FUZZY SOFT SETS: A MODEL FOR MAKING CHOICES IN AN INTERTEMPORAL FRAMEWORK

José Carlos Rodríguez Alcantud
Universidad de Salamanca (Spain)
jcr@usal.es

María José Muñoz Torrrecillas
Universidad de Almería (Spain)
mjmtorre@ual.es

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Abstract

This paper introduces a model where the options are characterized by one fuzzy soft set in each of an indefinite number of periods. This model extends the standard case of fuzzy soft sets. We explain how to associate a characteristic fuzzy soft set with each model. Finally, a decision making procedure for the selection of alternatives is proposed.

The target applications include portfolio selection in finance, environmental issues, et cetera.

Key-words: Fuzzy soft set, intergenerational choice, decision making.

Notation and definitions

- Let X denote a **set**. Then $\mathscr{P}$(X) is the set of all non-empty subsets of X.
- A **fuzzy subset** (also, FS) A of X is a function $\mu A : X \to [0, 1]$.
  For each $x \in X$, $\mu A(x) \in [0, 1]$ is the degree of membership of x in that subset.
- The set of **all fuzzy subsets** of X will be denoted by **FS**(X).
- Universe of objects: *U* ; universal set of parameters: *E*.

Definition 1. (Molodtsov, 1999). Let *A* be a subset of *E*. The pair (*F*, *A*) is a soft set over *U* if $F : A \to \mathscr{P}(U)$.

Notation and definitions

- Soft set based decision making was pioneered by Maji, Biswas and Roy (2002), and further applications of soft sets in decision making were given in Alcantud, Cruz and Muñoz (2017), Alcantud and Santos-García (2015, 2017), and Roy and Maji (2007).

Definition 2. (Maji, Biswas and Roy, 2002). The pair (F, A) is a **fuzzy soft set** (FSS) over *U* when $A \subseteq E$ and F : A $\rightarrow$ **FS**(*U*).

The set of all fuzzy soft sets over U will be denoted as $\mathscr{F}\mathscr{S}$(U).

✓Example: Films parameterized by attributes.

Notation and definitions

- In real practice both *U* and *A* use to be finite. Then let k and n denote the respective number of elements of *U* and *A*. These **soft sets** can be represented either by k x n matrices or by a tabular form.
- Both practical representations are binary, that is to say, all cells are either 0 or 1.
- One can proceed in a similar way in **fuzzy soft sets**, but now the possible values in the cells lie in [0, 1].

Notation and definitions

Ref.	Aggregation	Methodology	Solution	Other issues
[12]	Min operator	Scores from a comparison matrix	Unique	Many ties Information is lost by aggregation
[7]	Not provided	Choice value of level soft set	Not unique	Ties proliferate Richness introduces indeterminacy Additional inputs needed (e.g., threshold fuzzy set)
[1]	Product operator	Scores from new relative comparison matrix	Unique	Good power of discrimination
[8]	Not provided	Similarity measure	Unique	Use of subjective weights

Table 1: A list of the main fuzzy soft set based decision making procedures with their main characteristics.

[12]: Roy and Maji (2007)
[7]: Fen, Jun, Liu and Li (2010)
[1]: Alcantud (2016)
[8]: Liu, Qin and Pei (2017)

An intertemporal model for FSSs

- Let $\mathscr{S}$ denote the set of infinite sequences of the interval [0, 1] (also called infinite utility streams).

- Our intertemporal model of fuzzy soft sets over U is defined by

  F : A → ***S*** (*U*) where ***S*** (*U*) represents the mappings $U \to \mathscr{S}$.

- For each attribute and each alternative, we express the degree of belongingness of such an alternative in each period of time.

- In practical terms, where both $U = \{o_1, \ldots, o_m\}$ and $A = \{e_1, \ldots, e_m\}$ are finite, we can represent this information in a table where the cells are either finite or infinite sequences of membership degrees:

An intertemporal model for FSSs

	e_1	e_2	$\ldots\ldots$	e_n
o_1	$(u_{11}^1, u_{11}^2, \ldots, u_{11}^t, \ldots)$	$(u_{12}^1, u_{12}^2, \ldots, u_{12}^t, \ldots)$	$\ldots\ldots$	$(u_{1n}^1, u_{1n}^2, \ldots, u_{1n}^t, \ldots)$
$\vdots$				
o_m	$(u_{m1}^1, u_{m1}^2, \ldots, u_{m1}^t, \ldots)$	$(u_{m2}^1, u_{m2}^2, \ldots, u_{m2}^t, \ldots)$	$\ldots\ldots$	$(u_{mn}^1, u_{mn}^2, \ldots, u_{mn}^t, \ldots)$

Table 2: Tabular representation of our intertemporal model for fuzzy soft sets.

Decision making in intertemporal FSSs. Algorithm for decision making

- Inputs: An intertemporal table of fuzzy soft sets, a reduction mechanism, and a fuzzy soft set decision making procedure (e.g., from Table 1).

- Procedure for prioritizing the alternatives in Table 2:

  1. Associate a FSS with the original intertemporal information by means of the selected reduction mechanism.
  2. Prioritize the alternatives in the reduced FSS by the selected decision making procedure (Alcantud, 2016).
  3. The result of the decision is any object o_k that is at the top of the ranking in the previous step.

Decision making in intertemporal FSSs. Algorithm for decision making

Algorithm 1 (Alcantud [38])

Input: a fuzzy soft set (F, A), which we place in the form of a table. Its cell (i, j) is represented by t_{ij}

1: For each parameter j, let M_j be the maximum membership value of any object, i.e., $M_j = \max_{i=1,\dots,k} t_{ij}$ for each $j = 1, \dots, q$.

Now construct a $k \times k$ comparison matrix $A = (a_{ij})_{k\times k}$ where for each i, j, we let a_{ij} be the sum of the non-negative values in the following finite sequence:

$$\frac{t_{i1} - t_{j1}}{M_1}, \frac{t_{i2} - t_{j2}}{M_2}, \dots\dots, \frac{t_{iq} - t_{jq}}{M_q}.$$

We can display this matrix as a comparison table.

2: For each $i = 1, \dots, k$, calculate R_i as the sum of the elements in row i of A, and T_i as the sum of the elements in column i of A. For every $i = 1, \dots, k$, calculate the score $S_i = R_i - T_i$ of object i.

3: The result of the decision is any object o_k such that $S_k = \max_{i=1,\dots,k} S_i$.

Decision making in intertemporal FSSs. Example 1

	e_1	e_2	e_3	e_4
o_1	$(0.2, 0.3, 0.3, 0.4)$	$(0.5, 0.5, 0.6, 0.6)$	$(0.7, 0.7, 0.6, 0.6)$	$(0.6, 0.5, 0.5, 0.6)$
o_2	$(0.6, 0.5, 0.5, 0.4)$	$(0.3, 0.4, 0.6, 0.6)$	$(0.4, 0.5, 0.5, 0.5)$	$(0.6, 0.6, 0.4, 0.4)$
o_3	$(0.3, 0.3, 0.4, 0.5)$	$(0.5, 0.5, 0.7, 0.6)$	$(0.5, 0.6, 0.6, 0.5)$	$(0.5, 0.4, 0.6, 0.6)$

	e_1	e_2	e_3	e_4
o_1	0.3	0.55	0.65	0.55
o_2	0.5	0.475	0.475	0.5
o_3	0.375	0.575	0.55	0.525

Decision making in intertemporal FSSs. Example 1

	o_1	o_2	o_3
o_1	0	0.491	0.199
o_2	0.4	0	0.25
o_3	0.193	0.335	0

	Row-sum (R_i)	Column-sum (T_i)	Score (S_i)
o_1	0.69	0.593	0.097
o_2	0.65	0.825	−0.175
o_3	0.528	0.449	0.079

Decision making in intertemporal FSSs. Intertemporal choices

- "Most choices require decision-makers to trade-off costs and benefits at different points in time. Decisions with consequences in multiple time periods are referred to as **intertemporal choices**. Decisions about savings, work effort, education, nutrition, exercise, and health care are all intertemporal choices." (Chavris et al. 2008)

- Discounted utility models (Ramsey, 1928; Samuelson, 1937; Koopmans, 1960) consider decision-makers facing the selection of alternatives based on the **weighted addition of utilities**, being these weights **discount factors** based on temporal delays.

Decision making in intertemporal FSSs. Intertemporal choices

- The discounted utility theory states that individuals discount future events at a constant rate, so the value of an experience during a period of time [0,T] is given by:

$$U = \sum_{t=0}^{T} \delta^t u$$

- The discounted utility model is used in **project appraisals** where the expected cash-flows of a project are discounted to obtain its Net Present Value (NPV) at instant 0.

- The European Commission (2014) recommends the use of the exponential discounting model and a constant 5% European social discount rate for the Cohesion Fund eligible countries.

Decision making in intertemporal FSSs. Example 2

- Evaluation of **two alternative portfolios of projects** that a public administration may undertake.
- Objective: to choose the best alternative, in terms of utility.
- Projects with an unlimited time horizon that could be enjoyed by different generations.
- Each portfolio is composed by four elements, being:
  - ✓ different type of projects (e.g. city park, urban infrastructure, sports facilities, etc.) which are addressed to the population in general,
  - ✓ different projects which are addressed to different groups of population, depending on their special needs,
  - ✓ different/similar projects that will be carried out in different geographical areas.

Decision making in intertemporal FSSs. Example 2

Period	Attribute 1	Attribute 2	Attribute 3	Attribute 4
1	0.05	1.00	0.70	1.00
2	0.10	0.95	0.70	0.95
3	0.15	0.90	0.70	0.90
4	0.20	0.85	0.70	0.85
5	0.25	0.80	0.70	0.80
6	0.30	0.75	0.70	0.75
7	0.35	0.70	0.70	0.70
8	0.40	0.65	0.70	0.65
9	0.45	0.60	0.70	0.70
10	0.50	0.55	0.70	0.75
11	0.55	0.50	0.70	0.80
12	0.60	0.45	0.70	0.85
13	0.65	0.40	0.70	0.90
14	0.70	0.35	0.70	0.95
15	0.75	0.35	0.70	1.00
16	0.80	0.35	0.70	1.00
17	0.85	0.35	0.70	1.00
18	0.90	0.35	0.70	1.00
19	0.95	0.35	0.70	1.00
20	1.00	0.35	0.70	1.00
21	1.00	0.35	0.70	1.00
22	1.00	0.35	0.70	1.00
23 onwards	1.00	0.35	0.70	1.00
A_i	13.09	10.61	14.00	18.33
A_i/T	0.65	0.53	0.70	0.92

Table 3: Portfolio 1

Decision making in intertemporal FSSs. Example 2

Period	Attribute 1	Attribute 2	Attribute 3	Attribute 4
1	0.00	1.00	0.60	1.00
2	0.10	0.94	0.60	0.90
3	0.20	0.88	0.60	0.80
4	0.30	0.82	0.60	0.70
5	0.40	0.76	0.60	0.60
6	0.50	0.70	0.60	0.50
7	0.60	0.64	0.60	0.40
8	0.70	0.58	0.60	0.30
9	0.80	0.52	0.60	0.40
10	0.90	0.52	0.60	0.50
11	1.00	0.52	0.60	0.60
12	1.00	0.52	0.60	0.70
13	1.00	0.52	0.60	0.80
14	1.00	0.52	0.60	0.90
15	1.00	0.52	0.60	1.00
16	1.00	0.52	0.60	1.00
17	1.00	0.52	0.60	1.00
18	1.00	0.52	0.60	1.00
19	1.00	0.52	0.60	1.00
20	1.00	0.52	0.60	1.00
21	1.00	0.52	0.60	1.00
22	1.00	0.52	0.60	1.00
23 onwards	1.00	0.52	0.60	1.00
A_i	15.44	12.24	12.00	16.65
A_i/T	0.77	0.61	0.60	0.83

Table 4: Portfolio 2

Decision making in intertemporal FSSs. Example 2

	e_1	e_2	e_3	e_4
o_1	(0.05. 0.10. 0.15. 0.20. 0.25. …)	(1.00. 0.95. 0.90. 0.85. 0.80. …)	(0.70. 0.70. 0.70. 0.70. 0.70. …)	(1.00. 0.95. 0.90. 0.85. 0.80. …)
o_2	(0.00. 0.10. 0.20. 0.30. 0.40. …)	(1.00. 0.94. 0.88. 0.82. 0.76. …)	(0.60. 0.60. 0.60. 0.60. 0.60. …)	(1.00. 0.90. 0.80. 0.70. 0.60. …)

Table 5: Intertemporal fuzzy soft sets

	Attribute 1	Attribute 2	Attribute 3	Attribute 4
P_1	0.65	0.53	0.70	0.92
P_2	0.77	0.61	0.60	0.83

Table 6: Tabular representations of the reduced FSS (by discounted utility) associated with Table 5

Decision making in intertemporal FSSs. Example 2

	Att. 1	**Att. 2**	**Att. 3**	**Att. 4**
Diff. memberships P_1 vs. P_2	−0.12	−0.08	0.10	0.08
Diff. memberships P_2 vs. P_1	0.12	0.08	−0.10	−0.08
M_j	0.77	0.61	0.70	0.92

	P_1	P_2
P_1	0	0.29
P_2	0.23	0

	Row-sum (R_i)	Column-sum (T_i)	Score (S_i)
P_1	0.29	0.23	0.06
P_2	0.23	0.29	−0.06

Figure 1: Computing the Comparison table and scores of the reduced fuzzy soft set (S, P) through Algorithm 1.

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