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Macroeconomic effects of an indirect tax substitution

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Abstract We study an indirect tax reform in a general equilibrium model with imperfect competition for both the Cournot and the Free entry equilibria. We show that it is possible to attain a positive balanced budget multiplier by means of a substitution of specific by ad valorem taxation. Moreover, although any tax substitution causes higher prices and the flow up of firms in the long-run, the Free entry equilibrium output can increase with respect to that of the Cournot equilibrium. Finally, in contrast with the partial equilibrium, welfare decreasing tax reforms are likely to occur even when the balanced budget multiplier is positive.

Keywords Indirect taxation · Tax substitution · Balanced budget multiplier

JEL Classification E62 · H29

1 Introduction

Indirect tax comparison is a well-known issue in Public Economics since Wicksell (1896,1959) showed the different outcomes that specific and ad valorem taxes generate under the monopoly case. Later, Suits and Musgrave (1955) and Bishop (1968) established the superiority of ad valorem taxation over an equal-yield specific one. Such ad valorem upon specific taxation dominance has been exhaustively studied in several imperfect competition settings under partial equilibrium. For instance, Skeath and Trandel (1994), for the monopoly case; Dellipalla and Keen (1992),

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for the Cournot–Nash case with and without entry; Denicoló and Matteuzzi (2000), for asymmetric Cournot oligopolies; Anderson et al. (2001), for Cournot–Nash and Bertrand competition with product differentiation; and Dellipalla and Keen (2005), for a product quality model. Roughly speaking, the main conclusion of all these works is that under an equal-yield comparison, ad valorem Pareto dominates specific taxation and stems variations in tax revenue and profits. Under general equilibrium, the issue of indirect tax comparison has been studied in some imperfect competition settings, without consensus about the ad valorem upon specific dominance result. For instance, Schröder (2004) agrees with this dominance for a Dixit-Stiglitz monopolistic competition framework with differentiated products, entry–exit and love for variety, while Grazzini (2006) reaches an opposite result of dominance by means of an example using a strategic market game. At the same time, Blackorby and Murty (2007) show the equivalence of both taxes on the set of Pareto optima when a monopoly is imbedded in a general equilibrium framework and profits are taxed at 100%.

The purpose of this paper is to enhance the discussion about the indirect tax comparison under general equilibrium in one of the imperfect competition settings which still remains unexplored: the Cournot–Nash case with and without entry. For this task we use a model based on those which have been used to discuss some Keynesian features with fully flexible prices.[1] This approach allows us to study both the implications on welfare as well as some macroeconomic effects (the balanced budget multiplier) of such an indirect tax reform taking into account its wealth effects. In order to capture all possible cases, we address a general tax reform which shifts from specific to ad valorem taxation characterized by means of a rate of substitution between the taxes. This characterization departs from the usual treatment of the tax substitution problem based on the revenue-neutral criterion. Nevertheless, the approach allows us to set thresholds for which equilibrium outcomes change, making the analysis easier; to characterize the usually tackled revenue-neutral tax reform as a particular case and; to assume that the government lacks sufficient information or, simply, it has not any criteria about the rate at which taxes have to be substituted as such.

The model considered is composed of a representative household, a government and a large number of industries each one formed by a small number of non-competitive firms. This assumption about the size of industries and firms, introduced by Neary (2003), allows us to match up the maximizing behaviour of firms and shareholders without affecting factor prices.[2] When the government carries out the tax substitution, the subsequent changes to tax revenue and profits shift aggregated demand and total output. The final effect on prices, government purchases and welfare depends on the

[1] Outstanding contributions in this literature are Hart (1982), Blanchard and Kiyotaki (1987), Dixon (1987) and Mankiw (1988). Those papers are devoted to stating the effectiveness of the balanced budget fiscal policy in boosting output in these settings.

[2] Gabszewicz and Vial (1972) pointed out two difficulties of general equilibrium models with imperfect competition: the reliance of the oligopoly equilibrium with respect to the choice of numérarie (the normalization rule); and the incompatibility between profit and utility maximization rules (see also Dierker and Grodal, 1995 and 1998). Different approaches have been used to amend these problems; see for instance, Cordella and Gabszewicz (1997) or Dierker and Grodal (1999). Neary's (2003) approach solves the problem by assuming that firms are large in their own sector and small in the overall economy. Nevertheless, implementing this approach requires some limits as the specification of objective functions of agents.

rate at which the taxes are substituted. As we will see, although distortionary taxation is considered, it is possible to find rates of substitution between specific and ad valorem taxes for which the balanced budget multiplier is positive.[3] In addition, although any tax substitution causes higher prices and the flow up of firms in the long-run, the Free entry equilibrium output can increase with respect to that of the Cournot equilibrium. Finally, it is possible to find rates of substitution between specific and ad valorem taxes for which welfare declines, even when the balanced budget multiplier is positive. This result arises as a counter-example of the partial equilibrium statement about the positive effect of such a tax reform in welfare, and is due to the fact that wealth effects matters in the sign of the change in welfare under general equilibrium set-up.

The paper is structured as follows: Sect. 2 presents the model, the Cournot equilibrium outcomes and defines the tax substitution. Section 3 develops the results for total output, total profit and prices for the Cournot equilibrium. Section 4 analyses the tax substitution under Free entry equilibrium, in such a way that the effect of the tax substitution is expressed in connection with those yielded under the Cournot equilibrium. Section 5 is devoted to the effects on welfare. Finally, Sect. 6 comments the results.

2 Model and equilibrium

Following Caminal (1990), let us consider an economy with $h+1$ goods (leisure and h goods produced from labour) and $hn+2$ agents (a representative household, h symmetric industries each with n non-competitive firms, and the government) defined by the following:

(i) Household preferences are represented by a Cobb–Douglas utility function over the quantity L of leisure, the vector $X=(x_1, x_2, \ldots, x_h)$ of consumption of the produced goods and the vector $g=(g_1, g_2, \ldots, g_h)$ of publicly provided produced goods.[4]

$$u(X, L, g)=\sum_{i=1}^{h} \frac{\alpha}{h} \ln x_i+(1-\alpha) \ln L+\sum_{i=1}^{h} \frac{\beta}{h} \ln g_i, \tag{1}$$

where $\alpha, \beta \in (0,1)$. Cobb–Douglas utility over leisure and goods implies that the price elasticity of private-sector demand for each good is unity. Denoting W as the initial endowment of time and considering labour as the *numèraire*, let p_i and π_i be the price and the industry profit of the produced good $i=1,2,\ldots,h$, and

[3] I stress the tax distortion point because the positiveness of the balanced budget multiplier has been disclaimed when distortionary taxation is considered in these settings. See Molana and Moutos (1992), Heijdra et al. (1998), and Torregrosa (1998, 2003).

[4] g can also be understood as the inputs necessary for the production of a quantity $\zeta(g)$ of a public good, where $\zeta(g)=\sum_{i=1}^{h} \frac{\beta}{h} \ln g_i$.

$\pi = \sum_{i=1}^{h} \pi_i$ be industries' total profit. Thus, household's budget constraint is given by

$$\sum_{i=1}^{h} p_i x_i \leq W - L + \pi. \tag{2}$$

Maximizing (1) subject to (2) the household's optimal choice is

$$p_i x_i = \frac{\alpha}{h}(W + \pi), \quad i = 1, 2, \ldots, h, \tag{3}$$

$$L = (1 - \alpha)(W + \pi). \tag{4}$$

(ii) There are h industries, each one formed by $n > 1$ identical and non-competitive firms, producing an amount q_{ij} $(j = 1, 2, \ldots, n)$ of output from labour through the cost function

$$C(q_{ij}) = k + cq_{ij},$$

which exhibits decreasing average cost. Let us assume that h is large enough and n is small enough so that the labour market is perfectly competitive while the produced goods markets are imperfectly competitive. This assumption solves the issue of how to frame the firms' optimization problem, given that market demand depends on profits distributed to consumers, allowing each industry to take total expenditure $Y_i = p_i(x_i + g_i)$ as given, and making both firms and household choices independent (Neary 2003). It is also assumed that firms maximize profits and behave *à la Cournot*. Thus the representative firm of industry i faces the following unit isoelastic inverse demand

$$p_i = Y_i / Q_i, \quad \text{where } Q_i = \sum_{j=1}^{n} q_{ij}.$$

Finally, in each industry firms bear simultaneously an ad valorem tax rate $t \in (0, 1)$ and a specific tax rate $s > 0$ respectively. Therefore, the goal of the representative firm is to maximize

$$\left((1 - t)\frac{Y_i}{Q_i} - c - s\right) q_{ij} - k,$$

whose first order condition yields the symmetric equilibrium in each industry

$$Q_i(t, s) = \frac{(n-1)}{n} \frac{(1-t)}{(s+c)} Y_i, \tag{5}$$

$$p_i(t, s) = \frac{n}{(n-1)} \frac{(s+c)}{(1-t)}, \tag{6}$$

$$\pi_i(t, s) = (1 - t)\frac{Y_i}{n} - nk. \tag{7}$$

This partial equilibrium outcome reflects the different form in which each tax rate affects the firms' objective function. Ad valorem taxation affects total income while specific taxation increases marginal cost. Moreover, according to Eq. (4), leisure depends on taxes linearly through total profit. As a consequence, labour supply, and thus total output, are affected by changes in both tax rates in the opposite way to total profit. On the other hand when we consider Eq. (6) we can see that, due to the constant elasticity of demand, there is a fixed mark-up of the price over the marginal cost which depends on n, thus, throughout the paper, n represents the measure of the market power and, due to the symmetry of the model, $p_i(t, s)$ is identical in each industry $i = 1, 2, \ldots, h$; thus Eq. (6) is just the price index of the economy.

(iii) The government uses the tax revenue to finance the quantity g_i of government purchases in industry i. Thus, given the price p_i, the government budget constraint is

$$\sum_{i=1}^{h} p_i g_i = G(t, s), \tag{8}$$

where

$$G(t, s) = \sum_{i=1}^{h} t p_i(t, s) Q_i(t, s) + s \sum_{i=1}^{h} Q_i(t, s) \tag{9}$$

is government tax revenue. Substituting Eqs. (5) and (6) in Eq. (9), taking into account (8), and denoting by $Y = \sum_{i=1}^{h} Y_i$ the total expenditure in the economy, government expenditure is given by

$$G(t, s) = (1 - t) \left(\frac{t}{1 - t} + \frac{s}{(s + c)} \frac{(n - 1)}{n} \right) Y. \tag{10}$$

This endogenous government expenditure is taken to be fixed by industries, so that government purchases in each industry $g_i = \frac{G(t,s)}{ph}$ is also unit elastic (see Dixon and Rankin 1994).[5] Finally, Government policy consists of a substitution of specific by ad valorem taxation. Thus, starting from an initial situation given by the pair $(t, s) \in R_+^2$, let us write,

$$ds = -\gamma dt \quad \text{with } \gamma > 0. \tag{11}$$

where γ is the rate at which the government substitutes specific by ad valorem taxation and represents government policy. Moreover, as government purchases are determined in an endogenous way, this tax substitution policy entails an indirect expenditure policy. On the other hand, the tax substitution policy given in (11) is a generalization of all possible tax reforms and is not based on any particular basis of comparison. We could

[5] Caminal (1990) shows that, given the symmetry of the model, this uniformity in government purchases in each sector is the optimal fiscal policy.

interpret this characterization assuming that the government lacks sufficient information or, simply, it has not any criteria about γ as such. In any case, the characterization provided by Eq. (11) will allow us to study and calculate the set of all possible effects of such a tax substitution policy, allowing the study of any particular comparison criterion as a special case. For instance, Dellipalla and Keen's (1992) revenue-neutral rate of substitution[6] would be given by

$$\gamma_{DK} = \frac{n}{(n-1)} \frac{(s+c)}{(1-t)}. \tag{12}$$

In order to close the model and to determine the equilibrium, we can equalize demand and supply of labour or, simply, determine total expenditure endogenously. Following the second approach, and taking into account Eq. (3) and g_i, total expenditure in industry i is given by

$$Y_i = \frac{\alpha}{h}(W + \pi) + p_i g_i, \tag{13}$$

adding Eqs. (7) and (13) with respect to the number of industries, taking into account (8),

$$\pi(t, s) = \frac{(1-t)}{n} Y - hnk, \tag{14}$$

$$Y = \alpha\,[W + \pi(t, s)] + G(t, s). \tag{15}$$

The particular shape of the demand function allows us to interpret α as the marginal propensity to consume in Eq. (15) (Mankiw 1988). Substituting Eqs. (14) and (10) into (15), total expenditure in equilibrium is

$$Y(t, s) = \frac{\alpha n(s+c)(W - hnk)}{(1-t)\,[(n-\alpha)c + (1-\alpha)s]}, \tag{16}$$

calling $w = W/h$, expenditure in industry i can be written as

$$Y_i(t, s) = \frac{\alpha n(s+c)(w - nk)}{(1-t)\,[(n-\alpha)c + (1-\alpha)s]}, \tag{17}$$

and adding Eq. (5), taking into account (16), total output in equilibrium can be written as

$$Q(t, s) = \frac{\alpha(n-1)(W - hnk)}{(n-\alpha)c + (1-\alpha)s}. \tag{18}$$

[6] The so-called P-Shift tax reform, a local version of the Suits and Musgrave's (1955) "matched pairs", which is the rate of substitution between the ad valorem and the specific tax for which the (ex-ante) tax revenue remains unchanged, considering prices and output constant.

As we see, total output in equilibrium is independent of the ad valorem tax because this tax proportionally affects total expenditure. On the other hand, total output in equilibrium does depend on specific taxation due to the fact that this tax rate works as a large marginal cost. The different way in which the two tax rates affect total output and total expenditure (through profits and government expenditure) determines the outcome. Finally, from (16), (17) and (18) let us assume that $W > hnk$ (or $w > nk$) to ensure the existence of an interior equilibrium.

3 Short-run effects of a tax substitution

This section addresses the effect of the tax substitution given in (11) on total output, total profit, price and government expenditure under Cournot equilibrium. In our model this equilibrium represents the short-run since industry size is fixed. Hereinafter the upper script C refers to the Cournot equilibrium and will be useful for comparison with Free entry equilibrium.

Thus, the gradient of total output (Eq. 18) with respect to the vector of tax instruments is

$$\nabla Q^C(t,s) = \left(0, -\frac{\alpha(1-\alpha)(n-1)(W-hnk)}{[(n-\alpha)c+(1-\alpha)s]^2}\right) = \left(0, -\frac{(1-\alpha)Q}{(n-\alpha)c+(1-\alpha)s}\right). \tag{19}$$

Calculating the total differential taking into account (11), the effect on total output of the tax substitution is

$$dQ^C = \frac{\partial Q}{\partial t}dt + \frac{\partial Q}{\partial s}ds = -\gamma\frac{\partial Q}{\partial s}dt > 0, \tag{20}$$

due to the sign of the gradient given in (19).

Notice the different way in which both taxes affect total output in equilibrium. On the one hand, a change in ad valorem tax rate does not produce real effects, whereas an increase in the specific tax rate decreases total output. As the specific tax rate is shifted by the ad valorem one, the final effect on total output is positive for any tax substitution. Notice that the higher the rate of substitution between taxes γ the greater the increase in total output.

In relation with total profit, starting from Eq. (14), taking into account Eq. (16), the gradient with respect to the vector of tax instruments is

$$\nabla \pi^C(t,s) = \left(0, \frac{c\alpha(n-1)(W-hnk)}{[(n-\alpha)c+(1-\alpha)s]^2}\right) = \left(0, \frac{cQ}{(n-\alpha)c+(1-\alpha)s}\right). \tag{21}$$

Calculating the total differential taking into account (11), the effect on total profit of the tax substitution is

$$d\pi = \frac{\partial \pi}{\partial t}dt + \frac{\partial \pi}{\partial s}ds = -\gamma\frac{\partial \pi}{\partial s}dt < 0, \tag{22}$$

due to the sign of the gradient given in (21). Notice that, according to Eqs. (4) and (22), the tax substitution implies lower household leisure and, as happens with total profit, this drop is greater the greater the tax of substitution between taxes. On the other hand, Eq. (22) is similar to Proposition 3b of Dellipalla and Keen (1992) but for different reasons. On the one hand, Dellipalla and Keen's (1992) result refers to the P-Shift tax reform whereas Eq. (22) refers to any rate of substitution of specific by ad valorem taxation. On the other hand, while in Dellipalla and Keen's (1992) partial equilibrium set-up price effects are the only source of the reduction in profits, in our general equilibrium setting other effects appear. Let us assess the gradients of Eqs. (6), (10) and (16) with respect to the vector of tax instruments in order to set these effects,

$$\nabla p^C(t,s) = \left(\frac{n}{(n-1)} \frac{(s+c)}{(1-t)^2}, \frac{n}{(n-1)} \frac{1}{(1-t)} \right), \tag{23}$$

$$\nabla Y^C(t,s) = \left(\frac{Y}{1-t}, \frac{c(n-1)}{[(n-\alpha)c + (1-\alpha)s]} \frac{Y}{(s+c)} \right), \tag{24}$$

$$\nabla G^C(t,s) = \left(\frac{Y}{1-t}, \frac{[n-(1-t)\alpha]c(n-1)}{n\,[(n-\alpha)c + (1-\alpha)s]} \frac{Y}{(s+c)} \right). \tag{25}$$

As can be seen, price, total expenditure and government expenditure in equilibrium increase monotonically with respect to the vector of tax instruments. In particular, an increase in each tax rate pushes the price and shifts aggregated demand upwards, through the increase in total expenditure. These effects allow us to make clear the differences with the partial equilibrium approach by means of the full interpretation of Eqs. (19) and (21). In case of an increase in the ad valorem tax rate, the boost in aggregated demand is completely compensated by the increase in the price, yielding a crowding-out effect. However, for specific taxation, the boost in aggregated demand is not enough to compensate the increase in the price with the consequence of a drop in total output.[7] Finally, as the specific tax rate is shifted by the ad valorem one, the final effect on output is positive for any rate of substitution between taxes. Regarding profits, the explanation is similar provided that industries' total income is just total expenditure in equilibrium. So that, under ad valorem taxation, the shift in industries' total income is cancelled out by the increase in the price, leaving total profits unchanged. Meanwhile, under specific taxation, the increase in the price does not compensate the shift in industries' total income, yielding an increase in total profits in equilibrium.[8] Finally, the final effect on profits is negative for any rate of substitution between specific and ad valorem taxes.

The general equilibrium framework allows for an additional interpretation of condition (20) from the point of view of the labour market. According to (4), labour supply

[7] Torregrosa (2003) shows these results as counter-examples of positive balanced budget multipliers in imperfect competition general equilibrium models.

[8] There is no contradiction with the partial equilibrium result of Seade (1985) which states that provided that the demand is isoelastic, profits increase with a specific tax rate if and only if demand elasticity is less than unity. One can verify this fact through Eq. (5).

depends on taxes through total profit. As total profit does not change with changes in the ad valorem tax, labour supply remains constant with changes in this tax rate and, thereby, output does not change. On the other hand, as total profit increases when the specific tax rate increases, households substitute leisure for consumption, leading to a reduction in labour supply, and therefore in total output. Finally, as in the tax substitution the specific tax is shifted by the ad-valorem, the net effect on labour and output is positive.

In order to learn the effect of the tax substitution on price, total expenditure and government expenditure in equilibrium, let us equalize to zero their total differentials. Taking into account Eq. (11), we obtain those rates of substitution between taxes for which prices, total expenditure and government expenditure in equilibrium remain unchanged. That is, if

$$\left.\frac{ds}{dt}\right|_p = -\frac{s+c}{1-t} < 0,$$

the tax substitution does not change the equilibrium price; if

$$\left.\frac{ds}{dt}\right|_Y = -\frac{(s+c)}{(1-t)}\frac{[(n-\alpha)c+(1-\alpha)s]}{(n-1)c} < 0,$$

the tax substitution does not change total expenditure in equilibrium; and if

$$\left.\frac{ds}{dt}\right|_G = -\frac{(s+c)}{(1-t)}\frac{n}{(n-1)}\frac{[(n-\alpha)c+(1-\alpha)s]}{c[n-(1-t)\alpha]} < 0,$$

the tax substitution does not change government expenditure in equilibrium. Throughout the paper we shall refer to each one of these rates of substitution as the iso-price, the iso-total expenditure and the iso-government expenditure respectively. Moreover these rates of substitution fulfil the following properties: on the one hand, while the iso-price rate is constant with respect to the market power n, both the iso-total expenditure and the iso-government expenditure are decreasing with respect to this and converge at the iso-price rate of substitution as market power increases. On the other hand, these rates of substitution are ordered as

$$0 < -\left.\frac{ds}{dt}\right|_p < -\left.\frac{ds}{dt}\right|_Y < -\left.\frac{ds}{dt}\right|_G . \tag{26}$$

Recall that, as was commented in Sect. 2, the paper is concerned with the substitution between ad valorem and specific taxation without choosing a particular basis of comparison. In this trend, the above rates of substitution only show the thresholds for which those variables (which have a monotonically increasing behaviour with respect to the vector of tax instruments) change.

Therefore, given that the government makes a choice about γ and the monotonicity of prices, total expenditure and government expenditure-revenue, the following proposition holds.

Proposition 1 *The tax substitution given in* (11) *produces the following effects on price, total expenditure and government expenditure-revenue*:

(3.1) If $\gamma < -\frac{ds}{dt}\big|_p$ price, total expenditure and government expenditure increase.

(3.2) If $-\frac{ds}{dt}\big|_p \leq \gamma < -\frac{ds}{dt}\big|_Y$ price does not increase and total expenditure and government expenditure increase.

(3.3) If $-\frac{ds}{dt}\big|_Y \leq \gamma < -\frac{ds}{dt}\big|_G$ price decreases, total expenditure does not increase and government expenditure increases.

(3.4) If $-\frac{ds}{dt}\big|_G \leq \gamma$ price and total expenditure decrease and government expenditure does not increase.

Proof Taking into account the tax substitution (11), the total differentials of the price, total expenditure and government expenditure-revenue can be written as

$$dp^C = \left(\frac{\partial p}{\partial t} - \gamma \frac{\partial p}{\partial s}\right) dt = -\frac{\partial p}{\partial s}\left(\gamma + \frac{ds}{dt}\bigg|_p\right) dt, \tag{27}$$

$$dY^C = \left(\frac{\partial Y}{\partial t} - \gamma \frac{\partial Y}{\partial s}\right) dt = -\frac{\partial Y}{\partial s}\left(\gamma + \frac{ds}{dt}\bigg|_Y\right) dt, \tag{28}$$

$$dG^C = \left(\frac{\partial G}{\partial t} - \gamma \frac{\partial G}{\partial s}\right) dt = -\frac{\partial G}{\partial s}\left(\gamma + \frac{ds}{dt}\bigg|_G\right) dt. \tag{29}$$

Thus, when $\gamma < -\frac{ds}{dt}\big|_p$ by (26) $\gamma + \frac{ds}{dt}\big|_p < 0$, $\gamma + \frac{ds}{dt}\big|_Y < 0$ and $\gamma + \frac{ds}{dt}\big|_G < 0$ and due to (27), (28) and (29) $dp^C > 0$, $dY^C > 0$ and $dG^C > 0$, holding 3.1.

When $-\frac{ds}{dt}\big|_p \leq \gamma < -\frac{ds}{dt}\big|_Y$ by (26) $\gamma + \frac{ds}{dt}\big|_p \geq 0$, $\gamma + \frac{ds}{dt}\big|_Y < 0$ and $\gamma + \frac{ds}{dt}\big|_G < 0$ and due to (27), (28) and (29) $dp^C \leq 0$, $dY^C > 0$ and $dG^C > 0$, holding 3.2.

When $-\frac{ds}{dt}\big|_Y \leq \gamma < -\frac{ds}{dt}\big|_G$ by (26) $\gamma + \frac{ds}{dt}\big|_p > 0$, $\gamma + \frac{ds}{dt}\big|_Y \geq 0$ and $\gamma + \frac{ds}{dt}\big|_G < 0$ and due to (27), (28) and (29) $dp^C < 0$, $dY^C \leq 0$ and $dG^C > 0$, holding 3.3.

Finally, when $-\frac{ds}{dt}\big|_G \leq \gamma$ by (26) $\gamma + \frac{ds}{dt}\big|_p > 0$, $\gamma + \frac{ds}{dt}\big|_Y > 0$ and $\gamma + \frac{ds}{dt}\big|_G \geq 0$ and due to (27), (28) and (29) $dp^C < 0$, $dY^C < 0$ and $dG^C \leq 0$, holding 3.4. □

Proposition 1 states that the tax reform defined in (11) can produce different effects on price, total expenditure and government expenditure depending on the rate at which both tax rates are shifted. In particular, inflationary tax reforms are possible for small enough rates of substitution between taxes and even when the output always increases. Indeed, when $\gamma < -\frac{ds}{dt}\big|_p$ the increase in total expenditure is so large that the increase in total output is not enough to compensate the boost in aggregated demand with the consequential rise in equilibrium price. On the contrary, if the rate of substitution between both tax rates is large enough, that is when $-\frac{ds}{dt}\big|_p < \gamma$, the shift in aggregated demand is overtaken by the increase in total output and the tax substitution leads to a fall in equilibrium price. This is the case, for instance, of Dellipalla and Keen's (1992) P-shift tax reform (Eq. 12). In addition, according to 3.3 when $-\frac{ds}{dt}\big|_Y \leq \gamma < -\frac{ds}{dt}\big|_G$ total expenditure declines while government expenditure increases. This entails, according to (15), lower household consumption. Finally, when $-\frac{ds}{dt}\big|_p \leq$

$\gamma < -\frac{ds}{dt}\big|_G$ the equilibrium price declines while government expenditure rises; as we will see, this entails higher government purchases, thus, in such a case we can state the following result

Proposition 2 *If* $-\frac{ds}{dt}\big|_p \leq \gamma < -\frac{ds}{dt}\big|_G$ *the balanced budget multiplier is positive.*

Proof The balanced budget multiplier is given by the quotient dQ^C/dg^C, where $g^C = G^C/p^C$ represents the total government purchases. On the one hand, according to (20), $dQ^C > 0$. On the other hand, according to Proposition 1, when $-\frac{ds}{dt}\big|_p \leq \gamma < -\frac{ds}{dt}\big|_G$, $dp^C \leq 0$ and $dG^C > 0$, in this case $dg^C = \frac{1}{p}\left[dG^C - g^C dp^C\right] > 0$, and thus $dQ^C/dg^C > 0$. □

Although the fiscal policy is concerned with the substitution of specific by ad valorem taxation, the endogenous way in which government purchases $g^C = G^C/p^C$ are determined allows us to assess the rates of substitution between taxes for which g^C ends up getting boosted. Provided that any rate of substitution increases output, this is equivalent to obtaining those rates of substitution between taxes for which the balanced budget multiplier $\frac{dQ^C}{dg^C}$ is positive. Thus when $-\frac{ds}{dt}\big|_p \leq \gamma < -\frac{ds}{dt}\big|_G$ the tax substitution between taxes increases government expenditure beyond household expenditure, which can even decline, in such a way that the increase in government purchases shifts total output beyond the shift of the aggregated demand. In addition, according to the properties of the iso-price and the iso-government rates of substitution, it is readily seen that the higher the market power the higher the interval for which the balanced budget multiplier is positive.

4 Long-run effects of a tax substitution

This section analyzes how the equilibrium outcomes are affected by changes in the number of firms in each industry as a consequence of the indirect tax substitution. As is well known, this equilibrium is interpreted as a long-run situation after industry size has adjusted itself due to the disappearance of economic profits. We follow the usual practice of treating $n \in (1, W/hk)$[9] as a continuous variable. Substituting expenditure in equilibrium in industry i (Eq. 17), into Eq. (7) and imposing the zero profit condition we hold,

$$kcn^2 + ksn - \alpha(s + c)w = 0, \tag{30}$$

this equation has a unique positive solution which is

$$n(s, \alpha) = \frac{1}{2kc}\left(\sqrt{k^2s^2 + 4kc\alpha(s + c)w} - ks\right). \tag{31}$$

[9] The left boundary of this interval is open because the unit isoelasticity of the demand function impedes the monopoly case. The openness of the right boundary is necessary for the existence of the equilibria given in (16) and (18). To remark that $W = hw$.

Notice that the specific tax rate is the only one which affects industry size in equilibrium. This is due to the fact that the ad valorem tax rate is proportional to total expenditure in the industry and it is cancelled out when zero profit condition is held. Furthermore, in order to guarantee that $n(s, \alpha) \in (1, W/hk)$ in Eq. (31), it is necessary to assume that $\alpha \in (k/w, \min(1, \frac{cw+ks}{c+s}))$. This constraint reminds us of the importance of the marginal propensity to consume α in the determination of the equilibrium. For instance, Eq. (31) reveals that there is a monotonic-positive reliance between industry size in equilibrium and the marginal propensity to consume, that is, the larger α the larger $n(s, \alpha)$, a feature which plays an important role in our further results. Considering our utility function, the interpretation of this feature is straightforward: the greater α, the less leisure is preferred to consumption and household expenditure in each industry is larger (Eq. 3). In the absence of profits this means that the number of firms in equilibrium has to be larger as well (Eq. 7 or 14). Let us calculate the variation of industry size in equilibrium with respect to the specific tax rate:

$$\frac{\partial n}{\partial s} = \frac{1}{2c}\left(\frac{ks + 2c\alpha w}{\sqrt{k^2 s^2 + 4kc\alpha(s + c)w}} - 1\right) > 0. \tag{32}$$

The fact that (32) is positive is due to the assumption that $\alpha > k/w$.[10] This outcome is parallel to that achieved for profits in the previous section (Eq. 21), and its insight is related to the fact that incipient profits attract entry (Stern 1987). On the other hand, it is straightforward, from Eq. (31), that $\frac{\partial n}{\partial t} = 0$. Thus, taking into account (32), the effect of the substitution of specific by ad valorem taxation given by (11) on industry size under Free entry equilibrium is

$$dn = \frac{\partial n}{\partial t}dt + \frac{\partial n}{\partial s}ds = -\gamma \frac{\partial n}{\partial s}dt < 0. \tag{33}$$

This result is not surprising given how industry profit in equilibrium is affected by the tax substitution under the Cournot equilibrium (Eq. 22), and it entails that the tax substitution given by (11) drives our economy to a more non-competitive situation in the long-run. Moreover, the same as what happens between Eq. (22) of Sect. 1 and Proposition 3.b of Dellipalla and Keen (1992), Eq. (33) is similar to Proposition 4.b of that paper. This is obviously due to the parallelism between profits in the Cournot equilibrium and industry size in Free entry equilibrium. We address the comments on the differences between our approach and Dellipalla and Keen's (1992) result to that made in Sect. 3.

In what follows the upper script F refers to the Free entry equilibrium and it is useful for comparison with the Cournot equilibrium results (upper scripted by C).

[10] There is no contradiction with Besley (1989) because this author analizes how industry size changes with a specific tax rate in partial equilibrium. Here the positiveness of $\frac{\partial n}{\partial s}$ is due to the general equilibrium effects considered.

Regarding the price, the gradient of (6) with respect to the tax rates is

$$\nabla p^F(t,s) = \left(\frac{n}{(n-1)} \frac{(s+c)}{(1-t)^2}, \frac{1}{(n-1)} \frac{1}{(1-t)} \left[n - \frac{s+c}{n-1} \frac{\partial n}{\partial s} \right] \right). \tag{34}$$

Note that in this case, while the effect of a change in the ad valorem tax rate is the same as that produced under the Cournot equilibrium (Eq. 23), the effect of changes in the specific tax rate now transmits the changes in industry size in equilibrium. Thus, an increase in the specific tax rate has two opposite effects on price. On the one hand, as in Eq. (23), a rise in the specific tax rate increases the price for a given industry size, and on the other hand, it lowers the price as a consequence of the increase in the number of firms in equilibrium. Therefore, the effect on price of the tax substitution under Free entry equilibrium can be written, taking into account (27) and (34), as

$$dp^F = \left(\frac{\partial p^F}{\partial t} - \gamma \frac{\partial p^F}{\partial s} \right) dt = dp^C + \gamma \frac{(1+t)(s+c)}{(n-1)^2} \frac{\partial n}{\partial s} dt, \tag{35}$$

where $dp^F - dp^C > 0$, since the second term of (35) is positive. Thus, the variation in the price induced by the tax substitution is larger under Free entry equilibrium than under the Cournot equilibrium. For example, as has been shown in Sect. 3, if the iso-price rate of substitution between taxes is applied, that is if $\gamma = \frac{s+c}{1+t}$, the price does not change under Cournot equilibrium ($dp^C = 0$), however, under Free entry equilibrium, the price increases as a consequence of the fall in the industry size in equilibrium.

Regarding total output, in accordance with Eq. (18), it depends on both industry size and the specific tax rate and is independent of the ad valorem tax rate. So, the effect of a change in specific tax rate on total output is (see Sect. A1 of the Appendix)

$$\frac{\partial Q^F}{\partial s} = -\frac{(1-\alpha)Q}{(n-\alpha)c + (1-\alpha)s} - \frac{hk[(2\alpha-1)n - \alpha]}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s}. \tag{36}$$

Equation (36) has two terms; the first one is equal to (19), which is just the variation in total output due to a variation in the specific tax rate under the Cournot equilibrium. The second term captures the net effect of the change in industry size on output as a consequence of the change in the specific tax rate. Equations (5) and (18) reveal that an increase in n has, on the one hand, a direct boosting effect on industry output but, on the other hand, has an indirect decreasing effect due to the number of times that fixed costs incurred increase with the entry of firms. The balance of these two effects is captured in Eq. (36) through the sign of $(2\alpha-1)n-\alpha$. Hence, the fall in equilibrium output as a consequence of an increase in the specific tax rate can be lesser or greater in the long-run than in the short-run depending on the sign of $(2\alpha-1)n-\alpha$. As we will see, this sign depends on a threshold on α in such a way that it is positive (negative) if α is large (small) enough. Moreover, according to Eq. (31), $n(s, \alpha)$ is monotonic increasing with α. Thus, when the marginal propensity to consume is, for instance, large enough (and thereby the initial equilibrium size of industries) $(2\alpha-1)n-\alpha$ is positive and an increase in the specific tax rate yields a lesser output in the long-run with respect to the short-run equilibrium. In this case the increase in the number of firms has the effect of multiplying the number of times that the fixed costs associated with the existence of a distinct firm are incurred, causing an additional takeover of resources (see Keen 1998). The larger the initial equilibrium size in industries the larger this effect, and it can exceed the positive effect caused by the increase in the number of firms on equilibrium price. Then, taking into account (20), the effect on total output of the tax substitution given in Eq. (11) can be written as

$$dQ^F = \left(\frac{\partial Q^F}{\partial t} - \gamma \frac{\partial Q^F}{\partial s}\right) dt = dQ^C + \gamma \frac{hk[(2\alpha - 1)n - \alpha]}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s} dt. \quad (37)$$

This leads us to the following proposition,

Proposition 3 *The variation on total output induced by the tax substitution is larger under Free entry equilibrium than under Cournot equilibrium, i.e.* $dQ^F > dQ^C$, *if* $\alpha > \alpha^q$. *Otherwise* $dQ^F \leq dQ^C$, *if* $\alpha \in (k/w, \alpha^q)$, *where*

$$\alpha^q = \frac{1}{2} + \frac{(2s+c)k + \sqrt{(2s+c)^2k^2 + 8kwc(s+c)}}{8(s+c)w}.$$

Proof Section A2 of the Appendix. □

This result sets the value of α which determines the sign of $(2\alpha - 1)n - \alpha$. This term now operates in the opposite way as it does in Eq. (36) due to the fact that in the tax substitution the specific tax is shifted by the ad-valorem. This allows us to explain why the tax substitution can make output be large in the long-run rather than in the short-run, despite some firms flowing out and increases in price. As was claimed earlier, when α is large enough the initial equilibrium size of industries is large as well. Thus, the increase in equilibrium price is not so large (Eq. 35) and it is overtaken by the fall in the number of times that fixed costs are incurred, as a consequence of the exit of firms caused by the substitution of specific by ad valorem tax rate.

5 Welfare effects of a tax substitution

As we have seen in the previous sections, a substitution of specific by ad valorem taxation generates larger output and lower profits, industry size and household leisure for every rate of substitution between taxes as well as different effects (which depend on the value of the rate of substitution between taxes) on other variables which enter welfare, such as price and government expenditure. Within this trend, we are concerned with analyzing whether or not the tax substitution improves household welfare. Therefore, considering the long-run case, let us build the indirect utility function substituting the equilibrium values given by Eqs. (6), (3) and (4) into Eq. (1)

taking into account the zero profit condition[11]

$$V^F(t,s) = \ln \frac{\alpha^\alpha (1-\alpha)^{1-\alpha}}{h^{\alpha+\beta}} W - (\alpha+\beta)\ln p(t,s) + \beta \ln G(t,s).$$

Differentiating $V^F(t,s)$ with respect to $r = t, s$

$$\frac{\partial V^F(t,s)}{\partial r} = -(\alpha+\beta)\frac{\partial \ln p^F}{\partial r} + \beta \frac{\partial \ln G^F}{\partial r}. \tag{38}$$

Where upper script F refers to Free-entry equilibrium. Therefore, the total effect of the tax substitution on welfare can be obtained by totally differentiating $V^F(t,s)$ with respect (t,s) taking into account (11).

$$dV^F(t,s) = \left(\frac{\partial V^F(t,s)}{\partial t} - \gamma \frac{\partial V^F(t,s)}{\partial s}\right) dt, \tag{39}$$

substituting the partial derivatives given by Eq. (38) and arranging terms, the tax reform increases (decreases) welfare in the long-run if $\gamma < (>)\ \gamma(\alpha,\beta)$, where

$$\gamma(\alpha,\beta) = \frac{(\alpha+\beta)\frac{\partial \ln p^F}{\partial t} - \beta \frac{\partial \ln G^F}{\partial t}}{(\alpha+\beta)\frac{\partial \ln p^F}{\partial s} - \beta \frac{\partial \ln G^F}{\partial s}}. \tag{40}$$

By substituting Eqs. (6), (10), (34), and the value of $\frac{\partial \ln G^F}{\partial s}$ (calculated in Sect. A4 of the Appendix) in Eq. (40), $\gamma(\alpha,\beta)$ becomes an intricate expression which depends on α, β as well as on W, c, t, s and h. For this reason, in order to assess the change in welfare induced by the tax reform, let us plot Eq. (40) for a set of reasonable parameters. Provided the form in which consumption and government expenditure enter in the utility function, we address the welfare analysis to different values of α and β. In this trend, Fig. 1 depicts the shape of $\gamma(\alpha,\beta)$ as a function of α for different values of β, setting the value of the remaining parameters at $W = 1000$; $h = 100$, $k = 1$; $c = 1$, $t = 0.1$ and $s = 1$. According to (39), the locus downwards of each curve $\gamma(\alpha,\beta)$ represents increases in welfare for this value of parameter β, and the locus upwards a decrease. In addition, due to the monotonic-positive reliance between industry size in equilibrium and the marginal propensity to consume (Eq. 31), every α corresponds to a unique value of $n(s,\alpha)$. Therefore the iso-government expenditure and Dellipalla and Keen's (1992) P-shift (Eq. 12) rates of substitution (which depend on n) are depicted as well in Fig. 1 as a function of α. The iso-price rate of substitution (which does not depend on n) is also depicted. According to Proposition 1, the locus upwards of each of these curves in Fig. 1 contains the rates of substitution between taxes for which the variable in question decreases, and vice versa for the locus downwards. Therefore, according to Proposition 2, the locus between the iso-price and the

[11] The analysis of changes in welfare both in Cournot and Free entry equilibrium are quite similar in our model. However, the long-run point of view seems more relevant, provided that the tax substitution remains in time.

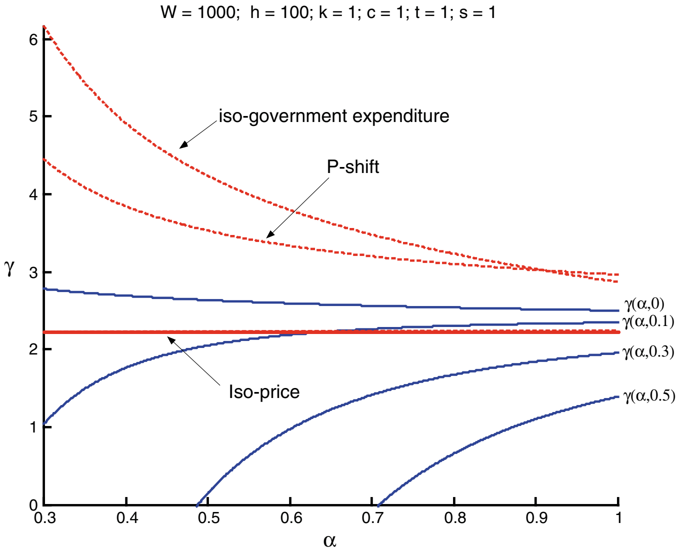

Fig. 1 $\gamma\ (\alpha, \beta)$

iso-government expenditure curves contains the rates of substitution for which the tax reform yields a positive balanced budget multiplier. We see that, for example, when $\alpha = 0.6$ ($n(0.6) = 3$), welfare declines when $\beta \geq 0.1$ for rates of substitution between taxes for which the balanced budget multiplier is positive [2.222, 3.794). The unique welfare-improving chance within this interval is in case in which $0 \leq \beta < 0.1$, that is, for a very low (and even for null) propensity to consume public purchases, but in this case even the P-shift tax reform (which is multiplier boosting), given by $\gamma_{DK} = 3.3333$, decreases welfare.

Therefore, the example shows that a fall in welfare can take place for both those rates of substitution between taxes for which the balanced budget multiplier is positive and the (usually tackled) neutral-revenue tax reform. Notably this case constitutes a counter-example of what occurs in partial equilibrium. Although the fall in welfare is most likely to occur the higher the propensity to consume public expenditure, it holds as well when this propensity is zero. This last point shows us that the (partial equilibrium neglected) wealth effects play an important role in the sign of the change in welfare under general equilibrium set-up. Indeed, the substitution of specific by ad valorem taxation implies a fall in profits and leisure (Eq. 22) which is equivalent to a fall in industry size in the long-run (Eq. 33), these effects having implications in household wealth and changes the composition of total expenditure. In addition, the fall in welfare is most likely to occur the higher the rate of substitution between taxes, because the fall in leisure is higher the higher γ. On the other hand, when the propensity to consume public expenditure β is positive, as is most preferred by households the rates of substitution between taxes γ needed to lead an increase in welfare have to be lower, that is, the interval of tax reforms which yields a welfare-improvement is smaller. This is because, according to Proposition 1, the lower the rate of substitution between taxes γ the higher the increase in government-expenditure.

On the other hand, given β, as consumption is most preferred by households, the rates of substitution between taxes γ needed to lead an increase in welfare can be higher, because higher rates of substitution between taxes correspond to lower increases, and even with decreases, in prices.

This case constitutes a counter-example to the partial equilibrium assertion that, in general, a substitution of specific by ad valorem taxation yields to Pareto-improvements. We show that in general equilibrium settings the wealth effects arising from the variations in tax revenue and profits caused by such a tax substitution can negatively affect welfare.

6 Conclusions

This paper addresses the macroeconomic effects of a substitution policy which shifts from specific to ad valorem taxation in a general equilibrium model with imperfect competition. This set-up allows us to analyze the short-run and the long-run effects of such a tax policy by means of the Cournot and the Free entry equilibria, respectively. The tax reform is characterized by means of a rate of substitution between the tax rates without choosing any particular basis of comparison. This allows us to lay down the set of all possible effects of such a tax reform on the equilibrium outcomes.

In the Cournot equilibrium case one finds that any tax substitution increases total output and decreases total profit. This outcome is similar to that obtained in partial equilibrium, although in our general equilibrium framework wealth effects appear to shift aggregated demand as a consequence of the changes in both government and household expenditures. If the rate of substitution between taxes is small enough, the increase in total expenditure is so large that the increase in total output is not enough to compensate the boost in aggregated demand with the consequential rise in equilibrium price. If the rate of substitution between both taxes is large enough, the shift in aggregated demand is overtaken by the increase in total output, leading to a fall in the equilibrium price. In this case, there is a wide range of substitution rates between taxes for which government purchases increase and, thus, the balanced budget multiplier is positive. In addition, this range of substitution rates is higher the higher the degree of market power.

In the Free entry Oligopoly case, one finds that the substitution of specific by ad valorem taxation decreases the number of firms in equilibrium. This occurs as a consequence of the decrease in total profit under the Cournot equilibrium. This adjustment in industry size increases the price with respect to the Cournot equilibrium. Furthermore, the adjustment has implications for the remaining variables which depend on the marginal propensity to consume. In this way if the marginal propensity to consume is large enough, total output increases beyond its Cournot equilibrium value. Thus, in this case, a rise in output is compatible with a rise in prices and a fall in industry size. This is due to the monotonic increasing relationship between industry size in equilibrium and the marginal propensity to consume. An economy with a large marginal propensity to consume is an economy with a large equilibrium number of firms. As the tax substitution decreases the number of firms in equilibrium, when the number of firms is initially high, the positive effect on output resulting from the fall in the fixed costs incurred by the firms is greater than the negative effect that causes the increase in the price.

Regarding welfare, as the effects of such a tax substitution change the composition of total expenditure and indirectly convert leisure into government expenditure, it is likely to find welfare decreasing tax reforms although total output always increases. This case deserves attention because, on the one hand, in contrast with the neo-Keynesian models, the fall in welfare takes place even when the balanced budget multiplier is positive, and on the other hand, in contrast with the partial equilibrium results, the revenue-neutral tax reform can provoke a fall in welfare.

Finally we must point out that our conclusions are related to the special assumptions and functional forms of our framework. However, the difficulties arising in modelling imperfect competition in general equilibrium settings often require of the use of a specific formulation. Further research should extend the analysis to other functional forms for households and firms, or to other contexts as the case of asymmetric industries.

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Appendix

(A1) Determination of $\frac{\partial Q}{\partial s}$ under free entry equilibrium

Taking into account that $W = hw$ Eq. (18) can be written as

$$Q(t, s) = \frac{\alpha h(n-1)(w-nk)}{[(n-\alpha)c + (1-\alpha)s]},$$

differentiating with respect to s

$$\frac{\partial Q^F}{\partial s} = \alpha h \left[\frac{(w-nk)}{[(n-\alpha)c+(1-\alpha)s]} \frac{\partial n}{\partial s} - \frac{(n-1)\,k}{[(n-\alpha)c+(1-\alpha)s]} \frac{\partial n}{\partial s} \cdots \right.$$
$$\left. \cdots - \frac{(n-1)\,(w-nk)}{[(n-\alpha)c+(1-\alpha)s]^2} \left(c\frac{\partial n}{\partial s} + 1 - \alpha \right) \right]$$

grouping terms and operating taking into account (18),

$$\frac{\partial Q^F}{\partial s} = -\frac{(1-\alpha)Q}{(n-\alpha)c+(1-\alpha)s} \cdots$$
$$\cdots + \frac{\alpha h}{(n-\alpha)c+(1-\alpha)s} \left[\frac{(1-\alpha)\,(s+c)\,(w-nk))}{(n-\alpha)c+(1-\alpha)s} - (n-1)\,k \right] \frac{\partial n}{\partial s},$$

writing from Eqs. (7) and (13) the zero profit condition as $\frac{\alpha(s+c)(w-nk)}{(n-\alpha)c+(1-\alpha)s} = nk$, and operating Eq. (36) holds.

(A2) Proof of proposition 3

According to (37)

$$dQ^F - dQ^C = \frac{k[(2\alpha - 1)n(\alpha) - \alpha]}{(n-\alpha)c + (1-\alpha)s}\frac{\partial n}{\partial s}dt,$$

thus the negativity or positivity of $dQ^F - dQ^C$ depends only on the sign of $\Gamma(\alpha) = (2\alpha - 1)n(s, \alpha) - \alpha$. As $n(s, \alpha) > 1$ for $\alpha \in (k/w, 1]$ it is readily seen that $\Gamma(\alpha) < 0$ for $\alpha \leq \frac{1}{2}$ and therefore $dQ^F - dQ^C < 0$. When $\alpha > \frac{1}{2}$ the sign of $\Gamma(\alpha)$ is positive if $n(s, \alpha) > \alpha/(2\alpha - 1)$.Taking into account Eq. (31), and operating, this condition can be written as

$$(2\alpha - 1)\left[\sqrt{k^2s^2 + 4kc\alpha(s+c)w} - ks\right] > 2kc\alpha,$$

or

$$(2\alpha - 1)\sqrt{k^2s^2 + 4kc\alpha(s+c)w} > 2kc\alpha + (2\alpha - 1)ks,$$

since both left and right side terms of the inequality are strictly positive it is true that

$$(2\alpha - 1)^2\left[k^2s^2 + 4kc\alpha(s+c)w\right] > (2kc\alpha + (2\alpha - 1)ks)^2,$$

developing terms and simplifying

$$(2\alpha - 1)^2 w(s+c) > kc\alpha + (2\alpha - 1)ks,$$

and developing $(2\alpha - 1)^2$ and grouping terms with respect to α this inequality can be written as

$$L(\alpha) = 4w(s+c)\alpha^2 - [(s+c)(k+4w) + sk]\alpha + (s+c)w + sk > 0.$$

$L(\alpha)$ is a convex parabola which reaches its minimum at

$$\alpha_{\min} = \frac{1}{2} + \frac{(2s+c)k}{8(s+c)w},$$

and has two roots such that only one belongs to the interval $(\frac{1}{2}, 1)$ (we are analyzing the positivity of $L(\alpha)$ for $\alpha > \frac{1}{2}$), this value is given by α^q. Therefore $L(\alpha) > 0$, i.e. $n(\alpha) > \alpha/(2\alpha - 1)$ for $\alpha > \alpha^q$. And $L(\alpha) \leq 0$ for $\frac{k}{w} < \alpha \leq \alpha^q$.

(A3) Calculation of $\frac{\partial Y^F}{\partial s}$

Taking into account that $W = hw$ Eq. (16) can be written as

$$Y(t,s) = \frac{\alpha h}{(1-t)} \frac{(s+c)n(w-nk)}{[(n-\alpha)c + (1-\alpha)s]},$$

differentiating with respect to s,

$$\frac{\partial Y^F}{\partial s} = \frac{\alpha h}{(1-t)} \left[\frac{n(w-nk)}{(n-\alpha)c + (1-\alpha)s} + \frac{(s+c)(w-nk)}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s} - \cdots \right.$$
$$\cdots - \frac{n(s+c)k}{(n-\alpha)c + (1-\alpha)s} \frac{\partial n}{\partial s} - \frac{n(s+c)(w-nk)}{[(n-\alpha)c + (1-\alpha)s]^2}$$
$$\left. \times \left(c\frac{\partial n}{\partial s} + 1 - \alpha \right) \right],$$

grouping terms and operating,

$$\frac{\partial Y^F}{\partial s} = \frac{\alpha h}{(1-t)[(n-\alpha)c + (1-\alpha)s]}$$
$$\left[(s+c) \left(w - nk - \frac{cn(w-nk)}{(n-\alpha)c + (1-\alpha)s} - nk \right) \frac{\partial n}{\partial s} \cdots + n(w-nk) \right.$$
$$\left. \times \left(1 - \frac{(1-\alpha)(s+c)}{(n-\alpha)c + (1-\alpha)s} \right) \right],$$

$$\frac{\partial Y^F}{\partial s} = \frac{\alpha h}{(1-t)[(n-\alpha)c + (1-\alpha)s]}$$
$$\times \left[(s+c) \left(\frac{(s - \alpha(s+c))(w-nk)}{[(n-\alpha)c + (1-\alpha)s]} - nk \right) \frac{\partial n}{\partial s} + \frac{n(w-nk)(n-1)c}{(n-\alpha)c + (1-\alpha)s} \right],$$

writing from Eqs. (7) and (13) the zero profit condition as $\frac{\alpha(s+c)(w-nk)}{(n-\alpha)c+(1-\alpha)s} = nk$, and operating

$$\frac{\partial Y^F}{\partial s} = \frac{\alpha h}{(1-t)[(n-\alpha)c + (1-\alpha)s]}$$
$$\times \left[(s+c) \left(\frac{s(w-nk)}{[(n-\alpha)c + (1-\alpha)s]} - 2nk \right) \frac{\partial n}{\partial s} + \frac{(1-t)(n-1)cY}{\alpha h(s+c)} \right],$$

$$\frac{\partial Y^F}{\partial s} = \frac{c(n-1)Y}{[(n-\alpha)c + (1-\alpha)s]\,(s+c)} + \frac{\alpha h(s+c)}{(1-t)[(n-\alpha)c + (1-\alpha)s]}$$
$$\times \left(\frac{s(w-nk)}{[(n-\alpha)c + (1-\alpha)s]} - 2nk \right) \frac{\partial n}{\partial s},$$

operating, taking into account (24)

$$\frac{\partial Y^F}{\partial s}=\frac{\partial Y^C}{\partial s}+\frac{nhk(s-2\alpha(s+c))}{(1-t)[(n-\alpha)c+(1-\alpha)s]}\frac{\partial n}{\partial s},$$

or

$$\frac{\partial Y^F}{\partial s}=\frac{\partial Y^C}{\partial s}+\frac{(s-2\alpha(s+c))}{(n-\alpha)c+(1-\alpha)s}\frac{Y}{n}\frac{\partial n}{\partial s}$$

(A4) Calculation of $\frac{\partial G^F}{\partial s}$ and $\frac{\partial \ln G^F}{\partial s}$

Differentiating Eq. (10) with respect to s

$$\frac{\partial G^F}{\partial s}=(1-t)\left[\left(\frac{c}{(s+c)^2}\frac{(n-1)}{n}+\frac{s}{(s+c)n^2}\frac{\partial n}{\partial s}\right)Y\right.$$
$$\left.+\left(\frac{t}{1-t}+\frac{s}{(s+c)}\frac{(n-1)}{n}\right)\frac{\partial Y}{\partial s}\right],$$

substituting the value of $\frac{\partial Y^F}{\partial s}$ obtained in A3

$$\frac{\partial G^F}{\partial s}=(1-t)\left[\left(\frac{c}{(s+c)^2}\frac{(n-1)}{n}+\frac{s}{(s+c)n^2}\frac{\partial n}{\partial s}\right)Y+\cdots\right.$$
$$\cdots+\left(\frac{t}{1-t}+\frac{s}{(s+c)}\frac{(n-1)}{n}\right)$$
$$\times\left[\frac{c(n-1)Y}{[(n-\alpha)c+(1-\alpha)s](s+c)}\right.$$
$$\left.\left.+\frac{s-2\alpha(s+c)}{[(n-\alpha)c+(1-\alpha)s]}\frac{Y}{n}\frac{\partial n}{\partial s}\right]\right]$$

grouping terms, taking into account that $\left(\frac{t}{1-t}+\frac{s}{(s+c)}\frac{(n-1)}{n}\right)=\frac{t(nc+s)+s(n-1)}{(1-t)(s+c)n}$

$$\frac{\partial G^F}{\partial s}=(1-t)\frac{Y}{n}\left[\frac{c(n-1)}{(s+c)^2}\left(1+\frac{t(nc+s)+s(n-1)}{(1-t)[(n-\alpha)c+(1-\alpha)s]}\right)+\cdots\right.$$
$$\left.\cdots+\left(s+\frac{[t(nc+s)+s(n-1)][s-2\alpha(s+c)]}{(1-t)[(n-\alpha)c+(1-\alpha)s]}\right)\frac{1}{n(s+c)}\frac{\partial n}{\partial s}\right]$$

$$\frac{\partial G^F}{\partial s} = \frac{(1-t)}{(s+c)}\frac{Y}{n}\left[\frac{[n-\alpha(1-t)]\,c(n-1)}{(1-t)\,[(n-\alpha)c+(1-\alpha)s]}\right.$$

$$\left. + \left(s + \frac{[t(nc+s)+s(n-1)]\,[s-2\alpha(s+c)]}{(1-t)\,[(n-\alpha)c+(1-\alpha)s]}\right)\frac{1}{n}\frac{\partial n}{\partial s}\right],$$

$$\frac{\partial G^F}{\partial s} = \frac{\partial G^C}{\partial s} + \left(s + \frac{[t(nc+s)+s(n-1)]\,[s-2\alpha(s+c)]}{(1-t)\,[(n-\alpha)c+(1-\alpha)s]}\right)\frac{(1-t)Y}{n^2(s+c)}\frac{\partial n}{\partial s}.$$

Finally, as $\frac{\partial \ln G^F}{\partial s} = \frac{\frac{\partial G^F}{\partial s}}{G}$ using Eq. (10) and operating we hold

$$\frac{\partial \ln G^F}{\partial s} = \frac{[n-\alpha(1-t)]\,c(n-1)}{[t(nc+s)+s(n-1)]\,[(n-\alpha)c+(1-\alpha)s]} + \cdots$$

$$\cdots + \left(s + \frac{[t(nc+s)+s(n-1)]\,[s-2\alpha(s+c)]}{(1-t)\,[(n-\alpha)c+(1-\alpha)s]}\right)$$

$$\times \frac{(1-t)}{n\,[t(nc+s)+s(n-1)]}\frac{\partial n}{\partial s}$$

References

Anderson SP, de Palma A, Kreider B (2001) The efficiency of indirect taxes under imperfect competition. J Publ Econ 81:231–251

Besley T (1989) Commodity taxation and imperfect competition. A note on the effects of entry. J Publ Econ 40:359–367

Blanchard O, Kiyotaki N (1987) Monopolistic competition and the effects of aggregated demand. Am Econ Rev 77:647–666

Blackorby C, Murty S (2007) Unit versus ad valorem taxes: monopoly in general equilibrium. J Publ Econ 91:817–822

Bishop RL (1968) The effects of specific and ad-valorem taxes. Q J Econ 82:198–218

Caminal R (1990) Consecuencias macroeconómicas de la competencia imperfecta: un ejemplo. Cuadernos Económicos del I.C.E 45:165–179

Cordella T, Gabszewicz JJ (1997) Comparative advantage under oligopoly. J Int Econ 43:333–346

Dellipalla S, Keen M (1992) The comparison between ad-valorem and specific taxation under imperfect competition. J Publ Econ 49:351–367

Dellipalla S, Keen M (2006) Product quality and the optimal structure of commodity taxes. J Publ Econ Theory 8:547–554

Denicoló V, Matteuzzi M (2000) Specific and Ad Valorem Taxation in Asymmetric Cournot Oligopolies. Int Tax Publ Fin 7:335–342

Dierker E, Grodal B (1995) Profit maximization mitigates competition. Econ Theory 7:139–160

Dierker E, Grodal B (1998) Modelling Policy Issues in a World of Imperfect Competition. Scandin J Econ 100:184–186

Dierker E, Grodal B (1999) The price normalization problem in imperfect competition and the objective of the firm. Econ Theory 14:257–284

Dixon H (1987) A simple model of imperfect competition with Walrasian features. Oxf Econ Pap 39:143–160

Dixon H, Rankin N (1994) Imperfect Competition and Macroeconomics: a survey. Oxf Econ Pap 46:171–199

Gabszewicz JJ, Vial JP (1972) Oligopoly à la Cournot in general equilibrium analysis. J Econ Theory 4:381–400

Grazzini L (2006) A note on ad valorem and per unit taxation in an oligopoly model. J Econ 89:59–74

Hart O (1982) A model of imperfect competition with Keynesian features. Q J Econ 97:109–138
Heijdra BJ, Ligthart JE, van der Ploeg F (1998) Fiscal policy, distortionary taxation, and direct crowding out under monopolistic competition. Oxf Econ Pap 50:79–88
Keen M (1998) The balance between specific and ad valorem taxation. Fisc Stud 19:1–37
Mankiw NG (1988) Imperfect competition and the Keynesian cross. Econ Lett 26:7–13
Molana H, Moutos T (1992) A note on taxation, imperfect competition and the balanced budget multiplier. Oxf Econ Pap 44:68–74
Neary JP (2003) The road less travelled: oligopoly and competition policy in a general equilibrium. In: Arnott R, Greenwald B, Kanbur R, Nalebuff B (eds) Economics for an imperfect world: essays in honor of Joseph E. Stiglitz. Mass. MIT Press, Cambridge, pp 485–500 (2003)
Seade J (1985) Profitable cost increases and the shifting of taxation: equilibrium responses of markets in oligopoly. Warwick economic research paper no. 260
Schröder PJH (2004) The comparison between ad valorem and unit taxes under monopolistic competition. J Econ 83:281–292
Skeath SE, Trandel GA (1994) A Pareto comparison of ad valorem and unit taxes in noncompetitive environments. J Publ Econ 53:53–71
Stern NH (1987) The effects of taxation, price control and government contracts in oligopoly and monopolistic competition. J Publ Econ 32:133–158
Suits DB, Musgrave RA (1955) Ad-valorem and unit taxes compared. Q J Econ 97:598–604
Torregrosa RJ (1998) On the monotonicity of balanced budget multiplier under imperfect competition. Econ Lett 59:331–335
Torregrosa RJ (2003) Balanced budget multiplier with indirect taxes under imperfect competition. Estudios Económicos 18:3–13
Wicksell K (1959) Taxation in the monopoly case translate from 1896 original. In: Musgrave A, Soup B (eds) Readings in the economics of taxation, Irwin, Momewood