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Analysis of Teacher-Student Interaction in the Joint Solving of Non-Routine Problems in Primary Education Classrooms

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Abstract: The analysis of teacher–student interaction when jointly solving routine problems in the primary education mathematics classroom has revealed that there is scarce reasoning and little participation on students' part. To analyze whether this fact is due to the routine nature of the problems, a sample of teachers who solved, together with their students, a routine problem involving three questions with different cognitive difficulty levels (task 1) was analyzed, describing on which part of the problem-solving process (selection of information or reasoning) they focused their interaction. Results showed that they barely focused the interaction on reasoning, and participation of students was scarce, regardless of the cognitive difficulty of the question to be answered. To check whether these results could be due to the routine nature of the problem, a nonroutine problem (task 2) was solved by the same sample of teachers and students. The results revealed an increase in both reasoning and participation of students in processes that required complex reasoning. This being so, the main conclusion of the present study is that including nonroutine problem solving in the primary education classroom as a challenging task is a reasonable way to increase students' ability to use their own reasoning to solve problems, and to promote greater teacher–student collaboration. These two aspects are relevant for students to become creative, critical, and reflective citizens.

Keywords: classroom interaction; mathematics education; primary education; problem solving; non-routine problem; creativity; critical thinking

1. Introduction

The analysis of the educational practice of teachers when jointly solving tasks with their students in the classroom has been a focus of interest in research in mathematics education (e.g., [1–3]). Knowledge of what happens in joint problem solving in the classroom can provide insight to promote learning.

One of the most important cognitive tasks to be undertaken in the mathematics classroom is problem solving (e.g., [4,5]). It is one of the most relevant aspects in the curriculum of many countries and an educational competency that is taken into account in international assessment frameworks (e.g., Trends in International Mathematics and Science Study, TIMSS; Program for International Student Assessment, PISA or National Assessment of Educational Progress, NAEP). It provides students with knowledge, abilities, and skills to cope with everyday problem situations [6].

According to [7], students' learning opportunities depend not only on the type of task under consideration but also on how teachers and students interact to address it. Specifically, most of the problems set out in the tasks that are solved in the classroom could be described as mechanical, since the solution is reached by simply selecting the appropriate data from the statement to choose the operation using textual cues (such as keywords) which may involve applying the knowledge that is to be acquired [3,8]. It is important to note how research shows that when teachers solve these problems together with their students in the classroom, they promote little reasoning and scarce student participation [3,9,10]. This result could be due to the essentially mechanistic and automated nature of the activities performed in mathematics classrooms [11].

The purpose of this article was to analyze whether the type of task used for teaching in mathematics classrooms influences the reasoning and collaboration that teachers and students develop while solving word problems. Problem solving can help students to reach this goal if it is focused on promoting creativity, critical thinking, communication, and collaboration [12].

As for the tasks that are carried out in the classrooms, their nature is varied, and they can be used to achieve different objectives. Researchers have analyzed these tasks in different ways, such as by establishing relationships between the task proposed and what it requires of those who are dealing with it to reach a solution [6]. Mathematics problems are an example of this. One of the most widely used problem types in primary education classrooms is word problems, understood as "verbal descriptions of problematic situations that give rise to one or more questions whose answers can be obtained by applying mathematical operations to the numerical data present in the problem" [13] (p. 641). These authors classified problems into two types according to their solving requirements: routine and nonroutine. Routine problems are those that are solved by mechanically using known arithmetic operations [6,14], that is, the solution is reached mechanically, since all that is required is to select the data and the operation. Nonroutine problems are those whose solution cannot be reached by directly applying a single or several arithmetic operations, but require knowledge, previous experience, and intuition (see examples in [15–18]), as well as analysis and creativity [19,20].

Mathematical tasks can be also analyzed in terms of difficulty according to the difficulty levels underlying each of the cognitive domains established by [21]. Such cognitive domains are, in increasing order of cognitive difficulty: knowing (applying mathematics about mathematical situations, depends on familiarity with mathematical concepts and fluency in mathematical skills); applying (involves the use of mathematics in a range of contexts in which the facts, concepts, and procedures as well as the problems are familiar to the student), and reasoning (involves logical, systematic thinking; it includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in novel or unfamiliar situations) [21].

Regarding interaction, it could be described as the exchange of information between teacher and students that allows them to negotiate meanings in the classroom [22]. This negotiation leads to the construction of knowledge as the interaction progresses [2,23]. One of the aspects that influences the development of interaction is the type of questions asked by the teacher, since this will have an impact on the cognitive level of students' answers (e.g., [24,25]). Questions can, in one sense, encourage the production of an elaborate answer where students can reason, argue, and express their ideas, so that they can express their opinions or, in another sense, demand a single and expected answer [9,26]. If teachers ask appropriate questions, they can guide student to develop their own strategies [27]. As regards the questions asked, quality is more important than number, which means that they should promote students' reasoning and reflection [28]. Indeed, classroom interaction where the teacher's role is limited to asking and the student's to answering is not desirable. According to [29], the questions involved in the type of interaction that can favor mathematics learning can be classified as follows: open-ended, meaning that there is no fixed answer, but that they can be answered correctly in different ways (e.g.,: "Why have you done it?"); closed-ended, where the answer only admits one correct possibility that is limited to yes or no or to choosing an option among several alternatives (e.g.,: "Have you taken data 6 and multiplied it by two?"); or invasive, when the answer is not limited to a one-syllable word or to choosing an option, but the key to the correct answer lies in the question (e.g.,: "So, 10 and 2, make . . . ?").

Educational research underlines how important it is for teachers to be capable of promoting students' autonomy in communicative exchanges, so that they should be able to pose questions and, through interaction, elicit from students adequate answers [30–32]. Learning to dialogue in this way leads them to progressively become more capable of asking open-ended questions and providing a reasoned and critical dialogue [32–34]. In this regard, several authors consider that the analysis of teacher–students interaction is one of the most useful elements to describe this dialogue between teachers and students and how students learn through it, since a large part of the knowledge acquired by students is learned in communicative contexts that involve interaction (for example [7,28,35]. Such interaction is especially effective when it fosters students' reasoning and negotiated participation [36].

One of the areas where teacher–student interactions in the mathematics classroom have been analyzed is routine problem solving. More specifically, certain authors have analyzed the cognitive processes that take place in the solving of routine problems in the primary education classroom (e.g., [2,18]) and others have focused on teacher–student participation in reaching the solution [29,37].

In terms of the cognitive processes that come into action when solving a routine problem in the classroom, it has been noticed that the promotion of reasoning in teacher–students interactions when solving these problems is scarce (e.g., [35]). Precisely, [3] analyzed the interaction that ten primary education teachers engaged in when solving a routine and a rewritten problem in their classroom. The rewritten problem included situational and mathematical elements aimed at promoting understanding of the problem [11]. The interaction that look place during the solving process was found to be similar for both problems, most of it being aimed at data selection and the implementation of algorithms, involving very little reasoning.

Regarding teachers' and students' participation in the solving process while solving routine problems in the classroom, the analysis of the interaction revealed that student participation was scarce (e.g., [3,35,38]).

In short, the results of previous studies that analyzed how teacher and students behave when they engage in joint routine problem solving in the primary education classroom show that teachers focused their attention on selecting data and implementing operations, using scarce reasoning [35] and promoting little student participation in reaching the solution [3,35,38]. Based on these studies, the purpose of this research was to analyze if the level of complexity of the tasks influences in how teachers and students interact when jointly solving two tasks: a routine problem (task 1) and a nonroutine problem (task 2). To do so, teacher–student interactions were analyzed considering two aspects: the cognitive processes developed, and, more specifically, the level of reasoning they generated [39–41] and the level of participation of teacher and students—that is, whether the ideas needed to solve the problem were mainly generated by the teacher, the students, or both [42].

Considering the results of previous studies, the hypotheses posed were that as the task's complexity level increases:

1. As regards cognitive processes, the greater the level of complexity of the task, the greater the proportion of the interaction devoted to reasoning.
2. As regards level of participation of teachers and students, there will be a high participation of students in the cognitive processes aimed at reasoning in the resolution process, especially in tasks with a higher level of complexity.

2. Materials and Methods

2.1. Participants

Ten mainstream teachers (5 male, 5 female; Table 1) and their students (age ranging from 10 to 12 years old; M = 11.18, SD = 0.47). The teachers had training in culturally relevant pedagogy and were selected at random from an initial pool of teachers from different schools of an urban and rural school in Spain (professional experience ranging from 13 to 31 years; M = 25, SD = 6.84). The schools where the teachers worked were medium size (average 331 students); the number of students in each classroom varied from 11 to 25 (M = 19.50, SD = 4.70). The students belonged to middle-class socioeconomic backgrounds and most of them were Spanish. The teachers reported that there were no noteworthy difficulties in terms of students' average achievement and that it did not deserve specific analysis. Neither teachers nor students received any specific training in problem solving. All participants entered into a data use agreement for the purposes of this research.

Table 1. Participants.

Teacher	Experience (Years)	Geographical Scope	Students in Each Class (Number)
Teacher 1	32	Rural 1	20
Teacher 2	33	Rural 2	22
Teacher 3	28	Urban 1	14
Teacher 4	23	Urban 2	23
Teacher 5	23	Urban 3	24
Teacher 6	13	Urban 4	11
Teacher 7	13	Urban 5	13
Teacher 8	30	Rural 3	21
Teacher 9	25	Rural 4	25
Teacher 10	30	Rural 5	22

2.2. Materials

Two different word problem were used:

A routine problem with three subtasks, each aimed at a different level of cognitive complexity (adapted from [21]): (S1) knowing, (S2) applying, and (S3) reasoning (subtasks B, A, and C, respectively, in Appendix A, Figure A1). Section (A) belonged to the cognitive domain "applying", since it involved the use of the information that was provided in the problem to subsequently implement an algorithm, such as adding the cost of each type of bicycle for each hour and reaching the solution. Section (B) belonged to the cognitive domain "knowing", since it was enough to look at the tables above and to choose the correct answer to the question of the problem. Finally, section (C) belonged to the cognitive domain "reasoning", since the requirements to solve this task involved analyzing the situation, linking the different elements of the problem, and assessing and implementing the best strategy to reach the solution, which involved developing a mathematical model of the type $c(h) = f + c{\cdot}h$, where $c\,(h)$ stands for total cost, f for fixed cost, c for cost per hour, and h for number of hours [21].

Additionally, a nonroutine problem was utilized, specifically (Figure 1). This problem is defined as nonroutine because there is no marked path to solve it.

One day on my way to the market with my father we met a neighbor, John, who, upon knowing it, asked us to sell the 30 melons he had at the price of 3 melons € 1. After, we met an acquaintance, Mary, who, taking advantage asked us if we could sell her 30 melons at 2 melons for € 1. We agreed in both cases and, in view of the differences in the melons' prices, my father came up with the idea of selling the melons in batches of 5 melons at € 2 per batch. We sold them all and my father asked me to put the money away.
On our way back, my father told me that we would organize the money earned to pay John and Mary. I looked at the money I had and, since we had sold 12 batches of 5 melons, checked that I had € 24. My father then said that we had to give John € 10 and Mary € 15, a total of € 25. Had I lost € 1 on the way?

Figure 1. Nonroutine problem (adapted from [43]).

2.3. Procedure and Analysis

The teachers and the students solved the problems (task 1 and task 2) in the classroom. A classroom observer took notes to supplement the audio-recorded discourse, so that each of the teachers' statements or actions could be correctly discerned. This resolution process was recorded in audio, which was transcribed and analyzed. The analysis unit used was the interaction cycle, understood as the segmentation of the actions involved in the interaction during the development of a classroom task. Cycles usually began with a question, either explicit or implicit, and ended when the question was answered or left unanswered [44]. For this purpose, public contents were considered, understood as the information that teacher and students openly shared, so that each cycle included one single item of public content (when a single cycle included more than one public content, the main content on which the other public content or contents depended was established as public content [3]).

After defining the cycles, each of them was categorized in two ways according, on the one hand, to the *processes* that are promoted in the solving process and, on the other hand, to the *level of participation* of teacher and students in the task:

a. Processes (adapted from [3]; Table 2), depending on different aspects where attention must be paid as follows:

Table 2. Processes categories system [42].

	Categories	Definition
Cognitive Processes	**Selection**	Aspects that are explicitly included in the problem's statement or that arise, unjustified, in the solving process.
	Integration	Aspects that relate or compare information or data that are explicitly included in the problem's statement or that arise in the solving process in an adequate and justified way.
Metacognitive Processes	**Generalization**	Aspects of the solving process that are more general than those considered in the problem.
	Regulation	Aspects of the solving process that are related to planning (organization of the process), monitoring (assessment and observation of the process), and evaluation (definition of the advance and progress in producing the solution as well as assessment of the development of the process).
Other Processes	**Control**	Aspects related to keeping attention focused and classroom order, or related to organization, with no relation whatsoever to the solving process.
	Reading	Aspects related to the reading of the problem, which includes vocabulary definition prior to the solving process.

As our goal was to describe specific cognitive processes, some other general processes, including *Metacognitive processes* (*Generalization* and *Regulation*, 21.66% of the total of cycles) and *Other processes* (*control* and *reading*, 12.78% of the cycles total) were not considered in this study, though they could be aims for future research.

b. Level of Participation of teacher and students (adapted from [3,42]), depending on who took most of the responsibility in the generation of the public content in each interaction cycle. To analyze this, the following categories (following [42]) were considered (Table 3):

Table 3. Level of participation categories system [42].

Categories		Indicators
Low Level	**T Level (teacher):** Teacher autonomously.	The teacher begins the cycle, develops it individually during the entire intervention and can end it.
	Ts Level (teacher–students): Teacher and student with greater teacher participation.	The teacher begins the cycle with a question or closed or invasive intervention. The teacher may end the cycle providing feedback.
High Level	**St Level (students–teacher):** Teacher and student with greater student participation.	The teacher begins the cycle with an open-ended question or intervention that is adequately answered by the student. If the teacher ends the building of the main idea of the cycle, it will be St Level. The teacher may end the cycle providing feedback.
	S Level (students): Student autonomously.	The student begins the cycle, although the teacher can also do so by returning to the participation of a student in a previous cycle and can end it.

The following is an analysis example (Table 4):

Table 4. Example of application of the system of analysis chosen (observer agreement for each category, measured by Cohen's kappa, is shown in brackets).

Cycles (0.99)	Transcription	Processes		Level of Participation (0.94)
		Public Content (0.84)	Category (0.94)	
1	Teacher: It is about completing with the tables, isn't it? Let's see. Student: Mountain bike rental.1 h, price 8; 2 h, price 11. Teacher: Price 11.	"Mountain bikes 1 h, €8; 2 h, €11, … "	*Selection*	*Low Level*
2	Teacher: Why is the price 11? Let's see, Pablo. How do we know that the price is 11? Student: Because you must add 8 plus 3. Teacher: 8 plus 3. Student: Yes. Teacher: That is, 8 that is the price of an hour, the first one.	"Two hours are €11 because the first hour costs €8 and the second €3, so €8 plus €3, €11"	*Integration*	*High Level*
3	Teacher: And the second hour, how much does it cost? Student: 3. Teacher: 3. So that you already pay €11, don't you? For two hours, €11.	"The second hour is €3, and two hours are €11"	*Selection*	*Low Level*
4	Teacher: Then, after 3 h? Student: 14 Teacher: 14.	"3 h are €14"	*Selection*	*Low Level*

2.4. Reliability

Two independent coders analyzed the transcriptions. Cohen's kappa statistic was used to measure interrater reliability for each of the dimensions of the analysis (varying from 0.84 to 0.99, see Table 4), so the agreement between the coders was considered appropriate.

2.5. Measures

Two different measures were taken: (a) cognitive processes promoted in the solving process by comparing the average percentage of the selection and integration cycles; (b) level of participation of teacher and students in the solving process, by comparing the average percentages of low-level and high-level participation cycles found, on the one hand, in the interaction cycles aimed at selection, and on the other, in the cycles devoted to integration.

For the analysis of the data, the IBM SPSS Statistic 25 software was used. The average percentages of cycles for each measure were compared using comparisons of proportions as simple columns (when the study is performed globally, between-task) and Chi-square statistics to verify the importance of the differences found (when the study is performed within each of the subtasks of the problem, intra-task).

3. Results

The results obtained from the analysis of the interaction of the ten teachers when solving a routine with three subtasks, each aimed at a different level of cognitive complexity (task 1), and a nonroutine problem (task 2) with their students in the classroom are outline in the following sections.

3.1. In Relation to Cognitive Processes

Regarding the routine problem, there was an overall predominance of selection over integration (66.31% vs. 33.69%; $\chi^2_{(2, 300)} = 2.017$, $p > 0.05$). The comparison of cognitive processes in each of the subtasks showed that the percentage of selection cycles was larger than that of integration, regardless of the subtask's cognitive complexity, with significant differences (S1: $\chi^2_{(1, 100)} = 10.240$, $p < 0.05$; S2: $\chi^2_{(1, 100)} = 17.640$, $p < 0.05$ and S3: $\chi^2_{(1, 100)} = 5.7600$, $p < 0.05$; Figure 2). These results did not support hypothesis 1.

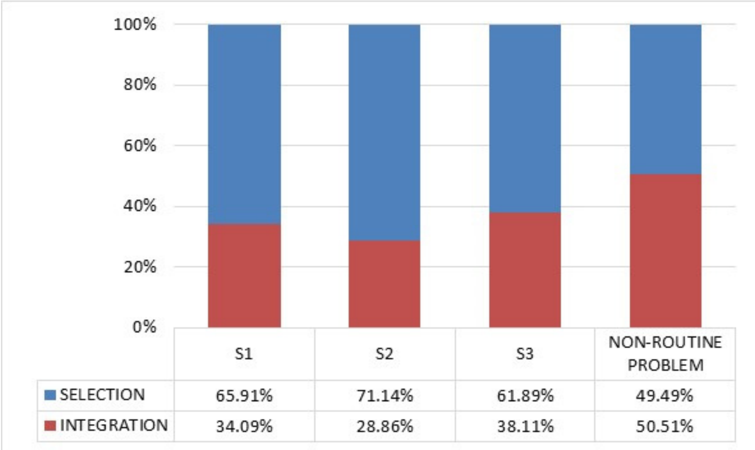

Figure 2. Total average percentages of cycles referred to cognitive processes in the routine and nonroutine problems.

Regarding the nonroutine problem, they were similar in selection and integration (49.49% vs. 50.51%; $\chi^2_{(1, 100)} = 0.040$, $p > 0.05$; Figure 2). As for the comparison of the nonroutine problem's cognitive processes with those of routine problem, taking the three subtasks together, it showed that there was a larger percentage of integration in the resolution of the nonroutine problems, with significant differences ($\chi^2_{(1, 100)} = 11.290$, $p < 0.05$). These results support hypothesis 1.

3.2. In Relation to Level of Participation

Regarding the routine problem, the global results revealed a low level of student participation (low level: 77.76% vs. high level: 22.24%). Considering the subtasks, low level prevailed over high L = level (S1: 81.06% vs. 18.94%; S2: 81.59% vs. 18.41%; S3: 73.21% vs. 26.79%; low level and high level, respectively). An analysis of the level of participation results according to cognitive processes showed that, generally, teacher participation accounted for the majority in the selection process, while there was a balance between teacher and students in the integration process, although, none of these statistical differences were significant. However these results varied considerably depending on the cognitive complexity of each subtask analyzed: low level was predominant in selection with significant differences in all three complexity levels (S1: $\chi^2_{(1, 100)} = 70.560$, $p < 0.05$; S2: $\chi^2_{(1, 100)} = 70.560$, $p < 0.05$ and S3: $\chi^2_{(1, 100)} = 57.760$, $p < 0.05$), while in integration, low level and high level tended to be equal with significant differences only in S1 (S1: $\chi^2_{(1, 100)} = 4.000$). This partially supports hypothesis 2, as student participation increased in high-level reasoning cognitive processes only in the subtask with low complexity level (Figure 3).

Regarding the overall level of participation in the nonroutine problem, there was lower teacher participation as compared to students (low level: 71.48% vs. high level: 28.52%). In relation to the two cognitive processes considered, low level prevailed over high level in selection, with significant differences, ($\chi^2_{(1, 100)} = 46.240$, $p < 0.05$), while in integration, participation was shared between teacher and students, although these differences were not significant differences (see Figure 3). These results support hypothesis 2.

However, in the comparison between level of participation in the nonroutine problem and the routine problem, considering the three sections jointly, the behavior observed was similar, and no significant differences were found. Furthermore, no significant differences were observed in levels of participation and in the cognitive complexity of the task, for each of the cognitive processes analyzed between the results of the routine problem taken jointly and the results of the nonroutine problem (selection ($\chi^2_{(3,\,400)} = 4.494$, $p > 0.05$) and integration ($\chi^2_{(3,\,400)} = 0.351$, $p > 0.05$)).

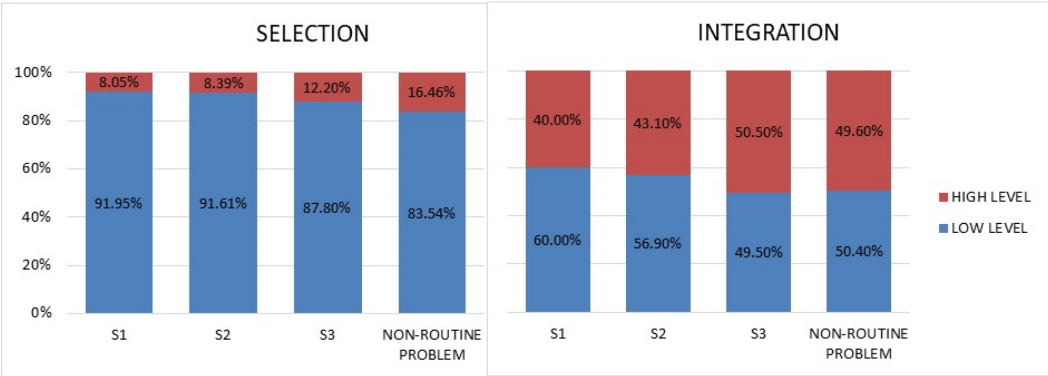

Figure 3. Total average percentages of cycles referred to level of participation considering the cognitive processes of the routine problem and those of the nonroutine problem.

4. Discussion

Different types of tasks can be approached in mathematics classrooms. In order to acquire adequate mathematical competence, students should develop skills, learn different types of mathematical contents with a variety of complexity levels [45], and use different types of mathematical tasks [46]. Among these types of tasks are mathematics problems, which may be of a routine or a nonroutine nature. To understand what takes place in the mathematics classroom, teacher–student interaction can be analyzed, for example, when they jointly solve word problems. Previous studies showed that there was little reasoning during the joint solving of routine or reworded problems in the primary education classroom [11,35,47] and a low level of student participation [3,37,38]. To explore the influence of the nature of the task on these previous results, it would be interesting to analyze the interaction that takes place in the classroom when solving a routine problem with different levels of cognitive complexity and when solving a nonroutine problem. Our study addressed the joint resolution of these problems, developed in the same classroom context and involving the same participants. The assumption was that as a task's complexity level increases, the greater the proportion of the interaction devoted to reasoning and there will be a high participation of students in the cognitive process aimed at reasoning in the resolution process.

4.1. In Relation to Process Cognitives

Regarding the routine problem, the results obtained revealed that the selection process prevailed in the three analyzed subtasks, despite the growing level of cognitive complexity involved in each of them [48]. These findings show that, even in higher cognitive complexity tasks, the behavior exhibited by the teachers participating in the study was like that of those in previous studies where teachers solved routine problems [2,3]. This means that the complexity level of each subtask had no impact on teacher and students' behavior in the interaction in terms of reasoning [3,49].

Regarding the solving of the nonroutine problem, the results revealed that teachers dedicated almost the same proportion of interaction cycles to promote selection and integration in the resolution. This could be explained by the fact that most of the tasks that are conducted in the classroom come from textbooks, where, in most cases, the solution can be reached mechanically through data selection [4,36,50]. As a result of this, teachers and students alike have internalized that all problems have a single solution that is reached by implementing arithmetic operations using all the data included in the statement, which leads teachers to develop an automated approach to a known form that students follow to reach the solution in a straightforward manner [18]. Nevertheless, when dealing with a nonroutine problem, the task involved a challenge for both students and teacher, so that it is likely why the teacher was unable to proceed in the same habitual way as when solving routine problems [2,3].

4.2. In Relation to Level of Participation

Regarding the teachers and students' level of participation in solving the routine problem, the results showed that teachers' participation in the generation of ideas was greater in the most basic process of selection but, as the cognitive complexity of the subtasks increased, so did students' participation. This suggests that students participated more when the task's complexity level was higher [7,37].

Regarding the nonroutine problem, students were more involved in the building of the main ideas of the interaction, especially in the integration cycles. These results could be explained because there is not only one path to solve a nonroutine problem. Moreover, not being able to follow a set path where data could be selected to reach the solution fast afforded students the possibility to go from being mere spectators to play a central role in the interaction with greater participation and reasoning [15], so that part of the responsibility for generating public content fell to the students. As a result, students questioned their ideas and argued their answers [29,51,52]. Nevertheless, more research is required to explore this explanation in greater depth.

In essence of the results obtained in this study, it must be noted that there must be a clear difference in the level of difficulty between routine and nonroutine problem. In the routine problem, teachers did not promote reasoning processes to a higher degree in the subtasks that entailed greater difficulty and there was only a very slight increase in student participation, with little differentiation among these aspects in the three subtasks, which could lead to the assumption that the nonroutine and challenging nature of the problem must be marked to achieve greater reasoning and participation. These findings suggest that nonroutine problem solving requires the implementation of advanced cognitive processes that go beyond data selection; because of this, students' interest and motivation may become awakened, allowing them to express and share their ideas [23,39,53] and that can encourage students to participate, share ideas, and develop creativity and critical thinking [4,39,47].

5. Conclusions

This study provides an analysis of the interaction of ten primary education teachers when solving problems together with their students in the classroom. The aim was to check whether the type of task used had an impact on the results considering the cognitive processes and the level of participation of teacher and students in the interaction. The findings of previous studies based on routine problems provide evidence of scarce reasoning and low student participation. As a result of this, the approach chosen for this research consisted of a first study using a routine problem with three subtasks with different cognitive complexity levels and of a second study using a nonroutine problem. The results revealed the following:

- Nonroutine problems pose a challenge for teachers and students as they are not used to working in the classroom with these types of problems [2,3]. When solving the nonroutine problem, reasoning was promoted to a greater extent by elaborating a more interesting interaction than when solving the routine problem [47]. In the latter, the differentiation established by the TIMSS to define its cognitive domains—knowing, applying, and reasoning—was so subtle that changes in teacher–students behavior in the solving process barely took place.
- In the solving of the nonroutine problem, students collaborated more. Furthermore, this collaboration was aimed at promoting reasoning. As has been said in the previous point, the concern or interest that this type of challenging task arouses causes the student's interest to materialize in a question aimed at the promotion of reasoning [2,3].

These results have educational implications. They suggest that, although more research would be advisable, if the aim is to indeed boost reasoning and collaborative work between teacher and students in the classroom, it would be necessary to reduce the use of automated and routine tasks in education and to increase the use of nonroutine tasks that are challenging enough to encourage students' ability to reason, integrate and associate data, take the initiative in the solving process, contribute ideas that they can justify and defend, and participate in the building of their own knowledge without the teacher leading the rhythm of the lesson [7,51,52]. This involves selecting or creating tasks that are often different from those included in textbooks, which are mostly automated or stereotyped tasks (e.g., [2,4,36]) that leave little room to reason and participate, and, under these conditions, little room to be creative and critical, as requested by PISA. One possibility is that routine problems could be used when a topic is first taught, and nonroutine problems could be used to enhance reasoning about the content that was previously explained [53]. Therefore, approaching different types of problems can promote the development of creativity and critical thinking, as well as cognitive processes and collaboration in students [20,54,55]. In addition, teachers and future teachers must be trained to perform different types of tasks that demand the development of different cognitive levels, improving their teaching practice, to help to form citizens with a critical and collaborative spirit [56,57].

The results of this work should be taken with caution as the study has certain limitations. First, the teachers who participated in the experience were from different geographical areas, with different training backgrounds and different numbers of students per classroom and, although this study was carried out in order to provide information on how these kinds of mathematics tasks influence interaction, it would be advisable to broaden the sample to increase the representativeness of the findings. Secondly, the results obtained could be further supplemented by interviewing the teachers involved in the experience to consider aspects that could have an impact on the results, such as their training or beliefs.

Hence, as a future prospect, the sample size could be increased including such interviews. Another possibility would be to analyze interactions solving different types of nonroutine problems, for example, realistic problems. Also, analyze the metacognitive processes that take place in the interactions included in this study, as well as other reading and control processes to assess whether a task's cognitive complexity may also modify the extent to which these processes are fostered. Likewise, a more in-depth analysis of the cycles started by the students could be undertaken, for example, as regards the cognitive and metacognitive processes they are targeted at, thus observing which processes are fostered when students play the main role in content building and where students' interest is focused when they come up with a question. Whether the teachers' teaching experience has an impact on the results could also be considered. Finally, to undertake a similar study with teachers of other educational levels such as secondary or university education, or even in the training of mathematics teachers, and compare the findings with the results of this study could provide relevant information.

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Appendix A

Posters for two sports clubs that rent bikes are shown below.

Mountain Bike Rentals

8 € por 1st hour
3 € for each additional hour

Roadrace Bike Rentals

10 € por 1st hour
2 € for each additional hour

A. Use the information in the posters to complete de tables.

Mountain Bike Rentals	
Hours	Cost (€)
1	8
2	11
3	
4	
5	
6	

Roadrace Bike Rentals	
Hours	Cost (€)
1	10
2	12
3	
4	
5	
6	

B. For what number of hours are the rental costs the same at the two clubs?
Answer: ______________________________

C. From which club does it cost less to rent a bike for 12 hours)
a. Mountain Bike Rentals
b. Roadrace Bike Rentals
c. They are both the same
d. It cannot be worked out

Figure A1. Routine problem with three subtasks (adapted to euro currency [56]).

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