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Cryptocurrency risk assessment under a semi-nonparametric approach, risk measures and backtesting techniques

DOCTORAL THESIS

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To David, Daniel y Arturo.

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ABSTRACT

The emergence of cryptocurrencies with a philosophy of independence from any institutions or central banks makes them a unique asset that has marked an important milestone in the financial and economic world. These assets also exhibit remarkable high volatility with frequent extreme values, which may cause instability in financial markets. Nor should we forget that cryptocurrencies are in their earliest stage and most probably its great volatility is due, in part, to this initial phase in which it finds itself. That is one of the purposes for carrying out a more comprehensive analysis on risk assessment of this industry.

The focus of this research is illustrated through the following four chapters, based on modeling the full density of cryptocurrency returns with particular emphasis on providing accurate risk measures, i.e., fitting the tails of the distributions. With this aim, we analyze the conditional variance, modeled under GARCH-type models, considering the semi-nonparametric (SNP) approach based on Gram Charlier (GC) and Positive Gram Charlier (PGC) series.

Firstly, this PhD Thesis explores the capacity of a method to approximate the cryptocurrency return conditional frequency distribution by endogenously selecting the best SNP expansion at any point in time compared to considering SNP expansions with a fixed length (number of parameters) in terms of the cumulative distribution function (cdf). The good performance of this new methodology compared to fixed-order GC expansion supports the thought about its usage in the future for forecasting risk.

Following the line of a univariate perspective and considering SNP distributions as well as others parametric (Gaussian, Student's t, Skewed-t), three different risk measures Value at Risk (VaR), Median Shortfall (MS) and Expected Shortfall (ES) have been assessed through backtesting techniques and a wide variety of tests. This comprehensive analysis for Bitcoin and five of the most representative altcoins[1] shows that flexible SNP approaches outperform risk measures of most crypto assets (especially Bitcoin) and tend to provide the most conservative risk assessment. Furthermore, MS seems to be a robust-to-outliers and reliable risk measure for cryptocurrencies and discuss

[1]The term 'altcoins' *(alternative coins*) refers to all cryptocurrencies different from Bitcoin.

on the choice of the appropriate probability levels according to the assumed distribution. The evidence supports that MS might be an accurate alternative to VaR and ES.

Under a multivariate analysis, it is implemented a flexible and accurate new methodology for portfolio risk management that consists of computing pairwise conditional correlations under bivariate marginal SNP distributions with PGC through the Dynamic Conditional Correlation (DCC) model. This method tries to solve the 'curse' of dimensionality triggered in DCC models when there are large portfolios compared with a parsimonious model as Dynamic Equicorrelation (DECO). Both models are tested for a portfolio of three cryptocurrencies (Bitcoin, Litecoin and Ripple) through backtesting techniques for VaR, MS and ES. In the light of the results, both models show good performance but the new method might be an excellent proposal for analyzing accurately large portfolios.

Finally, a summarize of all the conclusions are presented at the end of this PhD Thesis as well as some recommendations for futures research.

RESUMEN

La aparición de las criptomonedas con su filosofía de no dependencia de cualquier institución o banco central las convierte en un activo único que ha marcado un hito importante en el mundo financiero y económico. Estos activos también presentan una notable alta volatilidad con frecuentes valores extremos, lo que puede provocar inestabilidad en los mercados financieros. Tampoco hay que olvidar que las criptomonedas se encuentran en su etapa más temprana y muy probablemente su gran volatilidad se deba, en parte, a esta fase inicial en la que se encuentran. Por ello, es necesario realizar un análisis más exhaustivo sobre la evaluación del riesgo de esta industria.

El enfoque de esta investigación se muestra a través de los cuatro capítulos siguientes, basados en la modelización de la densidad completa de los rendimientos de las criptomonedas con especial énfasis en proporcionar medidas de riesgo precisas, es decir, ajustando las colas de las distribuciones. Con este objetivo se analiza la varianza condicional, modelada bajo modelos de tipo GARCH, considerando el enfoque semi-noparamétrico (SNP) basado en las series de Gram Charlier (GC) y Gram Charlier Positiva (PGC).

En primer lugar, esta tesis doctoral explora la capacidad de un método para aproximar la distribución de frecuencia condicional de la rentabilidad de las criptomonedas mediante la selección endógena de la mejor expansión GC en cualquier momento, en comparación con las expansiones GC con una longitud fija (por el número de parámetros) en términos de la función de distribución acumulativa (fda). El buen comportamiento de esta nueva metodología en comparación con la expansión GC de orden fijo apoya la idea de su uso en el futuro para la previsión del riesgo.

Siguiendo la línea de una perspectiva univariante y considerando las distribuciones SNP, así como otras paramétricas (Gausiana, t de Student y su versión asimétrica), se evalúan tres medidas de riesgo diferentes: Valor en Riesgo (VaR), Median Shortfall (MS) y Expected Shortfall (ES), a través de técnicas de backtesting y aplicando una amplia variedad de tests. Este análisis exhaustivo para Bitcoin y cinco de las altcoins[2]

[2] El término ‘altcoins’ *(alternative coins)* se refiere a todas las criptomonedas exceptuando Bitcoin.

más representativas, muestra que los enfoques flexibles SNP tienen buen comportamiento para las medidas de riesgo de la mayoría de las criptomonedas (especialmente Bitcoin) y tienden a proporcionar una evaluación de riesgo más conservadora. Además, MS parece ser una medida de riesgo robusta a los valores atípicos y fiable para las criptomonedas, y plantea la elección de los niveles de probabilidad adecuados según la distribución asumida. Los resultados apoyan que MS podrían ser una alternativa más precisa al VaR y al ES.

Bajo un análisis multivariante, se implementa una nueva metodología flexible y precisa para la gestión del riesgo de carteras, consistente en estimar correlaciones condicionales por pares bajo distribuciones marginales bivariantes SNP con la Gram Charlier Positiva a través del modelo de Correlación Condicional Dinámica (DCC). Este método trata de resolver la "maldición" de la dimensionalidad que se desencadena en los modelos DCC cuando hay carteras grandes y se compara con un modelo parsimonioso como el de Equicorrelación Dinámica (DECO). Ambos modelos se analizan para una cartera de tres criptomonedas (Bitcoin, Litecoin y Ripple) mediante técnicas de backtesting para VaR, MS y ES. A la luz de los resultados, ambos modelos muestran un buen comportamiento, pero el nuevo método podría ser una excelente propuesta para analizar con precisión grandes carteras.

Por último, se presenta un resumen de todas las conclusiones al final de esta tesis doctoral, así como algunas recomendaciones para investigaciones futuras.

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INTRODUCTION

Just a few years ago, the cryptocurrency industry hardly existed. It was in 2008 when Bitcoin emerged (Nakamoto, 2008), being totally unknown and only accessible to computer freaks or mathematicians, due to the complex technology behind it. After those years since their inception, no one could have imagined the paradigm shift what the cryptocurrency industry would become today: a fast-growing industry carefully studied by governments and institutions. It is a peer-to-peer (P2P) network that let interaction between people without an intermediary and is fully decentralized, i.e. it does not depend on any central bank or any institution to authenticate transactions. It is undeniable that this new conception challenges the current understanding of the monetary and financial system (Dyhrberg, 2016), which is based on the trust of a third party such as central banks and financial institutions. Perhaps, to some extent, it would be more appropriate to say that this omission of the trusted third party in cryptocurrencies is not such, but is replaced by cryptographic proof that ensures that the system is irreversible. In addition, Bitcoin uses a public ledger or blockchain accessible to everyone where all transactions are recorded and it is not located anywhere.

Nowadays, thousands of cryptocurrencies exist, each of them with its individual characteristics that make them different from one another but based mainly on the independence of a trusted third party and blockchain technology. Given its own features and their specific performance, maybe it would be accurate to refer to them such as *'cryptoassets'*, which is widely accepted for cryptocurrencies and all assets related to them (Rauchs et al., 2019). Its price depends on supply and demand and unlike other assets, they trade twenty-four seven. That is the reason why transactions are continuously making, regardless of the hour or any location worldwide. Being a decentralized system, no regulator imposes rules avoiding sudden ups and downs as happen in other markets.

One of the major areas of concern is the high volatility they exhibit maybe since they are in their early stage (Baek and Elbeck, 2015) and therefore, setting proper risk forecasting patterns are necessary to address certain economic behavior. On this account, this PhD Thesis is based on volatility analysis using risk measures under a semi-nonparametric (SNP) approach which seems to be more flexible and fit better to our data, in comparison to a parametric approach, mainly focusing on the far end of the tail distributions in order to gather the jumps they own, due to the high presence of extreme values.

Previously, there have been authors who have focused their research on analyzing cryptocurrencies volatility (Baek and Elbeck, 2015; Balcilar et al., 2017; Bouri et al., 2017; Blau, 2018; Ardia et al., 2019) considering a variety of conditional heteroskedastic GARCH processes. For this purpose, our proposal also lies on GARCH processes but modeling the full density through a semi-nonparametric (SNP) approach in order to model the probability density function (pdf) of the underlying stochastic distribution. This approach, based on Edgeworth and Gram-Charlier (GC) series, has the property of asymptotically approximating the true distribution in terms of the weighted derivatives of the standard Gaussian pdf or their corresponding Hermite polynomials, which form an orthonormal basis (Kendall and Stuart, 1977; Hald, 2000). This orthogonality property allows defining a proper family of density functions even when the expansion is truncated.

Given its ability to approximate finite sample distributions, the SNP approach was introduced in the seventies in econometrics (Sargan, 1976) but it was not until the end of the 20th Century when appeared the first applications to finance. Initially, only short GC expansions were considered to account for skewness and kurtosis in option pricing models (Corrado and Su, 1996). In turn, Mauleón and Perote (2000) were the first to propose large expansions, which they called Edgeworth-Sargan density, to model the full density of asset returns and extend such a distribution to a multivariate context (Mauleón, 2003; Perote, 2004). It is worth mentioning that this distribution is called the Edgeworth-Sargan density in recognition to Denis Sargan, who brought Edgeworth series into econometrics, but other authors preferred to call it the Gram-Charlier density as selected in this research. Recently, a growing interest in economics and finance has emerged, specifically in econometrics and asset pricing (Mauleón, 2010; Del Brio and Mora-Valencia, 2017; Del Brio et al., 2019, 2020; Trespalacios et al., 2020; Molina-Muñoz et al., 2020).

However, one well-known shortcoming of the finite order GC expansions is that they may not yield positive values for any combination of their parameters. To this end, some authors have considered implementing positive transformations (Gallant and

Nychka, 1987; León et al., 2009) or positive constraints (Jondeau and Rockinger, 2001), although most risk management applications implement controlled maximum likelihood (ML) optimization for ensuring positivity at the global optimum. For this research, two GC versions have been considered: Gram Charlier (Mauleón and Perote, 2000) and Positive Gram-Charlier (Ñíguez and Perote, 2012). The latter guaranteeing positivity in the whole parametric domain by squaring and normalizing the expansion, but at the expense of not presenting a linear link between its central moments and parameters and being a symmetrical distribution. For this reason, whenever this type of expansion is in this research, it incorporates an asymmetric conditional variance process as the GJR-GARCH developed by Glosten et al. (1993). To the best of our knowledge, this PhD Thesis is the first that introduces this kind of GC series for the modeling of cryptocurrencies.

Providing adequate risk forecasting models to prevent risky behaviors is the second main contribution in this research. For this purpose, three risk measures have been considered to have more precise information on which one of them offers the best alternative for assessing cryptocurrency risk. Model performance in terms of all the three risk measures is assessed through backtesting techniques, which are standard procedures to examine out-of-sample forecasting through a wide variety of tests.[3] In what follows the main characteristics of the three risk measures are briefly described.

Value at Risk (VaR) is perhaps the best known and most widely used for assessing financial risk, especially as it was applied by the Basel Committee on Banking Supervision (BCBS) for several years (BCBS, 2013). From a statistical point of view, it is nothing more than a quantile of the distribution of returns showing the maximum loss for a probability, over a period of time. Its main advantage is that it is easy to calculate and interpret, but it has some drawbacks. On the one hand, it does not consider the risk beyond the quantile, i.e. it does not capture the extreme values of the tail which, in our case, is particularly important, since cryptocurrencies exhibit extreme volatility. On the other hand, it is not a consistent risk measure, as it does not always meet the subadditivity property.

[3] R package software version 3.6.3 and its libraries have been used to implement algorithms and estimates.

Unlike VaR, Expected Shortfall (ES) takes into account tail risk beyond the quantile, which is the reason why Basel Committee decided to change 99%-VaR to 97.5%-ES (BCBS, 2013). This measure is defined as an expected loss given a VaR (quantile) considering tail risk. It is a consistent risk measure, as it satisfies the subadditivity property, but does not satisfy the elicitability property, which makes it difficult to apply backtesting techniques. A new contribution in this research has been to include another way of estimating ES proposed by (Emmer et al., 2015), which to our knowledge, has been very little used in cryptocurrencies (Acereda et al., 2019). It is a simple method that consists of approximating ES by averaging several quantiles (VaR) corresponding to different confidence levels, which can be considered consistent and elicitable in conjunction with VaR.

The last analyzed risk measure is Median Shortfall (MS). To the best of our knowledge, it has never been used in cryptocurrencies and is perhaps less well known than VaR and ES in the financial industry. This alternative is important as it considers risk beyond the quantile (VaR) and is not as sensitive to extreme values as ES, highlighting the robustness property of the statistic since it is the median, from statistically viewpoint. Note that for cryptocurrency analysis this feature seems particularly relevant due to the high frequency of extreme returns. How to estimate, is extremely simple as it is no more than the estimation of a quantile (VaR) with a higher confidence level, e.g. to estimate 99%-MS would be the same as estimating 99.5%-VaR.

The appropriate choice of the risk model in connection to the proper risk measure for every cryptocurrency is the key for providing accurate risk management decisions concerning cryptocurrency investments, e.g. for portfolio choice, capital buffer provision or hedging strategies. Given the high heterogeneity between cryptocurrencies, the best choice may not always be always the same and thus, the election of the aggregating risks confidence levels should be adequately selected. All these ideas are discussed in the following next chapters that coincide with four papers written independently. That is the reason why some models or fundamental concepts may be presented more than once.

Section One coincides with Chapter One entitled 'Dynamic selection of Gram-Charlier expansions with risk target: An application to cryptocurrencies'. As stated in the title, its main contribution is the proposal of an endogenous selection method for the GC expansion length based on the differences between the cumulative distribution functions with a particular focus on the fitting of the distribution left tail for risk assessment purposes, selecting the best-fit expansion at each time. The “incremental probability” criterium, i.e. the distance between probability distributions, is employed to discriminate between alternative expansions. To the best of our knowledge, this is the first time that this kind of procedure has been implemented with good performance compared to fixed-length expansions. The method is tested through backtesting techniques in terms of the probabilities, instead of the traditional quantile-based measures.

Section Two, which comprises Chapter Two and Chapter Three, is devoted to a comprehensive analysis of risk performance of different SNP and parametric models for cryptocurrencies in terms of the three considered risk measures, all focused under a univariate perspective.

'Risk quantification and validation for Bitcoin' (Chapter Two) is primarily orientated to the analysis of Bitcoin, by far the most influential cryptocurrency, under a univariate concept. A comparison of an AR-GARCH model with either a SNP (GC) or parametric (skewed and leptokurtic) distributions is performed, as well as other processes in comparative terms such as GAS and RobustGARCH. Backtesting techniques are used to analyze the performance of the three risk measures described above (VaR, ES and MS), highlighting the better performance of AR-GARCH with the SNP distribution compared to the others.

‘Semi-nonparametric risk assessment with cryptocurrencies' (Chapter Three) which analyses the implementation of the GC and the Positive GC expansion in comparison to others, with another GARCH-type, AR-GJR-GARCH, for VaR, MS and ES. This chapter extends the spectrum beyond Bitcoin, considering five relevant cryptocurrencies: Litecoin, Ripple, Monero, Stellar and Ethereum. The model performance is assessed with backtesting techniques taking into account a wide variety of tests and comparing the competing models using Diebold-Mariano tests and scoring functions through the Model Confidence Set selection methodology. The results show heterogeneity among cryptocurrencies, finding that there is a clear distinction between Bitcoin and these altcoins. However, both SNP approaches and MS may be good alternatives for this kind of asset.

The last section corresponds to Chapter Four, 'Portfolio risk assessment under dynamic (Equi)correlation and semi-nonparametric estimation: An application to Cryptocurrencies.' In this case, three cryptocurrencies have been considered: Bitcoin, Litecoin and Ripple. Unlike the other two chapters, the evolution of this research leads us to use a multivariate AR-GJR-GARCH structure and the positive transformation of the GC distribution under a Dynamic Conditional Correlation (DCC) and Dynamic Equicorrelation (DECO) model. The main contribution of this analysis is the implementation of a method for estimating a DCC model based on bivariate marginal SNP distributions and the introduction of these dynamic correlations into the univariate SNP distribution of the portfolio. It states that this method is feasible even for large portfolios, albeit at the cost of an increase in the number of correlations/distributions to be estimated. The performance of both SNP-DCC (staged procedure) and SNP-DECO (joint estimation) is tested for several portfolios with different weights on major cryptocurrencies by analyzing with VaR, ES and MS backtesting techniques. The results support that SNP-DCC model has a better performance for lower confidence levels than the SNP-DECO model and is more appropriate for portfolio diversification purposes.

Finally, the overall conclusions are summarized at the end of this PhD Thesis. In the same way some future research and policy recommendations are discussed.

SECTION 1: AN APPLICATION OF GRAM-CHARLIER EXPANSIONS

CHAPTER 1: Dynamic selection of Gram-Charlier expansions with risk targets: An application to cryptocurrencies

A version of this chapter has been accepted for publication: Jiménez, I., Mora-Valencia, A., and Perote, J. (2021). Dynamic selection of Gram-Charlier expansions with risk targets: An application to cryptocurrencies. *Risk Management.* (Forthcoming)

SECTION 2: UNIVARIATE VOLATILITY MODELS

CHAPTER 2: Risk quantification and validation for Bitcoin

A version of this chapter has been already published: Jiménez, I., Mora-Valencia, A., and Perote, J. (2020). Risk quantification and validation for Bitcoin. *Operations Research Letters*, 48(4), 534–541. https://doi.org/10.1016/j.orl.2020.06.004

CHAPTER 3: Semi-nonparametric risk assessment with cryptocurrencies

A version of this chapter is currently under second round of review in "Research in International Business and Finance".

SECTION 3: MULTIVARIATE VOLATILITY MODELS

CHAPTER 4: Portfolio Risk Assessment under Dynamic (Equi)Correlation and Semi-Nonparametric Estimation: An application to Cryptocurrencies

A version of this chapter has been already published: Jiménez, I., Mora-Valencia, A., Ñíguez, T.-M., and Perote, J. (2020). Portfolio Risk Assessment under Dynamic

(Equi)Correlation and Semi-Nonparametric Estimation: An Application to Cryptocurrencies. *Mathematics*, 8(12), 2110. https://doi.org/10.3390/math8122110

CONCLUSIONS

The new technology and paradigm shift caused by blockchain, out of the question a few years ago, shows that it is possible to establish a payment system, a currency or even an asset without the need of institutions. While writing these lines, the cryptocurrency industry is growing, not only in market capitalization but also in an endless number of concepts related to them, such as the promising DEFI, mining, proof-of-work, ICO, token, NFT and many more that will emerge gradually. This new sector has marked a turning point in the economy and this implies having a broader understanding about it.

Nowadays, the importance of risk management in this area is crucial, since one of the most common questions that governments, central banks and financial institutions are concerned about is, related to the aggressive up and downs movements in daily prices, due to the high volatility they present. Hence, this PhD Thesis tries to cover this area of concern, presenting comprehensive research in that field.

First of all, estimating risk is not enough to analyze risk. It should be linked to risk measures taking into account different properties they own. For this research has been selected, Value at Risk (VaR), Expected Shortall (ES) and Median Shortfall (MS). The two formers are the best-known in quantify risk from a financial viewpoint. Both have advantages and disadvantages: VaR is simple to compute and use backtesting techniques but does not meet subadditivity property as well as it does not consider the tail beyond the quantile. ES considers the tail risk beyond the quantile but without satisfying the elicitability property making difficult the usage of backtesting techniques. Moreover, it is considered, from a statistical point of view, the mean of the tail distribution and thus is very sensitive to extreme values.

To the best of our knowledge, it is the first time that MS has been applied for risk assessment for cryptocurrencies. It is an excellent alternative as a risk measure since it is not sensitive to extreme values because, from a statistical point of view, it is the median of the tail distribution beyond the quantile. Furthermore, the provided results in this research support the proposal to use this risk measure.

Choosing the best model to estimate risk is essential. For that purpose, the conditional variance should be modeled considering the heteroskedasticity effect applying GARCH-type processes (standard GARCH and GJR-GARCH) and other processes as GAS or RobustGARCH for comparative purposes. Furthermore, to obtain more accurate results, selecting the best underlying distribution should be carefully chosen. In this line we propose the application of the SNP approach, showing its advantages against other asymmetric and leptokurtic distributions such as Student's or its asymmetric version.

At this point, there is no doubt that the core of this PhD Thesis is the implementation process of the SNP approach to cryptocurrencies to obtain more precise estimates in combination with the most appropriate risk measure. As far as we know, it is the first time that this approach is applied to cryptocurrencies. For this purpose, it was considered two alternatives: Gram Charlier (GC) and Positive Gram Charlier (PGC). As it was mentioned in previous chapters, both are based on a standard Gaussian pdf and the corresponding Hermite polynomials. The former allows for the possibility of choosing the length of the expansion considering parameters not only to account for skewness and kurtosis (c_3 and c_4) but also making it easier to fit the jumps at the very far end distribution adding more parameters (c_6 and c_8). Alternatively, positive transformations have been also implemented to ensure positivity in the whole parametric domain with PGC at the expense of losing the relationship between parameters and central moments but also yielding good results.

Apart from risk measures and selecting the best model with the most accurate distribution, backtesting techniques are required to compared parametric distributions (Gaussian, Student's t and Skewed t) and SNP. It is focusing on out-of-sample performance, measuring the number of exceptions the model has in comparison to the empirical data and applying several tests. As mentioned above, the test for VaR and MS are the same, since MS is the same as VaR but with a higher confidence level, e.g., 99%-MS is the same as 99.5%-VaR (see Eq. 2.13 in Chapter Two). But in the case of ES, apart from the typical assessment with a 'direct ES', it has been considered a new alternative of computing as an approximation of ES (as Eq. 2.12 in Chapter Two) that allow avoiding the elicitability drawback averaging N levels of VaR. For that reason, a brand-new multinomial test has been kept in view, as well as other tests. Nor should we forget that scoring functions have been applied to rank competing models with Model Confidence

Set, to have more comprehensive results about the whole performance in cryptocurrencies.

The flexibility offered by the SNP approach selecting the expansion length and the order in the parameter allows the implementation of a new tool, as a novelty, for Gram Charlier outlined in Chapter One. This methodology selects the expansion order and the length dynamically and endogenously supporting by the properties of the cumulative distribution function providing a straightforward comparison of the effect of additional terms in the expansion (ten expansions have been proposed), considering the risk assessment in terms of probabilities rather than quantiles. For this purpose, four cryptocurrencies have been considered: Bitcoin, Litecoin, Ripple, Stellar. The assessment performance is through backtesting techniques focused on the left tail of the distribution's returns with a 1% probability. The results show that the risk forecasting may be improved by applying the strategy of selecting the most accurate expansion at each $t-$time compared to a fixed-order Gram Charlier expansion. This tool might be appropriate for those areas where the forecasting of frequency distributions is required.

Considering risk measures, the analysis of Bitcoin compared to the most relevant altcoins such as Litecoin, Ripple, Monero, Stellar and Ethereum show a great heterogeneity. The dominance of SNP over the rest of parametric distributions and even for the rest of the processes for Bitcoin is undeniable. It must also be admitted that the results for GAS and RobustGARCH processes are also appropriate for ES, but in the case of the latter, it is time-consuming compared to other processes. The rest of the analyzed cryptocurrencies have excellent results supporting the proposal of SNP especially for high confidence levels whereas for lower confidence levels it is difficult to find the best model since other parametric distributions have similar performance. However, model ranking MCS promotes SNP models, mostly in the first positions for risk forecasting ability. Furthermore, 99%-MS and 97.5%-MS might be a reasonable alternative to 99%-VaR or 97.5%-ES since it overcomes the sensitivity of ES to extreme values.

Extending the analysis under a multivariate perspective is essential to portfolio risk assessment. To the best of our knowledge, it is the first time this methodology is applied to a multivariate SNP model based on the Positive Gram Charlier expansion. It consisst of estimating the Dynamic Conditional Correlation (DCC) model considering the bivariate SNP marginal distributions and to plug these dynamic correlations into the univariate SNP distribution in order to beat the curse of dimensionality. For comparison purposes, Dynamic Equicorrelation (DECO) model is considered, which is a straightforward but at the same time computationally demanding alternative. The performance of these models is tested for several portfolios with different weights and considering three cryptocurrencies: Bitcoin, Litecoin and Ripple for 97.5% and 99% VaR and MS and 97.5% ES. The result for both procedures, DCC and DECO, seems to provide similar excellent performance, although the latter presents the best results for the higher confidence level whereas DCC show better results for smaller confidence level (97.5%-VaR). To this end, this new methodology may fulfil two main aspects of risk management: easy procedures and optimal decision making. On one side, it is a straightforward and accurate method that avoid complex mechanisms that enable handling large portfolios preventing dimensionality problems. On the other side, the proposal of this PhD Thesis seems to be useful in optimal decision making to risk managers, policymakers and regulator due to the good performance mentioned above even with assets with high volatility and extreme events such as cryptocurrencies.

There still exist new areas that must be explored and assessed within cryptocurrencies under a SNP approach. For future research, it may be appealing, but a difficult task, to extend to a wide variety of cryptocurrencies quantitative measures to set a proper classification in different risk assessment groups, considering the diverse features on the technology behind them. This is not a trivial matter since there is nothing like that at present. Another research line should be focused on analyzing intraday volatility to find the influence of every country in the tradeoff offer and demand owing to 24/7 trade.

The whole idea of using cryptocurrencies as a means of payment seems to have faded away over time to become a mere speculative asset. From my point of view, this is not correct, this industry goes far beyond, it seems to have widespread other many services owing to the technology behind them, thanks to the blockchain. Its decentralization is out of the control of governments or institutions, which will involve a deep change in the decision-making process in the economy and the standards. Thirteen years after Bitcoin inception, nobody could have pictured, in September 2021, how a country like El Salvador would have this cryptocurrency as legal tender. A madness for many, but a genius for others. Time will prove right or wrong.

CONCLUSIONES

La nueva tecnología y el cambio de paradigma provocado por el blockchain, impensable hace unos años, demuestra que es posible establecer un sistema de pagos, una moneda o incluso un activo sin necesidad de instituciones. Mientras se escriben estas líneas, la industria de las criptomonedas crece, no sólo en capitalización de mercado, sino también en un sinfín de conceptos relacionados con ellas, como la prometedora DEFI, minería, proof-of-work, ICO, token, NFT y muchos más que irán surgiendo poco a poco. Este nuevo sector ha marcado un punto de inflexión en la economía y esto implica tener un amplio conocimiento sobre ello.

En la actualidad, la importancia de la gestión del riesgo en este ámbito es crucial, ya que una de las cuestiones más habituales que preocupan a gobiernos, bancos centrales e instituciones financieras está relacionada con los agresivos movimientos de subida y bajada de los precios diarios debido a la alta volatilidad que presentan. Por ello, esta Tesis Doctoral trata de cubrir este ámbito de preocupación, presentando una investigación exhaustiva en ese campo.

En primer lugar, para analizar el riesgo no basta con estimarlo. Se debe vincular a las medidas de riesgo teniendo en cuenta las diferentes propiedades que poseen. Para esta investigación se ha seleccionado, Valor en Riesgo (VaR), Expected Shortall (ES) y el Median Shortfall (MS). Las dos primeras son los más conocidas para cuantificar el riesgo desde un punto de vista financiero. Ambas tienen ventajas e inconvenientes: VaR es sencillo de calcular y de utilizar técnicas de backtesting, pero no cumple la propiedad de subaditividad y no considera la cola más allá del cuantil. ES considera el riesgo de cola más allá del cuantil, pero no satisface la propiedad de elicitabilidad, lo que dificulta el uso de las técnicas de backtesting. Además, se considera, desde un punto de vista estadístico, la media de la distribución de la cola y, por tanto, es muy sensible a los valores extremos.

Hasta donde sabemos, es la primera vez que se aplica MS para la evaluación del riesgo de las criptomonedas. Es una excelente alternativa como medida de riesgo ya que no es sensible a los valores extremos porque, desde un punto de vista estadístico, es la mediana de la distribución de cola más allá del cuantil. Además, los resultados aportados en esta investigación apoyan la propuesta de utilizar esta medida de riesgo.

La elección del mejor modelo para estimar el riesgo es esencial. Para ello, la varianza condicional se ha modelizado teniendo en cuenta el efecto de la heteroscedasticidad aplicando procesos de tipo GARCH (GARCH estándar y GJR-GARCH) y otros procesos como GAS o RobustGARCH con fines comparativos. Además, para obtener resultados más precisos, se debe elegir cuidadosamente la mejor distribución subyacente. En esta línea proponemos la aplicación del enfoque SNP, mostrando sus ventajas frente a otras distribuciones asimétricas y leptocúrticas como la t de Student o su versión asimétrica.

Llegados a este punto, no cabe duda de que el núcleo de esta Tesis Doctoral es el proceso de aplicación del enfoque SNP a las criptomonedas para obtener estimaciones más precisas en combinación con la medida de riesgo más adecuada. Por lo que sabemos, es la primera vez que se aplica este enfoque a las criptomonedas. Para ello, se consideraron dos alternativas: Gram Charlier (GC) y Positive Gram Charlier (PGC). Como se ha mencionado en capítulos anteriores, ambas se basan en una función de probabilidad de densidad Gausiana estándar y los correspondientes polinomios de Hermite. La primera alternativa, GC, permite elegir la longitud de la expansión considerando parámetros no sólo para dar cuenta de la asimetría y la curtosis (c_3 y c_4) sino también para facilitar el ajuste de los saltos en la distribución más lejana añadiendo más parámetros (c_6 y c_8). Por otro lado, también se han implementado transformaciones positivas para asegurar la positividad en todo el dominio paramétrico con PGC a costa de perder la relación entre los parámetros y los momentos centrales, pero también dando buenos resultados.

Aparte de las medidas de riesgo y de la selección del mejor modelo con la distribución más precisa, se requieren técnicas de backtesting para comparar las distribuciones paramétricas (Gausiana, t de Student y su versión asimétrica) y SNP. Se centra en el rendimiento fuera de la muestra, midiendo el número de excepciones que tiene el modelo en comparación con los datos empíricos y aplicando varios tests. Como se ha mencionado anteriormente, los tests para VaR y MS son los mismos, ya que MS es lo mismo que VaR pero con un nivel de confianza más alto, por ejemplo, el 99%-MS es lo mismo que el 99,5%-VaR (véase la ecuación 2.13). Pero en el caso de ES, además de la valoración típica con un "ES directo", se ha considerado una nueva alternativa de cálculo como un ES aproximado (como la ecuación 2.12) que permite evitar el inconveniente de la elicitabilidad, promediando N niveles de VaR. Por ello, se ha tenido en cuenta un nuevo test multinomial, así como otros tests. Tampoco hay que olvidar que se han aplicado funciones scoring para clasificar los modelos MCS, para tener resultados más completos sobre el conjunto del comportamiento en criptomonedas.

La flexibilidad que ofrece el enfoque SNP seleccionando la longitud de la expansión y el orden en el parámetro permite la implementación de una nueva herramienta, como novedad, para Gram Charlier esbozada en el Capítulo Uno. Esta metodología selecciona el orden de expansión y la longitud de forma dinámica y endógena, apoyándose en las propiedades de la función de distribución acumulativa proporcionando una comparación directa del efecto de los términos adicionales en la expansión (se han propuesto diez expansiones), considerando la evaluación del riesgo en términos de probabilidades y no de cuantiles. Para ello, se han considerado cuatro criptomonedas: Bitcoin, Litecoin, Ripple y Stellar, realizándose mediante técnicas de backtesting centradas en la cola izquierda de la distribución con una probabilidad del 1%. Los resultados muestran que la previsión del riesgo puede mejorarse aplicando la estrategia de selección de la expansión más precisa en cada momento t en comparación con una expansión de GC de orden fijo. Esta herramienta podría ser apropiada para aquellos ámbitos en los que se requiere la previsión de distribuciones de frecuencia.

Considerando las medidas de riesgo, el análisis de Bitcoin comparado con las altcoins más relevantes como Litecoin, Ripple, Monero, Stellar y Ethereum muestran una gran heterogeneidad. Es innegable el dominio de SNP sobre el resto de distribuciones paramétricas e incluso para el resto de procesos para Bitcoin. También hay que admitir que los resultados para los procesos GAS y RobustGARCH son también apropiados para ES, pero en el caso de este último, es más lento en comparación con el resto de procesos. El resto de las criptomonedas analizadas tienen excelentes resultados que apoyan la propuesta de SNP especialmente para niveles de confianza altos mientras que para niveles de confianza más bajos es difícil encontrar el mejor modelo ya que otras distribuciones paramétricas tienen un rendimiento similar. Sin embargo, el ranking de modelos MCS promueve los modelos SNP, sobre todo en las primeras posiciones para la capacidad de previsión del riesgo. Además, el 99%-MS y el 97,5%-MS podrían ser una alternativa razonable al 99%-VaR o al 97,5%-ES, ya que supera la sensibilidad del ES a los valores extremos.

La ampliación del análisis bajo una perspectiva multivariante es esencial para la evaluación del riesgo de la cartera. Hasta donde sabemos, es la primera vez que se aplica esta metodología a un modelo SNP multivariante basado en la expansión GCP. Consiste en estimar el modelo de Correlación Condicional Dinámica (DCC) teniendo en cuenta las distribuciones marginales bivariadas de los SNP y en aplicar estas correlaciones dinámicas en la distribución univariada de los SNP para vencer la "maldición" de la dimensionalidad. A efectos de comparación, se considera el modelo de Equicorrelación Dinámica (DECO), que es una alternativa parsimoniosa, pero al mismo tiempo computacionalmente exigente. El rendimiento de estos modelos se prueba para varias carteras con diferentes pesos y considerando tres criptomonedas: Bitcoin, Litecoin y Ripple para un VaR del 97,5% y 99% y un ES del 97,5%. El resultado para ambos procedimientos, DCC y DECO, parece proporcionar un excelente rendimiento similar, aunque este último presenta los mejores resultados para el nivel de confianza más altos mientras que DCC muestra mejores resultados para el nivel de confianza más bajos (97,5%-VaR).

En este sentido, esta nueva metodología puede cumplir dos aspectos principales de la gestión de riesgos: la facilidad de los procedimientos y la optimización de la toma de decisiones. Por un lado, se trata de un método sencillo y preciso que evita mecanismos complejos que permiten manejar grandes carteras evitando problemas de dimensionalidad. Por otro lado, la propuesta de esta Tesis Doctoral parece ser útil en la toma de decisiones óptimas para los gestores de riesgo, los responsables políticos y los reguladores debido al buen rendimiento mencionado anteriormente incluso con activos con alta volatilidad y eventos extremos como las criptomonedas.

Todavía existen nuevas áreas que deben ser exploradas y evaluadas dentro de las criptomonedas bajo un enfoque SNP. Para futuras investigaciones, puede ser atractivo, pero una tarea difícil, extender a una amplia variedad de criptomonedas estas medidas cuantitativas para establecer una clasificación adecuada en diferentes grupos de evaluación de riesgo, teniendo en cuenta las diversas características en la tecnología detrás de ellas. No es una cuestión trivial, ya que no existe nada parecido en la actualidad. Otra línea de investigación debería centrarse en el análisis de la volatilidad intradia para encontrar la influencia de cada país en la oferta y la demanda debido al comercio abierto veinticuatro horas al día.

La idea de utilizar las criptomonedas como medio de pago parece haberse desvanecido con el tiempo para convertirse en un mero activo especulativo. Desde mi punto de vista, esto no cierto, esta industria va mucho más allá, parece haber generalizado otros muchos servicios gracias a la tecnología que las sustenta, el blockchain. Su descentralización hace que escape del control de los gobiernos o las instituciones, lo que implicará un profundo cambio en el proceso de toma de decisiones en la economía sus normas. Trece años después de la creación de Bitcoin, nadie podía imaginar que, en septiembre de 2021, un país como El Salvador adoptaría esta criptomoneda como moneda de curso legal. Una locura para muchos, pero una genialidad para otros. El tiempo dará la razón o no.

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