Decision making under incompleteness based on soft set theory [*]

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Abstract. Decision making with complete and accurate information is
ideal but infrequent. Unfortunately, in most cases the available infor-
mation is vague, imprecise, uncertain or unknown. The theory of soft
sets provides an appropriate framework for decision making that may be
used to deal with uncertain decisions. The aim of this paper is to propose
and analyze an effective algorithm for multiple attribute decision-making
based on soft set theory in an incomplete information environment, when
the distribution of incomplete data is unknown. This procedure provides
an accurate solution through a combinatorial study of possible cases
in the unknown data. Our theoretical development is complemented by
practical examples that show the feasibility and implementability of this
algorithm. Moreover, we review recent research on decision making from
the standpoint of the theory of soft sets under incomplete information.

Keywords: Soft sets; Decision making; Incomplete information; Choice
value; Combinatorics.

1 Introduction

The aim of this paper is to propose an effective algorithm that facilitates a mul-
tiple attribute decision-making based on the theory of soft sets under incomplete
information, under the general assumption that the distribution of the incom-
plete data is unknown. Han *et al.* [16], Qin *et al.* [26] and Zou and Xiao [37] laid
the foundation of soft-set based decision making under incomplete information.
Here we follow a novel and different approach.

   Complete and accurate information is ideal for decision making, but this
situation is rarely met in practice. In most cases the available information is
vague, imprecise, uncertain or unknown.

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Motivated by these concerns, Zadeh [35] marked the beginning of fuzzy sets theory. At its core, partial membership allows that imprecise information about more complex situations can be faithfylly represented and correctly handled. Many extensions of fuzzy sets broaden their scope (v., [4,10] for definitions and relationships). Relatedly, Molodtsov initiated the concept of soft set theory [25], whereas Aktaş and Çağman [1], Ali *et al.* [9], Maji *et al.* [24], Sezgin and Atagün [29] and Feng *et al.* [15] are further essential references. Concerning extensions of soft sets, Maji, Biswas and Roy [22] introduced fuzzy soft sets (v., [2,3,8,20] for decision making criteria in this model), and Wang, Li and Chen [30] introduced hesitant fuzzy soft sets. Ma *et al.* [21] provided a review of decision making methods based on hybrid soft set models.

Maji *et al.* [23] provides a criteria for selecting an object in a soft set scenario, which consists of maximization of the choice values of the problem. Applications of extended soft set theory include rule mining (Herawan and Deris [18]), data mining processes (Qin *et al.* [27]), international trade (Xiao *et al.* [31]), and medical diagnosis (cf., e.g., [7,11,12,17,34]). Recent advances in this field include the parameter reduction problem in soft set based decision making (cf., [36]).

Zou and Xiao [37] observed that in the process of collecting data, the practitioner often encounters unknown, missing or inexistent data, which suggests the concept of incomplete soft sets (v., Han *et al.* [16], Qin *et al.* [26], Lin *et al.* [19] and Alcantud and Santos-García [5,6] for additional analyses). Deng and Wang [14] extended this notion to incomplete fuzzy soft sets in order to predict unknown data in fuzzy soft sets.[3]

However, there are situations where there is perfect uncertainty about the real value of missing data, or we are sure that the alternatives or attributes are independent. In those cases we cannot presume that averages, probabilities or any other specific evaluations produce reliable estimations as in previous solutions. To deal with these situations Alcantud and Santos-García [6] presented a completely redesigned approach to soft set based decision making problems under incomplete information. It relies on the classical Laplacian argument of probability theory and consequently it suggests to examine all completed tables arising from the original incomplete table. All these tables are then evaluated as is standard, i.e., by their respective choice values (cf., [23]). The alternatives are ultimately ranked according to the number or proportion of tables where they are choice value maximizers. The computational costs of this procedure are examined in [6]. The conclusion is that with a large number of missing values, the problem cannot be efficiently solved by bruteforce. Hence in [5] these authors propose two modified algorithms that permit to tackle problems where the number of unknown values is larger.

In this paper we produce an exact solution to that problem based on the application of combinatorics. We provide an algorithm that implements the mathematical solution. The computational performance of the algorithm is then compared with prior solutions in the literature.

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[3] Although Yang *et al.* [32] showed some weaknesses of this approach, Deng and Chen [13] successfully resolved these conflicting issues.

The remainder of this paper is organized as follows. Section 2 briefly states some terminology and definitions from soft set theory. Then we review the relevant antecedents, define the technical notions that we need, and recall the *domination sieve* for incomplete soft sets. Section 3 shows the steps in the proposed algorithm, as well as a fully developed example that proves the feasibility and implementability of our proposal. Finally, Section 4 concludes the paper.

2   Definitions and Notation

2.1   Definitions: Soft Sets and Incomplete Soft Sets

In this section, we first introduce the standard definitions for complete and incomplete soft sets. There is a fixed universe U of objects, options or alternatives, and there is a universal set E of parameters, characteristics or attributes.

Definition 1 (Molodtsov [25]). A pair (F, A) is a *soft set* over U when $A \subseteq E$ and $F : A \longrightarrow \mathcal{P}(U)$, where $\mathcal{P}(U)$ denotes the set of all subsets of U.

Definition 2 (Han *et al.* [16]). A pair (F, A) is an *incomplete soft set* over U when $A \subseteq E$ and $F : A \longrightarrow \{0, 1, *\}^U$, where $\{0, 1, *\}^U$ is the set of all functions from U to $\{0, 1, *\}$.

The $*$ symbol in the previous definition represents an unknown data. In other words, if the membership of an element u in the subset of U approximated by e is unknown, then $F(e)(u) = *$. Of course, as in standard soft sets, if an object u is (resp. is not) an element of the subset of U approximated by e, then $F(e)(u) = 1$ (resp., $F(e)(u) = 0$). Thus any soft set can be regarded as an incomplete soft set in a natural way.

Henceforth we assume $U = \{u_1, u_2, \ldots u_N\}$ and $A = \{e_1, e_2, \ldots e_M\}$. Yao [33] explained that under this finiteness assumption, soft sets can be represented either by matrices or in tabular form. Rows correspond to the options, and columns correspond to the parameters. The same is true for incomplete soft sets. Suppose that $T = (t_{ij})_{N \times M}$ is the $N \times M$ matrix associated with the incomplete soft set (F, A). Then element t_{ij} is either one or zero or $*$, depending on whether object i verifies property j, does not verify it, or it is unknown whether i verifies property j, respectively.

Concerning the choice decision mechanism for soft sets, we agree with existing literature in that choice values should be used. As to incomplete soft sets, our proposal is original in that it does not discard any of the possible filled tables or completed soft sets.

2.2   Previous literature

Table 1 summarizes the main previous approaches to our problem.

Table 1. Summary of research studies about incomplete soft set based decision making. The indexes are named as in the original papers.

Methodology and references		Indexes
Weight-average of all possible choice values of objects. Weights of each choice value given by distribution of the other objects. Original approach [37]		d_i, d_{i-p}, $c_{i(0)}$, $c_{i(1)}$
Data filling based on association between parameters. Choice: choice values for completed set. It presumes that objects are cross-related. Inspired by above approach [26]		Q_i
Elicitation criterions for incomplete soft sets generated by restricted intersection. This problem is related although different [16]		-
Elimination of dominated options. Choice mechanism: choice values.	*Random sample*, thus results depend on sample [5]†	s_i^2
Laplacian argument: equal probability to all completed tables. Suitable when there is no guarantee that objects are related to each other.	*Brute force*, thus computationally costly [6]	s_i

2.3 Notation and Fundamentals of our Algorithm

From the input matrix T we calculate the number of ones and unknown values for every object in the soft set. These are fundamental elements for the analysis of the optimal solution in a fully uncertain environment.

Let v^1 be the vector of numbers of 1's (ones values) by rows in T, i.e., $v^1 = (v_1^1, \ldots v_N^1)$, where v_i^1 is the number of ones in object i. Observe that if $c_{i(0)}$ is i's choice value if all missing data are replaced with 0, then $v_i^1 = c_{i(0)} = |\{e_j \in A : F(e_j)(u_i) = 1\}|$.

The maximum value in vector v^1 is $c_0 = \max\{c_{i(0)} : i = 1, \ldots, N\} = \max\{v_i^1 : i = 1, \ldots, N\}$.

Let v^* be the vector of numbers of $*$'s (unknown values) by rows in T, i.e., $v^* = (v_1^*, \ldots v_N^*)$, where v_i^* is the number of unknown values in object i. This means $v_i^* = |\{e_j \in A : F(e_j)(u_i) = *\}|$. We let m_* denote the maximum value in vector v^*, i.e., $m_* = \max\{v_i^* : i = 1, \ldots, N\}$.

Observe that if $c_{i(1)}$ is i's choice value when all missing data are replaced with 1, then $c_{i(1)} = c_{i(0)} + v_i^*$ for each i.

The number of unknown values is $M^* = \sum_{i=1}^{N} v_i^*$. A simple combinatorial analysis shows that the number of *combinations* of unknown choice values is:

$$M^c = \prod_{i=1}^{N} (v_i^* + 1).$$

Example 1 below illustrates these notions.

2.4 Domination Sieve

In order to gain efficiency, our algorithm performs a pre-screening by removing the objects whose choice values can never be maximal. Put shortly, we eliminate dominated alternatives defined as follows:

Definition 3 (Alcantud and Santos-García [6]). Let (F, A) be an incomplete soft set over U. Option i *dominates* option k if and only if $c_{k(1)} < c_{i(0)}$.

Hence in order to check if an option i dominates an option k one needs to verify if $c_{k(0)} + v_k^* < c_{i(0)}$. Intuitively, no matter how we complete the soft set, option i has a choice value that is strictly higher than the choice value of option j. Therefore dominated options should be rejected in any choice-valued approach.

In order to simplify our problem we compute the maximum value $c_0 = 3$ of the choice values $v_i^1 = c_{i(0)}$ for every i. If c_0 is greater than the choice value $c_{k(1)}$ of object u_k, then object u_k can be dropped from the initial matrix because its choice value can never be maximum in any posible completed soft set. In this way, by reducing the number of rows in the matrix form of an incomplete soft set, there are less missing data, which reduces runtime and improves final results.

After dominated options are sieved out, we apply the remaining steps to the new trimmed matrix. To simplify the notation, this reduced matrix is also called T. Its objects and features will be reappointed N (also number of rows of T) and M (also number of columns of T), respectively.

Example 1. Let $T = (t_{ij})$ be the following 5×4 initial matrix, which represents an incomplete soft set over a universe of 5 objects with 4 relevant characteristics for evaluation of the alternatives:

$$T = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & * & 0 \\ 1 & * & * & 0 \\ 1 & * & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

It is easy to compute that $v^1 = (3, 2, 1, 1, 0), v^* = (0, 1, 2, 1, 0)$ therefore $m_1 = 3$, $m_* = 2$, $M^* = 4$, and $M^c = 12$.

We calculate the maximum value c_0 of all choice values $c_{i(0)}$. Observe that $c_{4(0)} + v_4^* = 1+1 < c_{1(0)} = c_0 = 3$ and $c_{5(0)} + v_5^* = 0+0 < c_{1(0)} = c_0 = 3$. Thus c_0 is unattainable for the choice values $c_{i(1)}$ of objects u_4 and u_5, and these objects

Table 2. Tabular representation of the soft set in Example 1, and noteworthy associated indices.

	e_1	e_2	e_3	e_4	$v_i^1 = c_{i(0)}$	$c_{i(1)}$	v_i^*
u_1	1	1	1	0	$c_0 = \underline{3}$	3	0
u_2	1	1	*	0	2	3	1
u_3	1	*	*	0	1	3	2
$\overline{(u_4)}$	1	*	0	0	1	$\underline{2}$	1
$\overline{(u_5)}$	0	0	0	0	0	$\underline{0}$	0

can be safely removed from the initial matrix (see Table 2, where the relevant items are underlined). Observe that no matter how the table is completed, the choice values of options u_4 and u_5 will be smaller than the choice value of u_1.

After removing objects u_4 and u_5, the new sieved matrix is:

$$T\,(\text{sieved}) = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & * & 0 \\ 1 & * & * & 0 \end{pmatrix}$$

For the new trimmed matrix the vector of numbers of one values v_1 and the vector of numbers of unknown values v^* are:

$$v^1 = (\,3,\,2,\,1\,); \qquad v^* = (\,0,\,1,\,2\,)$$

which can be drawn from the corresponding columns in Table 2. And now it is immediate to compute $m_1 = 3$, $m_* = 2$.

Moreover $M^* = 3$ (number of unknown values) and $M^c = 6$ (number of combinations of unknown choice values). This means that the four unknown values of the initial matrix T become three values for the new sieve matrix T, while the number of feasible states (in terms of configurations of choice values) is halved. These states are:

(1) the choice values can be 3 for option u_1, 2 for u_2 and 1 for u_3. This fact happens in exactly one completed table.
(2) the choice values can be 3 for option u_1, 2 for u_2 and 2 for u_3. This fact happens in exactly two completed tables.
(3) the choice values can be 3 for option u_1, 2 for u_2 and 3 for u_3. This fact happens in exactly one completed table.
(4) the choice values can be 3 for option u_1, 3 for u_2 and 1 for u_3. This fact happens in exactly one completed table.
(5) the choice values can be 3 for option u_1, 3 for u_2 and 2 for u_3. This fact happens in exactly two completed tables.
(6) the choice values can be 3 for option u_1, 3 for u_2 and 3 for u_3. This fact happens in exactly one completed table.

We can represent the completed soft sets for cases (1), (2) and (3) as follows:

	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
u_1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0
u_2	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0
u_3	1	0	0	0	1	1	0	0	1	0	1	0	1	1	1	0

And the completed soft sets for cases (4), (5) and (6) are represented as

	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4	e_1	e_2	e_3	e_4
u_1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0
u_2	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	0
u_3	1	0	0	0	1	1	0	0	1	0	1	0	1	1	1	0

3 Algorithm: Incomplete Soft Sets for Decision Making Problem

3.1 The Main Elements of the Algorithm

Our algorithm computes the number of tables that produce every possible configuration of choice values. Under the Laplacian assumption that they are equally probable, it chooses the alternative that is best in the highest proportion of completed soft sets. However our algorithm does not compute them explicitly as in Example 1, but instead we use a combinatorial analysis in order to define several auxiliary matrices corresponding to possible values and probabilities for each of the objects that we actually use. Let PV_1 (resp., PPV_1) be the $(N \times (m_* + 1))$-matrices of potential values (resp., proportion by rows of potential values) for each object. For computational purposes, the rows with a number of elements less than $m_* + 1$ are filled up with zero values. Its elements are calculated according to the following formulas:

$$PV_1(i,j) = v_i^1 + j - 1; \qquad PPV_1(i,j) = \frac{\binom{v_i^*}{j}}{2^{v_i^*}},$$

where i varies from 1 to N and j varies from 1 to $v_i^* + 1$.[4] These matrices are shown in Example 2.

Example 2. According to the data of Example 1, now we calculate matrices PV_1 and PPV_1:

$$PV_1 = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 2 & 3 \end{pmatrix}, \qquad PPV_1 = \begin{pmatrix} 1.00 & 0 & 0 \\ 0.50 & 0.50 & 0 \\ 0.25 & 0.50 & 0.25 \end{pmatrix}.$$

Hence for example, the fact $PV_1(1,1) = 3$ and $PPV_1(1,1) = 1$ means that in 100% of the 8 filled tables, the choice value of u_1 is 3. The fact $PV_1(2,2) = 3$

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[4] As usual, $\binom{n}{k}$ or "n choose k" returns the binomial coefficient, i.e., the number of combinations of n items taken k at a time, defined as $\dfrac{n!}{(n-k)!k!}$.

and $PPV_1(2,2) = 0.5$ means that in 50% of the 8 filled tables, the choice value of u_2 is 3. And $PV_1(3,2) = 2$ and $PPV_1(3,2) = 0.5$ means that in 50% of all filled tables, the choice value of u_3 is 2.

It is well-known that if two events A and B are independent then their joint probability equals the product of their probabilities, i.e., $P(A \cap B) = P(A) P(B)$. Because choice values are independent events, with probability $1 \cdot 0.5 \cdot 0.5 = 0.25$ the choice values of u_1, u_2 and u_3 are 3, 3, and 2, respectively. And in that case, both u_1 and u_2 are optimal because they are choice value maximizers.

After calculating the auxiliary matrices PV_1 and PPV_1, we analyze all feasible combinations of decision values. As shown in Example 2, we know the probabilities for each of the possible events. Computationally, we build all possible different vectors CV with N rows, in which each element will be a non-zero element of each row in PV_1 matrix. That is, the element CV_i for each object i has a value between $c_{i(0)}$ and $c_{i(0)} + v_i^*$.

For this particular case, we now calculate the probabilities of occurrences from matrix PPV_1. The joint probability XP is the product of all the individual probabilities. When the choice value of that particular case is maximal (may be several maximal), we add that probability XP to the decision values matrix DV.

We repeat the process for all possible vectors CV. The final decision of our Algorithm will consist of the object(s) with greater DV values.

Figure 1 shows a flowchart for the proposal of a decision making procedure under incomplete information that we have described throughout this section.
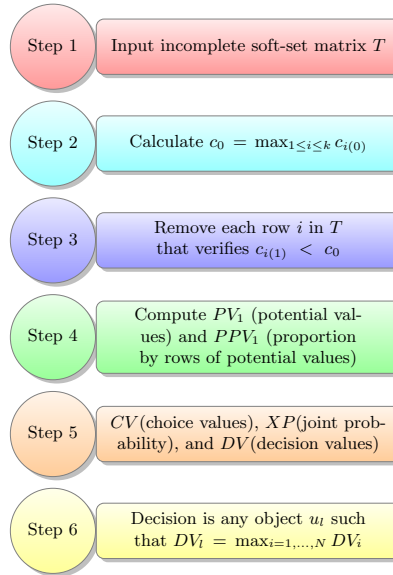
Step 1	Input incomplete soft-set matrix T
Step 2	Calculate $c_0 = \max_{1 \leq i \leq k} c_{i(0)}$
Step 3	Remove each row i in T that verifies $c_{i(1)} < c_0$
Step 4	Compute PV_1 (potential values) and PPV_1 (proportion by rows of potential values)
Step 5	CV(choice values), XP(joint probability), and DV(decision values)
Step 6	Decision is any object u_l such that $DV_l = \max_{i=1,\dots,N} DV_i$

Fig. 1. Step by step procedure for decision making under incompleteness.

Example 3. According to the data of Example 1 and 2, now we calculate final matrices CV, XP, and DV. In particular we obtain

$$DV = \begin{pmatrix} 1.00 \\ 0.50 \\ 0.25 \end{pmatrix}$$

which we interpret as follows. In 100% of the randomly filled tables, option 1 achieves the maximum choice value. In 50% of the randomly filled tables, option 2 achieves the maximum choice value. And in 25% of the randomly filled tables, option 3 achieves the maximum choice value. We recall that ties may happen, which explains why this figures sum up over 100%.

Table 3 contains the elements that produce our solution. In view of our arguments, option 1 should be selected.

Table 3. Combinations of choice values and their respective probabilities in Example 1.

Choice value			Probability	Optimal
u_1	u_2	u_3		solutions
3	2	1	1/8	u_1
3	2	2	1/4	u_1
3	2	3	1/8	u_1, u_3
3	3	1	1/8	u_1, u_2
3	3	2	1/4	u_1, u_2
3	3	3	1/8	u_1, u_2, u_3

3.2   Decision of the Algorithm

Intuitively, our practical implementation of the ideas above is based on the following arguments. According to Laplace's principle of indifference in probability theory, under complete ignorance we must assume that in all tables ∗'s are replaced in equiprobable manner with either 0 or 1. Hence the best we can do is consider in each of these cases that the objects should be selected according to soft-set based decision making procedures. Consequently we should opt for any object that is optimal in the highest proportion of cases with the completed information. The number of possible cases is exponential on the number of unknown values, however we only need to consider completed soft sets which are equivalent for purposes of decision making.

To do this we note that for any object i, if it has multiple unknown characteristics that are completed, we are only interested in the number, not in the

order, of 1's assigned to these unknown values. Thus, the calculation is reduced to a combinatorial analysis in each object.

In accordance with this idea, we endorse the following algorithm for the problems where both U and A are finite:

Algorithm 1 Incomplete Soft Sets Algorithm for Making Decision

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1: Input an incomplete soft set (F, E) with k objects and l parameters in a matrix form, where $t_{ij} \in \{0, 1, *\}$ denotes a cell (i, j).

2: Calculate $c_0 = \max_{1 \leq i \leq k} c_{i(0)}$ from the incomplete matrix, where $c_{i(0)}$ is the choice value for i if all missing data are assumed to be 0.

3: Remove each row i from the incomplete matrix that verifies $c_{i(1)} < c_0$, where $c_{i(1)}$ is the choice value for i if all missing data are assumed to be 1.

4: From the new reduced matrix, calculate the auxiliary matrices PV_1 (matrix of potential values) and PPV_1 (matrix of proportion by rows of potential values).

5: Compute the following matrices: choice values CV, joint probability XP, and decision values DV.

6: The result of the decision is any object u_l such that $DV_l = \max_{i=1,\ldots,N} DV_i$.

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4 Discusion and conclusions

The works of Han *et al.* [16], Qin *et al.* [26] and Zou and Xiao [37] present interesting approaches to incomplete soft set based decision making. These authors used averages, probabilities or other specific evaluations in order to estimate the real value of missing data in a general way and afterwards, they made a decision based on the complementary data.

In this paper we look at the problem from an altogether different perspective. Rather than filling the incomplete data table (see also Khan *et al.* [28]), we propose a combinatorial study through all possible filled tables that can be produced from the original incomplete table. Then the alternatives are ranked by the proportion of filled tables where they achieve the highest choice value. In other words, a final indicator for each of the objects by our algorithm is defined as the value of this ratio. And our decision making procedure consists of selecting alternatives that maximize this indicator. A classical Laplacian argument from probability theory justifies our research method. In general there is perfect uncertainty about the real value of missing data. Therefore, we cannot support the idea that other aspects would let us faithfully estimate these unknown values. Under Laplace's principle of indifference, due to our complete ignorance we are entitled to assume that all possible tables where the missing data are replaced with either 0 or 1 are equiprobable.

Our novel approach meets the following targets: (1) We do not need to assume any cross-relations among options. (2) We do not attempt to fill the tables with hypothesized values. (3) We adopt a Laplacian position and make use of combinatorics. (4) A unique, computationally tractable solution is provided.

An example illustrates the detailed implementation process of our approach and shows evidence of its potential applications in decision-making problems with incomplete information.

In practice, problems where all parameters describing compared options are equally important rarely exist. Our algorithm has been tightly designed for the exclusive purpose of decision making, hence the introduction of weighted parameters would not be trivial and as such deserves a separate analysis. Domination sieve is simple to read in that instance, though.

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