left-singular vectors of $X_{(2)}$ and $X_{(3)}$ (the same for $\mathbf{A}_{Y_1}$, $\mathbf{B}_{Y_1}$ and $\mathbf{C}_{Y_1}$).
2. $\mathbf{G}_{X_1}$ is calculated as the tensor product of X, $\mathbf{A}_{X_1}$, $\mathbf{B}_{X_1}$ and $\mathbf{C}_{X_n}$ (the same for $\mathbf{G}_{Y_1}$):

   $$\left[\mathbf{G}_{X_1}\right]_{pqr} = \left[\left(\left(X \times_1 \mathbf{A}_{X_1}\right) \times_2 \mathbf{B}_{X_1}\right) \times_3 \mathbf{C}_{X_1}\right]_{pqr} = \sum_{i=1}^{I_1} \sum_{j=1}^{J_1} \sum_{k=1}^{K_1} \left[\mathbf{A}_{X_1}\right]_{ip} \left[\mathbf{B}_{X_1}\right]_{jq} \left[\mathbf{C}_{X_1}\right]_{kr} x_{ijk} .$$

3. Iteration step, n=1,...:
   a) The optimum matrix $\mathbf{A}_{X_{n+1}}$ is calculated, fixing $\mathbf{B}_{X_n}$ and $\mathbf{C}_{X_n}$ (the same for $\mathbf{A}_{Y_{n+1}}$). We call optimum matrix in this case to the first P left-singular vectors of $\left(\left(X \times_2 \mathbf{B}_{X_n}\right) \times_3 \mathbf{C}_{X_n}\right)_{(1)}$.
   b) The optimum matrix $\mathbf{B}_{X_{n+1}}$ is calculated, fixing $\mathbf{A}_{X_{n+1}}$ and $\mathbf{C}_{X_n}$: the first Q left-singular vectors of $\left(\left(X \times_1 \mathbf{A}_{X_{n+1}}\right) \times_3 \mathbf{C}_{X_n}\right)_{(2)}$ (the same for $\mathbf{B}_{Y_{n+1}}$).
   c) The optimum matrix $\mathbf{C}_{X_{n+1}}$ is calculated, fixing $\mathbf{A}_{X_{n+1}}$ and $\mathbf{B}_{X_{n+1}}$: the first R left-singular vectors of $\left(\left(X \times_1 \mathbf{A}_{X_{n+1}}\right) \times_2 \mathbf{B}_{X_{n+1}}\right)_{(3)}$ (the same for $\mathbf{C}_{Y_{n+1}}$).
   d) The core array $\mathbf{G}_{X_{n+1}}$ is calculated as the tensor product of X, $\mathbf{A}_{X_{n+1}}$, $\mathbf{B}_{X_{n+1}}$ and $\mathbf{C}_{X_{n+1}}$ (the same for $\mathbf{G}_{Y_{n+1}}$).
   e) This step is stopped when the differences between $\mathbf{A}_{X_{n+1}}$, $\mathbf{B}_{X_{n+1}}$, $\mathbf{C}_{X_{n+1}}$, $\mathbf{G}_{X_{n+1}}$ and $\mathbf{A}_{X_n}$, $\mathbf{B}_{X_n}$, $\mathbf{C}_{X_n}$, $\mathbf{G}_{X_n}$ are lower than an initial established value (the same for the corresponding matrices of Y).
4. Then, the matrices $\mathbf{A}_X$, $\mathbf{B}_X$ and $\mathbf{C}_X$ and the core array $\mathbf{G}_X$ are defined as the obtained ones after the n-th iteration: $\mathbf{A}_X = \mathbf{A}_{X_{n+1}}$, $\mathbf{B}_X = \mathbf{B}_{X_{n+1}}$, $\mathbf{C}_X = \mathbf{C}_{X_{n+1}}$ and $\mathbf{G}_X = \mathbf{G}_{X_{n+1}}$ (the same for $\mathbf{A}_Y$, $\mathbf{B}_Y$, $\mathbf{C}_Y$ and $\mathbf{G}_Y$).

The way of choosing which model PxQxR will be considered is to calculate the decomposition above for every combination PxQxR with $P \leq min(I_{1,} I_2)$, $Q \leq min(J_{1,} J_2)$ and $R \leq min(K_{1,} K_2)$; but, moreover, we must take into account the maximum product rule, that says that the cases where the number of elements of one dimension is higher than the product of the other two, P>QR, Q>PR or R>PQ, can be ignored, because in these cases the model is equivalent to another more reduced:

If P>QR, the model PxQxR is equivalent to the model QRxQxR

If Q>PR, the model PxQxR is equivalent to the model PxPRxR

If R>PQ, the model PxQxR is equivalent to the model PxQxPQ.

When this has been done, it is studied which is the simplest among the most stable ones and those that reach an explained variation high enough, such as the Tucker3, where the explained variation of a model is the combination of the obtained ones for each one of the two sequences of matrices. Therefore, if the explained variation for one model given according to the first sequence of matrices is $\frac{x_1}{x_2}$ and the explained variation for the same model according to the second sequence of matrices is $\frac{y_1}{y_2}$ we call combined explained variation for that model according to both sequences of matrices to

$$\frac{x_1+y_1}{x_2+y_2}.$$

For every model, the sum of the number of components is calculated, S=P+Q+R, and for every value of S we choose that model that has a lower value of the residual sum of squares, or equivalently, a higher value of the explained variation. Therefore, we have a list of models, one for each value of S. Then, the difference ratio between the residual sum of squares and S is calculated for each one of the models above in increasing order of S, and we will pick those models whose difference ratio is similar to the next one, that is, the most stable models. Finally, the model that will be chosen for the analysis will be that among the most stable ones that has a lower value of S, that is, the simplest among the most stable ones.

Once we choose the model PxQxR, after the execution of the algorithm for that model for each one of the two sequences of matrices, we will obtain the matrices $\mathbf{A}_X$(I1xP), $\mathbf{B}_X$(J1xQ), $\mathbf{C}_X$(K1xR) and $\mathbf{A}_Y$(I2xP), $\mathbf{B}_Y$(J2xQ), $\mathbf{C}_Y$(K2xR), and the core arrays $\mathbf{G}_X$ and $\mathbf{G}_Y$, both of them of order PxQxR.

The core arrays can be interpreted as the strength of the relations between the components of the different dimensions of each one of the sequences of matrices, as well as the weight of the combinations of the components, or as a measure of the interactions, and the square of every element as the explained variation.

But the final interpretation of the individuals, the variables of both sequences of matrices and the repetitions for one combination of components, pxqxr, not only depends on if the element of $\mathbf{G}_X$ or $\mathbf{G}_Y$ has a high value, but also on the combination of the signs of the four factors of the term of the tensor product. For example, if g_{Xpqr} has a positive sign, the i-th individual has a positive sign in the p-th component, the j-th variable of the first sequence of matrices has a positive sign in the q-th component and the k-th repetition has a positive sign in the r-th component, the interaction between the i-th individual, the j-th variable of the first sequence of matrices and the k-th repetition will be positive: during the k-th repetition, the i-th individual has a high value in the j-th variable of the first sequence of matrices. If one element of some of the core arrays is low, the interpretation of such combination is not needed.

Most of the results of the first step of the Co-Tucker3 are focused in the interpretations of the interactions within each one of the sequences of matrices separately, according to the signs of the four factors, as we have just explained; however, these results can be difficult of interpreting. The pairs of plots for the three dimensions of both sequences of matrices, the joint biplots, are an easy way of visually understanding these interactions.

They are interpreted as the classic biplots (Gabriel, 1971), except that each biplot is built for two combinations of components, the horizontal one and the vertical one.

At the moment, we could study the obtained results such as they were two individual Tucker3. The two separate Tucker3 can be interpreted, such as we have explained before, using the core arrays, that they are defined as the strength of the relations between the components of the three dimensions, in this case, for those of each one of the separate sequences of data matrices, before we study the relations between the variables of one sequence of matrices and of the other one. Moreover, the interactions between the three dimensions of each one of the separate sequences of matrices can be interpreted through the different joint plots split into three, and taking into account the product of the four, positive or negative, signs of the individuals, variables, repetitions and the elements of the corresponding core array.

Therefore, every pair of plots has three sub-plots, one for each dimension, with two components in all of them, and the corresponding columns of the matrices $\mathbf{A}_X$, $\mathbf{B}_X$, $\mathbf{C}_X$ and $\mathbf{A}_Y$, $\mathbf{B}_Y$, $\mathbf{C}_Y$ from the decompositions of X and Y are represented in them. If we jointly represent the three dimensions in only one plot, visual interpretations of the relations of each separate sequence of matrices can be done.

4.2. Second step: three Co-Inertia Analysis

Then, the second step of the Co-Tucker3 method is to perform the three Co-Inertia Analysis for each pair of matrices for each one of the three dimensions: one Co-Inertia Analysis between $\mathbf{A}_X^t$ and

$\mathbf{A}_Y^t$, one between $\mathbf{B}_X^t$ and $\mathbf{B}_Y^t$ and another between $\mathbf{C}_X^t$ and $\mathbf{C}_Y^t$, because each pair has the same number of columns, P, Q and R respectively and they have to have the same rows for being able to perform the Co-Inertia Analysis. But in case of both sequences of matrices X and Y have the same rows (I_1=I_2), columns (J_1=J_2) or repetitions (K_1=K_2), it be taken advantage of this fact for directly performing the Co-Inertia Analysis between $\mathbf{A}_X$ and $\mathbf{A}_Y$, $\mathbf{B}_X$ and $\mathbf{B}_Y$ or $\mathbf{C}_X$ and $\mathbf{C}_Y$.

We will explain the two different cases, the first one, where we suppose that the two matrices of the same pair have the same rows (I_1=I_2), columns (J_1=J_2) or repetitions (K_1=K_2), and the second one, where the two matrices of the same pair have different rows, columns or repetitions. Without loss of generality, we will explain the Co-Inertia Analysis for the pair of matrices corresponding to the rows, obtained after the Tucker3 decompositions for each one of the sequences of matrices. For the other two dimensions the development is similar.

1. Case where X and Y have the same rows (see Figure 3):

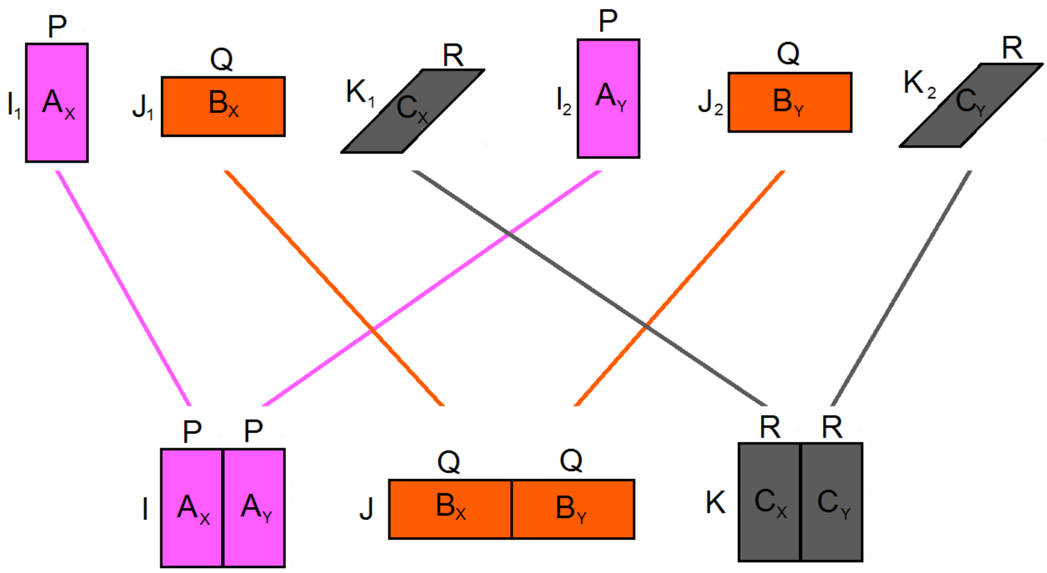


Figure 3: Scheme of the Co-Inertia Analysis from the Co-Tucker3 in case of X and Y have the same rows, columns or repetitions

Let $\mathbf{A}_X$ be the first table, with I=I_1 rows and P columns, and let $\mathbf{A}_Y$ be the second table, with the same I_2=I_1=I rows and P columns.

Let D_I be the diagonal matrix IxI with the weights for the rows:

$$D_I = diag(\omega_1, \dots, \omega_I)$$

and let D_P be a metric in the space of dimension P.

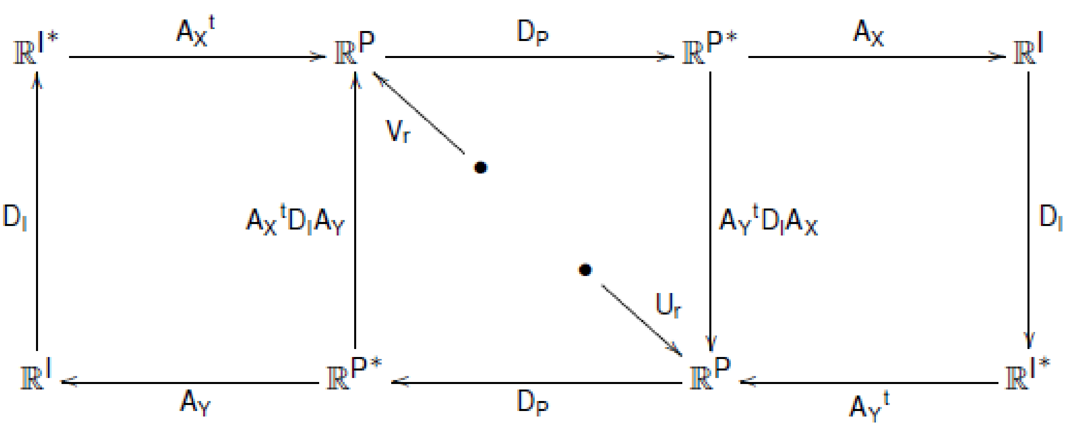
The Co-Inertia Analysis is the decomposition explained in the section of reduction of the dimensionality for $\mathbf{A_Y}^t D_I \mathbf{A_X}$. That is, the cross-diagram below:

$$
\begin{tikzcd}
\mathbb{R}^{I*} \arrow[r, "A_X^t"] & \mathbb{R}^{P} \arrow[r, "D_P"] & \mathbb{R}^{P*} \arrow[r, "A_X"] \arrow[d, "A_Y^t D_I A_X"] & \mathbb{R}^{I} \arrow[d, "D_I"] \\
\mathbb{R}^{I} \arrow[u, "D_I"] & \mathbb{R}^{P*} \arrow[l, "A_Y"] \arrow[u, "A_X^t D_I A_Y"'] & \mathbb{R}^{P} \arrow[l, "D_P"] & \mathbb{R}^{I*} \arrow[l, "A_Y^t"]
\end{tikzcd}
$$

If we are in case of D_I is the matrix with uniform weights for the rows and the metric D_P is euclidean, because of the columns of both tables are centered, then the total inertia of each table is simply the sum of variances. And the co-inertia between $\mathbf{A_X}$ and $\mathbf{A_Y}$ is, in this case, one sum of squares of covariances.

The Co-Inertia Analysis maximizes the covariance between the coordinates of the rows of both tables. The co-inertia is high when the values in both tables are simultaneously high (or when they vary inversely) and it is low when they vary independently or when they do not vary. This is the meaning of the co-structure between the two data tables.

Now, as a part of the Co-Tucker3, we can graphically represent the rows of both matrices $\mathbf{A_X}$ and $\mathbf{A_Y}$ in the subspace of dimension r obtained from the Co-Inertia Analysis, by calculating different coordinates:



the rows of the matrix $\mathbf{A}_X$, that is, the rows according to the sequence of matrices X have the coordinates

rows of X: $\mathbf{A}_X D_P V_r$ and the $\mathbf{A}_Y$'s, the same rows but according to the sequence of matrices Y rows of Y: $\mathbf{A}_Y D_P U_r$.

2. Case where X and Y have different rows (see Figure 4):

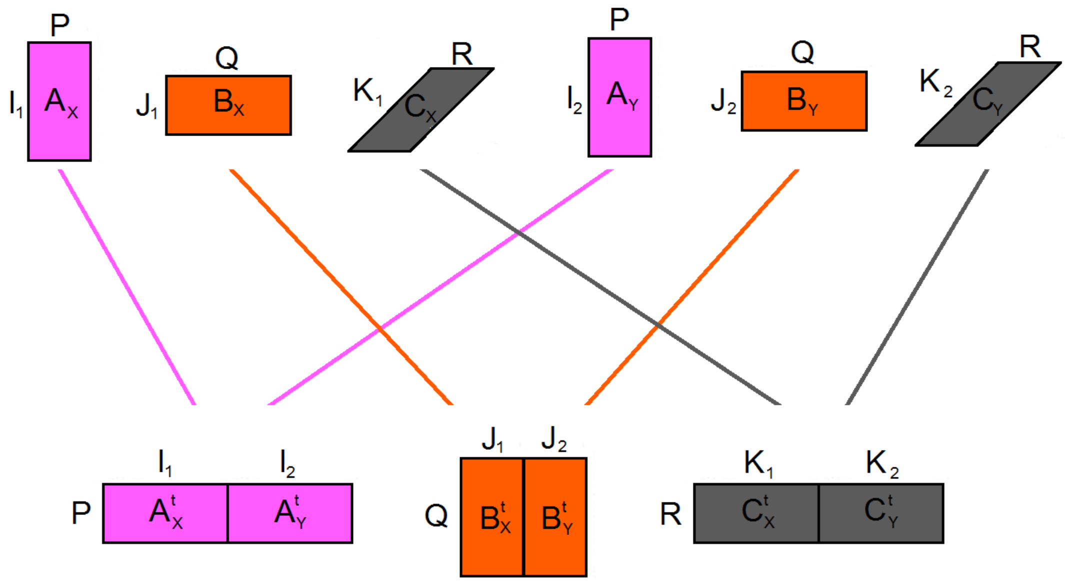


Figure 4: Scheme of the Co-Inertia Analysis from the Co-Tucker3 in case of X and Y have different rows, columns or repetitions

Let $\mathbf{A}_X^t$ be the first table, with P rows and I_1 columns (because of $\mathbf{A}_X$ has I_1 rows and P columns), and let $\mathbf{A}_Y^t$ be the second table, with the same P rows and I_2 columns, different to $\mathbf{A}_X^t$'s.

Because of, actually, the columns of $\mathbf{A}_X^t$ and $\mathbf{A}_Y^t$ represent the rows (of $\mathbf{A}_X$ and $\mathbf{A}_Y$), with the purpose of clearing the development, from now on we will call 'rows' to the I_1 columns of $\mathbf{A}_X^t$ and to the I_2 columns of $\mathbf{A}_Y^t$, and we will call 'components' to the P rows of $\mathbf{A}_X^t$ and $\mathbf{A}_Y^t$.

Let D_P be the diagonal matrix PxP with the weights for the 'components':

$D_P = diag(\omega_1, \ldots, \omega_P)$

and let D_{I_1} and D_{I_2} be two metrics in the spaces of dimension I_1 and I_2 respectively.

The Co-Inertia Analysis is the decomposition explained in the section reduction of the dimensionality for $\mathbf{A}_Y D_P \mathbf{A}_X^t$. That is, the cross-diagram below:

$$
\begin{tikzcd}
\mathbb{R}^{P*} \arrow[r, "A_X"] & \mathbb{R}^{I_1} \arrow[r, "D_{I_1}"] & \mathbb{R}^{I_1*} \arrow[r, "A_X^t"] \arrow[d, "A_Y D_P A_X^t"] & \mathbb{R}^{P} \arrow[d, "D_P"] \\
\mathbb{R}^{P} \arrow[u, "D_P"] & \mathbb{R}^{I_2*} \arrow[l, "A_Y^t"] \arrow[u, "A_X D_P A_Y^t"] & \mathbb{R}^{I_2} \arrow[l, "D_{I_2}"] & \mathbb{R}^{P*} \arrow[l, "A_Y"]
\end{tikzcd}
$$

If we are in case of D_P is the matrix with uniform weights for the 'components' and the metrics D_{I_1} and D_{I_2} are euclidean, because of the 'rows' of both tables are centered, then the total inertia of each table is simply the sum of variances.

And the co-inertia between $\mathbf{A}_X^t$ and $\mathbf{A}_Y^t$ is, in this case, one sum of squares of covariances.

Now, as a part of the Co-Tucker3, we can graphically represent the 'rows' of both matrices $\mathbf{A}_X^t$ and $\mathbf{A}_Y^t$ in the subspace of dimension r obtained from the Co-Inertia Analysis, by calculating different coordinates:

$$
\begin{tikzcd}
\mathbb{R}^{P*} \arrow[r, "A_X"] & \mathbb{R}^{I_1} \arrow[r, "D_{I_1}"] & \mathbb{R}^{I_1*} \arrow[r, "A_X^t"] \arrow[ddd, "A_Y D_P A_X^t"] & \mathbb{R}^{P} \arrow[ddd, "D_P"] \\
 & & \bullet \arrow[ul, "V_r"'] & \\
 & \bullet \arrow[dr, "U_r"] & & \\
\mathbb{R}^{P} \arrow[uuu, "D_P"] & \mathbb{R}^{I_2*} \arrow[l, "A_Y^t"] \arrow[uuu, "A_X D_P A_Y^t"] & \mathbb{R}^{I_2} \arrow[l, "D_{I_2}"] & \mathbb{R}^{P*} \arrow[l, "A_Y"]
\end{tikzcd}
$$

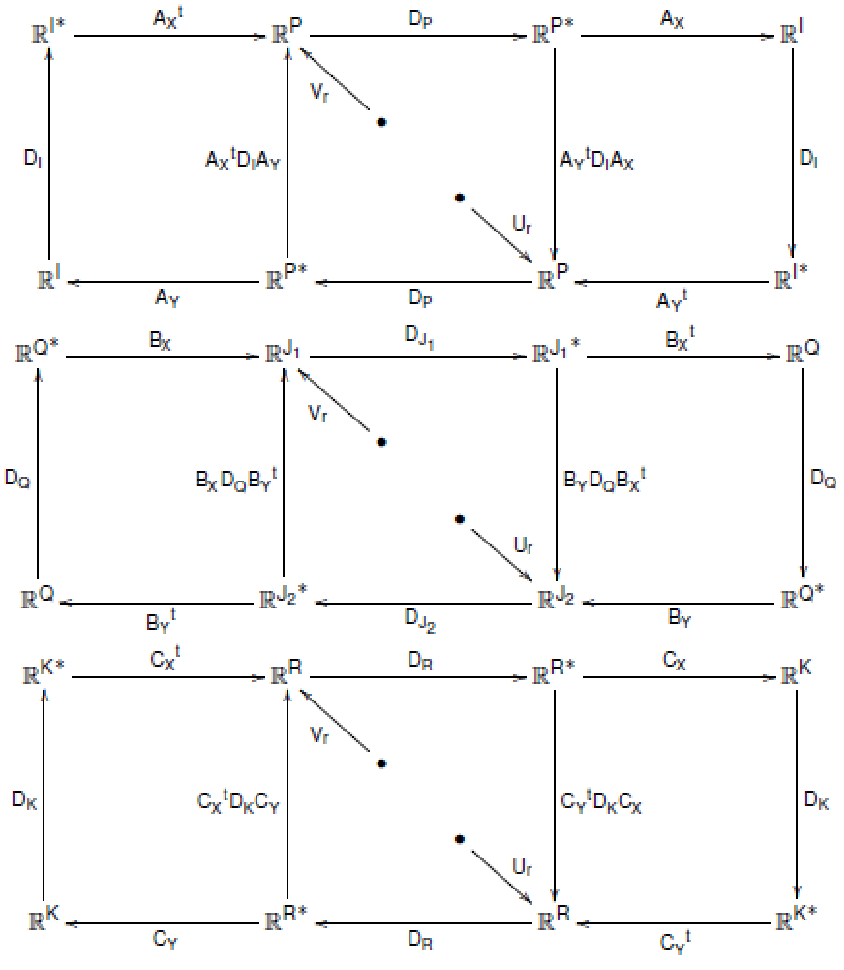
the 'rows' of the matrix $\mathbf{A}_X{}^t$, that is, the rows of the sequence of matrices X have the coordinates

rows of X: $\mathbf{A}_X D_P \mathbf{A}_Y{}^t D_{I_2} U_r$

and $\mathbf{A}_Y{}^t$'s, the rows of the sequence of matrices Y

rows of Y: $\mathbf{A}_Y D_P \mathbf{A}_X{}^t D_{I_1} V_r$.

Then, taking into account the two types of Co-Inertia Analysis that you can find, the second part of the Co-Tucker3 analysis is the analysis of three duality diagrams, one for the rows, another for the columns and the last one for the repetitions. Any combination of the two types is valid, one example could be:



That it corresponds to the case where the two sequences of data matrices X and Y have the same rows and repetitions, but different columns.

The complete algorithm of the Co-Tucker3 would be summarized in the next table (Table 1):

Table 1: Co-Tucker3 algorithm

X and Y are centered by columns and, if it is wanted, they are normalized by lateral slices.		
The following algorithm is performed for every combination PxQxR with $\mathrm{P} \le min(\mathrm{I}_{1,} \mathrm{I}_2)$, $\mathrm{Q} \le min(\mathrm{J}_{1,} \mathrm{J}_2)$ and $\mathrm{R} \le min(\mathrm{K}_{1,} \mathrm{K}_2)$, and the maximum product rule must be taken into account too.		
	$\mathbf{A}_{\mathrm{X}_1}$ is defined as the first P left-singular vectors of $\mathrm{X}_{(1)}$. Similarly, $\mathbf{B}_{\mathrm{X}_1}$ and $\mathbf{C}_{\mathrm{X}_1}$ are defined as the first Q and R left-singular vectors of $\mathrm{X}_{(2)}$ and $\mathrm{X}_{(3)}$ (the same for $\mathbf{A}_{\mathrm{Y}_1}$, $\mathbf{B}_{\mathrm{Y}_1}$ and $\mathbf{C}_{\mathrm{Y}_1}$).	
	$\mathbf{G}_{\mathrm{X}_1}$ is calculated as the tensor product of X, $\mathbf{A}_{\mathrm{X}_1}$, $\mathbf{B}_{\mathrm{X}_1}$ and $\mathbf{C}_{\mathrm{X}_n}$ (the same for $\mathbf{G}_{\mathrm{Y}_1}$): $$\left[\mathbf{G}_{\mathrm{X}_1}\right]_{pqr} = \left[\left(\left(\mathrm{X} \times_1 \mathbf{A}_{\mathrm{X}_1}\right) \times_2 \mathbf{B}_{\mathrm{X}_1}\right) \times_3 \mathbf{C}_{\mathrm{X}_1}\right]_{pqr} = \sum_{i=1}^{\mathrm{I}_1} \sum_{j=1}^{\mathrm{J}_1} \sum_{k=1}^{\mathrm{K}_1} \left[\mathbf{A}_{\mathrm{X}_1}\right]_{ip} \left[\mathbf{B}_{\mathrm{X}_1}\right]_{jq} \left[\mathbf{C}_{\mathrm{X}_1}\right]_{kr} x_{ijk} \ .$$	
	Iteration step, n=1,...:	
		The optimum matrix $\mathbf{A}_{\mathrm{X}_{n+1}}$ is calculated, fixing $\mathbf{B}_{\mathrm{X}_n}$ and $\mathbf{C}_{\mathrm{X}_n}$ (the same for $\mathbf{A}_{\mathrm{Y}_{n+1}}$). We call optimum matrix in this case to the first P left-singular vectors of $\left(\left(\mathrm{X} \times_2 \mathbf{B}_{\mathrm{X}_n}\right) \times_3 \mathbf{C}_{\mathrm{X}_n}\right)_{(1)}$.
		The optimum matrix $\mathbf{B}_{\mathrm{X}_{n+1}}$ is calculated, fixing $\mathbf{A}_{\mathrm{X}_{n+1}}$ and $\mathbf{C}_{\mathrm{X}_n}$: the first Q left-singular vectors of $\left(\left(\mathrm{X} \times_1 \mathbf{A}_{\mathrm{X}_{n+1}}\right) \times_3 \mathbf{C}_{\mathrm{X}_n}\right)_{(2)}$ (the same for $\mathbf{B}_{\mathrm{Y}_{n+1}}$).
		The optimum matrix $\mathbf{C}_{\mathrm{X}_{n+1}}$ is calculated, fixing $\mathbf{A}_{\mathrm{X}_{n+1}}$ and $\mathbf{B}_{\mathrm{X}_{n+1}}$: the first R left-singular vectors of $\left(\left(\mathrm{X} \times_1 \mathbf{A}_{\mathrm{X}_{n+1}}\right) \times_2 \mathbf{B}_{\mathrm{X}_{n+1}}\right)_{(3)}$ (the same for $\mathbf{C}_{\mathrm{Y}_{n+1}}$).
		The core array $\mathbf{G}_{\mathrm{X}_{n+1}}$ is calculated as the tensor product of X, $\mathbf{A}_{\mathrm{X}_{n+1}}$, $\mathbf{B}_{\mathrm{X}_{n+1}}$ and $\mathbf{C}_{\mathrm{X}_{n+1}}$ (the same for $\mathbf{G}_{\mathrm{Y}_{n+1}}$).
		This step is stopped when the differences between $\mathbf{A}_{\mathrm{X}_{n+1}}$, $\mathbf{B}_{\mathrm{X}_{n+1}}$, $\mathbf{C}_{\mathrm{X}_{n+1}}$, $\mathbf{G}_{\mathrm{X}_{n+1}}$ and $\mathbf{A}_{\mathrm{X}_n}$, $\mathbf{B}_{\mathrm{X}_n}$, $\mathbf{C}_{\mathrm{X}_n}$, $\mathbf{G}_{\mathrm{X}_n}$ are lower than a initial established value (the same for the corresponding matrices of Y).
	Then, the matrices $\mathbf{A}_\mathrm{X}$, $\mathbf{B}_\mathrm{X}$ and $\mathbf{C}_\mathrm{X}$ and the core array $\mathbf{G}_\mathrm{X}$ are defined as the obtained ones after the n-th iteration: $\mathbf{A}_\mathrm{X} = \mathbf{A}_{\mathrm{X}_{n+1}}$, $\mathbf{B}_\mathrm{X} = \mathbf{B}_{\mathrm{X}_{n+1}}$, $\mathbf{C}_\mathrm{X} = \mathbf{C}_{\mathrm{X}_{n+1}}$ and $\mathbf{G}_\mathrm{X} = \mathbf{G}_{\mathrm{X}_{n+1}}$ (the same for $\mathbf{A}_\mathrm{Y}$, $\mathbf{B}_\mathrm{Y}$, $\mathbf{C}_\mathrm{Y}$ and $\mathbf{G}_\mathrm{Y}$).	
The combined explained variation is calculated for every combination.		
The combinations with a better fit are chosen for every value of the sum of components S=P+Q+R.		
The combinations that belong to the convex hull of all of them are chosen		
All the most stable combinations except the simplest one among these ones are deleted.		
One combination of components is chosen.		
The three Co-Inertia Analysis for the three dimensions are performed.		
	Both data matrices are centered by columns and, if it is wanted, they are normalized by columns.	
	The left-singular and right-singular vectors are calculated for the Co-Inertia according to the method explained in section 3.	
	The coordinates are calculated for the rows and columns for the Co-Inertia according to the duality diagrams.	

This algorithm developed in order to apply the Co-Tucker3 will serve to better know the sustainability of the countries, we will be able to graphically visualize the differences and similarities among the studied countries, sustainability indicators and years, moreover, it will be possible to deduce if differences or similarities among the countries also exist according to if they behave in a similar or opposite way in each one of the types of sustainability indicators, and the same can be deduced for the years during the studied period of time (2006-2012). Therefore, this algorithm and the obtained graphics allow decision-makers providing information whenever they need this kind of help related to sustainability.

5. Results of the empirical analysis – Applicability to real sustainability data

Now, we present the results after the analysis with the Co-Tucker3 method whose mathematical model and algorithm was above-described. First of all, we must choose how many components we have to retain for each one of the dimensions, the countries, the indicators, and the years, for each one of the three analysis, in order to study the relations between the social issue and the environmental one, the relations between the social one and economic one and the relations between the environmental and economic aspects. To do that, we look at the tables that we obtain as results from each analysis, that show all the combinations of components that have a higher explained variation for every sum of components.

Table 2: Combinations with better fit from the Co-Tucker3 between social and environmental variables

Model	S	Residual Sum of Squares	Difference in the Fit	Percentage of Fit	Number of iterations
1x1x1	3	24569.000	53.007	53.007	5
2x2x1	5	14372.657	19.502	72.510	18
2x2x2	6	14355.007	0.034	72.543	21
3x3x1	**7**	**9041.929**	**10.162**	**82.706**	**12**
3x3x2	**8**	**9014.115**	**0.053**	**82.759**	**17**
4x4x1	9	5692.008	6.354	89.113	10
4x4x2	10	5629.892	0.119	89.232	10
5x5x1	11	3813.064	3.475	92.707	67
5x5x2	12	3737.423	0.145	92.851	75
5x5x3	13	3726.767	0.020	92.872	69
5x5x4	14	3718.881	0.015	92.887	70

Table 3: Combinations with better fit from the Co-Tucker3 between social and economic variables

Model	S	Residual Sum of Squares	Difference in the Fit	Percentage of Fit	Number of iterations
1x1x1	3	23658.352	49.521	49.521	13
2x2x1	5	15598.668	17.197	66.718	18
2x2x2	6	15396.049	0.432	67.150	21
3x3x1	**7**	**10608.009**	**10.216**	**77.366**	**36**
3x3x2	**8**	**10384.786**	**0.476**	**77.843**	**43**
4x4x1	9	6705.658	7.850	85.692	13
4x4x2	10	6438.726	0.570	86.262	18
5x5x1	11	4398.413	4.353	90.615	26
5x5x2	12	4046.941	0.750	91.365	27
5x5x3	13	4009.932	0.079	91.444	27
5x5x4	14	3995.516	0.031	91.475	27

Table 4: Combinations with better fit from the Co-Tucker3 between environmental and economic variables

Model	S	Residual Sum of Squares	Difference in the Fit	Percentage of Fit	Number of iterations
1x1x1	3	29469.251	41.499	41.499	13
2x2x1	5	16835.529	25.080	66.579	10
2x2x2	6	16647.516	0.373	66.952	11
3x3x1	**7**	**10429.422**	**12.344**	**79.296**	**36**
3x3x2	**8**	**10228.415**	**0.399**	**79.695**	**43**
4x4x1	9	5795.957	8.799	88.494	13
4x4x2	10	5582.846	0.423	88.917	18
5x5x1	11	3020.747	5.086	94.003	67
5x5x2	12	2732.469	0.572	94.576	75
5x5x3	13	2702.409	0.060	94.635	69
5x5x4	14	2694.285	0.016	94.651	70

From the analysis between social and environmental indicators (Table 2), we can see that the combinations 3x3x1 and 3x3x2 have percentages of fit of 82.706% and 82.759%, that they are already percentages high enough. Of course, the minimum percentage high enough to justify this choice is subjective, but we will observe in the graphics below that we have enough reasons for choosing these two combinations. Moreover, the increase in the explained variation if the next more complex model (4x4x1) was considered would only be of 6.354% as it can be seen in the column of Difference in the Fit, which is what it is already meant as insignificant from the statistical point of view.

For the analysis between the social and economic indicators, and between the environmental and economic ones, analogously, the better combinations are also the 3x3x1 and 3x3x2. At the moment of choosing the combination of components, the graphics that represent all the models according to the sum of the number of components against the residual sum of squares can be used too. In this case, they would be the following ones.

Figure 5: Sum of the number of components vs. Residual sum of squares from the Tucker3 in the Co-Tucker3 analysis between social and environmental variables

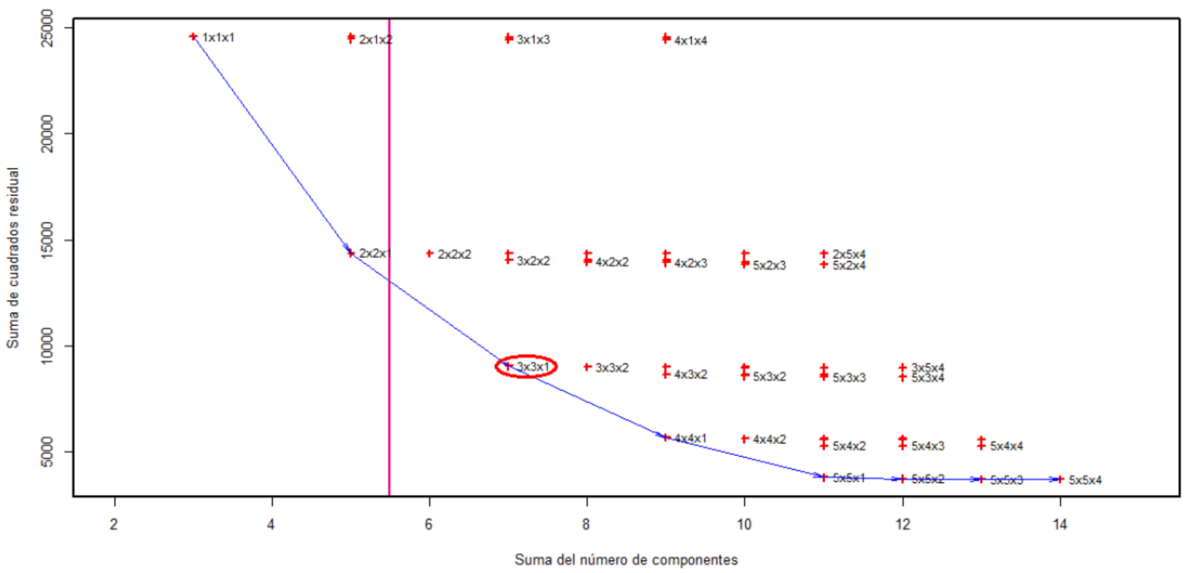


Figure 6: Sum of the number of components vs. Residual sum of squares from the Tucker3 in the Co-Tucker3 analysis between social and economic variables

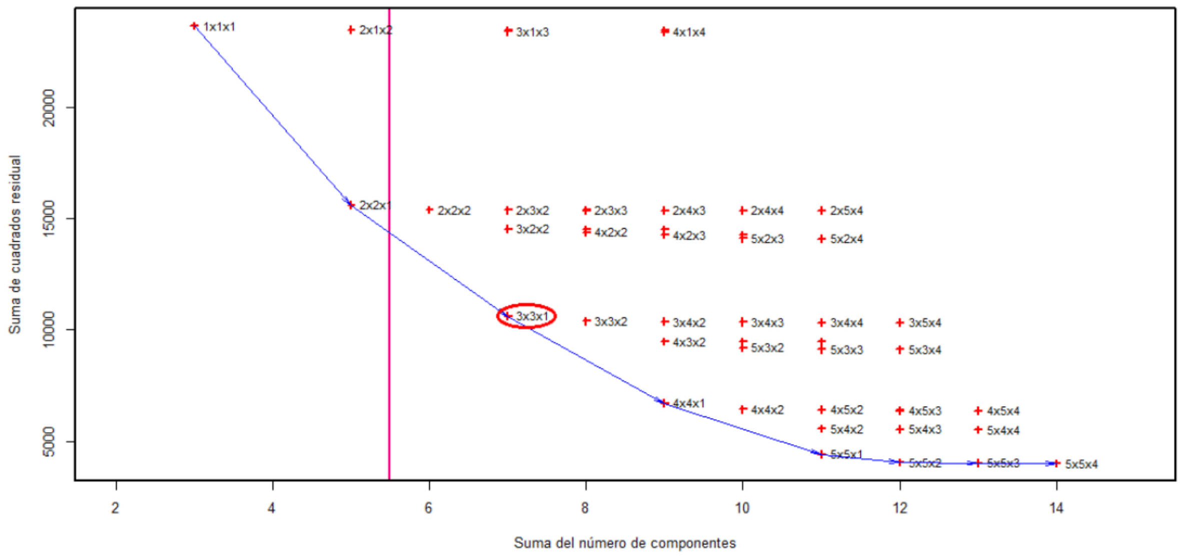
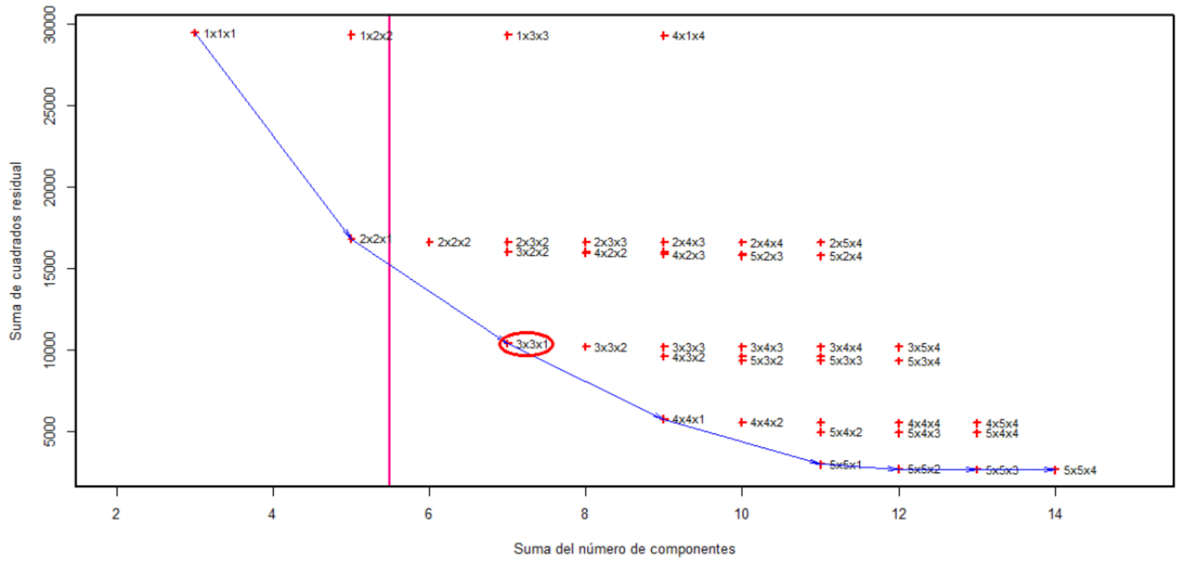


Figure 7: Sum of the number of components vs. Residual sum of squares from the Tucker3 in the Co-Tucker3 analysis between environmental and economic variables



We can observe in the three of them that the model 3x3x1 is one of the simplest combinations (it has the lowest residual sum of squares among the models that have the same sum of the number of components) and it is also the first of the most stable ones (the subsequent models have a decrease in the residual sum of squares insignificant from the statistical point of view). However, we will choose the model 3x3x2, because if we do not consider two components for the third dimension we would not see differences between the years, and seeing how our data evolve over the time is what we want.

Now, we see the graphics that we obtain after the last part of the analysis with the Co-Tucker3 method, the Co-Inertia Analysis between the countries, variables and years after we have chosen the model 3x3x2 for the three analysis, one for each pair of types of variables.

With these graphics we can find out the relations between deeper interactions than we can obtain from other methods like BGCOIA (Franquet et al., 1995), STATICO (Simier et al., 1999; Thioulouse et al. 2004) or COSTATIS (Thioulouse, 2011). The BGCOIA method should only be used when there are good reasons for giving more importance to only one dimension instead of all of them. The STATICO method is more appropriated when a description of the evolution of the relations between the variables of both sequences of matrices is looked for. Whereas the COSTATIS method must be preferred when the relations between the variables of the two sequences of matrices are strong and the chronological structures are not important.

Therefore, it has been proved from the performed analysis that this Co-Tucker3 method is a notable and useful technique whenever decision-makers or ordinary people need help at providing more systematic information, because with the graphics we can interpret if the countries, sustainability indicators or years of study are similar or different, if these differences vary over the time, and if the differences between the countries are because of their behavior in each one of the types of sustainability, the social, environmental or economic aspect.

The interpretation of these graphics is based on the position of the countries, variables and years in the different quadrants in order to study groups or interactions between the three dimensions; moreover, the length of the vectors that represent the three types of indicators and the length of the vectors that represent the Co-Inertia Analysis, that is, the difference between the countries or years according to one type of variables or another, can be studied too. A list with the name of all the countries and a code of three letters for each one of them can be found in the Appendix A.

Although the analysis was performed for all the countries, in order to facilitate the visualization and interpretations from the different plots, we represented only the countries with very long vectors (vectors longer than the average length, because all the vectors were long) and only those countries with a high quality of representation in the graphics (higher than 500 per mille), because we are retaining three axes for the dimension of the countries but we can only represent two of them, and the countries that are placed forming a small angle with one of the axes that we are not representing (in our case, we are only representing the first two axes, therefore we are not representing the third axis) cannot be efficiently interpreted.

For example, when we are studying the relations between the social variables and the environmental ones (Figure 8), firstly, we observe that there are countries that are placed in the second and third quadrants, that is, they are directly related to all the social variables and to Consumption, C., Greenhouse Gases, G.Ga., and Renewable Energy, R.E.; but we can only deduce this during years 2010 and 2012 because they are placed in the same quadrants. Whereas the opposite conclusion can be done for the countries, indicators and years placed in the first and fourth quadrants. Every plot is divided into three parts, the countries, the indicators of both sequences of matrices, and the years, that is, the three dimensions of our data. They can be represented in any order, but we have chosen this order because they are in rows, columns and repetitions respectively. In these figures 8, 9 and 10, the colours of the countries do not matter at this moment, we will explain them later.

Table 5: Relations for the social and environmental indicators for the first axis

Countries	Social indicators	Environmental indicators	Years
BLR, CZE, LUX, UKR	S.F., S.D., S.S., H.L., C.A., C.W., Ed., I.D., G.Go.	C., R.E., G.Ga.	2010, 2012
CUB, BHU, ETH, IND, MAR, MAW, PAN, PHL, SUD, TUN, YEM	-	B., R.W.R.	2006, 2008
CUB, TUN	S.F., S.D., S.S., H.L., C.A., C.W., Ed., I.D., G.Go.	-	2010, 2012
MGL, RSA	-	C., R.E., G.Ga.	2010, 2012

It is notorious that these conclusions can only be obtained after we know how the countries behave according to the different indicators over the years in a general way, and only because we are using the Co-Tucker3 method, whose mathematical model and algorithm were above-described, other methods like BGCOIA, STATIS or COSTATIS cannot obtain these conclusions, that is because now we are studying the deeper interactions, that is, these conclusions can be used by decision-makers in order to find out information related to the countries, different from the general knowledge about the countries.

If we were not able to observe which countries, indicators or years are in each quadrant because some of them had been placed near any axis, we could use the different tables we have obtained after the iteration step in the algorithm, that is $\mathbf{A}_X$, $\mathbf{B}_X$, $\mathbf{C}_X$ for the countries, indicators and years respectively from the first sequence of matrices, and $\mathbf{A}_Y$, $\mathbf{B}_Y$, $\mathbf{C}_Y$ from the second sequence of matrices. We could do that because these matrices have the coordinates of all the countries, indicators and years for the plots, so, according to their signs, we know in which demiplanes they are placed.

Moreover, we talk about the lengths of the vectors. Those variables that have a longer length are the ones that are useful to differentiate the different countries and years in quadrants, so, when we are jointly studying, for example, the social and economic variables (Figure 9), the countries in the first and fourth quadrants, during the years 2010 and 2012, that are also in the same quadrants, are mainly related to the indicators Income Distribution, I.D., and Public Debt, P.D. Whereas, if we talk about the vertical axis, the countries that are in the first and second quadrants, during the years 2006 and 2012 attach more importance to the social variables Clean Water, C.W., and Good Governance, G.Go., and attach less importance to the economic variables Organic Farming, O.F., and Gross Domestic Product, GDP, because these are in the third and fourth quadrants.

Table 6: Relations for the social and economic indicators for the first axis

Countries	Social indicators	Economic indicators	Years
BLR, CRO, EST, FIN, ISR, LAT, MAS, MNE, OMA, SLO, SVK, SWE	C.W., Ed., G.E., G.Go.	G.S., Em.	2006, 2008
BIH, RSA, SRB	C.W., Ed., G.E., G.Go.	-	2006, 2008
BIH, RSA, SRB	-	GDP, P.D.	2010, 2012
ANG, BHU, CGO, CHA, MLI, NCA, SEN, TAN, UGA, YEM	S.F., S.S., H.L., I.D.	GDP, P.D.	2010, 2012

According to the lengths of the vectors given by the Co-Inertia Analysis, for the countries, we can observe that all the vectors, in every of the three figures are long, that makes sense, because that means that the behaviors of the countries according to the social, environmental and economic issue, are not similar, but it is only because with this analysis of the Co-Tucker3 we are studying the deeper interactions, deeper than the stable ones. We understand for deeper interactions those that can be used for describing the interactions between the different rows, columns and repetitions in more detail than the explanation we can do by visualizing and interpreting the first groups obtained after the compromise and trajectories analysis from methods like BGCOIA, STATICO and STATIS. With the Co-Tucker3 method, whose mathematical model and algorithm were above-described, we can talk about the general results and about those deeper interactions, like the countries that, even though they are similar, they behave in a slightly different way according to some indicators, and depending on the year too. It must be clear that these deeper interpretations can only be possible due to the mathematical model and the algorithm described in this research, in particular, the employment of the matrices $\mathbf{A}_X$, $\mathbf{B}_X$, $\mathbf{C}_X$ and $\mathbf{A}_Y$, $\mathbf{B}_Y$, $\mathbf{C}_Y$ to knowing in which quadrants the countries, indicators and years are placed.

For the lengths of the vectors from the Co-Inertia Analysis between the years, in the analysis between the social and economic variables (Figure 9), it can be observed that they are short, what mean that, even in a deeper level, during all the years there is a strong relations between these two types of variables. Whereas for the analysis between the social and environmental variables (Figure 8), or between the environmental and economic ones (Figure 10), only during the last two biennia, years 2010 and 2012, this relation exists, because during the other years the vectors are longer.

All these interpretations related to the lengths of the vectors lead us to think that among the causes of this significant difference between the years 2006, 2008 and 2010, 2012 may be because of the financial crisis, that has had much more importance on some aspects of sustainability, such as the Public Debt and other economic factors as labour market reforms, economic freedom, education and part-time employment (Bertola et al., 2007; Choudhry et al., 2012). In addition, as Van de Kerk and Manuel (2012) point out, economic well-being is a precondition for the other two dimensions of well-being (environmental and social indicators).

Figure 8: Co-Inertia Analysis from the Co-Tucker3 between social and environmental variables

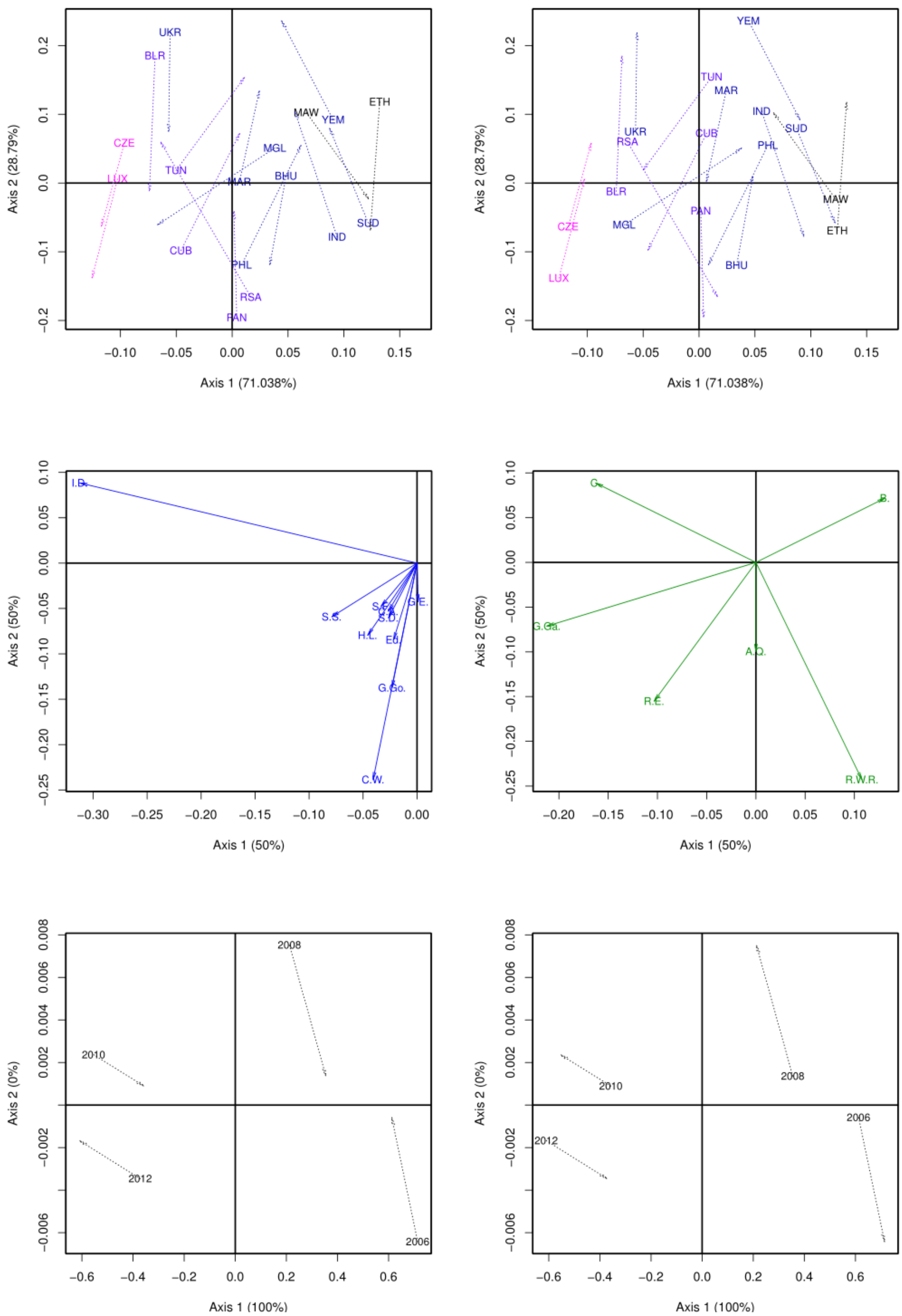


Figure 9: Co-Inertia Analysis from the Co-Tucker3 between social and economic variables

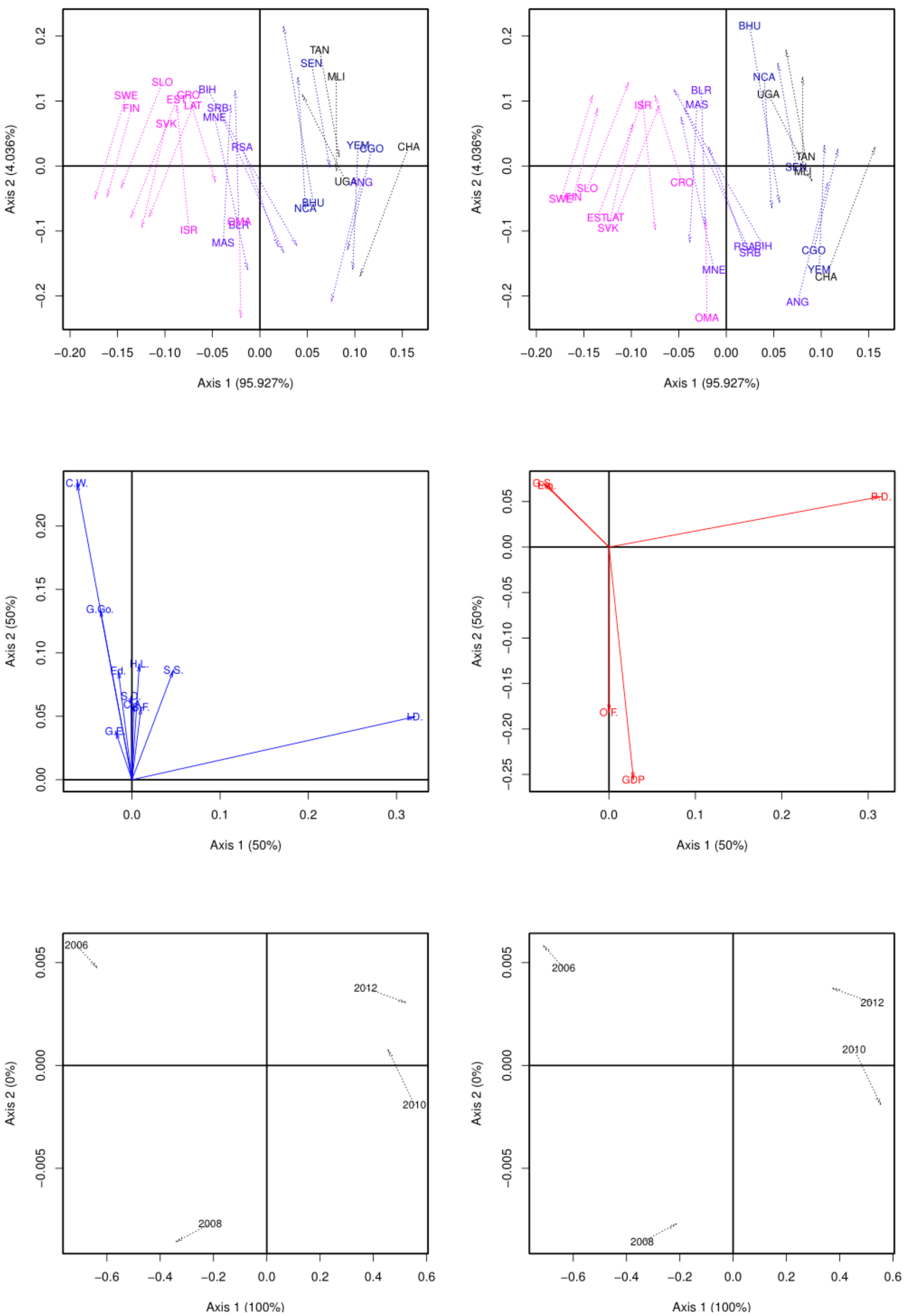
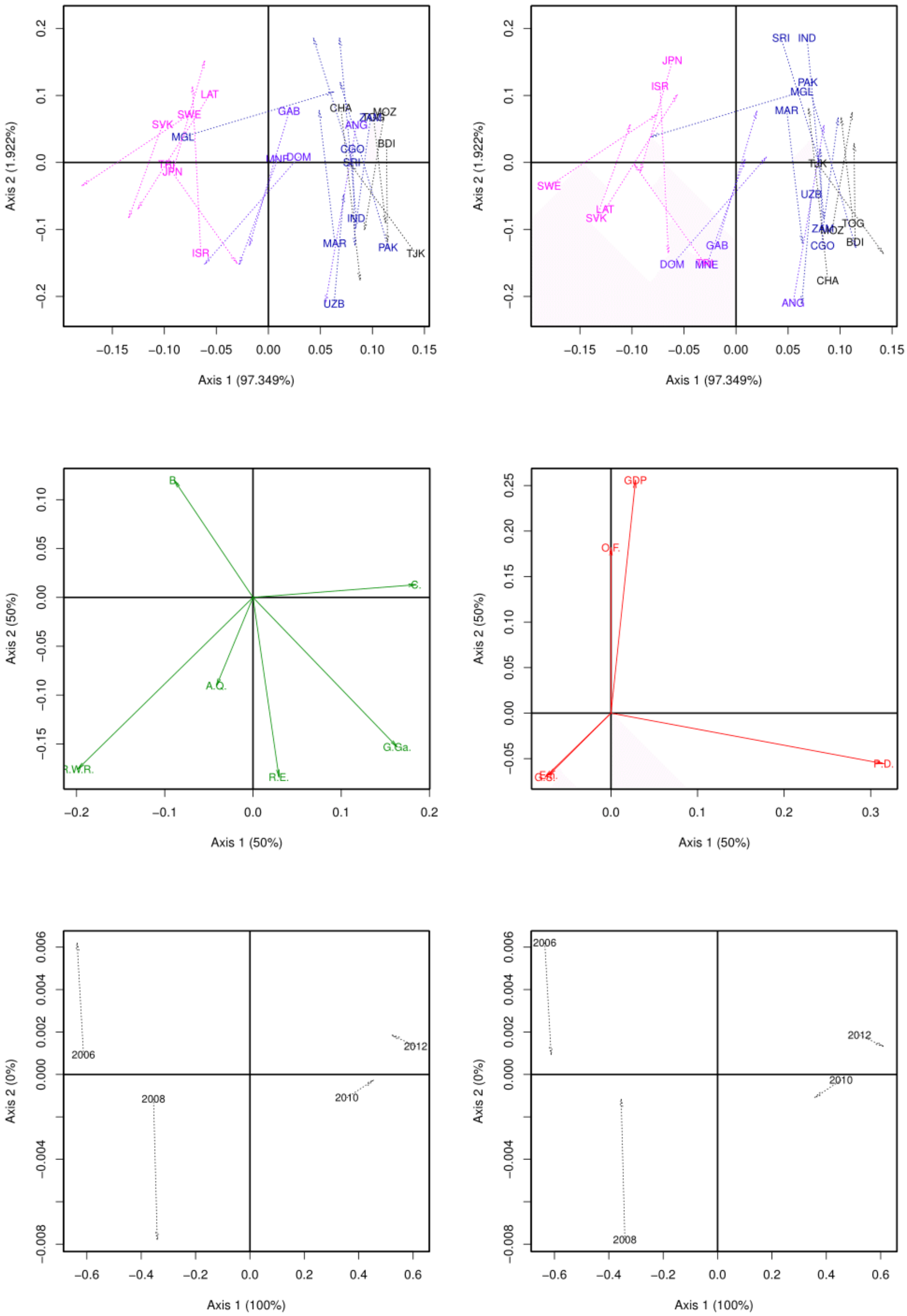


Figure 10: Co-Inertia Analysis from the Co-Tucker3 between environmental and economic variables



6. Discussion and conclusions

Nowadays, the awareness of that is necessary to preserve and keep the sustainability, is already reflected in some scopes of work, and not only at the environmental level. Collaborations between organizations, scientific institutions and even the citizens are caring out new measures for solving the global problems, particularly the economy-related ones (Robinson, 2004).

Techniques for long time periods assessment need to be developed (Hodge and Hardi, 1997), because there is no doubt that there has been a globally significant economic evolution, although with many regional differences, even the distribution is nothing homogeneous within the same region, with differences between the countries belonging to it. Developing a method that serves to help policy and decision-makers has been necessary.

For this reason, with this paper we have tried to find out what countries all over the world and in what ways they deal with the different issues of the sustainability. In this case, we have studied the Sustainable Society Index, SSI, for 151 countries, along four biennia, the period from 2006 to 2012.

A new mathematical model and its corresponding algorithm that serve to the needs given by the type of the data have been created. The Co-Tucker3 mathematical modelling can be used for every kind of data, in our case, sustainability data, but it is also useful for analyzing any data that have the shape of two sequences of matrices. Once we have obtained the graphics, in addition to only talk about all the countries studied in this research, they can also be, for example, grouped by the income levels they have (we have represented this with the colours we mentioned before: countries with high incomes in pink, countries with upper-middle incomes in violet, countries with lower-middle incomes in blue, and countries with low incomes in black), in order to observe that our questions and answers can be directly dependent on this aspect, although our analysis have been without taking into account these income levels, this is in agreement with Wilkinson and Pickett (2010).

The different countries are grouped according to their incomes levels by the World Bank (2013) and the classification is as follows:

Low incomes: $1,035 or less,

Lower-middle incomes: $1,036-4,085,

Upper-middle incomes: $4,086-12,615,

High incomes: $12,616 or more.

After analyzing the data with the Co-Tucker3 methodology (that is, the above-mentioned mathematical model and algorithm are used), the first thing we can deduce concerns the years similarities and differences, the plots show that the four years are two-by-two related, that is, on the one hand, 2006 and 2008 are similar, and on the other hand, so 2010 and 2012, this is due to the global financial crisis during 2009 (Authers, 2010; Jlassi, Naoui and Mansour, 2014).

The mathematical explaination of why the years are related like we have written in the previous paragraph, that is, 2006 and 2008 on the one hand, and 2010 and 2012 on the other hand, it is due to, firstly, the way the countries prioritize some indicators, and secondly, how they evolve along the studied period, because with the Co-Tucker3 method, if there are differences between the values that the countries take in the indicators along the four years, it will finally result in a plot where the years will be placed separately or jointly. In our case, 2006 and 2008 are placed together in the plots, and 2010 and 2012 are placed together, too and very separately from 2006 and 2008.

The relations between the countries and the indicators can be visualized in the same plots for the retained components during the Co-Tucker3 analysis. The principal interpretation we could see in almost every of the graphics is the difference between the four groups of countries by income levels.

When we are studying the relations between the interactions of the social variables and the economic ones, firstly, we observe that the countries that are placed in the second and third quadrants (Table 7 and Figure 9), the countries with high or upper-middle incomes as Sweden, Finland, Israel, Slovenia, among others are directly related to Clean Water, Good Governance, Education and Gender Equality and to Genuine Savings and Employment; but we can only deduce this during years 2006 and 2008 because they are placed in the same quadrants. Whereas the opposite conclusion can be done for the countries with low or lower-middle incomes (Tanzania, Yemen, Mali, Chad or Uganda), the other indicators (Sufficient Food, Safe Sanitation, Healthy Life, Income Distribution) and the years placed in the first and fourth quadrants.

Table 7: Relations for the social and economic indicators for the first axis with the clasification by income levels

Countries	Social indicators	Economic indicators	Years
High incomes: CRO, EST, FIN, ISR, LAT, OMA, SLO, SVK, SWE Upper-middle incomes: BLR, MAS, MNE	C.W., Ed., G.E., G.Go.	G.S., Em.	2006, 2008
Upper-middle incomes: BIH, RSA, SRB	C.W., Ed., G.E., G.Go.	-	2006, 2008
Upper-middle incomes: BIH, RSA, SRB	-	GDP, P.D.	2010, 2012
Low incomes: CHA, MLI, TAN, UGA Lower-middle incomes: BHU, CGO, NCA, SEN, YEM Upper-middle incomes: ANG	S.F., S.S., H.L., I.D.	GDP, P.D.	2010, 2012

After the interpretation of the countries, indicators and years according to the first axis, where we have obtained the first conclusions explained in the paragraphs above, we could deepen more according to the next axes, that is, we could build different sub-groups that, although they have the same characteristics as each other, they can behave differently in some of the indicators, and then more important conclusions can be deduced similarly to the previous ones, but only by understanding that these ones can represent a lower explained variation.

For example, in the same plot (Figure 9) we can differentiate the countries more detailedly according to the second (vertical) axis. Countries such as Russia, Saudi Arabia or Chile, that they already belong to the same group according to Essers (2013) (that they still having high values in the above-mentioned social and economic indicators) are slightly more related with the variables Organic Farming and Gross Domestic Product, while the countries of Austria, Germany or Denmark, that they already group together pursuant to Rusu (2013) (they take high values in some of the social and economic indicators too) pay more attention to Clean Water, Education, Gender Equality and Good Governance. This two deductions can only be done by seeing the original plot, where all the countries are represented.

As a general conclusion, the countries with a high and upper-middle level of incomes are positively related to the social and economic indicators, whereas the countries with low and lower-middle level of incomes do not pay enough attention to the same indicators, as it is reflected in the Table 5 and in the Figure 8, where the environmental indicators are represented, in countries like: Ethiopia, Bhutan, India, Sudan or Yemen among others, the environmental aspects like Biodiversity or Renewable Water Resources have the highest priority.

Unlike the STATIS methods (Structuration des Tableaux À Trois Indices de la Statistique), with the Tucker3 step in the Co-Tucker3 methodology we can still observing the stable behavior of the countries, the indicators and the years, but the evolving behavior of them too. Furthermore, in a STATIS method, only those combinations whose components are equal for the three dimensions (1x1x1, 2x2x2, 3x3x3,..…) can be retained, while with the 3 method we can highlight every combination, because we can retain every combination of components for the three dimensions.

Moreover, with the Co-Tucker3 method, apart from the possibility of using it for looking for a description of the stable part or of the evolution of the dimensions or the relations between the two sequences of matrices, that is what the BGCOIA, STATICO and COSTATIS methods perform, it serve to find deeper interactions than the stable ones obtained with those methods, that is, it can be used for describing the interactions between the different rows, columns and repetitions in a more specialized way than the reached one by visualizing and interpreting the groups obtained after the compromise and trajectories analysis, that is, we can find and study one interaction for which different numbers of components had been retained for the three dimensions, for example, the combination: one component for the rows, three for the columns, and two for the repetitions.

Appendix A

Table 8: Countries in the sample

ALB	Albania	ECU	Ecuador	KUW	Kuwait	POR	Portugal
ALG	Algeria	EGY	Egypt	LAO	Laos	PRK	Korea, North
ANG	Angola	ESA	El Salvador	LAT	Latvia	QAT	Qatar
ARG	Argentina	ESP	Spain	LBA	Libya	ROU	Romania
ARM	Armenia	EST	Estonia	LBR	Liberia	RSA	South Africa
AUS	Australia	ETH	Ethiopia	LIB	Lebanon	RUS	Russia
AUT	Austria	FIN	Finland	LTU	Lithuania	RWA	Rwanda
AZE	Azerbaijan	FRA	France	LUX	Luxembourg	SEN	Senegal
BAN	Bangladesh	GAB	Gabon	MAD	Madagascar	SLE	Sierra Leone
BDI	Burundi	GAM	Gambia	MAR	Morocco	SLO	Slovenia
BEL	Belgium	GBR	United Kingdom	MAS	Malaysia	SRB	Serbia
BEN	Benin	GBS	Guinea-Bissau	MAW	Malawi	SRI	Sri Lanka
BHU	Bhutan	GEO	Georgia	MDA	Moldova	SUD	Sudan
BIH	Bosnia-Herzegovina	GER	Germany	MEX	Mexico	SUI	Switzerland
BLR	Belarus	GHA	Ghana	MGL	Mongolia	SVK	Slovak Republic
BOL	Bolivia	GRE	Greece	MKD	Macedonia	SWE	Sweden
BOT	Botswana	GUA	Guatemala	MLI	Mali	SYR	Syria
BRA	Brazil	GUI	Guinea	MLT	Malta	TAN	Tanzania
BUL	Bulgaria	GUY	Guyana	MNE	Montenegro	THA	Thailand
BUR	Burkina Faso	HAI	Haiti	MOZ	Mozambique	TJK	Tajikistan
CAF	Central African Republic	HON	Honduras	MTN	Mauritania	TKM	Turkmenistan
CAM	Cambodia	HUN	Hungary	MYA	Myanmar	TOG	Togo
CAN	Canada	INA	Indonesia	NAM	Namibia	TPE	Taiwan
CGO	Congo	IND	India	NCA	Nicaragua	TRI	Trinidad and Tobago
CHA	Chad	IRI	Iran	NED	Netherlands	TUN	Tunisia
CHI	Chile	IRL	Ireland	NEP	Nepal	TUR	Turkey
CHN	China	IRQ	Iraq	NGR	Nigeria	UAE	United Arab Emirates
CIV	Cote d'Ivoire	ISL	Iceland	NIG	Niger	UGA	Uganda
CMR	Cameroon	ISR	Israel	NOR	Norway	UKR	Ukraine
COD	Congo. Dem. Rep.	ITA	Italy	NZL	New Zealand	URU	Uruguay
COL	Colombia	JAM	Jamaica	OMA	Oman	USA	United States
CRC	Costa Rica	JOR	Jordan	PAK	Pakistan	UZB	Uzbekistan
CRO	Croatia	JPN	Japan	PAN	Panama	VEN	Venezuela
CUB	Cuba	KAZ	Kazakhstan	PAR	Paraguay	VIE	Vietnam
CYP	Cyprus	KEN	Kenya	PER	Peru	YEM	Yemen
CZE	Czech Republic	KGZ	Kyrgyz Republic	PHL	Philippines	ZAM	Zambia
DEN	Denmark	KOR	Kprea, South	PNG	Papua New Guinea	ZIM	Zimbabwe
DOM	Dominican Republic	KSA	Saudi Arabia	POL	Poland		

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Apéndice B

Países en la muestra

Tabla B.1: Países en la muestra

Ingresos bajos $1035 o menos		**Ingresos medio-bajos** $1036-$4085		**Ingresos medio-altos** $4086-$12615		**Ingresos altos** $12616 o más	
BAN	Bangladesh	ARM	Armenia	ALB	Albania	AUS	Australia
BDI	Burundi	BHU	Bhutan	ALG	Algeria	AUT	Austria
BEN	Benin	BOL	Bolivia	ANG	Angola	BEL	Belgium
BUR	Burkina Faso	CGO	Congo	ARG	Argentina	CAN	Canada
CAF	Central African Republic	CIV	Cote d'Ivoire	AZE	Azerbaijan	CHI	Chile
CAM	Cambodia	CMR	Cameroon	BIH	Bosnia-Herzegovina	CRO	Croatia
CHA	Chad	EGY	Egypt	BLR	Belarus	CYP	Cyprus
COD	Congo. Dem. Rep.	ESA	El Salvador	BOT	Botswana	CZE	Czech Republic
ETH	Ethiopia	GEO	Georgia	BRA	Brazil	DEN	Denmark
GAM	Gambia	GHA	Ghana	BUL	Bulgaria	ESP	Spain
GBS	Guinea-Bissau	GUA	Guatemala	CHN	China	EST	Estonia
GUI	Guinea	GUY	Guyana	COL	Colombia	FIN	Finland
HAI	Haiti	HON	Honduras	CRC	Costa Rica	FRA	France
KEN	Kenya	INA	Indonesia	CUB	Cuba	GBR	United Kingdom
KGZ	Kyrgyz Republic	IND	India	DOM	Dominican Republic	GER	Germany
LBR	Liberia	LAO	Laos	ECU	Ecuador	GRE	Greece
MAD	Madagascar	MAR	Morocco	GAB	Gabon	IRL	Ireland
MAW	Malawi	MDA	Moldova	HUN	Hungary	ISL	Iceland
MLI	Mali	MGL	Mongolia	IRI	Iran	ISR	Israel
MOZ	Mozambique	MTN	Mauritania	IRQ	Iraq	ITA	Italy
MYA	Myanmar	NCA	Nicaragua	JAM	Jamaica	JPN	Japan
NEP	Nepal	NGR	Nigeria	JOR	Jordan	KOR	Korea, South
NIG	Niger	PAK	Pakistan	KAZ	Kazakhstan	KSA	Saudi Arabia
PRK	Korea, North	PAR	Paraguay	LBA	Libya	KUW	Kuwait
RWA	Rwanda	PHL	Philippines	LIB	Lebanon	LAT	Latvia
SLE	Sierra Leone	PNG	Papua New Guinea	MAS	Malaysia	LTU	Lithuania
TAN	Tanzania	SEN	Senegal	MEX	Mexico	LUX	Luxembourg
TJK	Tajikistan	SRI	Sri Lanka	MKD	Macedonia	MLT	Malta
TOG	Togo	SUD	Sudan	MNE	Montenegro	NED	Netherlands
UGA	Uganda	SYR	Syria	NAM	Namibia	NOR	Norway
ZIM	Zimbabwe	UKR	Ukraine	PAN	Panama	NZL	New Zealand
		UZB	Uzbekistan	PER	Peru	OMA	Oman
		VIE	Vietnam	ROU	Romania	POL	Poland
		YEM	Yemen	RSA	South Africa	POR	Portugal
		ZAM	Zambia	SRB	Serbia	QAT	Qatar
				THA	Thailand	RUS	Russia
				TKM	Turkmenistan	SLO	Slovenia
				TUN	Tunisia	SUI	Switzerland
				TUR	Turkey	SVK	Slovak Republic
				VEN	Venezuela	SWE	Sweden
						TPE	Taiwan
						TRI	Trinidad and Tobago
						UAE	United Arab Emirates
						URU	Uruguay
						USA	United States

Apéndice C

Países por niveles de ingresos y variables

Tabla C.1: Países por niveles de ingresos y variables (año 2006)

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
BAN	7.00	7.50	4.80	5.72	1.67	8.72	4.78	6.27	7.95	2.96	6.37	1.00	9.71	8.32	3.51	9.74	1.00	9.20	1.00	6.51	7.47
BDI	3.20	7.90	3.60	2.52	4.77	3.98	3.75	6.50	2.28	2.22	4.77	2.43	9.77	6.92	8.69	9.86	1.00	1.00	1.00	1.00	1.00
BEN	8.50	6.80	3.20	4.00	3.61	3.72	5.67	5.78	1.88	4.43	7.05	10.00	9.95	7.42	6.69	9.65	1.00	7.48	1.00	9.32	9.70
BUR	8.10	5.10	1.20	2.60	3.27	5.05	2.30	5.85	1.14	4.24	6.37	7.12	9.21	5.05	8.69	9.86	1.00	7.01	1.00	7.63	9.46
CAF	5.70	7.50	2.70	2.90	4.51	4.52	3.15	5.70	1.00	2.31	5.40	8.87	10.00	5.39	8.69	9.86	1.00	2.63	1.00	4.49	1.00
CAM	6.70	3.40	1.60	4.58	3.81	9.00	5.94	6.29	4.92	3.45	6.44	10.00	9.95	7.22	7.26	9.80	1.00	7.95	1.07	8.44	8.88
CHA	6.60	3.40	1.00	3.45	3.86	4.47	3.92	5.25	1.88	2.81	5.40	4.70	9.91	3.57	8.69	9.86	1.00	1.00	1.19	1.81	9.08
COD	2.90	4.60	2.90	2.85	2.61	4.79	3.44	5.70	1.88	1.64	6.78	5.00	10.00	7.35	9.72	9.96	1.00	2.74	1.00	1.81	1.00
ETH	5.40	2.20	1.00	3.53	4.27	4.28	3.79	5.95	8.11	3.01	6.85	8.86	9.54	6.35	9.31	9.94	1.00	6.49	1.00	5.83	3.16
GAM	7.30	8.20	5.30	4.92	1.00	4.86	5.45	6.45	2.08	4.37	6.37	1.00	9.91	5.76	8.69	9.86	1.00	7.10	1.13	4.54	1.00
GBS	6.70	5.90	3.40	3.42	5.01	4.55	5.76	6.10	2.35	3.44	6.37	10.00	9.94	6.48	8.69	9.86	1.00	7.47	1.00	3.61	1.00
GUI	7.40	5.10	1.30	4.13	4.45	4.55	4.29	6.10	4.58	3.11	6.37	3.21	9.93	4.77	8.69	9.86	1.00	1.88	1.00	3.61	1.00
HAI	5.30	7.10	3.40	3.97	5.04	3.97	5.84	6.70	1.88	1.96	6.77	1.00	9.14	8.33	7.43	9.79	1.00	9.33	1.00	1.00	8.35
KEN	6.70	6.20	4.80	4.07	5.15	5.79	5.87	6.49	4.03	3.50	5.85	5.87	9.11	6.90	8.51	9.80	1.00	8.76	1.03	1.00	7.37
KGZ	9.40	7.60	6.00	5.88	4.88	4.13	7.89	6.74	6.64	3.36	3.82	3.47	7.94	7.57	4.63	9.02	1.00	6.25	1.23	4.45	2.25
LBR	5.40	6.20	3.60	2.55	4.01	4.85	6.32	6.10	1.88	2.08	6.37	1.00	9.99	5.92	8.69	9.86	1.00	1.09	1.00	5.71	1.00
MAD	6.30	4.50	3.30	4.77	5.01	4.76	6.10	6.39	2.30	4.85	4.77	1.27	9.56	6.37	8.69	9.86	1.00	6.83	1.00	7.71	9.83
MAW	6.70	6.70	4.60	2.48	5.01	3.03	6.59	6.44	1.62	3.88	4.77	7.51	9.44	7.69	8.69	9.86	1.00	3.14	1.00	1.00	8.91
MLI	7.10	4.80	4.50	2.98	3.47	7.86	3.65	6.00	1.56	4.59	6.37	1.22	9.35	4.32	8.69	9.86	1.00	6.91	1.00	4.15	9.52
MOZ	5.30	4.20	2.70	2.82	5.02	4.66	4.83	6.88	4.49	4.08	5.02	7.41	9.97	7.44	9.64	9.93	1.00	4.07	1.00	1.22	6.08
MYA	9.40	8.00	7.30	5.28	3.45	4.57	5.17	6.80	1.88	1.55	7.02	2.61	9.72	5.47	7.04	9.77	1.00	5.00	1.00	6.69	2.82
NEP	8.30	8.40	2.70	5.30	2.08	4.60	5.76	5.48	6.19	3.21	5.52	8.50	9.53	7.51	8.92	9.89	1.00	9.34	1.00	1.00	6.93
NIG	6.60	4.60	1.20	2.58	3.47	4.22	2.40	6.10	1.00	3.68	6.37	3.54	9.30	3.18	8.69	9.86	1.00	8.99	1.00	3.61	9.67
PRK	6.40	10.00	7.87	6.47	7.16	4.41	5.95	6.50	6.07	2.05	3.91	1.00	8.88	7.79	1.02	6.89	1.00	5.00	1.28	2.37	9.05
RWA	6.30	7.30	4.10	3.05	3.90	4.61	5.39	6.50	8.78	3.65	4.77	5.00	9.84	7.44	8.69	9.86	1.00	8.26	1.00	1.00	9.24
SLE	5.00	5.70	3.90	1.43	4.47	4.90	4.67	6.10	1.00	3.34	6.37	2.15	9.97	6.45	8.69	9.86	1.00	7.19	1.00	3.61	1.00
TAN	5.60	7.30	4.60	3.40	5.03	8.50	5.13	7.04	5.33	4.03	6.37	10.00	9.46	6.15	8.98	9.87	1.00	8.80	1.00	1.00	3.93
TJK	3.90	5.80	5.30	5.78	4.19	4.51	7.16	6.58	7.19	2.71	3.80	2.07	2.52	8.05	6.23	9.64	1.00	1.00	1.03	3.01	8.68
TOG	7.40	5.10	3.40	4.10	3.74	4.42	6.26	6.10	1.88	3.20	7.00	5.52	9.89	6.60	8.43	9.82	1.00	1.31	1.00	3.61	1.03
UGA	8.10	5.60	4.10	3.78	4.88	3.95	6.82	6.80	3.53	3.65	4.77	5.13	9.95	4.76	8.69	9.86	3.01	8.70	1.00	8.19	2.35
ZIM	5.60	8.30	5.70	2.27	5.24	7.19	5.70	6.46	1.74	1.99	3.64	10.00	7.90	6.80	6.72	9.18	1.00	1.41	1.00	1.00	5.02
ARM	6.60	9.20	8.40	6.83	4.15	5.10	7.06	6.65	4.97	4.08	3.41	4.00	7.77	7.38	1.00	8.66	1.00	8.99	2.49	1.00	9.66
BHU	6.70	6.20	7.00	5.48	5.47	4.28	5.12	5.70	1.88	5.44	5.52	10.00	9.96	1.00	5.33	9.65	1.00	9.55	2.29	7.95	1.06
BOL	7.90	8.50	4.50	5.73	5.89	8.34	8.63	6.34	1.34	4.09	5.63	9.26	9.97	2.52	1.85	8.97	1.00	1.52	2.36	5.83	5.72
CGO	6.30	4.60	1.00	4.38	2.95	4.90	6.12	5.70	1.88	2.81	2.70	4.84	10.00	7.09	6.41	9.77	1.00	1.00	2.31	1.81	1.00
CIV	8.60	8.40	4.00	3.25	4.05	5.09	3.92	5.69	2.98	2.33	7.11	10.00	9.83	6.87	7.57	9.68	1.00	4.80	1.15	3.61	1.10
CMR	7.50	6.30	4.80	3.58	2.96	5.29	5.35	5.87	3.26	3.22	5.77	4.50	9.97	6.74	8.45	9.83	1.00	6.35	1.38	6.44	9.67

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	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
EGY	10.00	9.80	6.80	6.50	7.43	6.24	7.37	5.79	7.05	4.07	4.21	3.04	1.00	6.66	1.00	7.94	1.64	6.10	2.93	3.18	1.00
ESA	8.90	8.20	6.30	6.62	5.98	4.98	7.27	6.84	1.00	4.82	5.19	1.00	9.45	5.88	5.52	8.99	1.00	7.21	3.70	4.85	8.34
GEO	7.30	7.60	8.30	7.40	5.62	4.02	7.46	6.70	4.72	3.36	5.64	1.70	9.71	7.20	3.15	9.01	1.00	8.85	2.35	2.52	9.20
GHA	8.70	7.90	5.80	4.97	3.52	7.78	4.99	6.65	3.83	4.84	5.94	6.98	9.82	5.15	7.46	9.70	1.00	8.68	1.42	3.33	9.26
GUA	7.60	9.50	6.10	6.23	5.35	5.99	6.41	6.07	1.00	3.75	4.90	10.00	9.74	5.57	5.65	9.17	1.00	7.65	2.64	7.33	9.46
GUY	9.10	8.30	7.00	5.87	6.74	4.28	7.57	7.11	2.81	4.59	4.49	2.38	9.93	3.05	1.34	5.34	1.00	6.17	3.16	3.07	1.00
HON	7.80	9.00	6.80	6.40	5.47	4.97	7.21	6.48	1.00	3.98	4.26	6.93	9.88	5.48	4.44	8.99	1.00	9.28	2.31	6.70	8.94
INA	9.40	7.80	5.20	6.35	5.22	6.22	6.56	6.54	7.19	3.35	3.89	3.20	9.44	7.12	3.47	8.52	1.00	8.19	2.10	3.25	8.35
IND	7.90	8.60	3.00	5.58	1.00	7.89	5.90	6.01	7.56	4.39	3.89	2.41	6.81	8.16	3.12	8.94	1.00	9.29	1.52	6.44	1.57
LAO	7.80	4.30	2.40	4.50	3.86	8.51	5.92	6.80	5.95	2.63	4.87	8.31	9.87	5.99	5.33	9.65	1.00	4.31	1.19	8.69	2.41
MAR	9.30	8.00	6.10	6.70	6.86	6.29	5.82	5.83	4.87	4.62	3.16	1.00	5.66	6.67	1.00	8.68	1.00	9.39	2.32	3.31	4.76
MDA	8.90	9.20	6.80	6.63	6.37	4.88	7.16	7.13	5.60	3.68	4.51	1.00	8.36	7.50	1.00	8.11	1.05	8.98	1.63	4.82	8.99
MGL	7.20	6.20	5.90	5.93	5.19	4.50	7.82	6.82	2.64	5.09	1.30	6.70	9.86	1.00	1.00	6.28	1.00	9.24	1.92	7.20	1.00
MTN	9.00	5.60	4.20	4.08	4.57	4.58	4.67	5.83	4.72	4.52	6.37	1.00	8.60	1.00	8.69	9.86	1.00	1.00	1.26	1.00	1.00
NCA	7.30	8.10	6.60	6.90	5.37	4.23	7.01	6.57	3.33	4.48	4.26	10.00	9.93	5.18	5.88	9.26	2.14	8.13	1.74	5.74	1.00
NGR	9.10	6.00	3.80	3.58	2.63	4.48	5.50	6.10	1.30	2.55	6.26	6.30	9.64	5.84	7.92	9.61	1.00	9.34	1.28	3.04	9.78
PAK	8.00	9.00	5.40	5.55	1.88	6.26	3.94	5.43	7.33	2.85	4.12	4.91	3.01	8.42	3.84	9.26	1.00	8.71	1.55	4.63	5.19
PAR	8.60	8.30	7.80	6.98	5.50	5.18	7.25	6.56	1.00	3.46	6.23	2.72	9.99	1.00	10.00	9.42	1.00	8.62	2.53	5.60	9.18
PHL	7.80	8.50	7.30	6.55	5.53	8.93	7.92	7.52	3.01	4.08	3.91	2.52	8.35	6.98	4.23	9.17	1.00	9.05	2.03	3.21	6.50
PNG	6.70	3.90	4.50	5.32	5.08	3.96	3.70	7.00	1.45	3.59	5.96	1.00	10.00	4.21	3.32	9.65	1.00	5.00	1.27	8.35	7.31
SEN	7.60	7.20	5.20	4.67	2.78	8.36	4.12	6.43	4.36	4.56	4.86	10.00	9.43	5.73	4.35	9.57	1.00	8.82	1.14	3.68	9.41
SRI	7.80	7.80	9.10	6.93	5.22	9.17	5.90	7.20	6.98	4.55	3.65	7.48	7.54	6.76	5.47	9.32	1.00	9.25	2.30	4.63	1.00
SUD	7.30	6.90	3.40	4.75	3.98	6.52	3.52	6.00	1.88	1.96	6.71	2.09	4.24	4.22	7.59	9.76	1.00	4.17	1.22	1.83	1.15
SYR	10.00	7.90	7.70	6.95	7.48	4.50	6.49	6.22	3.04	3.18	2.74	1.00	1.00	6.87	1.00	7.03	1.00	1.09	2.54	4.49	7.34
UKR	10.00	9.80	9.90	6.53	6.74	2.98	8.62	6.80	8.27	3.67	1.88	1.80	7.24	6.90	1.00	3.51	1.36	8.59	3.34	4.87	9.70
UZB	7.40	8.90	5.70	6.57	4.24	3.80	7.48	6.89	8.52	2.02	3.06	1.13	1.00	8.05	1.00	5.88	1.00	1.00	1.38	9.70	9.48
VIE	8.10	7.30	4.10	6.88	2.87	7.27	6.48	6.89	6.13	3.66	4.38	2.29	9.07	7.17	3.92	9.03	1.00	9.09	1.49	5.88	8.72
YEM	6.40	6.90	3.00	4.88	3.60	4.48	5.46	4.59	6.64	2.90	2.80	1.00	1.00	7.83	1.00	9.09	1.00	1.00	1.53	2.00	8.16
ZAM	5.10	5.50	4.50	2.48	5.08	4.17	5.84	6.36	1.00	3.99	1.00	10.00	9.83	7.44	9.07	9.82	1.00	5.56	1.00	2.04	9.06
ALB	9.40	9.70	8.90	6.90	10.00	8.25	6.79	6.61	8.69	3.91	5.75	3.37	9.56	6.81	3.15	8.69	1.00	8.14	3.22	2.35	5.39
ALG	10.00	8.70	9.20	6.77	8.13	5.83	7.24	6.02	6.00	3.34	6.17	3.12	4.74	6.38	1.00	7.58	1.00	9.08	3.56	2.17	9.23
ANG	6.00	5.00	3.00	2.23	4.48	5.18	4.94	6.04	1.88	2.69	7.22	6.03	9.96	7.49	7.29	9.56	1.00	1.00	2.18	1.00	9.50
ARG	10.00	9.40	8.20	7.55	9.05	8.43	9.01	6.83	2.57	4.28	6.17	2.63	9.60	2.93	1.00	6.10	4.00	7.49	5.42	3.14	1.77
AZE	8.50	7.70	5.50	6.20	6.55	4.43	7.29	6.78	5.95	3.00	2.80	3.55	7.10	7.07	1.00	6.09	1.01	1.67	2.78	4.66	9.83
BIH	9.20	9.80	9.30	7.38	5.52	9.35	7.06	6.80	9.14	3.84	1.00	1.00	9.91	5.48	1.39	5.87	1.00	5.00	3.54	1.00	9.46
BLR	10.00	10.00	7.87	6.78	7.51	4.43	8.84	7.11	7.87	2.77	3.91	3.61	9.25	5.01	1.00	3.65	1.00	9.29	4.61	8.61	9.74
BOT	6.80	9.50	4.10	2.62	5.59	4.29	7.15	6.90	1.00	6.61	1.75	10.00	9.84	3.97	2.36	7.64	1.00	9.58	5.94	1.00	9.94
BRA	9.10	8.90	7.50	6.63	6.69	8.54	8.78	6.54	1.00	4.96	4.61	10.00	9.93	1.47	4.29	8.27	1.00	7.99	4.62	3.74	3.19
BUL	8.90	10.00	10.00	7.47	10.00	8.11	7.61	6.87	5.83	5.43	3.18	4.37	7.12	5.49	1.00	4.06	1.00	6.44	5.11	3.28	9.39

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
CHN	8.90	7.70	4.40	7.35	1.89	6.80	6.71	6.56	2.49	3.82	1.82	8.02	8.05	6.90	1.38	5.86	1.03	9.52	2.60	6.57	9.66
COL	8.70	9.20	8.60	7.00	6.37	5.46	7.65	7.05	1.00	3.90	6.11	9.66	9.94	5.12	2.46	8.66	1.00	7.08	4.12	3.08	8.56
CRC	10.00	9.70	9.20	7.87	6.48	4.77	7.48	6.94	1.27	6.63	6.02	8.82	9.76	4.72	5.31	8.68	1.00	8.95	4.68	5.15	8.83
CUB	10.00	9.10	9.80	8.05	6.97	8.86	8.36	7.17	3.04	3.04	1.93	2.21	8.02	6.23	2.00	7.77	1.00	5.00	4.51	8.27	1.38
DOM	7.50	9.30	5.70	6.60	6.47	4.65	7.32	6.64	2.67	4.48	4.08	10.00	8.35	7.49	2.16	8.11	6.06	6.80	3.62	1.67	9.30
ECU	10.00	8.60	7.20	6.98	6.68	8.34	8.00	6.43	1.00	3.72	4.72	10.00	9.64	4.69	1.06	8.20	1.18	4.45	3.70	3.43	9.14
GAB	9.40	8.70	3.60	6.65	5.90	4.21	7.58	5.70	1.00	4.05	4.51	7.29	9.99	3.36	5.92	8.43	1.00	5.98	6.10	1.22	8.01
HUN	10.00	9.90	9.50	7.48	10.00	7.40	8.73	6.70	9.05	6.79	4.14	2.56	9.44	4.93	1.00	4.41	5.29	7.90	7.05	4.82	3.35
IRI	10.00	9.30	8.40	6.27	8.40	4.98	6.51	5.80	2.81	3.13	2.76	3.44	3.48	6.62	1.00	3.95	1.00	5.38	5.09	2.98	9.59
IRQ	8.81	8.10	7.87	5.02	4.49	4.27	6.27	6.10	6.07	1.25	2.15	1.00	1.27	8.21	1.00	7.37	1.00	5.00	1.97	1.65	1.00
JAM	9.00	9.30	8.00	7.52	6.32	4.62	7.12	7.01	5.02	4.68	2.25	3.67	9.38	6.29	1.09	6.06	1.00	8.79	4.45	3.25	1.00
JOR	9.30	9.10	9.30	6.83	9.31	3.00	8.03	6.11	6.31	5.11	2.67	1.00	1.00	5.57	1.00	6.67	1.00	8.50	2.69	2.27	1.79
KAZ	8.70	8.60	7.20	5.98	6.20	4.34	8.89	6.93	7.41	3.20	3.09	1.26	7.11	6.34	1.00	1.00	1.00	1.10	4.64	4.45	9.91
LBA	10.00	7.20	9.70	7.28	9.55	4.94	9.27	6.00	6.07	3.07	4.15	1.00	1.00	6.99	1.00	2.64	1.00	5.00	5.65	1.00	9.88
LIB	10.00	10.00	9.80	6.73	10.00	4.06	7.53	6.08	6.38	3.84	2.61	1.00	7.18	5.05	1.00	6.43	1.00	2.56	5.26	4.54	1.00
MAS	10.00	9.50	9.60	7.20	7.80	5.46	7.31	6.51	1.72	5.52	4.15	6.84	9.77	3.66	1.00	4.17	1.00	9.01	5.59	7.02	7.88
MEX	10.00	9.10	7.70	7.57	6.01	6.14	7.65	6.46	1.00	4.99	3.26	5.14	8.42	4.16	1.04	6.29	3.67	8.76	5.93	6.98	8.42
MKD	8.90	9.30	9.00	7.23	5.74	5.97	6.94	6.98	7.95	4.09	2.04	2.43	8.92	5.32	1.01	5.69	1.00	7.29	4.23	1.00	8.93
MNE	8.90	9.30	9.00	7.30	5.82	8.36	7.15	6.80	6.38	3.92	3.38	5.72	9.76	6.68	2.48	7.77	7.03	4.07	4.37	1.00	8.88
NAM	7.80	8.00	3.00	3.88	5.41	4.57	7.09	6.86	1.00	5.65	1.00	6.96	9.83	4.04	2.50	8.81	1.00	9.30	3.31	1.12	9.37
PAN	7.40	9.10	7.20	7.70	6.24	9.22	7.95	6.93	1.00	5.30	4.31	5.75	9.97	4.30	2.68	7.89	1.00	9.07	4.53	3.56	5.81
PER	8.70	8.10	6.20	6.83	5.76	8.34	8.23	6.62	1.00	4.37	1.39	6.12	9.90	5.19	2.84	8.95	1.00	8.16	3.74	3.84	8.85
ROU	10.00	5.70	5.10	7.18	6.06	8.15	7.63	6.80	6.98	4.95	2.14	2.75	9.58	4.78	1.28	5.66	1.54	7.30	4.92	4.88	9.76
RSA	8.60	8.70	6.70	4.05	6.20	8.42	7.99	7.13	1.00	5.78	1.20	3.44	7.50	6.56	1.07	3.03	1.00	6.95	4.64	1.00	8.88
SRB	8.90	9.30	9.00	7.30	5.45	8.36	7.78	6.80	6.38	3.92	3.38	2.77	9.76	6.68	1.14	3.40	1.00	4.07	4.51	1.13	7.90
THA	8.00	8.50	9.90	6.68	3.57	8.27	7.00	6.83	4.11	4.84	4.29	8.67	8.69	4.87	1.78	6.75	1.00	9.12	4.03	8.31	8.02
TKM	9.10	7.10	6.20	5.73	9.77	4.50	6.33	6.80	4.58	1.95	1.68	1.50	1.00	5.07	1.00	1.00	1.00	9.38	2.57	1.00	9.98
TUN	10.00	8.20	8.00	7.08	8.33	6.30	7.69	6.29	4.11	4.88	2.72	1.00	3.87	5.57	1.37	7.99	3.06	8.08	4.05	2.78	7.03
TUR	10.00	9.30	8.30	7.00	6.59	5.79	7.24	5.85	4.15	4.59	3.06	1.00	8.03	5.49	1.20	6.85	1.00	7.65	5.44	3.47	7.40
VEN	8.30	8.30	6.80	7.05	10.00	4.05	7.58	6.66	1.00	3.07	3.94	10.00	9.93	5.04	1.13	4.43	1.00	6.68	5.13	2.94	9.15
AUS	10.00	10.00	10.00	8.77	10.00	6.17	10.00	7.16	4.49	8.30	1.08	6.12	9.54	1.00	1.00	1.00	4.82	7.39	9.15	6.03	9.83
AUT	10.00	10.00	10.00	8.57	10.00	9.51	8.78	6.99	7.33	8.27	7.04	10.00	9.53	3.97	2.10	1.00	9.85	8.95	9.18	5.95	4.09
BEL	10.00	10.00	10.00	8.52	10.00	6.63	9.52	7.08	7.19	7.69	5.37	6.56	5.88	1.00	1.00	1.00	3.40	8.94	9.06	4.28	1.00
CAN	10.00	10.00	10.00	8.67	10.00	9.31	8.95	7.16	5.71	8.27	2.24	2.76	9.84	1.00	1.59	1.00	1.93	8.60	9.25	5.09	2.58
CHI	10.00	9.50	9.20	7.88	10.00	5.26	8.11	6.45	1.00	7.55	1.00	6.64	9.88	1.29	2.51	6.42	1.00	7.89	6.00	3.95	9.94
CRO	9.30	9.30	9.00	7.77	6.52	9.25	7.40	7.14	7.56	5.41	3.96	3.57	9.94	4.10	1.01	5.33	1.00	8.36	6.66	2.81	8.67
CYP	9.40	10.00	10.00	7.93	10.00	7.53	7.85	6.43	6.38	6.67	2.82	2.27	8.01	3.91	1.00	1.00	2.44	8.44	8.33	5.86	3.59
CZE	10.00	9.30	9.00	8.07	8.90	7.45	7.97	6.71	9.32	6.38	3.64	7.53	8.67	3.72	1.00	1.00	7.76	8.45	7.83	4.53	9.15
DEN	10.00	10.00	10.00	8.30	10.00	7.49	10.00	7.46	6.98	8.74	6.73	1.95	8.89	1.00	1.51	1.09	7.26	8.94	9.14	6.17	8.14

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
ESP	10.00	10.00	10.00	8.77	8.90	8.31	9.42	7.32	6.38	7.29	3.38	3.82	6.72	1.23	1.00	2.18	4.65	8.60	8.64	4.00	8.28
EST	10.00	9.30	9.00	7.35	6.31	9.43	9.22	6.94	3.53	7.07	1.00	10.00	8.86	1.00	1.14	1.00	8.35	8.98	6.98	4.53	9.96
FIN	10.00	10.00	10.00	8.52	10.00	8.76	10.00	7.96	8.96	8.99	4.87	4.22	9.79	4.41	2.36	1.00	8.12	9.01	8.91	4.32	8.29
FRA	10.00	10.00	10.00	8.67	9.70	8.65	9.10	6.52	6.31	7.30	5.49	8.24	8.45	1.81	1.00	3.83	3.93	8.66	8.91	3.95	3.74
GBR	10.00	10.00	10.00	8.43	10.00	8.16	9.24	7.36	3.95	8.20	5.13	8.96	8.94	3.11	1.00	1.15	6.17	7.86	9.04	6.19	7.89
GER	10.00	10.00	10.00	8.63	10.00	7.86	9.26	7.52	7.87	8.02	5.89	10.00	7.46	3.75	1.00	1.00	6.94	8.64	8.89	3.26	2.96
GRE	9.40	9.30	9.00	8.50	10.00	7.71	9.34	6.54	5.77	6.39	2.35	3.64	8.67	1.88	1.00	1.44	5.79	6.25	8.37	3.72	1.00
IRL	10.00	10.00	10.00	8.30	10.00	9.19	9.59	7.33	5.95	8.00	4.61	1.00	9.76	2.31	1.00	1.00	1.89	9.30	9.43	6.46	9.33
ISL	10.00	10.00	10.00	8.80	10.00	10.00	9.43	7.81	5.49	8.96	1.10	3.99	9.99	1.00	7.57	2.64	1.00	7.48	9.25	8.14	9.04
ISR	10.00	10.00	10.00	8.57	9.96	5.77	9.16	6.89	4.87	5.83	1.77	7.54	1.33	3.93	1.00	1.56	1.99	8.93	8.26	4.08	1.07
ITA	10.00	10.00	10.00	8.78	10.00	8.22	8.84	6.46	4.92	6.47	5.81	7.93	7.63	2.71	1.00	2.14	8.78	8.29	8.70	4.63	1.00
JPN	10.00	10.00	10.00	9.17	10.00	8.78	8.58	6.45	10.00	7.26	5.96	5.46	7.91	4.78	1.00	1.00	1.00	8.35	8.91	6.42	1.00
KOR	10.00	9.20	10.00	7.97	8.72	8.49	9.66	6.16	7.19	5.96	5.37	2.09	6.35	4.21	1.00	1.00	1.00	9.28	8.07	6.88	8.98
KSA	10.00	8.58	7.87	6.90	8.71	4.24	7.67	5.24	6.07	4.31	1.88	10.00	1.00	4.70	1.00	1.00	1.00	7.36	7.84	3.16	9.20
KUW	10.00	8.58	7.87	7.85	9.38	4.48	8.31	6.34	6.07	5.66	1.00	1.00	1.00	5.48	1.00	1.00	1.00	8.87	9.35	8.74	9.82
LAT	10.00	9.30	9.00	7.13	6.65	9.05	9.09	7.09	6.25	6.29	7.44	8.07	9.88	1.00	3.34	6.71	7.83	8.43	6.13	4.14	9.83
LTU	10.00	9.30	9.00	7.22	10.00	8.59	9.28	7.08	7.12	6.43	4.28	7.06	8.89	1.93	1.00	6.03	4.39	8.17	6.42	4.37	9.60
LUX	10.00	10.00	10.00	8.58	10.00	7.03	7.63	6.67	7.19	8.79	6.25	10.00	9.81	1.00	1.00	1.00	4.66	9.40	10.03	6.51	9.91
MLT	8.50	10.00	6.70	8.57	10.00	2.39	7.85	6.52	1.00	7.58	2.36	1.00	2.87	2.61	1.00	3.30	1.00	5.25	7.75	4.83	3.65
NED	10.00	10.00	10.00	8.53	10.00	7.32	9.69	7.25	6.25	8.42	6.77	7.27	9.02	1.00	1.00	1.00	4.70	8.90	9.24	5.89	7.27
NOR	10.00	10.00	10.00	8.67	10.00	9.51	10.00	7.99	8.52	8.52	6.60	4.57	9.94	1.00	4.85	2.14	6.46	9.02	9.75	6.30	4.86
NZL	10.00	10.00	10.00	8.47	10.00	9.92	10.00	7.51	4.49	8.82	3.92	9.27	9.85	1.00	3.17	1.83	1.00	8.35	8.35	6.82	9.55
OMA	8.81	7.90	8.90	7.33	9.20	4.42	6.92	5.90	6.07	5.94	1.79	4.66	1.00	3.79	1.00	1.00	1.00	1.41	7.55	2.23	9.86
POL	10.00	9.30	9.00	7.63	10.00	8.16	8.62	6.80	6.64	5.98	2.54	10.00	7.92	4.82	1.00	2.32	2.37	8.16	6.23	1.70	7.21
POR	10.00	10.00	10.00	8.20	10.00	7.79	8.80	6.92	3.50	7.38	3.77	2.80	8.77	1.97	1.31	4.05	7.62	6.17	7.75	4.68	3.79
QAT	8.81	10.00	10.00	7.53	10.00	4.48	7.77	6.04	6.07	5.62	3.27	1.00	1.00	1.62	1.00	1.00	1.00	5.00	10.00	8.61	9.78
RUS	10.00	9.60	8.70	6.40	6.92	8.24	8.70	6.77	7.71	3.56	1.93	4.60	9.85	5.06	1.00	1.00	1.00	6.53	5.75	4.68	9.85
SLO	10.00	10.00	10.00	8.25	6.85	9.30	9.29	6.75	8.69	6.96	3.73	6.17	9.71	3.83	1.06	2.21	7.02	8.98	8.15	5.20	9.25
SUI	10.00	10.00	10.00	8.87	10.00	8.69	8.53	7.00	5.83	8.60	7.26	10.00	9.51	5.23	1.60	4.06	9.36	9.22	9.28	7.13	3.60
SVK	10.00	10.00	10.00	7.70	8.76	8.92	7.46	6.76	8.03	6.39	3.95	10.00	9.77	5.22	1.00	2.93	6.99	7.51	6.84	1.99	9.02
SWE	10.00	10.00	10.00	8.88	10.00	9.62	9.61	8.13	8.52	8.56	6.61	4.90	9.85	1.00	2.87	4.42	8.25	9.18	9.12	4.66	7.59
TPE	10.00	10.00	10.00	8.53	10.00	6.26	9.01	6.50	6.25	6.62	5.09	2.45	7.42	2.40	1.00	1.00	1.00	5.00	8.55	6.62	8.77
TRI	8.80	9.10	10.00	7.00	7.99	4.63	6.41	6.80	3.72	5.45	4.45	4.80	9.40	5.71	1.00	1.00	1.00	2.69	6.73	4.50	8.89
UAE	10.00	8.58	10.00	7.32	8.74	4.48	6.72	5.92	6.07	6.39	3.25	2.36	1.00	1.00	1.00	1.00	1.00	9.49	9.66	7.33	9.91
URU	10.00	9.80	9.40	7.70	7.68	6.34	8.82	6.55	5.54	6.07	5.14	1.00	9.74	1.00	3.45	8.40	7.24	8.02	5.03	2.97	2.44
USA	10.00	10.00	10.00	8.22	10.00	7.75	9.38	7.04	3.20	7.71	3.04	6.78	8.46	1.39	1.00	1.00	1.20	7.84	9.59	6.02	3.20

S.F. Sufficient Food; **S.D.** Sufficient to Drink; **S.S.** Safe Sanitation; **H.L.** Healthy Life; **C.A.** Clean Air; **C.W.** Clean Water; **Ed.** Education;
G.E. Gender Equality; **I.D.** Income Distribution; **G.Go.** Good Governance; **A.Q.** Air Quality; **B.** Biodiversity; **R.W.R.** Renewable Water Resources; **C.** Consumption;
R.E. Renewable Energy; **G.Ga.** Greenhouse Gases; **O.F.** Organic Farming; **G.S.** Genuine Savings; **GDP** Gross Domestic Product; **Em.** Employment; **P.D.** Public Debt.

Tabla C.2: Países por niveles de ingresos y variables (año 2008)

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
BAN	7.00	8.00	3.60	6.02	1.57	8.72	4.85	6.53	7.41	3.13	6.37	1.00	9.71	8.36	3.35	9.72	1.00	9.32	1.00	6.51	8.50
BDI	3.40	7.10	4.10	2.79	4.73	3.98	4.80	6.60	2.28	2.67	4.77	2.43	9.77	6.94	8.63	9.86	1.00	1.00	1.00	1.00	1.00
BEN	8.80	6.50	3.00	4.31	3.63	3.72	5.78	5.58	6.13	4.55	7.05	10.00	9.95	6.46	6.19	9.59	1.00	6.51	1.01	9.32	9.23
BUR	8.50	7.20	1.30	2.79	3.39	5.05	2.95	6.03	4.92	4.24	6.37	7.12	9.21	4.85	8.63	9.86	1.00	7.01	1.00	7.19	9.37
CAF	5.60	6.60	3.10	3.10	4.75	4.52	3.06	5.80	1.00	2.42	5.40	8.87	10.00	5.45	8.63	9.86	1.00	3.33	1.00	4.49	1.52
CAM	6.70	6.50	2.80	4.99	4.00	9.00	6.04	6.47	4.63	3.41	6.44	10.00	9.95	6.99	7.03	9.78	1.00	8.39	1.30	8.44	9.20
CHA	6.50	4.80	1.00	3.48	3.89	4.47	3.81	5.29	1.72	2.13	5.40	4.70	9.91	3.59	8.63	9.86	1.00	1.00	1.20	2.17	9.38
COD	2.60	4.60	3.10	3.06	2.87	4.79	4.57	5.80	1.72	1.77	6.78	5.00	10.00	7.44	9.69	9.96	1.00	4.49	1.00	2.17	1.00
ETH	5.40	4.20	1.10	3.84	4.12	4.28	4.75	5.87	8.11	3.17	6.85	8.98	9.54	6.32	9.28	9.94	1.00	8.80	1.00	5.83	8.86
GAM	7.10	8.60	5.20	5.13	1.00	4.86	5.45	6.62	2.08	3.94	6.37	1.00	9.91	6.15	8.63	9.86	1.00	7.67	1.18	6.25	2.51
GBS	6.10	5.70	3.30	3.55	5.01	4.55	6.54	6.20	2.35	3.10	6.37	10.00	9.94	6.42	8.63	9.86	1.00	9.10	1.00	2.49	1.00
GUI	7.60	7.00	1.90	4.45	4.49	4.55	4.89	6.20	1.00	2.11	6.37	3.21	9.93	4.67	8.63	9.86	1.00	1.00	1.00	2.49	1.00
HAI	5.40	5.80	1.90	4.32	5.05	3.97	5.86	6.90	1.00	2.67	6.77	1.00	9.14	8.32	7.43	9.79	1.00	9.35	1.00	1.00	8.47
KEN	6.90	5.70	4.20	4.52	5.21	5.79	5.97	6.55	4.03	3.71	5.85	5.87	9.11	6.85	8.34	9.77	1.00	8.80	1.16	1.00	7.56
KGZ	10.00	8.90	9.30	5.95	4.75	4.13	7.90	7.05	8.27	3.28	3.82	3.47	7.94	7.41	4.77	9.07	1.00	8.87	1.40	4.40	7.09
LBR	6.90	6.40	3.20	2.81	4.40	4.85	6.32	6.20	1.72	3.20	6.37	1.00	9.99	5.88	8.63	9.86	1.00	1.09	1.00	5.71	1.00
MAD	6.20	4.70	1.20	5.06	5.01	4.76	6.27	6.74	2.30	4.63	4.77	1.27	9.56	6.21	8.63	9.86	1.00	6.83	1.00	7.71	9.92
MAW	6.50	7.60	6.00	2.91	5.01	3.03	6.49	6.66	5.27	4.17	4.77	7.51	9.44	8.14	8.63	9.86	1.00	8.77	1.00	1.00	8.12
MLI	7.10	6.00	4.50	3.31	3.54	7.86	4.04	6.12	4.49	4.48	6.37	1.22	9.35	4.47	8.63	9.86	1.00	8.11	1.00	2.49	9.47
MOZ	5.60	4.20	3.10	2.55	5.02	4.66	5.47	7.27	2.39	4.38	5.02	7.41	9.97	7.52	9.67	9.92	1.00	3.55	1.00	1.22	8.01
MYA	10.00	8.00	8.20	5.60	3.46	4.57	5.64	6.80	1.72	1.62	7.02	2.61	9.72	4.79	7.00	9.79	1.00	5.00	1.00	6.69	7.97
NEP	8.30	8.90	2.70	5.65	1.79	4.60	5.76	5.94	3.23	3.07	5.52	8.50	9.53	7.57	9.08	9.91	1.00	9.28	1.00	1.00	8.11
NIG	6.80	4.20	1.00	2.84	3.58	4.22	2.63	6.20	1.00	3.63	6.37	3.54	9.30	3.04	8.63	9.86	1.00	8.99	1.00	2.49	9.73
PRK	6.60	10.00	5.90	6.55	7.10	4.41	5.95	6.60	6.44	1.99	3.91	1.00	8.88	7.93	1.00	6.86	1.00	5.00	1.28	3.36	9.05
RWA	6.70	6.50	2.30	3.46	3.84	4.61	6.06	6.60	2.44	3.94	4.77	5.00	9.84	7.48	8.63	9.86	1.00	8.31	1.00	1.00	9.47
SLE	4.90	5.30	1.10	1.62	4.64	4.90	4.67	6.20	1.00	3.38	6.37	2.15	9.97	6.43	8.63	9.86	1.00	7.15	1.00	2.49	6.05
TAN	5.60	5.50	3.30	3.78	5.02	8.50	5.36	7.07	6.25	4.36	6.37	10.00	9.46	5.99	8.91	9.86	1.00	8.39	1.00	1.00	8.71
TJK	4.40	6.70	9.20	5.89	4.22	4.51	7.14	6.54	7.19	2.96	3.80	2.07	2.52	7.87	5.95	9.61	1.00	8.08	1.19	7.95	9.06
TOG	7.60	5.90	1.20	4.20	3.80	4.42	6.60	6.20	1.72	2.98	7.00	5.52	9.89	6.97	8.61	9.84	1.00	1.31	1.00	2.49	1.17
UGA	8.10	6.40	3.30	4.33	4.80	3.95	6.48	6.98	2.98	3.83	4.77	5.13	9.95	4.73	8.63	9.86	4.39	8.12	1.00	8.19	9.43
ZIM	5.30	8.10	4.60	2.76	5.25	7.19	5.70	6.49	1.74	1.84	3.64	10.00	7.90	7.08	6.69	9.21	1.00	1.41	1.00	1.00	1.00
ARM	7.60	9.80	9.10	6.92	4.46	5.10	7.31	6.68	7.05	4.38	3.41	4.00	6.36	6.94	1.00	8.65	1.00	9.15	3.21	1.00	9.71
BHU	6.70	8.10	5.20	5.77	5.64	4.28	5.62	6.00	1.72	5.18	5.52	10.00	9.96	1.00	5.40	9.64	1.00	9.74	2.82	6.91	3.29
BOL	7.70	8.60	4.30	5.97	5.89	8.34	8.63	6.67	1.00	3.52	5.63	9.26	9.97	2.58	1.55	8.90	1.00	5.26	2.56	5.95	8.46
CGO	6.70	7.10	2.00	4.72	3.21	4.90	6.12	5.80	1.72	2.74	2.70	4.84	10.00	6.94	6.19	9.73	1.00	1.00	2.40	2.17	2.94
CIV	8.70	8.10	2.40	3.46	4.32	5.09	3.92	5.69	2.98	2.21	7.11	10.00	9.83	7.07	7.89	9.68	1.00	3.42	1.17	2.49	1.90
CMR	7.40	7.00	5.10	3.65	3.19	5.29	5.18	6.02	3.26	3.36	5.77	4.50	9.97	6.73	8.30	9.83	1.00	7.55	1.46	7.48	9.84

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
EGY	10.00	9.80	6.60	6.71	8.34	6.24	7.29	5.83	7.05	3.84	4.21	3.04	1.00	6.44	1.00	7.87	1.27	8.00	3.30	3.98	2.59
ESA	8.90	8.40	8.60	6.79	6.09	4.98	7.16	6.88	1.00	4.75	5.19	1.00	9.45	5.65	5.57	8.95	1.17	6.10	4.06	5.31	8.33
GEO	9.10	9.90	9.30	7.48	5.61	4.02	7.50	6.65	3.36	4.46	5.64	1.70	9.71	6.71	2.78	8.91	1.00	6.95	2.89	2.65	9.19
GHA	8.90	8.00	1.00	5.18	3.68	7.78	5.61	6.68	3.83	5.14	5.94	6.98	9.82	4.97	6.84	9.65	1.00	6.49	1.60	6.98	8.81
GUA	7.80	9.60	8.40	6.41	5.35	5.99	6.77	6.07	1.00	3.78	4.90	10.00	9.74	5.43	5.70	9.19	1.00	7.95	2.92	8.35	9.52
GUY	9.20	9.30	8.10	6.33	6.59	4.28	7.78	7.11	5.49	4.33	4.49	2.38	9.93	3.24	1.35	5.34	1.00	4.76	3.62	3.33	4.25
HON	7.70	8.40	6.60	6.63	5.41	4.97	7.21	6.96	1.00	4.01	4.26	6.93	9.88	5.20	4.95	9.10	1.00	9.09	2.59	6.77	9.53
INA	9.40	8.00	5.20	6.65	5.43	6.22	6.66	6.47	7.19	3.85	3.89	3.20	9.44	7.08	3.44	8.46	1.00	8.44	2.38	4.02	8.84
IND	8.00	8.90	2.80	5.84	1.00	7.89	6.09	6.06	6.64	4.63	3.89	2.41	6.81	8.22	3.03	8.87	1.34	9.36	1.84	6.44	1.97
LAO	8.10	6.00	4.80	4.82	3.98	8.51	5.99	6.80	6.84	3.16	4.87	8.31	9.87	6.01	5.40	9.64	1.00	6.31	1.39	8.69	4.64
MAR	9.40	8.30	7.20	6.93	6.93	6.29	5.90	5.76	2.67	4.43	3.16	1.00	5.66	6.91	1.00	8.68	1.00	9.40	2.61	3.75	7.13
MDA	8.90	9.00	7.90	6.78	6.38	4.88	7.19	7.24	6.91	3.97	4.51	1.00	8.36	6.57	1.00	8.18	1.11	9.23	1.84	6.00	9.55
MGL	7.30	7.20	5.00	6.19	5.19	4.50	7.84	7.05	6.91	4.58	1.30	6.70	9.85	1.00	1.00	5.94	1.00	9.09	2.30	7.56	6.69
MTN	9.00	6.00	2.40	4.31	4.58	4.58	4.83	6.12	4.72	3.93	6.37	1.00	8.60	1.00	8.63	9.86	1.00	5.75	1.42	1.00	1.00
NCA	7.30	7.90	4.80	7.20	5.42	4.23	7.01	6.75	3.33	3.90	4.26	10.00	9.93	5.28	6.05	9.23	2.89	8.26	1.91	5.52	1.76
NGR	9.10	4.70	3.00	3.63	2.74	4.48	5.58	6.34	2.64	2.79	6.26	6.30	9.64	5.63	8.07	9.65	1.00	9.34	1.44	2.81	9.79
PAK	7.60	9.00	5.80	5.82	1.81	6.26	3.97	5.55	8.19	2.86	4.12	4.91	3.01	8.30	3.75	9.21	1.00	8.69	1.75	5.87	4.71
PAR	8.50	7.70	7.00	7.13	5.53	5.18	7.22	6.38	1.00	3.60	6.23	2.72	9.99	1.31	10.00	9.39	1.00	8.48	2.81	5.71	9.56
PHL	8.20	9.30	7.80	6.75	5.54	8.93	7.74	7.57	3.33	3.98	3.91	2.52	8.35	7.00	4.39	9.26	1.00	8.95	2.28	4.81	7.75
PNG	6.90	4.00	4.50	5.39	5.08	3.96	3.70	7.00	1.45	3.74	5.96	1.00	10.00	2.95	3.32	9.64	1.00	5.00	1.39	8.19	8.02
SEN	8.00	7.70	2.80	4.86	2.72	8.36	4.33	6.43	4.58	4.40	4.86	10.00	9.43	5.73	4.50	9.60	1.00	8.67	1.23	1.00	9.33
SRI	7.80	8.20	8.60	7.17	5.18	9.17	6.09	7.37	5.17	4.06	3.65	7.48	7.55	7.20	5.66	9.41	1.63	9.22	2.69	5.38	1.13
SUD	7.40	7.00	3.50	5.07	3.91	6.52	3.88	6.00	1.72	1.94	6.71	2.09	4.24	4.32	7.46	9.72	1.00	2.43	1.47	1.87	1.91
SYR	10.00	8.90	9.20	7.09	8.05	4.50	6.56	6.18	2.84	3.03	2.74	1.00	1.36	6.50	1.00	6.93	1.00	5.91	2.80	3.99	8.50
UKR	10.00	9.70	9.30	6.58	6.92	2.98	8.88	6.86	8.69	4.21	1.88	1.80	7.24	6.04	1.00	3.37	1.40	8.74	3.96	5.30	9.51
UZB	7.50	8.80	9.60	6.67	3.97	3.80	7.36	6.91	5.43	2.49	3.06	1.13	1.00	8.02	1.00	5.78	1.00	2.16	1.63	9.80	9.76
VIE	8.40	9.20	6.50	7.07	2.91	7.27	6.46	6.78	7.87	3.88	4.38	2.29	9.07	7.04	3.89	9.01	1.00	9.07	1.77	6.29	8.93
YEM	6.20	6.60	4.60	5.21	3.41	4.48	5.47	4.66	6.64	3.06	2.80	1.00	1.00	7.96	1.00	9.09	1.00	1.00	1.62	2.14	8.59
ZAM	5.40	5.80	5.20	2.96	5.02	4.17	5.88	6.20	1.00	4.22	1.00	10.00	9.83	7.42	9.15	9.83	1.00	4.68	1.00	2.47	9.39
ALB	9.40	9.70	9.70	6.99	10.00	8.25	6.79	6.59	7.63	4.40	5.75	4.19	9.56	6.54	3.34	8.73	1.00	8.37	3.67	2.59	5.75
ALG	10.00	8.50	9.40	6.95	10.00	5.83	7.24	6.11	6.00	3.48	6.17	3.12	4.74	6.14	1.00	7.52	1.00	9.39	3.79	2.52	9.88
ANG	6.50	5.10	5.00	2.46	4.86	5.18	5.52	6.03	1.72	2.93	7.22	6.03	9.96	7.62	6.94	9.46	1.00	1.00	3.03	1.00	8.96
ARG	10.00	9.60	9.10	7.70	10.00	8.43	9.03	7.21	1.00	4.53	6.17	2.63	9.60	4.00	1.00	5.90	4.06	8.53	6.17	4.28	4.96
AZE	9.30	7.80	8.00	6.30	6.30	4.43	7.14	6.86	5.95	3.39	2.80	3.58	6.53	6.87	1.00	6.07	1.06	6.92	4.30	5.20	9.90
BIH	9.10	9.90	9.50	7.50	5.52	9.35	7.24	6.90	9.14	4.18	1.00	1.00	9.91	4.91	1.30	5.45	1.00	5.00	4.06	1.00	8.98
BLR	10.00	10.00	9.30	6.87	10.00	4.43	8.93	7.10	7.87	2.87	3.91	3.61	9.25	4.41	1.00	3.20	1.00	9.18	5.45	9.05	9.46
BOT	6.80	9.60	4.70	3.14	5.59	4.29	6.98	6.84	1.00	6.36	1.75	10.00	9.84	3.83	2.36	7.67	1.00	9.51	6.44	4.72	9.92
BRA	9.30	9.10	7.70	6.83	6.92	8.54	8.75	6.74	1.00	4.78	4.61	10.00	9.93	2.21	4.33	8.26	1.54	8.15	5.10	3.95	3.82
BUL	9.20	9.90	9.90	7.56	10.00	8.11	7.66	7.08	7.79	5.47	3.18	4.38	7.05	5.11	1.00	3.86	1.06	2.30	5.81	4.43	9.68

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
CHN	8.80	8.80	6.50	7.50	1.72	6.80	6.71	6.88	1.81	3.92	1.82	8.02	8.05	6.76	1.29	5.48	1.00	9.56	3.32	6.70	9.63
COL	8.70	9.30	7.80	7.17	6.47	5.46	8.09	6.94	1.00	4.15	6.11	10.00	9.94	5.34	2.52	8.68	1.00	6.83	4.65	3.28	9.00
CRC	10.00	9.80	9.60	7.97	6.49	4.77	7.41	7.11	1.00	6.14	6.02	8.82	9.76	4.86	5.21	8.65	1.00	9.11	5.29	6.31	9.33
CUB	10.00	9.10	9.80	8.21	7.51	8.86	9.33	7.20	2.84	3.31	1.93	2.21	8.02	6.17	1.65	7.73	1.00	5.00	4.66	8.35	8.56
DOM	7.10	9.50	7.90	6.78	6.66	4.65	7.32	6.74	1.00	4.50	4.08	10.00	8.35	7.57	2.33	8.00	7.94	5.60	4.24	2.12	9.33
ECU	9.40	9.50	8.40	7.09	10.00	8.34	8.00	7.09	1.00	3.27	4.72	10.00	9.64	4.89	1.09	8.15	1.53	5.08	3.98	4.15	9.48
GAB	10.00	8.70	3.60	6.63	5.95	4.21	7.58	5.80	1.00	3.92	4.51	7.29	9.99	3.41	6.10	8.53	1.00	5.98	6.38	1.22	9.49
HUN	10.00	10.00	10.00	7.62	10.00	7.40	9.02	6.87	9.05	6.59	4.14	2.57	9.46	3.23	1.00	4.45	5.15	7.64	7.42	4.63	2.20
IRI	10.00	9.10	8.30	6.47	8.16	4.98	7.11	6.02	2.81	2.79	2.76	3.44	3.23	6.79	1.00	3.55	1.00	8.09	5.63	3.48	9.74
IRQ	9.20	7.70	7.60	5.38	3.97	4.27	6.27	6.20	6.44	1.53	2.15	1.00	1.27	8.21	1.00	7.99	1.00	5.00	2.10	1.74	1.00
JAM	9.10	9.30	8.30	7.59	6.32	4.62	8.07	6.98	2.78	4.97	2.25	3.67	9.38	6.32	1.00	5.56	1.00	8.17	4.75	3.73	1.00
JOR	9.40	9.80	8.50	7.00	9.26	3.00	7.99	6.28	5.07	5.17	2.67	1.00	1.00	5.39	1.00	6.67	1.00	7.00	3.10	2.70	4.55
KAZ	9.40	9.60	9.70	6.28	6.53	4.34	9.19	6.98	6.70	3.85	3.09	1.26	7.11	5.63	1.00	1.00	1.00	1.78	5.42	4.84	9.91
LBA	10.00	9.10	9.70	7.47	10.00	4.94	9.27	6.00	6.44	3.34	4.15	1.00	1.00	5.77	1.00	2.78	1.00	5.00	6.22	1.00	9.95
LIB	10.00	10.00	9.80	6.88	10.00	4.06	7.84	6.08	5.95	3.44	2.61	1.00	8.14	4.89	1.00	6.67	1.00	7.96	5.75	4.54	1.00
MAS	10.00	9.90	9.40	7.37	9.04	5.46	7.13	6.44	1.72	5.66	4.15	6.84	9.77	3.35	1.00	4.03	1.00	9.19	6.15	7.26	7.93
MEX	10.00	9.50	8.10	7.76	5.97	6.14	7.78	6.44	1.34	4.67	3.26	5.52	8.27	4.50	1.00	6.23	5.11	8.87	6.34	6.90	7.89
MKD	10.00	10.00	8.90	7.35	5.73	5.97	7.01	6.91	4.49	4.60	2.04	2.43	8.39	5.08	1.09	5.70	1.00	3.92	4.76	1.00	9.50
MNE	10.00	9.80	9.10	7.37	5.83	8.36	7.62	6.90	5.95	4.57	3.38	5.74	9.76	5.87	2.04	7.07	7.04	4.07	5.11	1.00	8.93
NAM	7.60	9.30	3.50	4.08	5.41	4.57	6.94	7.14	1.00	5.66	1.00	6.96	9.83	2.06	2.41	8.84	1.00	9.39	3.71	1.12	9.61
PAN	7.70	9.20	7.40	7.82	6.32	9.22	7.95	7.10	1.00	5.26	4.31	5.75	9.97	3.49	2.58	7.81	1.00	9.32	5.28	5.07	8.13
PER	8.80	8.40	7.20	7.05	5.72	8.34	8.23	6.96	1.00	4.28	1.39	6.54	9.90	4.65	3.11	8.98	1.15	7.80	4.30	4.31	9.31
ROU	10.00	8.80	7.20	7.36	6.06	8.15	7.88	6.76	7.41	5.23	2.14	3.88	9.60	4.11	1.20	5.53	2.12	8.53	5.63	5.27	9.73
RSA	10.00	9.30	5.90	3.48	6.28	8.42	7.99	7.23	1.00	5.86	1.20	3.44	7.50	6.44	1.10	3.09	1.00	5.70	5.11	1.08	9.20
SRB	10.00	9.90	9.20	7.42	5.60	8.36	7.78	6.90	5.95	4.26	3.38	2.98	9.76	5.87	1.02	3.05	1.00	4.07	5.04	1.53	8.77
THA	7.80	9.80	9.60	6.98	3.95	8.27	7.15	6.92	4.45	4.36	4.29	8.67	8.69	4.70	1.80	6.74	1.00	9.34	4.50	8.71	8.52
TKM	9.30	9.10	8.80	5.85	9.96	4.50	6.51	6.90	4.58	2.32	1.68	1.50	1.00	4.63	1.00	1.00	1.00	9.38	3.12	1.00	9.99
TUN	10.00	9.40	8.50	7.21	10.00	6.30	7.82	6.29	4.11	4.96	2.72	1.00	3.87	5.84	1.34	7.97	3.26	7.86	4.55	2.89	7.86
TUR	10.00	9.70	8.80	7.13	6.39	5.79	7.52	5.85	2.92	4.85	3.06	1.00	8.17	5.14	1.11	6.55	1.16	7.47	5.97	3.59	8.25
VEN	8.20	9.10	8.60	7.19	10.00	4.05	8.23	6.88	1.00	2.75	3.94	10.00	9.93	5.41	1.18	3.70	1.00	7.40	5.84	4.27	9.25
AUS	10.00	10.00	10.00	8.89	10.00	6.17	10.00	7.24	4.49	8.24	1.08	6.20	9.54	1.00	1.00	1.00	5.06	7.50	9.35	6.46	9.79
AUT	10.00	10.00	10.00	8.70	10.00	9.51	8.94	7.15	7.87	8.30	7.04	10.00	9.53	3.62	2.21	1.24	9.86	9.09	9.43	6.44	3.76
BEL	10.00	10.00	10.00	8.70	10.00	6.63	9.59	7.16	6.91	7.74	5.37	6.57	6.60	1.00	1.00	1.00	4.48	9.02	9.28	4.74	1.00
CAN	10.00	10.00	10.00	8.79	10.00	9.31	8.95	7.14	6.13	8.29	2.24	2.85	9.84	1.00	1.57	1.00	1.86	8.65	9.42	5.46	2.45
CHI	10.00	9.50	9.40	7.98	10.00	5.26	8.26	6.82	1.00	7.24	1.00	6.64	9.88	1.00	2.53	6.36	1.00	6.87	6.54	4.96	9.95
CRO	9.30	9.90	9.90	7.90	6.51	9.25	7.85	6.97	7.56	5.67	3.96	4.03	9.94	3.47	1.00	5.32	1.46	8.42	7.24	3.90	9.10
CYP	10.00	10.00	10.00	7.93	10.00	7.53	7.76	6.69	5.95	6.99	2.82	2.27	7.66	4.13	1.00	1.00	3.28	7.42	8.65	6.77	7.05
CZE	10.00	10.00	9.90	8.23	10.00	7.45	8.22	6.77	9.32	6.60	3.64	7.53	8.71	2.74	1.00	1.00	8.41	8.65	8.40	5.87	9.13
DEN	10.00	10.00	10.00	8.45	10.00	7.49	10.00	7.54	6.98	8.67	6.73	1.96	9.08	1.00	1.43	1.00	7.39	8.89	9.36	6.84	8.04

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
ESP	10.00	10.00	10.00	8.90	10.00	8.31	9.65	7.28	5.60	6.84	3.38	3.82	7.08	2.19	1.00	2.47	5.54	8.41	8.92	4.38	8.23
EST	10.00	10.00	9.50	7.43	6.30	9.43	9.15	7.08	5.33	7.12	1.00	10.00	8.60	1.00	1.05	1.00	8.88	8.94	7.80	6.28	9.95
FIN	10.00	10.00	10.00	8.65	10.00	8.76	10.00	8.20	8.96	8.66	4.87	4.25	9.85	1.26	2.33	1.00	8.03	9.10	9.25	5.03	8.79
FRA	10.00	10.00	10.00	8.83	9.69	8.65	9.47	7.34	6.31	7.29	5.49	8.24	8.50	1.54	1.00	4.01	3.97	8.77	9.14	4.33	2.90
GBR	10.00	10.00	10.00	8.57	10.00	8.16	8.91	7.37	3.95	8.07	5.13	9.02	9.12	2.95	1.00	1.17	6.53	8.24	9.28	5.83	6.32
GER	10.00	10.00	10.00	8.74	10.00	7.86	9.45	7.39	7.87	8.04	5.89	10.00	7.90	3.45	1.00	1.00	7.21	9.01	9.21	4.15	3.19
GRE	10.00	10.00	9.80	8.68	10.00	7.71	10.00	6.73	5.66	6.22	2.35	4.20	8.73	1.91	1.00	1.56	5.70	5.01	8.75	4.36	1.00
IRL	10.00	10.00	10.00	8.46	10.00	9.19	9.69	7.52	6.13	8.19	4.61	1.00	9.76	1.89	1.00	1.00	2.20	9.16	9.62	6.33	7.74
ISL	10.00	10.00	10.00	8.92	10.00	10.00	9.58	8.00	5.95	8.76	1.10	4.12	9.99	1.00	7.83	2.69	1.00	7.70	9.48	9.04	2.57
ISR	10.00	10.00	10.00	8.73	9.68	5.77	9.13	6.90	4.11	6.12	1.77	7.54	1.00	4.13	1.00	1.25	2.40	9.00	8.64	4.81	1.72
ITA	10.00	10.00	10.00	8.89	10.00	8.22	9.08	6.79	4.92	6.19	5.81	7.93	7.63	2.43	1.00	2.13	8.95	8.38	8.93	5.42	1.00
JPN	10.00	10.00	10.00	9.27	10.00	8.78	8.74	6.43	10.00	7.30	5.96	5.46	7.91	5.09	1.00	1.00	1.00	8.53	9.15	6.81	1.00
KOR	10.00	10.00	10.00	8.19	8.61	8.49	9.89	6.15	7.19	6.48	5.37	2.50	6.35	4.50	1.00	1.00	1.24	9.25	8.54	7.23	9.05
KSA	10.00	9.10	8.80	7.07	8.75	4.24	7.67	5.54	6.44	4.24	1.88	10.00	1.00	4.83	1.00	1.00	1.00	5.70	7.96	3.33	9.75
KUW	10.00	9.10	8.80	7.95	8.72	4.48	8.31	6.36	6.44	5.54	1.00	1.00	1.00	4.03	1.00	1.00	1.00	9.05	9.40	8.44	9.84
LAT	10.00	9.90	7.80	7.32	6.64	9.05	9.02	7.40	4.92	6.36	7.44	8.07	9.88	1.00	3.14	6.50	8.80	8.54	7.09	5.38	9.62
LTU	10.00	10.00	9.40	7.35	10.00	8.59	9.28	7.22	5.54	6.43	4.28	7.20	9.05	1.64	1.07	5.97	6.79	8.26	7.30	6.51	9.68
LUX	10.00	10.00	10.00	8.66	10.00	7.03	7.64	6.80	6.19	8.62	6.25	10.00	9.81	1.00	1.00	1.00	4.76	9.42	10.00	6.44	9.73
MLT	10.00	10.00	7.10	8.69	10.00	2.39	7.85	6.63	1.00	7.61	2.36	1.00	2.87	2.90	1.00	3.65	1.00	5.25	8.14	5.23	4.08
NED	10.00	10.00	10.00	8.70	10.00	7.32	9.77	7.40	6.25	8.32	6.77	7.28	8.74	1.00	1.00	1.00	4.59	9.07	9.48	6.99	4.97
NOR	10.00	10.00	10.00	8.80	10.00	9.51	9.93	8.24	8.52	8.47	6.60	5.40	9.92	1.00	4.26	1.98	6.93	9.15	9.85	7.78	5.93
NZL	10.00	10.00	10.00	8.62	10.00	9.92	10.00	7.86	4.49	8.55	3.92	9.27	9.85	1.00	3.20	1.89	1.22	8.17	8.58	6.92	9.52
OMA	9.20	9.10	8.80	7.53	8.95	4.42	7.23	5.96	6.44	5.70	1.79	4.66	1.61	2.83	1.00	1.00	1.00	1.81	8.02	2.23	9.95
POL	10.00	10.00	9.40	7.78	10.00	8.16	8.73	6.95	6.51	5.97	2.54	10.00	7.94	4.35	1.00	2.02	3.70	8.49	6.92	3.83	7.31
POR	10.00	9.90	9.90	8.33	10.00	7.79	8.94	7.05	3.50	7.01	3.77	2.88	8.77	2.27	1.71	4.67	8.09	5.63	8.06	4.49	2.38
QAT	10.00	10.00	10.00	7.72	10.00	4.48	7.77	5.95	6.44	5.89	3.27	1.00	1.00	1.00	1.00	1.00	1.00	5.00	10.00	9.51	9.79
RUS	10.00	9.70	8.70	6.51	6.92	8.24	8.17	6.99	4.40	3.50	1.93	4.60	9.85	4.85	1.00	1.00	1.00	6.72	6.58	5.43	9.88
SLO	10.00	10.00	10.00	8.42	6.78	9.30	9.26	6.94	8.69	6.91	3.73	6.17	9.71	4.05	1.05	2.08	7.77	9.11	8.69	6.15	9.45
SUI	10.00	10.00	10.00	9.02	10.00	8.69	8.54	7.36	6.38	8.69	7.26	10.00	9.51	5.08	1.55	4.15	9.36	9.15	9.50	7.87	6.30
SVK	9.30	10.00	10.00	7.83	10.00	8.92	7.72	6.82	8.03	6.43	3.95	10.00	9.86	3.61	1.00	3.05	7.82	8.41	7.69	3.32	9.17
SWE	10.00	10.00	10.00	9.01	10.00	9.62	9.46	8.14	8.44	8.57	6.61	4.94	9.85	1.00	2.87	4.71	9.16	9.29	9.37	5.42	8.37
TPE	10.00	10.00	10.00	8.53	10.00	6.26	9.31	6.60	6.19	6.43	5.09	2.45	7.42	2.51	1.00	1.00	1.00	5.00	8.99	6.76	8.73
TRI	9.00	9.40	9.20	7.12	10.00	4.63	6.41	7.24	4.32	5.43	4.45	4.80	9.40	5.33	1.00	1.00	1.00	1.03	7.54	5.74	9.32
UAE	10.00	10.00	9.70	7.44	8.50	4.48	6.72	6.22	6.44	6.03	3.25	2.36	1.00	1.93	1.00	1.00	1.00	9.49	9.80	7.33	9.77
URU	10.00	10.00	10.00	7.86	7.31	6.34	9.08	6.91	2.62	6.34	5.14	1.00	9.74	1.00	2.47	8.13	7.97	7.65	5.58	4.00	3.87
USA	10.00	9.90	10.00	8.34	10.00	7.75	9.48	7.18	3.20	7.50	3.04	6.80	8.44	1.00	1.00	1.00	1.26	7.52	9.71	6.30	1.81

S.F. Sufficient Food; **S.D.** Sufficient to Drink; **S.S.** Safe Sanitation; **H.L.** Healthy Life; **C.A.** Clean Air; **C.W.** Clean Water; **Ed.** Education;
G.E. Gender Equality; **I.D.** Income Distribution; **G.Go.** Good Governance; **A.Q.** Air Quality; **B.** Biodiversity; **R.W.R.** Renewable Water Resources; **C.** Consumption;
R.E. Renewable Energy; **G.Ga.** Greenhouse Gases; **O.F.** Organic Farming; **G.S.** Genuine Savings; **GDP** Gross Domestic Product; **Em.** Employment; **P.D.** Public Debt.

Tabla C.3: Países por niveles de ingresos y variables (año 2010)

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
BAN	7.30	8.00	5.30	6.32	1.37	8.72	4.87	6.70	8.45	3.15	6.37	1.00	9.71	8.25	3.12	9.68	1.00	9.30	1.09	6.51	8.37
BDI	3.80	7.20	4.60	2.89	4.85	3.98	5.58	6.70	7.92	2.83	4.77	2.43	9.77	7.05	8.60	9.85	1.00	1.00	1.00	1.00	8.56
BEN	8.80	7.50	1.20	4.94	3.94	3.72	5.78	5.72	5.39	4.58	7.05	10.00	9.95	6.53	5.91	9.55	1.00	7.43	1.05	9.32	9.05
BUR	9.10	7.60	1.10	2.87	3.50	5.05	3.52	6.16	5.33	4.28	6.37	7.12	9.21	5.82	8.60	9.85	1.00	7.01	1.00	7.19	9.21
CAF	6.00	6.70	3.40	3.43	4.83	4.52	3.90	6.10	3.26	2.38	5.40	8.87	10.00	5.54	8.60	9.85	1.00	2.63	1.00	4.49	8.49
CAM	7.80	6.10	2.90	5.17	4.20	9.00	6.17	6.48	4.00	3.43	6.44	10.00	9.95	6.94	6.48	9.74	1.00	8.56	1.37	8.44	9.11
CHA	6.30	5.00	1.00	3.21	3.99	4.47	4.38	5.33	4.80	1.95	5.40	4.70	9.91	3.85	8.60	9.85	1.00	1.00	1.19	2.42	8.89
COD	3.10	4.60	2.30	3.21	2.91	4.79	5.31	6.10	3.47	1.63	6.78	5.00	10.00	7.54	9.63	9.95	1.00	3.52	1.00	2.42	8.99
ETH	5.90	3.80	1.20	4.14	4.20	4.28	5.50	6.02	8.40	3.12	6.85	9.20	9.54	6.52	9.40	9.93	1.00	8.25	1.00	1.29	8.57
GAM	8.10	9.20	6.70	4.62	1.00	4.86	5.45	6.76	2.08	3.98	6.37	1.00	9.91	6.32	8.60	9.85	1.00	7.08	1.30	2.52	3.06
GBS	7.80	6.00	2.10	3.76	5.01	4.55	6.54	6.20	5.97	2.92	6.37	10.00	9.94	6.63	8.60	9.85	1.00	9.07	1.00	1.77	6.98
GUI	8.30	7.10	1.90	4.70	4.73	4.55	5.23	6.20	3.74	2.09	6.37	3.21	9.93	4.74	8.60	9.85	1.00	1.00	1.00	1.77	1.40
HAI	4.30	6.30	1.70	4.36	5.05	3.97	5.87	6.90	1.00	2.72	6.77	1.00	9.14	8.19	7.17	9.76	1.00	9.06	1.00	1.00	9.63
KEN	6.90	5.90	3.10	4.53	5.21	5.79	6.33	6.50	1.92	3.64	5.85	5.87	9.11	6.84	8.33	9.78	1.00	8.56	1.17	1.00	6.84
KGZ	9.00	9.00	9.30	6.17	4.87	4.13	7.76	6.97	7.73	3.42	3.82	3.47	7.94	7.32	3.40	8.88	1.00	7.14	1.54	4.32	4.54
LBR	6.70	6.80	1.70	4.50	4.60	4.85	6.32	6.20	4.47	3.07	6.37	1.00	9.99	6.00	8.60	9.85	1.00	1.09	1.00	5.71	9.69
MAD	7.50	4.10	1.10	5.19	5.01	4.76	6.46	6.71	3.18	4.31	4.77	1.27	9.56	6.31	8.60	9.85	1.00	6.83	1.00	7.71	9.92
MAW	7.20	8.00	5.60	3.53	5.01	3.03	6.33	6.82	5.26	4.32	4.77	7.51	9.44	7.55	8.60	9.85	1.00	7.47	1.00	1.00	8.70
MLI	8.80	5.60	3.60	2.57	3.54	7.86	4.49	5.68	5.07	4.38	6.37	1.22	9.35	4.45	8.60	9.85	1.00	8.11	1.00	1.77	9.08
MOZ	6.20	4.70	1.70	3.36	5.02	4.66	5.89	7.33	2.43	4.40	5.02	7.41	9.97	7.66	9.59	9.91	1.00	1.93	1.00	1.22	8.30
MYA	8.40	7.10	8.10	5.53	3.38	4.57	5.74	6.80	1.72	1.37	7.02	2.61	9.72	4.33	7.14	9.84	1.00	5.00	1.00	6.69	7.92
NEP	8.40	8.80	3.10	6.06	1.80	4.60	5.76	6.08	3.51	3.22	5.52	8.50	9.53	7.68	9.00	9.90	1.00	9.45	1.00	1.00	8.62
NIG	8.00	4.80	1.00	2.25	3.64	4.22	2.93	6.20	3.32	3.64	6.37	3.54	9.30	2.76	8.60	9.85	1.00	8.99	1.00	1.77	9.65
PRK	6.70	10.00	5.90	6.54	7.21	4.41	5.95	6.70	6.44	1.98	3.91	1.00	8.88	8.13	1.12	7.14	1.00	5.00	1.28	3.36	9.05
RWA	6.60	6.50	5.40	4.04	4.00	4.61	6.36	6.70	2.98	4.15	4.77	5.00	9.84	6.89	8.60	9.85	1.00	8.37	1.00	1.00	9.40
SLE	6.50	4.90	1.30	2.20	4.74	4.90	4.67	6.20	4.31	3.47	6.37	2.15	9.97	6.56	8.60	9.85	1.00	4.76	1.00	1.77	3.43
TAN	6.60	5.40	2.40	4.20	5.02	8.50	5.66	6.83	6.56	4.43	6.37	10.00	9.46	6.12	8.94	9.86	1.00	8.65	1.00	1.00	8.25
TJK	7.00	7.00	9.40	5.89	4.17	4.51	7.21	6.60	6.83	2.99	3.80	2.07	2.52	7.73	5.50	9.56	1.00	7.95	1.30	8.03	8.60
TOG	7.00	6.00	1.20	4.73	4.12	4.42	6.27	6.20	3.28	3.17	7.00	5.52	9.89	7.17	8.34	9.81	1.00	1.31	1.00	1.77	8.92
UGA	7.90	6.70	4.80	4.51	4.93	3.95	6.75	7.17	4.23	3.92	4.77	5.13	9.95	4.82	8.60	9.85	3.20	8.76	1.00	6.57	9.39
ZIM	7.00	8.20	4.40	2.85	5.25	7.19	5.70	6.57	1.67	1.67	3.64	10.00	7.90	6.88	7.02	9.36	1.00	1.41	1.00	1.00	5.43
ARM	7.80	9.60	9.00	7.13	4.60	5.10	7.51	6.67	7.94	4.56	3.41	4.00	6.36	6.51	1.00	8.29	1.00	9.08	3.05	1.54	8.83
BHU	6.70	9.20	6.50	5.84	5.64	4.28	6.02	6.10	2.85	5.17	5.52	10.00	9.96	1.00	5.29	9.63	1.00	9.69	3.09	6.70	2.53
BOL	7.30	8.60	2.50	5.99	5.89	8.34	8.12	6.75	1.00	3.52	5.63	9.26	9.97	2.53	2.11	8.74	1.00	3.96	2.75	5.95	8.35
CGO	8.50	7.10	3.00	4.47	2.82	4.90	6.12	6.10	2.68	2.79	2.70	4.84	10.00	6.96	5.98	9.65	1.00	1.00	2.62	2.42	9.37
CIV	8.60	8.00	2.30	4.74	4.53	5.09	3.92	5.69	2.17	2.28	7.11	10.00	9.83	6.82	7.55	9.66	1.00	5.21	1.21	1.77	3.24
CMR	7.90	7.40	4.70	3.75	3.36	5.29	5.89	6.11	3.57	3.36	5.77	4.50	9.97	6.70	7.24	9.77	1.00	7.55	1.49	7.48	9.78

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
EGY	10.00	9.90	9.40	6.54	8.47	6.24	7.20	5.90	7.05	3.94	4.21	3.04	1.00	6.25	1.00	7.76	2.46	6.97	3.58	3.86	2.16
ESA	9.10	8.70	8.70	6.70	6.09	4.98	7.06	6.60	1.00	4.82	5.19	1.00	9.45	5.14	5.54	8.99	1.03	5.07	4.06	4.45	6.78
GEO	8.80	9.80	9.50	7.55	5.61	4.02	7.49	6.60	3.13	4.65	5.64	1.70	9.71	6.55	3.35	8.91	1.00	2.32	2.94	1.85	8.34
GHA	10.00	8.20	1.30	4.70	4.06	7.78	6.40	6.78	2.84	5.13	5.94	6.98	9.82	5.22	7.10	9.68	1.00	5.02	1.74	1.77	7.47
GUA	7.90	9.40	8.10	6.41	5.35	5.99	7.07	6.24	1.00	3.91	4.90	10.00	9.74	5.54	5.97	9.26	1.00	6.69	2.97	8.35	9.36
GUY	9.30	9.40	8.10	6.37	6.59	4.28	7.20	7.09	1.00	4.24	4.49	2.38	9.93	3.90	1.31	5.15	1.00	2.62	3.84	3.33	4.57
HON	8.80	8.60	7.10	6.91	5.41	4.97	7.21	6.93	1.00	3.93	4.26	6.93	9.88	5.54	4.57	8.93	1.00	8.90	2.61	6.44	9.25
INA	8.70	8.00	5.20	6.66	5.43	6.22	7.11	6.61	5.94	4.00	3.89	3.21	9.44	6.99	3.61	8.45	1.00	8.18	2.60	4.55	9.20
IND	7.90	8.80	3.10	5.70	1.00	7.89	6.46	6.15	6.61	4.66	3.89	2.41	6.61	8.12	2.82	8.74	1.32	9.11	2.05	6.44	2.71
LAO	7.70	5.70	5.30	4.89	3.95	8.51	6.09	6.80	7.56	3.02	4.87	8.31	9.87	6.07	5.29	9.63	1.00	4.35	1.58	7.79	4.15
MAR	10.00	8.10	6.90	6.95	6.93	6.29	6.04	5.77	4.59	4.45	3.16	1.00	5.66	7.10	1.00	8.61	1.00	9.30	2.86	4.03	6.56
MDA	9.40	9.00	7.90	6.71	6.38	4.88	7.05	7.16	5.38	4.10	4.51	1.00	8.36	8.02	1.00	8.22	1.11	9.12	1.92	5.27	9.24
MGL	7.40	7.60	5.00	6.16	5.19	4.50	8.24	7.19	5.91	4.49	1.30	6.70	9.85	1.00	1.00	5.80	1.00	7.85	2.43	7.19	6.69
MTN	9.30	4.90	2.60	3.42	4.57	4.58	4.65	6.15	4.80	3.26	6.37	1.00	8.60	1.26	8.60	9.85	1.00	5.75	1.42	1.00	1.00
NCA	8.10	8.50	5.20	7.23	5.42	4.23	7.01	7.18	1.00	3.82	4.26	10.00	9.93	5.75	5.55	9.26	2.92	6.66	1.94	4.42	1.44
NGR	9.40	5.80	3.20	3.78	2.96	4.48	5.58	6.06	3.10	2.92	6.26	6.30	9.64	5.41	8.15	9.67	1.00	9.34	1.57	1.39	9.60
PAK	7.40	9.00	4.50	5.96	1.88	6.26	4.29	5.46	7.95	2.83	4.12	4.91	2.57	8.27	3.70	9.20	1.00	7.67	1.79	5.79	4.27
PAR	8.90	8.60	7.00	7.14	5.53	5.18	7.09	6.80	1.00	3.61	6.23	2.72	9.99	1.00	10.00	9.40	1.00	7.88	2.84	5.27	9.69
PHL	8.50	9.10	7.60	6.76	5.54	8.93	7.88	7.65	3.82	4.03	3.91	2.52	8.30	6.73	4.25	9.22	1.00	8.98	2.37	4.74	8.00
PNG	6.90	4.00	4.50	5.99	5.08	3.96	3.70	7.10	1.82	3.69	5.96	1.00	10.00	3.73	3.45	9.63	1.00	5.00	1.53	8.27	9.04
SEN	8.30	6.90	5.10	3.89	2.76	8.36	4.76	6.41	4.70	4.50	4.86	10.00	9.43	6.65	4.61	9.57	1.00	8.26	1.27	1.00	8.64
SRI	8.10	9.00	9.10	7.41	5.18	9.17	6.46	7.46	4.97	3.93	3.65	7.48	7.55	6.92	5.62	9.40	1.90	8.95	2.94	5.54	1.00
SUD	7.80	5.70	3.40	4.99	3.87	6.52	3.86	6.00	1.72	1.68	6.71	2.09	4.24	4.61	7.05	9.70	1.00	2.00	1.52	2.26	2.37
SYR	10.00	8.90	9.60	7.10	8.08	4.50	6.64	5.93	2.84	3.08	2.74	1.00	1.36	7.80	1.00	6.81	1.00	6.58	3.00	4.45	9.09
UKR	10.00	9.80	9.50	6.64	6.92	2.98	9.12	6.87	8.81	4.21	1.88	1.80	7.24	6.42	1.00	3.30	1.51	8.60	3.68	4.13	8.24
UZB	8.90	8.70	10.00	6.76	3.97	3.80	7.17	6.91	5.67	2.61	3.06	1.13	1.00	7.86	1.00	5.80	1.00	1.08	1.88	9.80	9.83
VIE	8.90	9.40	7.50	7.08	3.10	7.27	6.49	6.78	6.00	3.89	4.38	2.29	9.07	6.84	3.46	8.80	1.00	8.39	1.96	6.31	8.42
YEM	6.90	6.20	5.20	5.24	3.47	4.48	5.47	4.60	5.42	2.86	2.80	1.00	1.00	7.89	1.00	9.07	1.00	1.00	1.68	2.32	8.15
ZAM	5.70	6.00	4.90	3.40	5.12	4.17	5.91	6.29	1.00	4.41	1.00	10.00	9.83	7.55	9.25	9.87	1.00	4.47	1.05	2.47	9.28
ALB	10.00	9.70	9.80	7.02	10.00	8.25	6.79	6.73	6.98	4.64	5.75	4.21	9.56	6.44	2.63	8.77	1.00	7.97	4.05	2.70	5.01
ALG	10.00	8.30	9.50	6.97	10.00	5.83	7.24	6.05	6.00	3.45	6.17	3.12	4.74	6.54	1.00	7.39	1.00	9.31	3.94	3.62	9.80
ANG	5.90	5.00	5.70	3.05	4.87	5.18	5.81	6.71	1.00	3.02	7.22	6.03	9.96	7.43	6.35	9.29	1.00	1.00	3.37	1.00	8.50
ARG	10.00	9.70	9.00	7.70	10.00	8.43	9.18	7.19	1.00	4.47	6.17	2.63	9.60	3.72	1.00	5.68	5.30	8.16	6.49	4.20	6.97
AZE	8.90	8.00	4.50	6.69	6.32	4.43	6.59	6.45	10.00	3.52	2.80	3.58	6.53	6.75	1.00	6.63	1.06	7.99	4.98	5.46	9.80
BIH	10.00	9.90	9.50	7.53	5.52	9.35	7.42	7.00	5.53	4.34	1.00	1.00	9.91	5.07	1.01	5.84	1.00	5.00	4.25	1.00	8.29
BLR	10.00	10.00	9.30	7.11	10.00	4.43	9.04	7.10	8.28	3.24	3.91	3.61	9.25	3.94	1.00	3.28	1.00	9.23	6.00	9.14	8.14
BOT	7.50	9.50	6.00	3.59	5.59	4.29	6.98	6.88	1.00	6.44	1.75	10.00	9.84	4.71	2.21	7.69	1.00	9.44	6.39	4.72	9.61
BRA	9.40	9.70	8.00	6.83	6.92	8.54	8.75	6.65	1.00	5.08	4.61	10.00	9.93	2.15	4.44	8.11	1.54	8.12	5.30	4.46	3.49
BUL	9.00	10.00	10.00	7.60	10.00	8.11	7.78	6.98	7.95	5.51	3.18	4.43	7.13	5.68	1.00	3.57	1.28	6.76	5.98	4.40	9.64

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
CHN	9.00	8.90	5.50	7.51	1.97	6.80	6.82	6.88	4.24	4.07	1.82	8.02	8.05	6.81	1.26	5.10	1.00	9.54	3.88	6.51	8.82
COL	9.00	9.20	7.40	7.19	6.47	5.46	8.33	6.93	1.00	4.23	6.11	10.00	9.94	5.38	2.58	8.68	1.00	5.62	4.86	3.01	8.62
CRC	10.00	9.70	9.50	7.98	6.49	4.77	7.41	7.19	1.41	6.11	6.02	8.82	9.76	4.37	5.12	8.54	1.05	9.03	5.42	4.32	9.10
CUB	10.00	9.40	9.10	8.27	7.80	8.86	10.00	7.25	2.84	3.39	1.93	2.21	8.02	6.42	1.38	7.79	1.00	5.00	4.96	8.52	8.74
DOM	7.60	8.60	8.30	6.84	6.66	4.65	7.32	6.77	1.49	4.46	4.08	10.00	8.35	7.42	2.33	8.02	7.94	3.81	4.50	2.26	9.13
ECU	8.50	9.40	9.20	7.11	10.00	8.34	8.00	7.07	1.00	3.27	4.72	10.00	9.64	5.01	1.47	8.11	2.12	6.19	4.22	4.27	9.66
GAB	10.00	8.70	3.30	7.28	5.95	4.21	7.58	6.10	4.24	3.78	4.51	7.29	9.99	4.39	5.86	8.39	1.00	5.98	6.42	1.22	9.32
HUN	10.00	10.00	10.00	7.72	10.00	7.40	8.99	6.72	7.88	6.63	4.14	2.57	9.46	5.61	1.00	4.72	5.16	7.89	7.33	3.65	1.32
IRI	10.00	9.10	8.30	6.53	8.08	4.98	7.00	5.93	5.02	2.86	2.76	3.44	3.23	6.71	1.00	3.12	1.00	8.09	5.80	3.04	9.64
IRQ	9.20	7.90	7.30	6.60	3.84	4.27	6.27	6.20	6.44	1.74	2.15	1.00	1.27	8.44	1.00	7.61	1.00	5.00	2.31	1.83	1.00
JAM	10.00	9.40	8.30	7.47	6.32	4.62	8.71	7.04	2.88	4.92	2.25	3.67	9.38	6.03	1.16	5.60	1.00	6.72	4.70	3.20	1.00
JOR	10.00	9.60	9.80	7.03	9.35	3.00	7.95	6.05	5.64	5.16	2.67	1.00	1.00	5.15	1.00	6.80	1.00	7.12	3.39	2.74	3.16
KAZ	10.00	9.50	9.70	6.20	6.53	4.34	9.05	7.06	7.81	3.98	3.09	1.26	7.11	4.94	1.00	1.00	1.00	1.72	5.55	5.18	9.81
LBA	10.00	9.10	9.70	7.52	10.00	4.94	9.27	6.00	6.44	3.46	4.15	1.00	1.00	5.68	1.00	2.36	1.00	5.00	6.39	1.00	9.97
LIB	10.00	10.00	9.80	6.88	10.00	4.06	8.01	6.08	5.95	3.42	2.61	1.00	8.14	4.76	1.00	6.20	1.00	5.17	6.40	4.54	1.00
MAS	10.00	10.00	9.60	7.40	9.73	5.46	7.13	6.48	5.24	5.53	4.15	6.84	9.77	3.23	1.00	3.31	1.00	9.07	6.29	6.91	6.23
MEX	10.00	9.40	8.50	7.60	6.27	6.14	8.00	6.58	1.00	4.73	3.26	5.81	8.26	4.40	1.01	6.21	3.21	8.80	6.14	5.80	7.92
MKD	10.00	10.00	8.90	7.35	5.73	5.97	7.16	7.00	2.87	4.83	2.04	2.44	8.39	4.81	1.00	5.60	1.00	4.15	4.99	1.00	9.33
MNE	10.00	9.80	9.20	7.31	5.83	8.36	8.19	7.00	5.22	5.14	3.38	5.74	9.75	7.16	1.86	6.95	1.00	4.07	5.23	1.00	7.98
NAM	8.10	9.20	3.30	5.21	5.41	4.57	6.94	7.24	1.00	5.97	1.00	6.96	9.83	4.84	1.98	8.39	1.00	9.40	3.78	1.00	9.67
PAN	8.50	9.30	6.90	7.85	6.32	9.22	7.86	7.07	1.00	5.38	4.31	5.75	9.97	3.52	2.85	8.06	1.00	9.38	5.75	6.07	8.33
PER	8.50	8.20	6.80	7.33	5.72	8.34	8.26	6.90	1.00	4.40	1.39	6.54	9.90	4.57	2.35	8.75	1.57	7.79	4.64	4.32	9.34
ROU	10.00	8.80	7.20	7.49	6.06	8.15	8.34	6.83	7.05	5.36	2.14	3.88	9.68	5.75	1.35	5.69	2.25	8.68	5.77	5.04	8.98
RSA	10.00	9.10	7.70	3.79	6.28	8.42	7.99	7.53	1.00	5.81	1.20	3.44	7.50	6.51	1.00	2.07	1.00	3.00	5.22	1.00	8.69
SRB	9.20	9.90	9.20	7.36	5.77	8.36	7.84	7.00	8.63	4.53	3.38	2.99	9.75	6.47	1.00	3.21	1.00	4.07	5.15	1.76	7.66
THA	8.40	9.80	9.60	6.73	4.03	8.27	7.24	6.91	4.29	4.40	4.29	8.67	8.69	5.23	1.94	6.63	1.00	9.16	4.59	8.61	7.95
TKM	9.40	9.10	9.80	6.08	9.76	4.50	6.64	6.90	4.40	2.45	1.68	1.50	1.00	4.79	1.00	1.00	1.00	9.38	3.65	1.00	9.79
TUN	10.00	9.40	8.50	7.27	10.00	6.30	7.86	6.27	4.20	4.89	2.72	1.00	3.87	5.93	1.33	7.92	3.57	6.41	4.84	2.65	8.20
TUR	10.00	9.90	9.00	7.15	6.48	5.79	7.71	5.88	3.28	4.89	3.06	1.00	8.17	5.30	1.00	6.29	1.03	7.67	5.84	2.46	8.00
VEN	9.20	9.10	8.60	7.15	10.00	4.05	8.89	6.86	2.29	2.71	3.94	10.00	9.93	4.82	1.20	3.97	1.00	7.16	5.88	4.55	8.23
AUS	10.00	10.00	10.00	8.92	10.00	6.17	10.00	7.27	4.40	8.30	1.08	6.24	9.54	1.00	1.00	1.00	5.12	7.37	9.41	5.72	9.51
AUT	10.00	10.00	10.00	8.79	10.00	9.51	9.05	7.09	7.81	8.25	7.04	10.00	9.53	2.91	2.53	1.53	9.87	9.10	9.43	6.19	2.35
BEL	10.00	10.00	10.00	8.80	10.00	6.63	9.60	7.51	6.86	7.52	5.37	6.58	6.60	1.00	1.00	1.00	4.78	8.89	9.28	4.54	1.00
CAN	10.00	10.00	10.00	8.83	10.00	9.31	8.95	7.37	6.04	8.30	2.24	2.94	9.84	1.62	1.68	1.00	2.07	8.42	9.40	4.36	1.05
CHI	10.00	9.60	9.60	7.98	10.00	5.26	8.48	7.01	1.16	7.31	1.00	6.64	9.88	1.40	2.44	5.91	1.00	5.78	6.65	3.39	9.86
CRO	10.00	9.90	9.90	7.91	6.51	9.25	8.00	6.94	8.26	5.76	3.96	4.78	9.94	3.76	1.00	5.27	1.77	8.67	7.22	4.04	8.11
CYP	10.00	10.00	10.00	8.03	10.00	7.53	8.13	6.64	5.95	7.03	2.82	2.27	8.16	3.78	1.00	1.00	3.28	6.56	8.73	5.86	4.32
CZE	10.00	10.00	9.80	8.34	10.00	7.45	8.42	6.85	9.25	6.78	3.64	7.53	8.71	1.90	1.00	1.00	8.66	8.63	8.39	5.14	8.49
DEN	10.00	10.00	10.00	8.51	10.00	7.49	9.97	7.72	6.91	8.59	6.73	2.04	8.93	1.00	1.69	1.18	7.56	8.85	9.26	5.45	7.86

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
ESP	10.00	10.00	10.00	8.93	10.00	8.31	9.87	7.55	5.64	6.90	3.38	3.82	7.10	1.36	1.00	3.04	6.78	8.10	8.85	1.65	4.34
EST	10.00	9.80	9.50	7.82	6.30	9.43	8.94	7.02	5.62	7.07	1.00	10.00	8.60	1.00	1.19	1.00	9.10	8.75	7.25	2.53	9.91
FIN	10.00	10.00	10.00	8.73	10.00	8.76	10.00	8.26	8.91	8.53	4.87	4.25	9.85	4.17	2.58	1.00	8.06	8.99	9.12	4.39	7.10
FRA	10.00	10.00	10.00	8.95	9.95	8.65	9.45	7.03	6.40	7.48	5.49	8.55	8.50	1.39	1.00	4.23	4.12	8.67	9.12	3.87	1.24
GBR	10.00	10.00	10.00	8.64	10.00	8.16	8.89	7.46	4.04	7.96	5.13	9.03	9.12	3.00	1.00	1.65	6.81	8.14	9.20	4.74	1.92
GER	10.00	10.00	10.00	8.85	10.00	7.86	9.45	7.53	7.86	8.01	5.89	10.00	7.90	2.55	1.00	1.00	7.38	8.94	9.20	4.61	1.18
GRE	10.00	10.00	9.80	8.76	10.00	7.71	9.86	6.91	5.54	6.14	2.35	4.80	8.73	1.93	1.00	1.61	6.17	3.11	8.73	3.92	1.00
IRL	10.00	10.00	9.90	8.61	10.00	9.19	10.00	7.77	6.14	8.20	4.61	1.00	9.76	1.93	1.00	1.00	2.37	8.91	9.46	3.07	1.00
ISL	10.00	10.00	10.00	8.89	10.00	10.00	9.62	8.50	5.95	8.20	1.10	6.59	9.99	1.00	8.13	3.43	1.00	1.58	9.40	4.49	1.00
ISR	10.00	10.00	10.00	8.76	9.81	5.77	9.14	6.96	3.98	6.18	1.77	7.54	1.00	4.32	1.00	1.24	2.67	8.73	8.74	4.69	1.82
ITA	10.00	10.00	10.00	9.09	10.00	8.22	9.10	6.77	4.89	6.09	5.81	7.93	7.63	2.66	1.00	2.73	8.60	7.95	8.80	4.59	1.00
JPN	10.00	10.00	10.00	9.27	10.00	8.78	8.80	6.52	9.98	7.41	5.96	5.46	7.91	5.35	1.00	1.00	1.00	8.18	9.07	6.02	1.00
KOR	10.00	9.80	10.00	8.36	8.90	8.49	10.00	6.34	7.22	6.41	5.37	2.51	6.35	4.20	1.00	1.00	1.54	9.25	8.69	6.94	8.83
KSA	10.00	9.10	8.80	7.11	8.63	4.24	7.88	5.71	6.44	4.50	1.88	10.00	1.00	4.26	1.00	1.00	1.00	4.35	7.98	3.51	9.83
KUW	10.00	9.90	10.00	8.01	8.53	4.48	8.31	6.32	6.44	5.42	1.00	1.00	1.00	3.22	1.00	1.00	1.00	8.97	9.41	8.49	9.82
LAT	10.00	9.90	7.80	7.25	6.64	9.05	8.79	7.43	5.38	6.30	7.44	8.07	9.88	1.00	3.07	6.51	8.98	8.45	6.40	1.77	8.26
LTU	10.00	10.00	9.40	7.17	10.00	8.59	9.14	7.13	5.68	6.37	4.28	7.20	9.05	1.00	1.08	5.74	6.84	7.65	6.98	2.54	8.45
LUX	10.00	10.00	10.00	8.93	10.00	7.03	7.55	7.23	6.19	8.41	6.25	10.00	9.81	1.00	1.00	1.00	4.91	9.20	10.00	5.60	9.56
MLT	10.00	10.00	10.00	8.72	10.00	2.39	7.63	6.70	1.00	7.51	2.36	1.00	2.87	2.61	1.00	3.77	1.00	5.25	8.23	5.00	2.71
NED	10.00	10.00	10.00	8.79	10.00	7.32	9.92	7.44	6.28	8.35	6.77	7.58	8.83	1.00	1.00	1.00	4.80	8.81	9.49	6.89	3.97
NOR	10.00	10.00	10.00	8.88	10.00	9.51	9.73	8.40	8.61	8.33	6.60	5.43	9.92	1.00	4.49	2.13	7.17	9.07	9.84	7.29	6.88
NZL	10.00	10.00	10.00	8.62	10.00	9.92	10.00	7.81	4.43	8.43	3.92	10.00	9.85	1.00	3.29	2.06	1.97	7.03	8.55	5.42	8.91
OMA	9.20	8.80	8.80	7.58	8.85	4.42	7.44	5.95	6.44	5.79	1.79	4.66	1.61	2.40	1.00	1.00	1.00	1.69	8.37	2.23	9.94
POL	10.00	10.00	9.00	7.77	10.00	8.16	8.76	7.04	6.33	6.26	2.54	10.00	8.06	3.43	1.00	2.17	3.98	8.45	7.27	4.42	5.80
POR	10.00	9.90	10.00	8.50	10.00	7.79	9.44	7.17	3.53	7.17	3.77	3.07	8.77	2.55	1.77	4.99	7.81	3.82	8.06	3.88	1.00
QAT	10.00	10.00	10.00	7.77	10.00	4.48	6.05	6.06	1.00	6.23	3.27	1.00	1.00	1.50	1.00	1.00	1.00	5.00	10.00	9.51	8.99
RUS	10.00	9.60	8.70	6.85	6.92	8.24	8.37	7.04	3.30	3.55	1.93	4.60	9.85	4.41	1.00	1.00	1.00	8.57	6.59	4.32	9.79
SLO	10.00	9.90	10.00	8.58	6.78	9.30	9.37	7.05	7.61	6.95	3.73	6.54	9.70	3.35	1.10	1.72	7.83	8.97	8.64	5.55	8.37
SUI	10.00	10.00	10.00	9.09	10.00	8.69	8.54	7.56	6.42	8.47	7.26	10.00	9.51	3.78	1.78	4.32	9.36	8.69	9.52	6.90	6.79
SVK	10.00	10.00	10.00	7.85	10.00	8.92	7.93	6.78	8.02	6.56	3.95	10.00	9.86	3.90	1.00	3.30	8.37	8.19	7.81	3.00	8.13
SWE	10.00	10.00	10.00	9.05	10.00	9.62	9.27	8.02	8.46	8.49	6.61	5.01	9.85	1.00	3.15	5.18	9.33	9.27	9.29	4.36	8.31
TPE	10.00	10.00	10.00	8.53	10.00	6.26	9.22	6.70	6.19	6.56	5.09	2.45	7.42	2.36	1.00	1.00	1.00	5.00	9.02	5.57	8.39
TRI	8.90	9.40	9.20	7.04	10.00	4.63	6.41	7.35	3.78	5.37	4.45	4.80	9.40	5.35	1.00	1.00	1.00	1.00	7.61	5.89	8.63
UAE	10.00	10.00	9.70	7.30	8.29	4.48	6.72	6.40	6.44	5.98	3.25	2.36	1.00	1.00	1.00	1.00	1.00	9.49	9.72	6.70	9.48
URU	10.00	10.00	10.00	7.80	7.31	6.34	9.04	6.90	1.71	6.35	5.14	1.00	9.74	1.00	3.34	7.69	7.91	7.11	6.02	4.81	4.99
USA	10.00	9.90	10.00	8.37	10.00	7.75	9.65	7.41	3.25	7.71	3.04	6.80	8.44	1.73	1.00	1.00	1.40	6.41	9.68	3.96	1.00

S.F. Sufficient Food; **S.D.** Sufficient to Drink; **S.S.** Safe Sanitation; **H.L.** Healthy Life; **C.A.** Clean Air; **C.W.** Clean Water; **Ed.** Education;
G.E. Gender Equality; **I.D.** Income Distribution; **G.Go.** Good Governance; **A.Q.** Air Quality; **B.** Biodiversity; **R.W.R.** Renewable Water Resources; **C.** Consumption;
R.E. Renewable Energy; **G.Ga.** Greenhouse Gases; **O.F.** Organic Farming; **G.S.** Genuine Savings; **GDP** Gross Domestic Product; **Em.** Employment; **P.D.** Public Debt.

Tabla C.4: Países por niveles de ingresos y variables (año 2012)

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
BAN	7.40	8.10	5.60	6.08	1.37	8.72	4.87	6.81	8.44	3.28	6.37	1.00	9.71	8.31	2.86	9.64	1.00	9.40	1.22	6.07	8.57
BDI	3.80	7.20	4.60	3.97	4.85	3.98	6.43	7.27	7.95	2.67	4.77	2.42	9.77	7.24	8.58	9.86	1.00	1.00	1.00	1.00	8.69
BEN	8.80	7.50	1.30	5.12	3.94	3.72	5.78	5.83	5.54	4.42	7.05	10.00	9.95	6.49	5.62	9.49	1.00	8.12	1.08	9.32	8.97
BUR	9.20	7.90	1.70	3.97	3.50	5.05	3.85	6.15	5.33	4.46	6.37	7.12	9.21	5.11	8.58	9.86	1.00	7.01	1.08	7.19	9.09
CAF	6.00	6.70	3.40	3.88	4.84	4.52	4.08	6.00	1.00	2.38	5.40	8.87	10.00	5.57	8.58	9.86	1.00	2.84	1.00	4.49	8.15
CAM	7.50	6.40	3.10	5.64	4.20	9.00	6.17	6.46	4.45	3.30	6.44	10.00	9.95	6.94	7.21	9.73	1.00	7.53	1.54	8.44	9.13
CHA	6.10	5.10	1.30	3.40	3.99	4.47	4.71	5.33	4.82	2.23	5.40	4.69	9.91	3.73	8.58	9.86	1.00	1.00	1.32	2.42	8.92
COD	3.10	4.50	2.40	4.27	2.91	4.79	5.38	6.00	3.46	1.69	6.78	4.99	10.00	7.56	9.63	9.95	1.00	3.51	1.00	2.42	8.93
ETH	5.90	4.40	2.10	5.21	4.20	4.28	5.51	6.14	8.35	3.10	6.85	9.20	9.54	6.37	9.47	9.94	1.00	8.13	1.00	1.29	8.52
GAM	8.10	8.90	6.80	5.26	1.00	4.86	5.45	6.76	2.37	3.94	6.37	1.00	9.91	5.97	8.58	9.86	1.00	7.95	1.37	1.85	2.82
GBS	7.80	6.40	2.00	3.77	5.02	4.55	6.54	6.20	6.25	2.94	6.37	10.00	9.94	6.56	8.58	9.86	1.00	8.83	1.00	1.64	7.60
GUI	8.40	7.40	1.80	4.65	4.73	4.55	5.23	6.20	5.02	2.45	6.37	3.21	9.93	4.62	8.58	9.86	1.00	1.41	1.00	1.64	2.30
HAI	4.30	6.90	1.70	5.77	5.05	3.97	5.84	7.10	1.00	2.69	6.77	1.00	9.14	8.30	7.05	9.79	1.00	8.98	1.00	1.00	9.82
KEN	6.70	5.90	3.20	4.87	5.21	5.79	6.70	6.49	2.25	3.66	5.85	5.86	9.11	7.20	8.07	9.73	1.00	8.84	1.25	1.00	7.00
KGZ	8.90	9.00	9.30	6.33	4.87	4.13	7.60	7.04	7.87	3.28	3.82	3.47	7.94	7.08	3.06	8.70	1.00	8.45	1.63	4.53	6.33
LBR	6.80	7.30	1.80	4.90	4.60	4.85	6.32	6.20	4.36	3.50	6.37	1.00	9.99	6.14	8.58	9.86	1.00	1.09	1.00	6.91	9.73
MAD	7.50	4.60	1.50	5.44	5.01	4.76	6.75	6.80	3.20	3.51	4.77	1.27	9.56	6.36	8.58	9.86	1.00	6.83	1.00	7.71	9.93
MAW	7.30	8.30	5.10	4.26	5.01	3.03	6.77	6.85	5.49	4.47	4.77	7.51	9.44	7.59	8.58	9.86	1.00	8.12	1.00	1.00	7.97
MLI	8.80	6.40	2.20	3.78	3.54	7.86	4.83	5.75	5.12	4.14	6.37	1.22	9.35	4.10	8.58	9.86	1.00	8.11	1.00	1.64	9.01
MOZ	6.20	4.70	1.80	3.77	5.02	4.66	5.89	7.25	2.37	4.49	5.02	7.40	9.97	7.66	9.56	9.89	1.00	7.04	1.00	1.22	8.84
MYA	8.40	8.30	7.60	5.11	3.38	4.57	5.74	6.80	1.72	1.51	7.02	2.61	9.72	3.80	7.84	9.83	1.00	5.00	1.00	6.69	7.73
NEP	8.30	8.90	3.10	5.97	1.80	4.60	5.76	5.89	3.33	3.19	5.52	8.50	9.53	7.69	8.78	9.88	1.00	9.46	1.00	1.00	8.78
NIG	8.40	4.90	1.00	4.14	3.64	4.22	3.39	6.20	7.19	3.62	6.37	3.53	9.30	1.50	8.58	9.86	1.00	8.99	1.00	1.64	9.57
PRK	6.50	9.80	8.00	6.51	7.21	4.41	5.95	6.70	6.44	1.80	3.91	1.00	8.88	8.15	1.20	7.41	1.00	5.00	1.28	3.36	9.05
RWA	6.80	6.50	5.50	3.97	4.00	4.61	6.84	6.80	1.09	4.50	4.77	4.99	9.84	7.80	8.58	9.86	1.00	8.16	1.00	1.00	9.39
SLE	6.50	5.50	1.30	2.65	4.74	4.90	4.67	6.20	4.36	3.67	6.37	2.15	9.97	6.43	8.58	9.86	3.80	7.63	1.00	1.64	4.62
TAN	6.60	5.30	1.00	4.40	5.02	8.50	5.66	6.90	5.49	4.33	6.37	10.00	9.46	6.30	8.95	9.87	1.00	8.76	1.11	1.00	7.72
TJK	7.40	6.40	9.40	6.27	4.17	4.51	7.18	6.53	6.98	2.80	3.80	2.07	2.52	7.69	5.90	9.60	1.00	2.75	1.45	8.03	8.68
TOG	7.00	6.10	1.30	5.26	4.12	4.42	6.27	6.20	6.84	3.23	7.00	5.52	9.89	6.99	8.30	9.81	1.00	1.31	1.00	1.64	9.00
UGA	7.80	7.20	3.40	3.86	4.93	3.95	6.85	7.22	3.36	3.81	4.77	5.13	9.95	4.97	8.58	9.86	3.36	8.44	1.00	6.57	9.10
ZIM	7.00	8.00	4.00	3.55	5.25	7.19	5.71	6.61	1.67	1.84	3.64	10.00	7.90	6.94	6.88	9.28	1.00	1.41	1.00	1.00	2.57
ARM	7.90	9.80	9.00	6.87	4.60	5.10	7.68	6.65	7.95	4.39	3.41	4.00	6.36	6.27	1.00	8.69	1.00	7.98	3.24	1.50	8.70
BHU	6.70	9.60	4.40	5.93	5.64	4.28	6.56	6.30	3.07	5.20	5.52	10.00	9.96	1.00	4.76	9.57	1.00	9.67	3.58	6.70	1.27
BOL	7.30	8.80	2.70	6.44	5.89	8.34	8.12	6.86	1.00	3.90	5.63	9.25	9.97	2.47	2.69	8.58	1.00	8.09	2.95	5.77	8.87
CGO	8.70	7.10	1.80	4.75	2.82	4.90	6.12	6.00	2.64	2.98	2.70	4.84	10.00	6.80	5.42	9.59	1.00	1.00	2.85	2.42	9.44
CIV	8.60	8.00	2.40	4.71	4.53	5.09	3.92	5.77	3.79	2.58	7.11	10.00	9.83	6.68	7.72	9.71	1.00	7.58	1.15	1.64	1.00
CMR	7.80	7.70	4.90	4.29	3.36	5.29	6.35	6.07	5.54	3.25	5.77	4.50	9.97	6.72	6.75	9.74	1.00	7.55	1.56	7.48	9.76

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
EGY	10.00	9.90	9.50	6.74	8.47	6.24	7.12	5.93	7.63	3.87	4.21	3.04	1.00	6.32	1.00	7.81	4.27	6.99	3.77	3.54	1.77
ESA	9.10	8.80	8.70	6.89	6.09	4.98	7.09	6.57	2.21	4.80	5.19	1.00	9.45	5.27	5.39	9.05	1.03	6.60	4.21	5.58	6.66
GEO	9.40	9.80	9.50	7.38	5.61	4.02	7.19	6.62	3.26	4.88	5.64	1.69	9.71	6.82	3.88	8.89	1.00	5.42	3.29	2.25	8.79
GHA	10.00	8.60	1.40	5.15	4.06	7.78	6.46	6.81	3.13	5.21	5.94	6.98	9.82	4.90	7.03	9.61	1.00	8.33	2.04	1.64	7.85
GUA	7.80	9.20	7.80	6.74	5.35	5.99	7.07	6.23	1.00	3.80	4.90	10.00	9.74	5.63	6.70	9.28	1.00	6.91	3.09	6.64	9.36
GUY	9.20	9.40	8.40	5.62	6.59	4.28	6.92	7.08	1.00	4.27	4.49	2.38	9.93	3.90	1.28	5.04	1.00	6.04	4.17	3.33	4.21
HON	8.80	8.70	7.70	7.07	5.41	4.97	7.21	6.95	1.00	3.79	4.26	6.93	9.88	5.81	4.86	9.04	1.30	8.41	2.72	6.44	9.16
INA	8.70	8.20	5.40	6.78	5.43	6.22	7.56	6.59	6.31	4.04	3.89	3.21	9.44	6.99	3.45	8.29	1.00	9.17	2.89	5.19	9.32
IND	8.10	9.20	3.40	6.10	1.00	7.89	6.46	6.19	6.64	4.38	3.89	2.41	6.61	8.15	2.63	8.61	1.03	9.28	2.38	3.75	2.94
LAO	7.80	6.70	6.30	5.80	3.95	8.51	6.30	6.80	6.31	3.07	4.87	8.31	9.87	5.92	4.76	9.57	1.00	6.66	1.80	7.79	5.23
MAR	10.00	8.30	7.00	7.09	6.93	6.29	6.04	5.80	4.49	4.36	3.16	1.00	5.66	6.81	1.00	8.56	1.00	9.35	3.08	4.07	5.91
MDA	10.00	9.60	8.50	6.91	6.38	4.88	6.90	7.08	5.49	4.21	4.51	1.00	8.36	5.58	1.00	8.28	2.77	8.74	2.20	5.12	9.39
MGL	7.30	8.20	5.10	6.45	5.19	4.50	8.67	7.14	6.64	4.49	1.30	6.69	9.85	1.00	1.00	5.69	1.00	1.20	2.93	7.41	6.69
MTN	9.20	5.00	2.60	5.23	4.57	4.58	5.01	6.16	4.92	3.19	6.37	1.00	8.60	1.45	8.58	9.86	1.00	5.75	1.51	1.00	1.00
NCA	8.10	8.50	5.20	7.42	5.42	4.23	7.01	7.25	1.00	3.66	4.26	10.00	9.93	5.91	5.53	9.23	1.51	7.45	2.11	4.57	2.32
NGR	9.40	5.80	3.10	3.79	2.96	4.48	5.58	6.01	3.07	2.65	6.26	6.29	9.64	5.70	8.44	9.71	1.00	9.34	1.75	1.00	9.60
PAK	7.50	9.20	4.80	5.90	1.88	6.26	4.37	5.58	7.71	2.74	4.12	4.91	2.57	8.28	3.73	9.22	1.00	8.45	1.87	5.52	4.58
PAR	9.00	8.60	7.10	7.39	5.53	5.18	7.05	6.82	1.00	3.68	6.23	2.72	9.99	1.63	10.00	9.27	1.00	8.98	3.25	5.71	9.73
PHL	8.70	9.20	7.40	7.06	5.54	8.93	7.88	7.69	3.83	3.87	3.91	2.52	8.30	6.72	3.98	9.18	1.54	9.10	2.58	4.95	8.20
PNG	6.90	4.00	4.50	6.12	5.08	3.96	3.70	7.10	1.83	3.62	5.96	1.00	10.00	3.86	3.67	9.57	1.00	5.00	1.72	8.27	9.44
SEN	8.10	7.20	5.20	5.25	2.77	8.36	4.97	6.57	4.77	4.14	4.86	10.00	9.43	5.60	4.70	9.56	1.00	8.88	1.33	1.00	8.18
SRI	8.00	9.10	9.20	7.21	5.18	9.17	6.46	7.21	5.33	4.26	3.65	7.48	7.55	6.94	5.61	9.36	3.42	9.03	3.38	6.13	1.57
SUD	7.80	5.80	2.60	5.08	3.87	6.52	3.86	6.00	1.72	1.74	6.71	2.09	4.24	4.86	7.07	9.69	1.00	3.00	1.84	3.00	2.17
SYR	10.00	9.00	9.50	7.21	8.08	4.50	6.64	5.90	6.64	3.14	2.74	1.00	1.36	7.53	1.00	7.18	1.00	2.55	2.95	4.32	8.75
UKR	10.00	9.80	9.40	6.81	6.92	2.98	9.20	6.86	9.05	3.88	1.88	1.80	7.24	4.96	1.00	4.19	1.51	8.11	4.07	4.41	8.58
UZB	8.90	8.70	10.00	6.56	3.97	3.80	7.15	6.90	5.60	2.38	3.06	1.13	1.00	7.58	1.00	6.44	1.00	1.00	2.16	9.80	9.85
VIE	8.90	9.50	7.60	7.39	3.10	7.27	6.58	6.73	6.13	3.92	4.38	2.29	9.07	6.81	2.88	8.50	1.00	9.02	2.20	6.37	8.45
YEM	7.00	5.50	5.30	5.79	3.47	4.48	5.47	4.87	5.43	2.56	2.80	1.00	1.00	8.18	1.00	9.10	1.00	1.30	1.59	2.32	7.96
ZAM	5.60	6.10	4.80	3.56	5.12	4.17	5.90	6.30	1.00	4.28	1.00	10.00	9.83	7.52	9.22	9.85	1.00	3.37	1.17	2.47	9.27
ALB	10.00	9.50	9.40	7.38	10.00	8.25	6.79	6.75	6.91	4.67	5.75	4.21	9.56	6.33	4.16	8.83	1.00	6.57	4.28	3.17	4.86
ALG	10.00	8.30	9.50	7.06	10.00	5.83	7.81	5.99	6.00	3.26	6.17	3.12	4.74	6.56	1.00	7.22	1.00	9.47	4.11	3.68	9.83
ANG	5.90	5.10	5.80	4.30	4.87	5.18	6.54	6.62	1.00	2.97	7.22	6.03	9.96	7.35	5.84	9.13	1.00	1.00	3.48	1.00	9.00
ARG	10.00	9.70	9.00	7.89	10.00	8.43	9.36	7.24	1.39	4.46	6.17	2.63	9.60	3.53	1.00	5.79	5.24	8.53	7.17	4.89	7.74
AZE	8.90	8.00	8.20	6.58	6.32	4.43	6.49	6.58	7.05	3.36	2.80	3.57	6.53	6.63	1.00	7.27	1.06	7.54	5.20	5.46	9.83
BIH	10.00	9.90	9.50	7.88	5.52	9.35	7.60	7.00	5.71	4.23	1.00	1.00	9.91	4.75	1.36	4.71	1.00	5.00	4.44	1.00	8.18
BLR	10.00	10.00	9.30	7.03	10.00	4.43	9.04	7.10	8.78	3.06	3.91	3.61	9.25	3.30	1.00	3.12	1.00	9.07	6.61	9.42	6.76
BOT	7.50	9.60	6.20	4.97	5.59	4.29	6.98	6.83	1.00	6.32	1.75	10.00	9.84	3.64	2.15	7.71	1.00	9.16	6.84	4.72	9.62
BRA	9.40	9.80	7.90	7.44	6.92	8.54	8.75	6.68	1.00	5.29	4.61	10.00	9.93	1.82	4.39	8.01	1.57	7.81	5.71	5.50	3.28
BUL	9.00	10.00	10.00	7.77	10.00	8.11	7.99	6.99	5.97	5.40	3.18	4.43	7.13	3.73	1.00	4.19	1.90	8.57	6.24	2.88	9.63

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
CHN	9.00	9.10	6.40	7.74	1.97	6.80	6.96	6.87	4.19	3.84	1.82	8.02	8.05	6.72	1.16	4.57	1.00	9.56	4.54	6.70	9.28
COL	9.10	9.20	7.70	7.73	6.47	5.46	8.48	6.71	1.00	4.35	6.11	10.00	9.94	5.42	2.11	8.69	1.00	7.09	5.22	3.39	8.73
CRC	10.00	9.70	9.50	8.21	6.49	4.77	7.41	7.27	1.48	6.22	6.02	8.82	9.76	4.68	5.24	8.60	1.43	8.48	5.76	4.36	9.01
CUB	10.00	9.40	9.10	8.24	7.80	8.86	9.56	7.39	2.84	3.93	1.93	2.67	8.02	6.30	1.17	7.33	1.00	5.00	5.10	6.84	8.72
DOM	7.60	8.60	8.30	7.24	6.66	4.65	7.32	6.68	1.76	4.20	4.08	10.00	8.35	7.42	2.32	8.13	8.80	3.50	4.88	2.32	9.10
ECU	8.50	9.40	9.20	7.38	10.00	8.34	8.00	7.04	1.41	3.41	4.72	10.00	9.64	4.18	1.15	7.92	1.93	5.11	4.58	5.49	9.60
GAB	10.00	8.70	3.30	5.46	5.95	4.21	7.58	6.00	4.36	3.85	4.51	7.29	9.99	3.95	5.62	8.24	1.00	5.98	6.88	1.22	9.51
HUN	10.00	10.00	10.00	7.77	10.00	7.40	8.94	6.64	9.70	6.49	4.14	2.57	9.46	3.47	1.00	5.11	5.30	8.82	7.56	3.35	1.39
IRI	10.00	9.60	10.00	6.92	8.08	4.98	7.77	5.89	5.07	2.65	2.76	3.44	3.23	7.05	1.00	3.12	1.00	7.52	6.09	2.22	9.76
IRQ	9.20	7.90	7.30	5.69	3.84	4.27	6.27	6.20	8.03	2.15	2.15	1.00	1.27	8.47	1.00	6.77	1.00	5.00	2.48	2.23	1.00
JAM	10.00	9.30	8.00	7.42	6.32	4.62	8.27	7.03	3.17	4.88	2.25	3.67	9.38	6.37	1.52	7.06	1.00	7.68	4.78	2.78	1.00
JOR	10.00	9.70	9.80	7.21	9.35	3.00	7.52	6.12	5.89	4.83	2.67	1.00	1.00	5.35	1.00	6.92	1.00	6.22	3.48	2.75	2.65
KAZ	10.00	9.50	9.70	6.27	6.53	4.34	8.96	7.01	8.03	4.03	3.09	1.26	7.11	6.01	1.00	1.00	1.00	3.61	6.08	5.83	9.81
LBA	10.00	9.10	9.70	7.39	10.00	4.94	9.27	6.00	6.44	2.81	4.15	1.00	1.00	5.46	1.00	1.88	1.00	5.00	3.43	1.00	9.95
LIB	10.00	10.00	9.80	7.04	10.00	4.06	8.14	6.08	5.95	3.78	2.61	1.00	8.14	4.94	1.00	5.60	1.00	5.72	6.73	4.54	1.00
MAS	10.00	10.00	9.60	7.39	9.73	5.46	7.13	6.53	2.23	5.64	4.15	6.84	9.77	3.73	1.00	3.49	1.00	9.16	6.74	7.26	6.30
MEX	10.00	9.60	8.50	7.91	6.27	6.14	8.09	6.60	1.00	4.60	3.26	5.93	8.26	4.66	1.00	6.15	3.21	8.64	6.50	5.93	7.80
MKD	10.00	10.00	8.80	7.71	5.73	5.97	7.13	6.97	3.33	4.79	2.04	2.44	8.39	5.02	1.48	6.01	5.60	8.71	5.26	1.00	9.16
MNE	10.00	9.80	9.00	7.53	5.83	8.36	8.58	7.00	7.95	5.13	3.38	5.74	9.75	5.93	3.18	6.69	1.59	4.07	5.64	3.17	7.51
NAM	8.20	9.30	3.20	5.53	5.41	4.57	6.94	7.18	1.00	5.61	1.00	7.34	9.83	4.48	1.97	8.54	1.00	9.48	4.13	1.00	9.45
PAN	8.50	9.30	6.90	7.88	6.32	9.22	7.83	7.04	1.00	5.18	4.31	5.75	9.97	3.30	2.23	7.61	1.00	8.57	6.37	6.57	8.46
PER	8.40	8.50	7.10	7.89	5.72	8.34	8.29	6.80	1.20	4.52	1.39	6.54	9.90	4.23	2.52	8.56	2.23	7.77	5.16	4.72	9.46
ROU	10.00	8.80	7.20	7.60	6.06	8.15	8.37	6.81	5.67	5.37	2.14	3.88	9.68	4.64	1.67	6.48	2.83	9.05	5.92	4.86	8.86
RSA	10.00	9.10	7.90	4.76	6.28	8.42	7.99	7.48	1.00	5.47	1.20	3.44	7.50	6.59	1.07	3.06	1.00	5.97	5.46	1.00	8.37
SRB	10.00	9.90	9.20	7.54	5.77	8.36	7.85	7.00	8.69	4.72	3.38	2.98	9.75	5.61	1.32	3.69	1.00	4.07	5.35	1.00	7.18
THA	8.40	9.60	9.60	7.04	4.03	8.27	7.19	6.89	4.40	4.32	4.29	8.67	8.69	4.94	1.97	6.41	1.00	9.27	4.92	9.34	8.06
TKM	9.30	9.10	9.80	5.86	9.76	4.50	6.76	6.90	4.40	2.22	1.68	1.49	1.00	4.64	1.00	1.00	1.00	9.38	4.33	1.00	9.68
TUN	10.00	9.40	8.50	7.71	10.00	6.30	7.80	6.26	4.32	4.64	2.72	1.00	3.87	6.31	1.42	7.92	3.61	8.56	4.95	1.51	7.97
TUR	10.00	10.00	9.00	7.76	6.48	5.79	7.56	5.95	2.87	4.90	3.06	1.00	8.17	5.39	1.11	6.35	3.26	6.60	6.48	3.72	8.31
VEN	9.30	9.10	8.60	7.71	10.00	4.05	8.92	6.86	2.64	2.41	3.94	10.00	9.93	4.32	1.00	3.65	1.00	8.54	5.95	4.45	7.56
AUS	10.00	10.00	10.00	9.03	10.00	6.17	10.00	7.29	4.40	8.18	1.08	6.23	9.54	1.00	1.00	1.00	5.20	8.26	9.50	6.00	9.41
AUT	10.00	10.00	10.00	8.68	10.00	9.51	9.13	7.17	9.05	8.13	7.04	10.00	9.53	2.54	2.68	1.73	9.93	9.01	9.56	6.57	2.29
BEL	10.00	10.00	10.00	8.70	10.00	6.63	9.68	7.53	8.66	7.67	5.37	6.58	6.60	1.00	1.00	1.00	5.90	8.95	9.39	4.86	1.00
CAN	10.00	10.00	10.00	8.82	10.00	9.31	8.95	7.41	6.13	8.24	2.24	3.09	9.84	1.00	1.71	1.00	2.29	7.86	9.51	4.74	1.06
CHI	10.00	9.60	9.60	8.38	10.00	5.26	8.47	7.03	1.00	7.35	1.00	6.63	9.88	1.65	2.20	5.92	1.00	6.71	7.10	4.90	9.83
CRO	10.00	9.90	9.90	8.07	6.51	9.25	8.18	7.01	6.15	5.82	3.96	4.77	9.94	2.33	1.33	5.70	3.62	8.80	7.30	2.66	7.55
CYP	10.00	10.00	10.00	8.38	10.00	7.53	8.60	6.57	8.14	7.20	2.82	2.27	8.16	3.68	1.00	1.01	4.58	6.48	8.79	4.60	2.34
CZE	10.00	10.00	9.80	8.39	10.00	7.45	8.48	6.79	9.22	6.83	3.64	7.53	8.71	2.06	1.00	1.00	9.28	8.70	8.60	5.12	8.09
DEN	10.00	10.00	10.00	8.73	10.00	7.49	9.91	7.78	5.25	8.64	6.73	2.04	8.93	1.00	2.03	1.52	7.83	8.90	9.36	5.42	7.42

Sigue en la página siguiente.

	S.F.	S.D.	S.S.	H.L.	C.A.	C.W.	Ed.	G.E.	I.D.	G.Go.	A.Q.	B.	R.W.R.	C.	R.E.	G.Ga.	O.F.	G.S.	GDP	Em.	P.D.
ESP	10.00	10.00	10.00	9.09	10.00	8.31	10.00	7.58	2.80	6.78	3.38	3.82	7.10	2.16	1.18	4.18	7.68	8.42	8.93	1.15	2.87
EST	10.00	9.80	9.50	7.97	6.31	9.43	8.95	6.98	6.95	7.16	1.00	10.00	8.60	1.00	1.52	1.00	9.56	9.01	7.70	2.87	9.92
FIN	10.00	10.00	10.00	8.73	10.00	8.76	10.00	8.38	9.31	8.70	4.87	4.24	9.85	3.14	2.53	1.00	8.42	8.49	9.31	4.60	7.07
FRA	10.00	10.00	10.00	8.88	9.95	8.65	9.44	7.02	7.72	7.54	5.49	8.55	8.50	1.10	1.00	4.48	5.37	8.31	9.25	3.80	1.00
GBR	10.00	10.00	10.00	8.77	10.00	8.16	9.01	7.46	6.24	7.76	5.13	9.03	9.12	3.12	1.00	2.22	6.62	6.11	9.30	4.49	1.23
GER	10.00	10.00	10.00	8.88	10.00	7.86	9.52	7.59	7.72	7.87	5.89	10.00	7.90	3.08	1.00	1.00	7.73	8.85	9.40	5.50	1.31
GRE	10.00	10.00	9.80	8.79	10.00	7.71	9.86	6.92	5.90	5.82	2.35	4.95	8.73	2.02	1.00	2.55	6.08	2.04	8.51	1.77	1.00
IRL	10.00	10.00	9.90	8.91	10.00	9.19	10.00	7.83	6.15	7.91	4.61	1.00	9.76	1.77	1.00	1.36	2.51	6.93	9.48	2.37	1.00
ISL	10.00	10.00	10.00	9.06	10.00	10.00	9.59	8.53	8.98	7.84	1.10	6.59	9.99	1.00	8.25	3.96	1.00	2.03	9.40	4.76	1.00
ISR	10.00	10.00	10.00	8.98	9.81	5.77	9.21	6.93	4.07	6.04	1.77	7.54	1.00	4.57	1.00	1.07	3.43	8.56	8.96	5.70	2.02
ITA	10.00	10.00	10.00	9.02	10.00	8.22	9.08	6.80	6.30	6.03	5.81	7.93	7.63	2.89	1.06	3.41	8.87	8.12	8.91	4.33	1.00
JPN	10.00	10.00	10.00	9.39	10.00	8.78	8.92	6.51	10.00	7.38	5.96	5.46	7.91	5.54	1.00	1.03	1.00	8.77	9.22	6.35	1.00
KOR	10.00	9.80	10.00	8.65	8.90	8.49	10.00	6.28	7.22	6.44	5.37	2.50	6.35	4.36	1.00	1.00	1.89	9.29	9.01	7.11	8.77
KSA	10.00	9.10	8.80	7.06	8.63	4.24	8.43	5.75	6.44	4.46	1.88	10.00	1.00	4.83	1.00	1.00	1.00	3.01	8.27	3.68	9.89
KUW	10.00	9.90	10.00	8.20	8.53	4.48	8.31	6.32	6.44	5.36	1.00	1.00	1.00	3.25	1.00	1.00	1.00	8.97	9.56	8.13	9.89
LAT	10.00	9.90	7.80	7.65	6.64	9.05	8.24	7.40	4.17	6.33	7.44	8.19	9.88	1.76	3.56	6.40	9.04	8.59	6.76	2.09	8.47
LTU	10.00	10.00	9.40	7.46	10.00	8.59	9.07	7.13	3.71	6.45	4.28	7.20	9.05	1.00	1.54	5.98	7.42	8.57	7.43	2.12	8.36
LUX	10.00	10.00	10.00	8.83	10.00	7.03	7.55	7.22	8.44	8.42	6.25	10.00	9.81	1.00	1.00	1.00	5.09	8.41	10.00	5.47	9.49
MLT	10.00	10.00	10.00	8.74	10.00	2.39	8.19	6.66	8.22	7.42	2.36	1.00	2.87	3.89	1.00	4.01	1.00	5.25	8.41	5.27	2.47
NED	10.00	10.00	10.00	8.89	10.00	7.32	9.94	7.47	8.95	8.30	6.77	7.58	8.83	1.00	1.00	1.00	4.51	8.82	9.58	6.38	3.27
NOR	10.00	10.00	10.00	8.89	10.00	9.51	9.80	8.40	9.11	8.39	6.60	5.43	9.92	1.00	3.61	1.99	7.49	9.06	9.86	7.20	6.88
NZL	10.00	10.00	10.00	8.85	10.00	9.92	10.00	7.81	4.45	8.52	3.92	10.00	9.85	1.00	3.90	2.96	2.37	8.31	8.66	5.21	8.54
OMA	9.20	8.90	9.90	7.49	8.85	4.42	7.54	5.87	6.44	5.59	1.79	4.66	1.61	1.95	1.00	1.00	1.00	1.69	8.54	2.23	9.94
POL	10.00	10.00	9.00	7.91	10.00	8.16	8.80	7.04	7.16	6.61	2.54	10.00	8.06	3.59	1.00	2.01	5.70	8.10	7.69	3.81	5.68
POR	10.00	9.90	10.00	8.56	10.00	7.79	9.56	7.14	6.27	6.91	3.77	3.06	8.77	2.96	2.33	5.47	7.65	3.25	8.15	2.80	1.00
QAT	10.00	10.00	10.00	7.88	10.00	4.48	5.74	6.23	1.00	6.32	3.27	1.00	1.00	1.00	1.00	1.00	1.00	5.00	10.00	9.61	8.96
RUS	10.00	9.70	7.00	6.83	6.92	8.24	8.53	7.04	4.23	3.51	1.93	4.60	9.85	3.84	1.00	1.00	1.00	7.32	7.00	5.22	9.84
SLO	10.00	9.90	10.00	8.56	6.78	9.30	9.42	7.04	9.58	6.83	3.73	6.53	9.70	3.35	1.48	2.52	7.92	8.85	8.75	4.46	7.28
SUI	10.00	10.00	10.00	9.21	10.00	8.69	8.69	7.63	7.60	8.41	7.26	10.00	9.51	4.15	1.90	4.37	9.42	9.37	9.62	7.33	7.05
SVK	10.00	10.00	10.00	7.94	10.00	8.92	8.02	6.80	8.56	6.56	3.95	10.00	9.86	2.06	1.00	3.55	8.95	8.57	8.14	2.62	7.68
SWE	10.00	10.00	10.00	9.08	10.00	9.62	9.28	8.04	9.05	8.53	6.61	5.01	9.85	1.00	3.39	4.93	9.70	9.09	9.51	4.74	8.50
TPE	10.00	10.00	10.00	8.53	10.00	6.26	9.51	6.70	6.19	6.94	5.09	2.45	7.42	2.37	1.00	1.00	1.00	5.00	9.39	6.45	8.16
TRI	8.90	9.40	9.20	7.06	10.00	4.63	6.41	7.37	3.79	5.22	4.45	4.80	9.40	5.01	1.00	1.00	1.00	1.00	7.64	5.60	8.90
UAE	10.00	10.00	9.80	8.06	8.29	4.48	6.72	6.45	6.44	5.82	3.25	2.35	1.00	1.76	1.00	1.00	1.00	9.49	9.76	6.70	9.63
URU	10.00	10.00	10.00	7.86	7.31	6.34	9.05	6.91	3.75	6.70	5.14	1.00	9.74	1.00	4.90	8.08	7.92	7.28	6.63	5.43	5.95
USA	10.00	9.90	10.00	8.38	10.00	7.75	9.83	7.41	3.20	7.38	3.04	6.83	8.44	2.28	1.00	1.00	1.40	5.26	9.76	4.09	1.00

S.F. Sufficient Food; **S.D.** Sufficient to Drink; **S.S.** Safe Sanitation; **H.L.** Healthy Life; **C.A.** Clean Air; **C.W.** Clean Water; **Ed.** Education;
G.E. Gender Equality; **I.D.** Income Distribution; **G.Go.** Good Governance; **A.Q.** Air Quality; **B.** Biodiversity; **R.W.R.** Renewable Water Resources; **C.** Consumption;
R.E. Renewable Energy; **G.Ga.** Greenhouse Gases; **O.F.** Organic Farming; **G.S.** Genuine Savings; **GDP** Gross Domestic Product; **Em.** Employment; **P.D.** Public Debt.

Apéndice D

Resultados gráficos

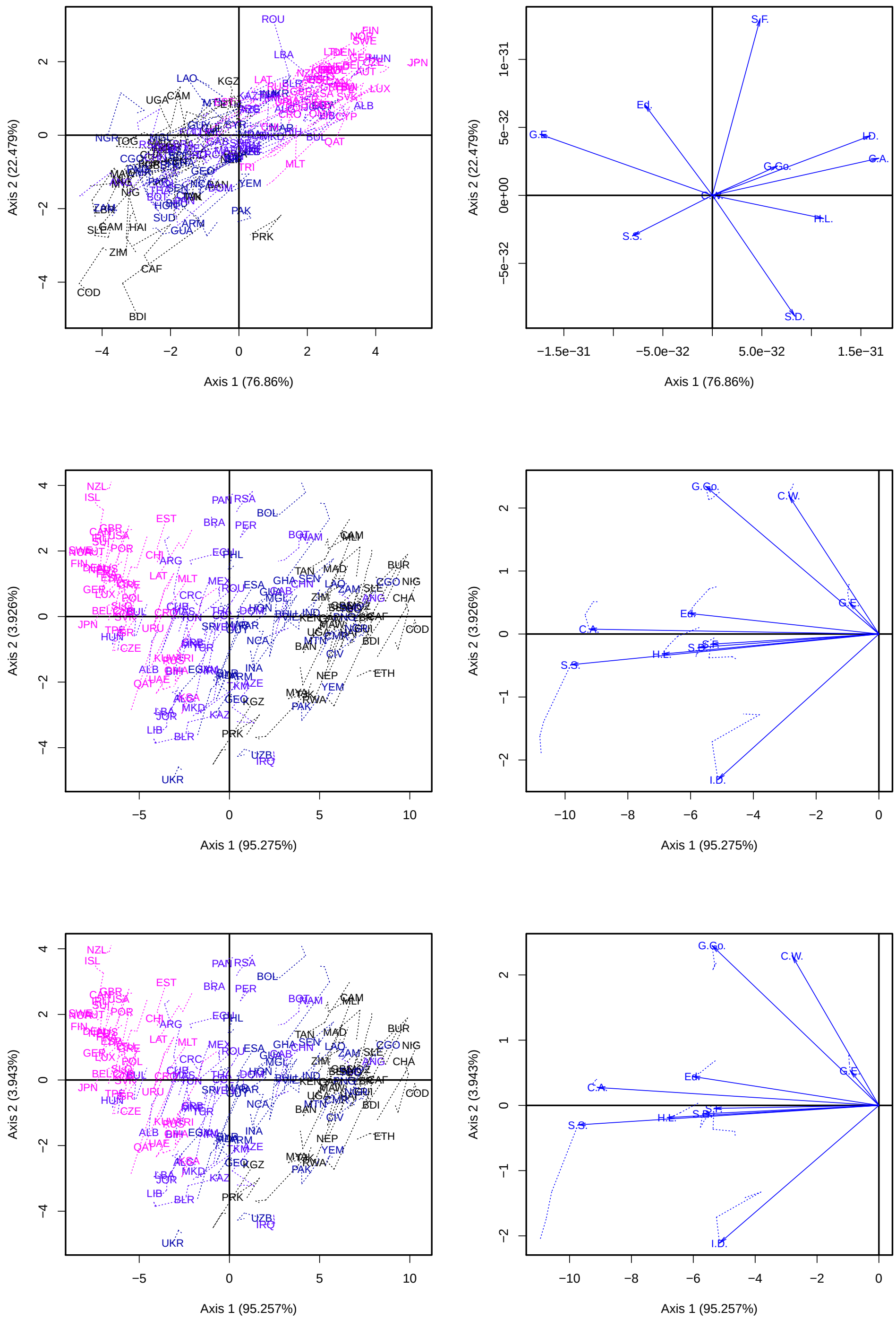

Figura D.1: *Trayectorias de los análisis entre variables sociales y medioambientales*

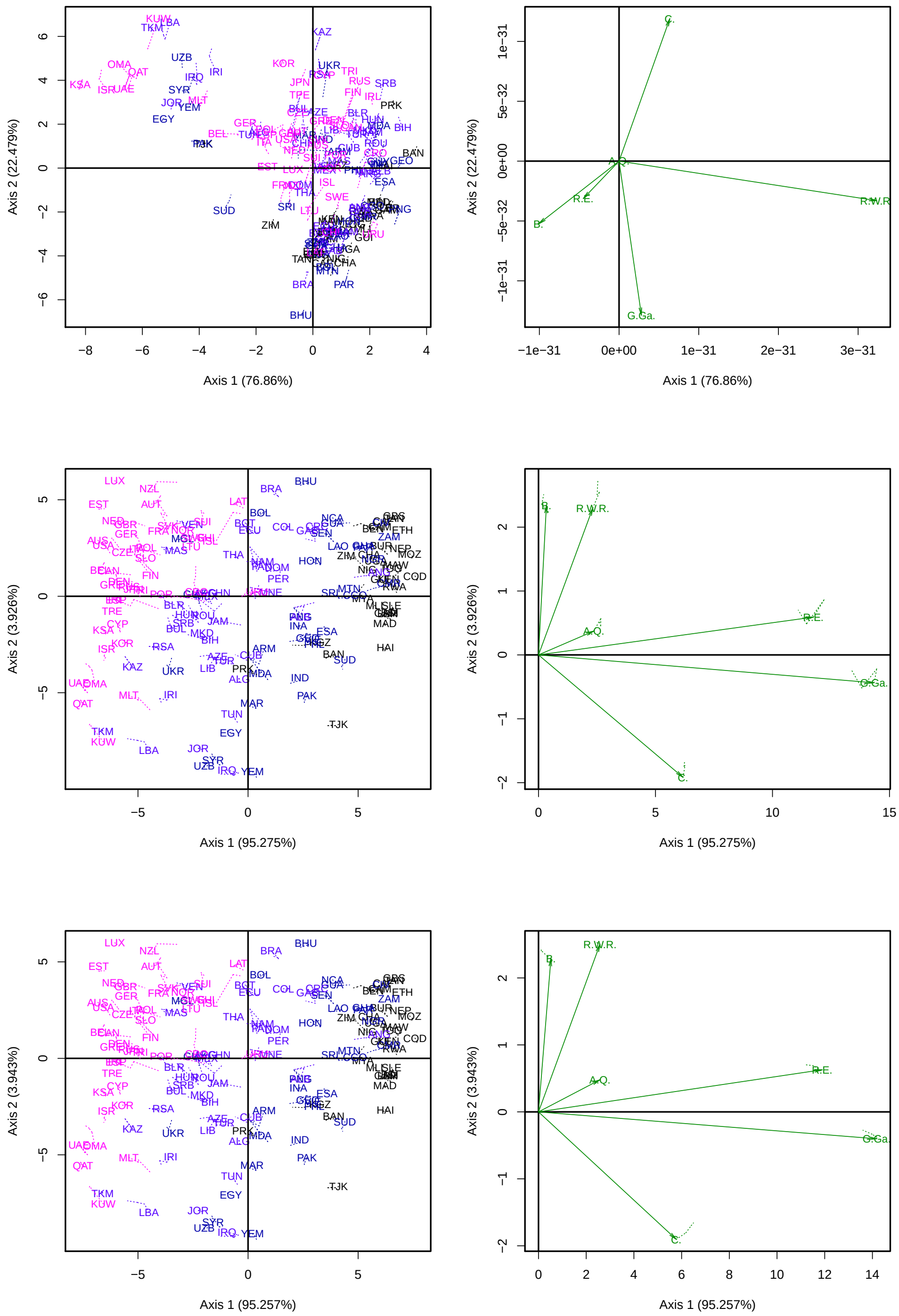

Figura D.2: *Trayectorias de los análisis entre variables medioambientales y sociales*

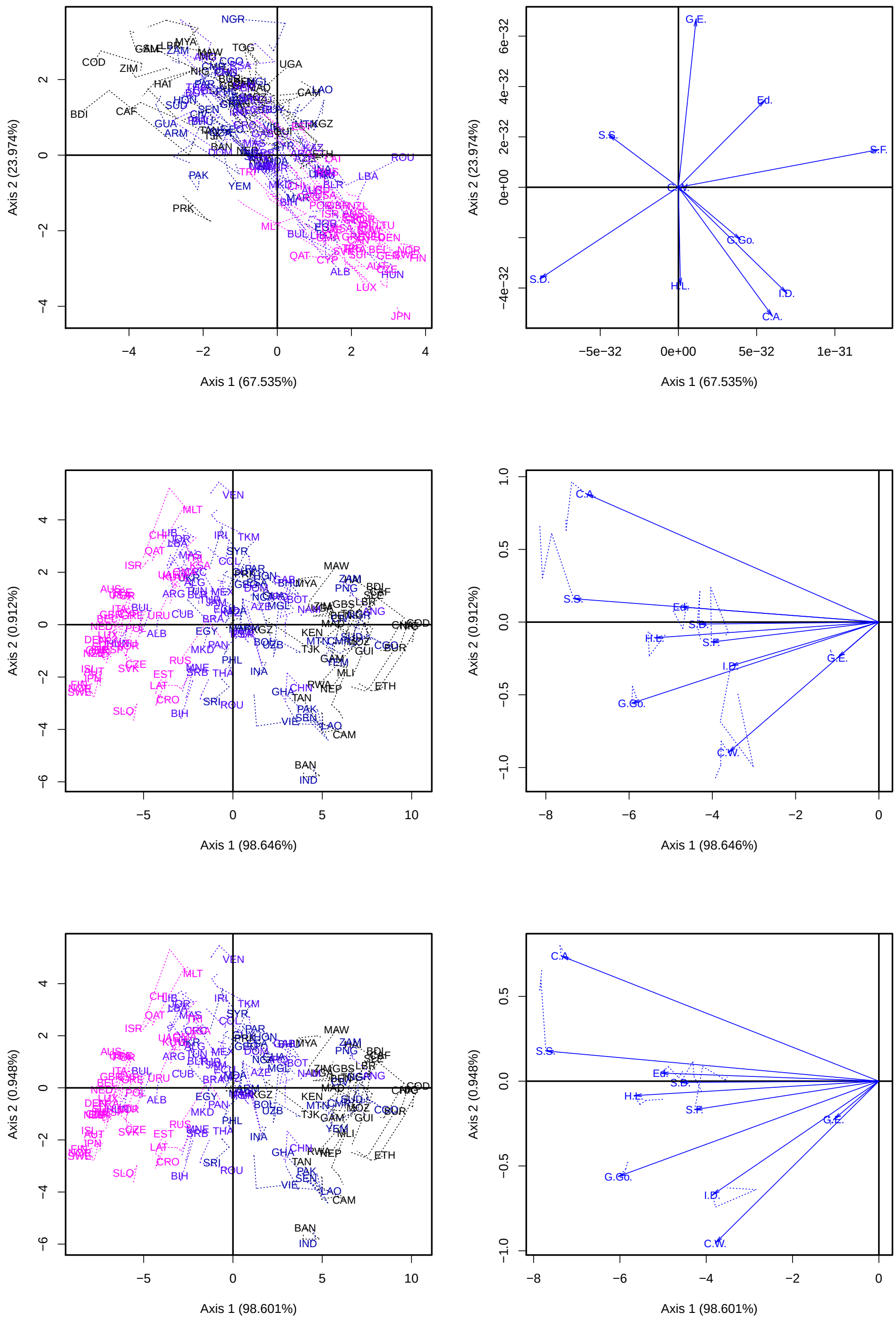

Figura D.3: *Trayectorias de los análisis entre variables sociales y económicas*

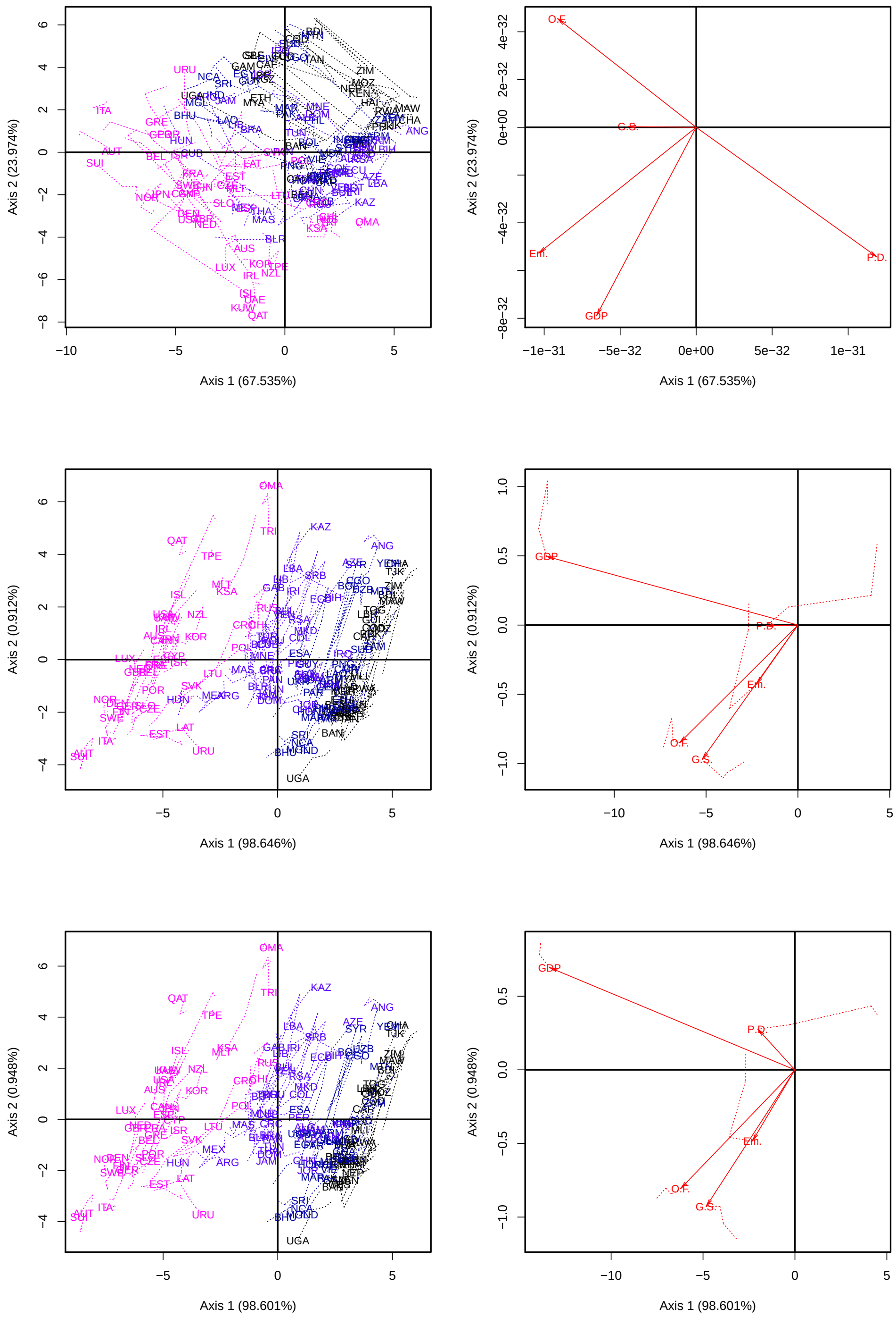

Figura D.4: *Trayectorias de los análisis entre variables económicas y sociales*

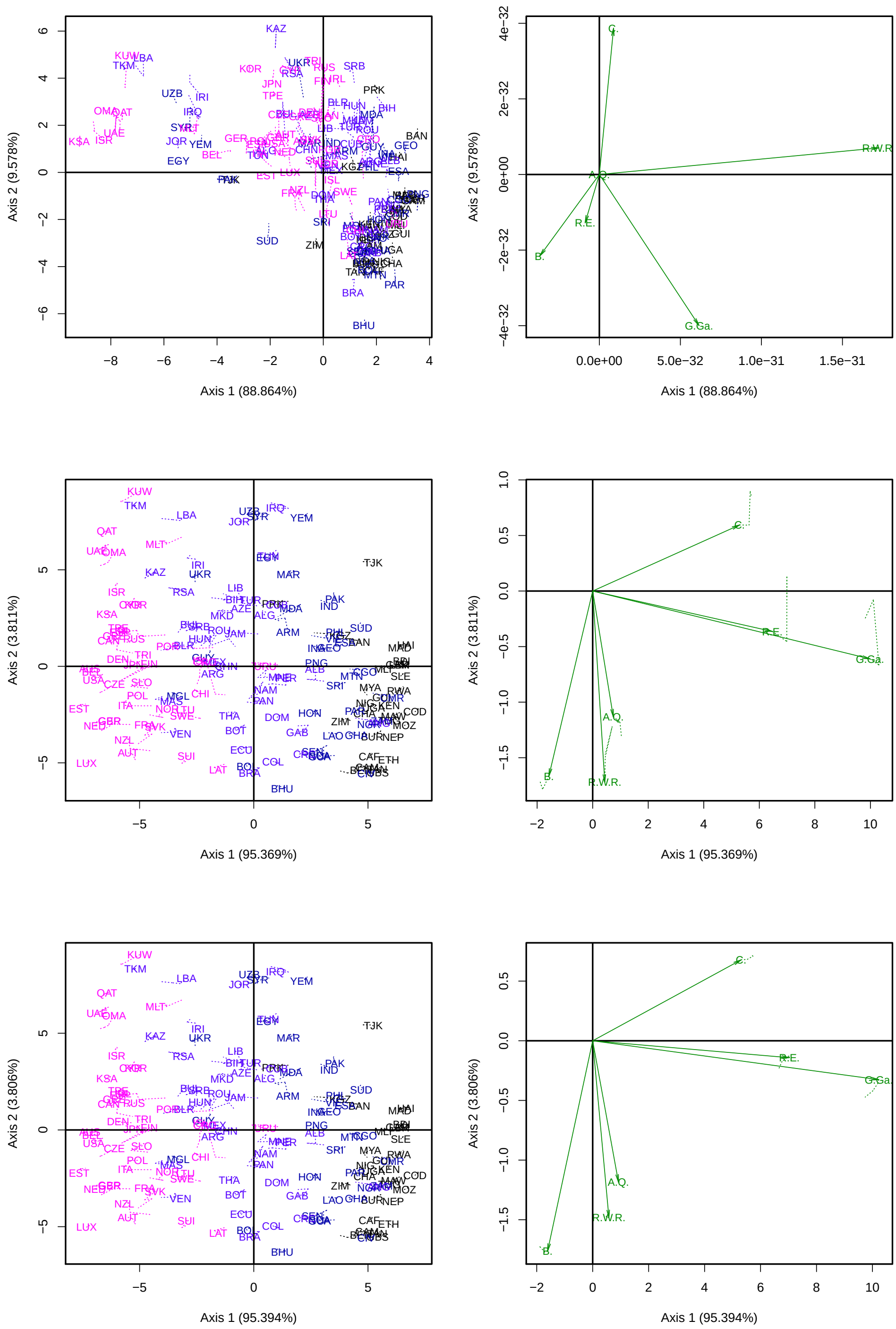

Figura D.5: *Trayectorias de los análisis entre variables medioambientales y económicas*

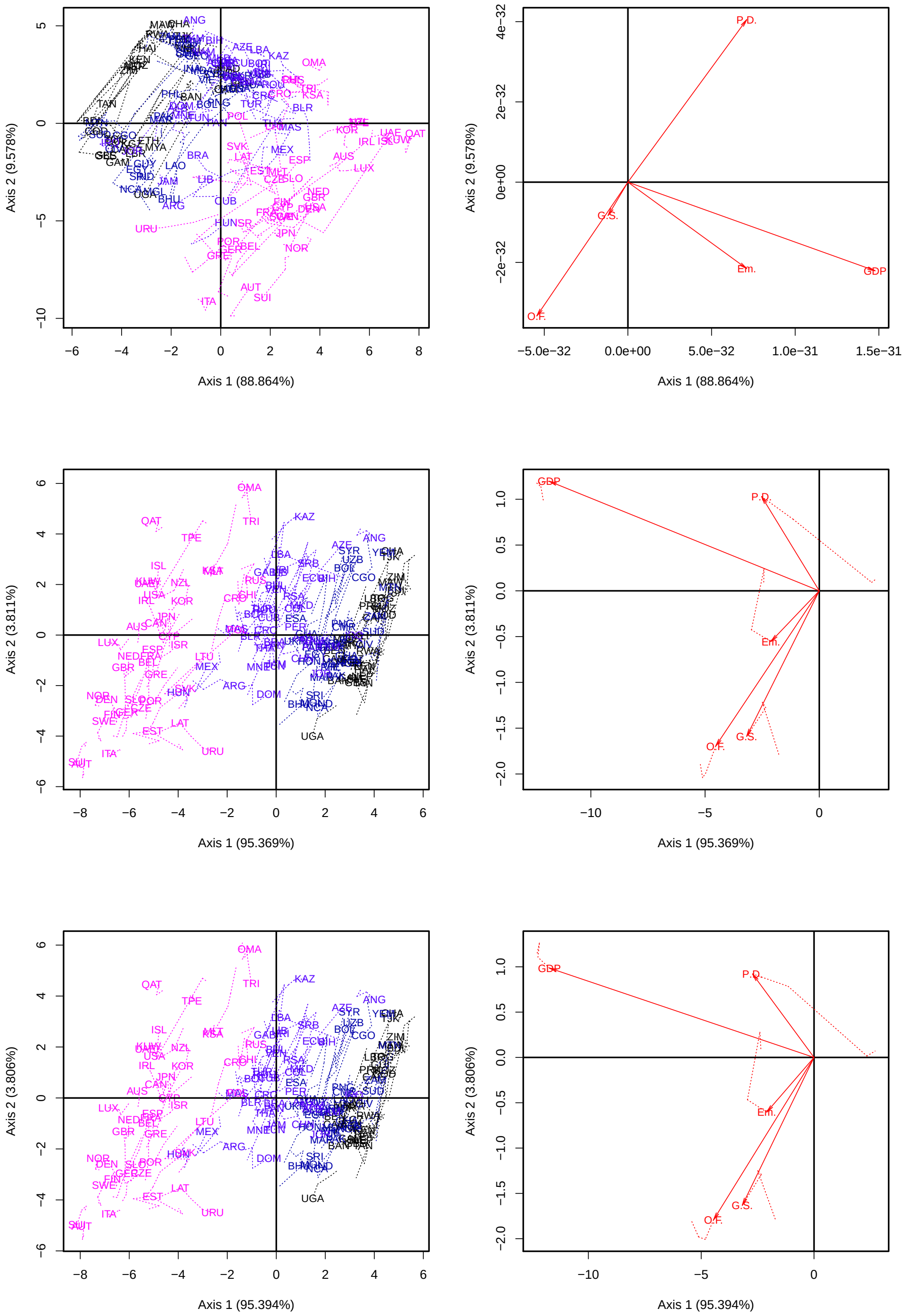

Figura D.6: *Trayectorias de los análisis entre variables económicas y medioambientales*