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Francisco Sánchez and the *Quaestio de certitudine mathematicarum*: A sceptical approach

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A B S T R A C T

In this paper I analyse Francisco Sánchez's role in the *Quaestio de certitudine mathematicarum* debate. Despite some studies on the philosophical and medical scepticism of Sánchez and, his extant letter with Christopher Clavius, a participant in the debate, we have few analyses about Sánchez's position regarding the certainty of mathematics. Sánchez discussed some problems that Clavius analysed in his *Prolegomena* to propose an empirical basis for mathematics through a questioning of its certainty. I will trace the conceptual connections between Sánchez's 1589 letter to Clavius and the *Quaestio* debate, to introduce Sánchez's sceptical approach to analysing the certainty of mathematics.

Introduction

In his *Prolegomena* to the 1589 edition of Euclid's *Elementorum* (*Elements*), the German Jesuit mathematician Christopher Clavius (1538–1612) gave his position in the debate, *De quaestio de certitudine mathematicarum* ("on the question of mathematical certainty"; Clavius, 1589), a subject that provoked a response from the Spanish Jewish sceptic philosopher Francisco Sánchez (1550–1623). In this article, I examine Sánchez's ideas regarding the certainty of mathematics in light of his position in the debate, articulated in a letter to Clavius.[1] Sánchez discussed the problem of the certainty of mathematical proofs with Clavius, offering a refutation of what Clavius had argued (to be explained further below). *De Quaestio de certitudine mathematicarum* was a debate that emerged in Padua in the sixteenth and seventeenth centuries. The central concern was the possibility of causal demonstrations in mathematics: this involved the logical structure of demonstrations, the conceptual foundations of explanations, and the status of mathematical certainty.

After discovering Sánchez's letter in archives, the Jesuit philosopher Joaquín Iriarte transcribed, introduced, and published the letter in 1940. The letter has 18 paragraphs in which Sánchez critiqued the certainty of mathematics, refuted Proclus's variant,[2] commented on the classification of the sciences, and critiqued the reliability of astronomy focusing on the Copernican hypothesis (of a heliocentric system).[3] In each of these paragraphs, Sánchez highlights his sceptical position, which González (1986) claimed has its antecedents in an empirical approach to mathematics characterised by common-sense statements. Nevertheless, scholars including Buccolini (2017) and García (2020) have defended that the position of Sánchez is framed in Academic scepticism, which fits with the idea of an inductive method that brings us closer to the knowledge of the world.

The *Quaestio* debate has been analysed by philosophers and historians of science as a relevant antecedent to the emergence of modern science, as the debate advanced a set of epistemological and institutional

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[1] There are two versions of the letter. The first is presented in Latin by Iriarte (1940). The second one is an edition in Spanish by Mellizo and Cunningham (1978). Throughout this article, quoted excepts of texts in languages other than English have been translated into English by the author.

[2] This is a new interpretation presented by Proclus about the Proposition 14 of Euclid's book I.

[3] Sánchez criticizes the concentric and eccentric spheres, the epicycles, and the plurality of the heavens, which are only extra-scientific representations.

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demarcations that established the disciplinary boundaries between mathematics and natural philosophy (Carolino, 2007; De Pace, 1993; Dear, 1995; Giacobbe, 1974; Higashi, 2018; Jardine, 1988; Mancosu, 1992, 1996; Schöttler, 2012; Velilla, 2015). The historiography on the *Quaestio* has focused on the logical structure of mathematics, its causality, its certainty, and the problem of the ontology of mathematical objects. Moreover, scholars have focused on the traditional centres of knowledge production,[4] namely, England, the north of Italy, France, and the Netherlands. These emphases have led to the omission of lesser-known subjects and authors whose works could help us to understand new facets of the philosophy of mathematics in the late sixteenth and early seventeenth centuries. I propose to analyse Francisco Sánchez's position on the *Quaestio* to help bridge this gap. In particular, I will focus on his sceptical approach as presented in his letter to Clavius, in which Sánchez addressed the certitude of mathematics. The relationships between the *Quaestio* and the letter to Clavius were previously mentioned by Jardine (1979, p. 168), in an investigation of the issue of certainty in mathematics in modernity; Jardine argued that Sánchez misunderstood Clavius's position in attributing to him the view that certainty is not achievable in astronomy, given that epicycles and eccentrics are products of the mind. Clearly, Clavius did not hold such statements, but Sánchez stressed the fictitious character of epicycles and eccentrics. Mammola (2010, p. 216) also analysed Sánchez's position with respect to the controversy, underscoring Sánchez's claim that mathematics could not be considered *scientiae* because it does not meet the Aristotelian criteria of *cognoscere per causas*. Likewise, Buccolini (in Sánchez, 2011) pointed out (in his Italian translation of *Opera Philosophica*), that Sánchez's letter to Clavius offered a polemical position in the debate; Sánchez considered he could omit both syllogism and Euclidean demonstrations, because mathematics is empirical and based on the senses (Sánchez, 2011, p. LXXXVI).

This debate may be divided into three stages. In the first stage, the participants included mathematicians and natural philosophers such as Girolamo Balduino (1549–1573), Alessandro Piccolomini (1508–1579), Francesco Barozzi (1537–1604), Benito Pereyra (1535–1610), and João Delgado (1553–1612). In this early stage, we find defenders of the causal status of mathematics, such as Barozzi and Delgado, but also critics of this status, such as Piccolomini, Pereyra, and Pietro *Catena* (1501–1576). In the intermediate stage further scholars participated, including Martynas Smigleckis (1564–1618), who followed Piccolomini's approach, Clavius, Pierre Gassendi (1592–1655), and Giuseppe Biancani (1566–1624), who followed Barozzi's approach. Finally, in a late stage we have scholars such as Thomas Hobbes (1588–1679), John Wallis (1616–1703), and Isaac Barrow (1630–1677). The participants in the debate had varying positions on causality in mathematics. Indeed, as Higashi (2018) has shown, even the Jesuit philosophers took divergent, even opposite, points of view on the *Quaestio.*

Sánchez had Clavius's *Elements* as a reference for the debate, and in the letter, Sánchez aims at Clavius's *Prolegomena* to show that we can doubt mathematical proofs. Clavius argued that mathematical proof leaves no room for doubt by highlighting the anti-sceptical function of mathematics, whose proofs are superior to those of physics and also of metaphysics. This means that, for Clavius, mathematical proofs have maximal certainty. Although Sánchez does not engage directly with Piccolomini and Barozzi, he quotes in his letter the technical concepts and geometrical demonstrations, which Clavius used in his intervention in the controversy. One example of this is his use of the method, "*scire per causas* or *demonstratio propter quid*," as it was known in the debate,[5] derived from Aristotle's *Posterior Analytics* (Aristotle, 2002, bk. I, 11, 71a 12). Sánchez's analysis of Clavius's position took an empirical approach, arguing that mathematics, like medical practice, has an empirical basis that requires senses and artifacts. Therefore, although Sánchez did not participate in the controversy explicitly, he nevertheless contributed to it through his analysis of Clavius's response, and by advocating for an understanding of mathematical knowledge through a sceptical approach.

I argue that Sánchez's considerations in this regard refute crucial features of the *Quaestio*, such as the certainty of mathematical demonstrations linked to causal demonstrations previously discussed in the Middle Ages and known as *demonstratio quia, demonstratio propter quid,* and *demonstratio potissima.*[6] Sánchez criticized causal mathematical proofs and the possibility of absolute knowledge in his response to Clavius. To demonstrate this, I proceed in three sections. In my first section, I present the characteristics of the *Quaestio*'s dissemination in Europe to shed light on the problems and questions of the debate that Sánchez also analysed. I will focus on Clavius's *Prolegomena* (1589) because it was a crucial text to understand the overall controversy and was used by Sánchez as the basis for the views he articulated in his letter to Clavius. Then, in the next section I analyse Sánchez's position—as given in his letter—within the *Quaestio* debate. In my conclusion, I suggest how my analysis offers a new approach to analysing the certainty of mathematics in the *Quaestio* debate.

Dissemination of the *Quaestio* de certitudine mathematicarum in Europe

The *Quaestio* debate over the certitude of mathematics was picked up by members of the Catholic order, the Society of Jesus (the Jesuits), and it impacted university curricula throughout Europe.[7] As Harris (1996) argued, the Jesuit networks provided the infrastructure "for the Society's entry into so many branches of early modern science" (p. 289). Likewise, these networks gathered natural knowledge from different parts of the world and universities for its dissemination. In this sense, the Jesuit order enabled networks around Europe and other geographical contexts (see Fig. 1); and together with the scholars and universities, these were the principal vehicles for the propagation of the *Quaestio* controversy. As we can see on the map in Fig. 1, the Jesuit schools and universities were concentrated in the centre and north of Europe, but the school founded in Messina, Sicily, was the model for this dissemination in other regions. Likewise, although we observe a few schools in Spain, it is important to highlight that the leadership of the Jesuit order was under Spain's direction in 1547, and places like Naples were under the Spanish government. Messina promoted the creation of university

[4] I mean by traditional centres of knowledge production those that the historiography about the Scientific Revolution has highlighted. They are the base to build an epistemological structure based on, for instance, the mathematization of nature, which is useful to validate what a scientific practice is. This approach does not allow us to observe other practices and contexts, such as the Spanish Monarchy and the Americas. Indeed, Cañizares-Esguerra (2017) points out that "the Spanish Monarchy was forged in a very different system, one of rewards and legislation in which most activities were transacted through one-on-one epistolary correspondence and intimate transference of information in translation workshops" (p. 1).

[5] For instance, Pereyra argued that mathematics does not follow the criteria *cognoscere per causas* because science knows the thing through to know the cause for which the thing is "Scire est rem per caussam cognoscere propter quam res est" (Pereyra, 1585, p. Book I, chap. 12, p. 26). Sánchez picked up this syntagma in his letter, as I point out in the next section of this article.

[6] The *demonstratio quia* proceeds from known effects to causes and the *demonstratio propter quid* explains the effects through causes. Averroes (1489) incorporated a third demonstration so-called "*demonstratio potissima"* which was considered a perfect type of demonstration and, as Mancosu (1996) points out, a scientific syllogism. This demonstration explains both causes and effects (*simul et quia et propter quid*).

[7] As Dear (1995) explained, one of the Jesuit's aims in this connection was to keep the separation between natural philosophy and mathematics. The separation was useful "to protect the use of artificial contrivance in the mathematical sciences from the criticism that, being unnatural, it could not contribute to knowledge of nature" (Dear, 1995, p. 161).

Fig. 1. Map of Jesuit schools and universities, by Paul F. Grendler, in *Jesuit Schools and Universities in Europe 1548–1773*, Brill's Research Perspectives in Jesuit Studies (Leiden: Brill, 2018), pp. 2–3, available https://doi.org/10.1163/9789004391123_002 (accessed 12 December 2022). License: CC BY-NC-ND 4.0.

schools across Europe and highlighted the academic network before the consolidation of systems of public education. Messina is particularly relevant because its curriculum enabled the creation of the *Ratio Studiorum* of 1599.[8] Moreover, the educational treatises written by Jerónimo Nadal (1507–1580) and Pieter Kanis (1521–1597) influenced the Jesuits' subsequent curricula (Grendler, 2019).

On the other hand, the Society of Jesus promoted the teaching of mathematics and astronomy, and as Udías (2014, p. v) pointed out, this interest in mathematical disciplines coincided with the origin of modern science, with some Jesuit professors engaging with many of its key figures, including Galileo, Kepler, Huygens, and Newton. Additionally, most activities relating to the publication of natural knowledge were placed in "the largest Jesuit universities and colleges in provincial capitals of the Italian, French, and German assistancies [regional provinces]" (Harris, 1999, p. 224). Indeed, as we can see on the map, the Society had established a compound of one form or another materialised in some 800 towns around the world. These universities and colleges placed around Europe enabled the production and circulation of natural knowledge with the exchange of personnel, texts, and natural objects, triggering "a sort of circulation between local sites and distributed practices" (Harris, 1999, p. 226). In this regard, I adopt the notion of circulation of knowledge as a historiographical category that enables analyses of the dissemination of knowledge taking into consideration political and economic positions, the uses and transformations of the ideas and artifacts, as well as the growth of disciplines and their transformations, rendering dissemination as much more than the simple mobility, appropriation, or reproduction of knowledge (Markovits et al., 2006; Östling, Heidenblad, Sandmo, Hammar, & Nordberg, 2018).

In this connection, the *Quaestio* had a long dissemination on the European continent during the sixteenth and seventeenth centuries. Several Jesuit scholars moved between different territories, which allowed the circulation of ideas and their transformations. Although there are different historical discussions of the *Quaestio*,[9] here I focus on an analysis of Clavius's position, as it was key not only to the general polemic but also to the problem that Sánchez addressed in his letter. Clavius focused on both epistemological and curricular topics of the debate, the latter of relevance to his design of curriculum focusing on mathematics, astronomy, and geography. Likewise, Clavius considered the epistemological dimensions in relation to the utility of mathematics for historians, political leaders, physicists, and theologians (Clavius, 2002a, pp. 459–60). In fact, Clavius's approach considered the interaction between mathematics and natural philosophy because the

[8] The study of the *Ratio Studiorum* allows us to approach the process of education and specialization in mathematics gradually implemented within the Company, which, as we know, was at the centre of several debates on the importance of mathematics and geometry. Clavius' participation in these debates and the *Ratio Studiorum* show, moreover, his epistemological approach to the certainty of mathematics and its importance in Jesuit curricula (Romano, 1999).

[9] See Giacobbe (1974); Jardine (1988); Mancosu (1992); De Pace (1993); Dear (1995); Mancosu (1996); Romano (1999); Carolino (2007); Schöttler (2012); Velilla (2015; 2018); and Higashi (2018).

understanding of natural phenomena like "heavenly bodies (orbes), the ebb and flow of the sea, winds, comets, the rainbow, the halo [of the sun and moon], and other meteorological phenomena oppositions" (Clavius, 2002b, p. 466) require knowledge of mathematics.

As is well known, Clavius was a key figure within the Jesuit order, and he addressed several debates on the status and role of mathematics with other members of the order in the years Sánchez was in Rome. As Romano (1999, p. 135) observes, a crucial text containing Clavius's epistemological position is the *Prolegomena.* In particular, Clavius defended in this text "*nobilitas atque praestantia scientiarum mathematicarum*" (the nobility and excellence of the mathematical sciences) and "*utilitates variae mathematicarum disciplinarum* (the various uses of mathematical disciplines) (Clavius, 1589, pp. 9v–11v). Through such statements Clavius imbued mathematics with the highest level of certainty, applicable in studies independent of sensible matter, in contrast to other, lower-level sciences. Thus, mathematics can demonstrate the effects by which things are produced without any doubt. This certainty allows using mathematics in other fields to obtain reliable knowledge. Clavius illustrated this through Euclid's theorems, which "to this day retain the same truth, the certainty of their objects and strength in their demonstrations" (Clavius, 1589, p. 10r). Clavius considered the problem of superposition (which he discussed with Peletier and which Sánchez also addressed in his letter): "a legitimate way of proving the equality of two figures which have the necessary parts respectively equal, or, in other words, to serve as an axiom of congruence" (Euclid, 1956, p. 225). Superposition is introduced in proposition I. 4 of Euclid's *Elements*:

> If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend (Euclid, 1956, p. 247).

As Euclid pointed out, things that coincide with each other are equal to each other. It means that the equality of various parts of the triangle may be deduced from their coincidence. Peletier (1557) argued that Euclid's demonstration is not mathematical but mechanical,[10] and therefore lacks the dignity (*dignitate*) of geometrical methods. In response, Clavius (1589, III. 110–16) defended the mathematical nature of Euclid's principle of superposition, arguing that it possesses the structure of legitimate mathematical demonstrations. The debate between Clavius and Peletier is crucial because superposition, as Axworthy (2018, p. 34) argued, led mathematicians to consider the requirements of geometrical methods and to define the nature of geometrical knowledge and its practice. As will be discussed in section 3, Sánchez examined these topics (superposition, contact angle, and Clavius's controversy with Peletier) in his letter to Clavius.

The stay of Sánchez in Rome coincided with the discussions about the status and role of mathematics between Clavius and other members of the Jesuit order. As Mammola (2010, p. 218) has shown, Sánchez's sojourn in Italy influenced his education and sceptical attitude, and his position in the letter to Clavius could have reflected his familiarity with the epistemological debates in Rome. For instance, by 1570, Clavius found other opponents to the certainty of mathematics within the order such as Benito Pereyra, who held that mathematics does not provide causal demonstrations (Pereyra, 1585). This position challenged the inclusion of mathematics in the Jesuit curriculum. In defence, members of the order argued for the teaching of mathematics, as materialised in the *Ratio Studiorum*; and in this context, Clavius's *Prolegomena* fundamentally served to theoretically uphold the importance of mathematics in the curriculum of the Jesuit order (Romano, 1999, p. 146).

The dissemination of the question over the certitude of mathematics within the Jesuit order can help us to understand how Sánchez encountered the problem and what sources he used for his analysis. In this context, while Sánchez was in the Collège de Guyenne he received training based on Scholastics and Aristotelian rhetoric. His training included Aristotelian logic and natural philosophy, specifically, the *Physica* and on *De Caelo*—both required lectures in students' first years of study in the college (Pérez, 2020). However, at this time the colleges and universities were undertaking a critical review of Aristotelian thought, with a particular focus on the Aristotelian medieval commentaries. In this sense, Lohr (1982, p. 81) claimed that the reception of Aristotelian science in the Latin West is related to three stages. First, Boethius translated Aristotle's treatises on logic and made adaptations of various other works on logic and rhetoric. Second, a gradual translation of the entire corpus of Aristotle's works occurred in the twelfth century. In this stage, the reception of Aristotle was focused not only on philosophy, medicine, astrology, and natural science from ancient Greece, but also on past and contemporary Judaism and Islam. Third, scholars produced new editions of the Greek texts, new Latin and vernacular translations and commentaries, and Latin versions of hitherto untranslated commentaries of Averroes. These stages made it possible to highlight Aristotle's positions, as well as the different interpretations made of his works (Schmitt, 2004, p. 44). The epistemological topics about mathematical demonstrations, their logical structure, and their certainty were at the core of the critique. In this context, we can understand Sánchez's criticism of the Aristotelian approach. Furthermore, Sánchez's stay at La Sapienza allowed him to focus on practical topics based on Galenic medicine, which led him to criticise the Aristotelian logic and natural philosophy of scholastics (Sánchez, 1988). This context sheds light on Sánchez's adoption of the problem of the certitude of mathematics in his own thought. Although he is not a direct participant in the controversy, the topics present in his letter to Clavius are concerned with the same problem of mathematical certainty. Thus, I suggest that Sánchez's sceptical position with respect to certainty, as articulated in his letter to Clavius, be considered in connection with the debate and indeed as a component of it.

Francisco Sánchez "*the sceptic*" and the problem of mathematical certainty

Francisco Sánchez (1551–1623; Fig. 2), known as "the sceptic," was a philosopher and physician born in Tuy, Spain.[11] Among his works, we have a record of the following: *De multum nobili et prima universali scentia. Quod nihil scitur* (1581); *Opera medica. His iuncti sunt tractatus quidam philosophici non insubtiles* (1636); and *Tractatus Philosophici* (1636), which include the following works: *De longitudine et brevitate vitae*; *In librum Aristotelis Physiognomicon commentarius*; *De divinatione per somnum ad Aristotelem*; and *Quod nihil scitur*.

[10] It is important to note that Newton later distinguished practical mechanics from rational mechanics. The deductive consequences of rational mechanics were empirically important because its primary concepts and laws were situated in natural reality (Pulte, 2012).

[11] Sánchez's birthplace is a topic of discussion for some Spanish and Portuguese historians. Cazac (1903) presented in 1903 a document in which Sánchez noted that he had been born in Tuy. This position was supported by Álvarez (1964), who was a defender of the Spanish nationality of Sánchez. On the other hand, Machado (1920) presented a baptismal certificate in which it mentioned that Sánchez was baptised in Braga, and in this connection, Sergio Da Silva Pinto (1946) defended that Sánchez was Portuguese (Orden, 2012). There seems to be an agreement in recent research about Sánchez and his Spanish nationality (Bermúdez, 2019; García, 2020; Orden, 2012). Despite these discussions, Sánchez developed his career in France and Italy. He emigrated to France to study at the Collège de Guyenne in 1562, and around 1571 he moved to Rome to study at La Sapienza, where according to Limbrick (Sánchez, 1988), he had contact with Cardano and Clavius. He then returned to France to teach some lectures and to complete his doctorate in Montpellier. Sánchez died in 1623 in Toulouse (Orden, 2012; Sánchez, 1988).

Fig. 2. Portrait of Francisco Sánchez, c. 1600, engraved by Michael Lasne. Image from the holdings of the Biblioteca Nacional de España, https://bdh-rd.bne.es/viewer.vm?id=0000037136. ©Biblioteca Nacional de España. Used by permission.

Scholars have shown that was influenced by two sceptical approaches, namely, the Academic and Pyrrhonian (Buccolini, 2017; Caluori, 2007, 2018; García, 2020; Orden, 2012; Popkin, 1979). His debt to the Academics is confirmed by his letter to Clavius, in which Sánchez presents the inconsistency of Proclus's demonstration in order to prove that we do not have certainty in mathematics.[12] Moreover, as Buccolini (2017) argued, Sánchez also drew on "conjectural empiricism" to defend (in his letter to Clavius) that both mathematics and medicine are supported by empirical knowledge and that empirical cases can resolve de facto controversies by avoiding metaphysical arguments.[13]

Likewise, as Limbrick (Sánchez, 1988) pointed out, Sánchez wrote his letter to Clavius while rewriting the *Quod nihil scitur* manuscript. Accordingly, the sceptical thesis regarding the impossibility of knowledge by abstraction presented in *Quod nihil scitur* is consistent with the position he articulated to Clavius, in which the sensory knowledge applied to mathematics could offer partially reliable knowledge. In addition, Sánchez debated the Aristotelian notion of causality in his *Quod nihil scitur*, indeed relevant to critiques of causality in the *Quaestio* debate. In his interrogation of Aristotle, Sánchez questioned the status of knowledge based on material, efficient, formal, and final causes by considering the difficulty of understanding a cause and the complexity of establishing causal relations. For instance, Sánchez asked:

> Is it necessary to know all the causes to know one thing? For example, the efficient cause, in any way; what does my father contribute to my knowledge of myself? What does the final cause also contribute? (Sánchez, 1955, pp. 41, 20–25)[14]

Sánchez criticised causal knowledge because it is insufficient to know natural phenomena. In particular, he questioned the efficient and final cause because if any cause explains an effect, then that cause should be explained by another cause, and so forth, which would involve an undefined series of causal relations (Orden, 2003).

The letter to Clavius is probably Sánchez's second in a longer correspondence between the two scholars, in which the central topic is Proclus's interpretation of Proposition 14 of Euclid's first book. Based on the contents of Sánchez's second letter, it seems Clavius's reply to the first letter did not was not convincing. Now, I will consider Sánchez's position in this letter with respect to the *Quaestio* debate.

By the time of the letter's date, 1589, we have the following publications about the *Quaestio*:

- Piccolomini, *Commentarium de Certitudine Mathematicarum Disciplinarum* (1547);
- Barozzi, *Opusculum, in quo una Oratio, et duas quaestiones: altera de certitudine et altera de medietate mathematicarum Continentur* (1560);
- Pereyra, *De communibus omnium rerum naturalium principiis et affectionibus* (1585);
- Clavius, sections on mathematics from the various editions of the *Ratio Studiorum* (1586);
- Clavius, *Euclidis elementorum libri XV: Accessit XVI. De solidorum regularium comparatione: Omnes perspicuis demonstrationibus accuratisque scholijs illustrati* (1589).

As we can see, the letter was written in the first stage of the debate when the certainty of mathematics was analysed by Piccolomini, Barozzi, Pereyra, and Clavius. Moreover, the context of the debate with Clavius, the internal clash within the Jesuit order on the statute, and the role of mathematics coincided with the years Sánchez was in Rome. I now turn to Sánchez's position on the role of mathematics in natural philosophy as given in his letter to Clavius.

First, there is an argument connected with the core of the debate, namely, the causality of mathematics. Despite Sánchez arguing that mathematics is not a scientific discipline, he accepted that mathematics could achieve maximum certainty:

> I would not dare to say that mathematics is a scientific discipline because it [mathematics] needs more of the senses than reason. However, mathematics can achieve a higher degree of certainty if there is anything in this world that is true (qtd. in Mellizo and Cunnigham, 1978, p. 395).

For Sánchez, true science focuses on the knowledge of God. It is a reason why mathematics is not a scientific discipline. Indeed, geometrical procedures like the comparison of angles with angles, figures with figures, or quantitative analysis are intellectual but not scientific works.[15] One of the problems of the *Quaestio*—derived from the classification of sciences—was the relationship between mathematics, sense, and reason. In this connection, Sánchez proposed that mathematics needs the senses to explain its objects. Precisely, for Sánchez, error in mathematics arises when mathematicians do not observe the elements and principles in nature. This position is related to the mathematical demonstration, which was a central topic of the debate in Padua. In Sánchez's case, the demonstration needs to be accompanied by an argument and a proof because only with these elements can a correct, albeit imperfect (in terms of absolute truth), judgment be made in mathematics. Indeed, Piccolomini rejected the causality of mathematics based on its logical structure, while Sánchez used a sceptical argument:

> I began my approach to mathematics by hesitating before studying it and being distrustful and fearful of being a victim of the deception that could provoke me from anywhere. That distrust served me well. Without it, I would have fallen into the many traps that undermine the field of Mathematics, traps that are not as many or as obvious as those found in Physics and Metaphysics but which, for that very reason, are more dangerous and more difficult to avoid (qtd. in Mellizo & Cunningham, 1978, p. 395).

Sánchez's position about the demonstration in mathematics contrasts with Piccolomini's and Barozzi's approaches because Sánchez was influenced by scepticism, which defended the (non-absolute) importance of experience in agriculture, navigation, and medicine (Caluori, 2007). Namely, Sánchez's position is articulated in terms of scepticism, applied not only to mathematics but to knowledge in general. Thus, his approach supersedes epistemological interest and shifts toward practical interests. Indeed, the importance of scepticism was well-known in the French context. This topic is relevant to analyse the characteristics in which the problem about the certitude of mathematics was received, mainly if we keep in mind that Sánchez studied in this context, among others, the French edition of Sextus Empiricus (1569) used by Gentian

[12] Sánchez signed this letter under the pseudonymous "Carneades philosophus," a representative of Academic scepticism, and according to Limbrick (Sánchez, 1988), the fact that Sánchez had signed his letter with Carneades's name placed him as an Academic sceptic. Probably, Sánchez's familiarity with Carneades was through Cicero's *Academic Questions* in which Carneades was presented as a critic of the certainty of mathematics (García, 2020). Moreover, many of Sánchez's claims could be taken from ancient sources, especially Sextus Empiricus. However, there is no conclusive evidence of a reading of Sextus by Sánchez. Authors such as Senchet (1904), Villey-Desmeserets (1968), Carvalho (1981), Popkin (2003), Bermúdez (2006), have traced the influence of Sextus on Sánchez.

[13] Sánchez argues this position in his *Opera medica* (1636, p. 589).

[14] I am using the bilingual edition entitled *Tratados Filosóficos* which picks up the translation of the *Quod nihil scitur* from Latin to Portuguese by Vasconcelos and Pinto (1955). Likewise, I contrast this edition with Limbrick's English translation (Sánchez, 1988), and M. A. Sánchez's Spanish translation (Sánchez, 2020).

[15] According to Limbrick (Sánchez, 1988), Sánchez criticised the Aristotelian system regarding knowledge and supported a scientific methodology based on empirical observation and experimentation, which is a result of his training in Galenic medicine.

Hervet in his lectures at the Collège de Guyenne (Bermúdez, 2019; González, 1986). Of course, Sánchez's criticism of mathematics attacked epistemological aspects such as its demonstrations and its certainty, but with fields such as agriculture, navigation, and medicine in mind, Sánchez linked mathematics with experience, positioning it as a set of practical method that could achieve better performance in problem-solving.

In this light, we can place Sánchez as a critic of the causality of mathematics. Moreover, he used the argument regarding the division of sciences and the technical syntagma "*cognoscere per causas*," which is the basis of the debate:

> True science consists, in the first place, in knowing God, and then in knowing his servant Nature, outside and inside, as it is usually said; or, as Aristotle said, in knowing reality by their causes. However, to compare sides with sides, angles with angles, figures with figures, the whole with the parts, and proportions with proportions, and to inscribe some figures in others and examine the various properties of this or that quantity is absolutely intelligent and acute work, but not scientific (qtd. in Mellizo & Cunningham, 1978, p. 396).

In this fragment, Sánchez confirms that true science is related to theology, and its aim is the knowledge of God. This position follows his approach in the *Quod nihil scitur*. By contrast, mathematics is considered intellectual work. If we contrast this position with Piccolomini's approach, then we find that for Sánchez, mathematics could help us to improve our cognition and serve the purpose of understanding God. Nevertheless, there is an important aspect connected with the relationship between mathematics and religion in Sánchez's position:

> All that refers to Astronomy—concentric and eccentric spheres, epicycles, augmentations, the agitation, the plurality of heavens multiplying to minuteness, and other similar things—are representations which you, distinguished man, have admitted, and which we always understood as such, though necessary and useful for our observations, for showing phenomena, and for preserving the economy and organization of the Church (qtd. in Mellizo & Cunningham, 1978, p. 396).

It can be seen in this fragment that astronomy could help us to understand some natural phenomena, but it is more effective in tasks like the calendar reform,[16] which was difficult to understand—according to Sánchez—because Copernicus was in an error when he proposed the earth's motion: "Copernicus tangled it up with splendid and absolute certainty by establishing that the earth moved, and the heavens remained motionless" (qtd. in Mellizo & Cunningham, 1978, p. 396). We do not have more details regarding Sánchez's criticism of astronomy in his 1589 letter to Clavius. Nevertheless, Sánchez's fuller position regarding astronomy is given in his earlier work *Carmen de Cometa* (1578), in which Copernicanism is considered akin to fiction. Indeed, both in *Carmen del Comenta* and in his letter to Clavius, Sánchez argued for the uselessness of many epicycles and spheres to explain the motion of celestial bodies. Accordingly, while Sánchez accepted the use of astronomy to understand cosmological problems, he did not classify astronomy as a scientific discipline.

An important aspect of this letter is the use of Euclid's *Elements* to analyse the demonstrative character of mathematics, which allows us to observe the position of Sánchez in the *Quaestio*, not only for his correspondence with Clavius about the central topic of the debate, but also because Sánchez used the concepts and problems of the *Quaestio* as it happened in Padua. Let us look at, for instance, the reconstruction of an equilateral triangle by Sánchez:

> If I wanted to construct an equilateral triangle starting from a given segment AB, an open compass would suffice; after having measured the length of the given segment, I would draw a circle, with the centre at A, passing through point B; and another, with centre at B, to C (the same could be done starting from the other end), or, with centre at C, passing through A, until I found the angle ACB. And once this is achieved, which is extremely easy, even on a first attempt, I would draw the triangle. But I would not rely on that process to prove that the triangle is equilateral, even if the use of the compass proved it to me. It would be more comfortable to believe in the equality of the three sides than in the composition of two circles and then of the sides (qtd. in Mellizo & Cunningham, 1978, p. 400; see Fig. 3).

As we can see, we have new elements in Sánchez's position that are not present in the debates of Piccolomini or Barozzi about causal demonstrations in mathematics, namely, the compass and the ruler.[17] The use of these instruments is relevant because one of the ways to understand the relationship between mathematics and natural philosophy is through the intellectual and material manipulation of the objects (Bertoloni-Meli, 2006). In this light, these implements are important means for establishing evidence in a mathematical demonstration, through the required use of the senses. Sánchez criticized the central topic of the *Quaestio* regarding demonstration in mathematics, which does not allow immediate knowledge:

> But this is what I think: we can make measurements with a compass and a ruler in a way accessible to our eyes; let us do it simply without resorting to more complicated demonstrations unless the senses cannot come to our aid (qtd. in Mellizo & Cunningham, 1978, p. 400).

Specifically, we need immediate knowledge about the object, but the mathematical demonstrations do not allow this because it increases the difficulty of relating the subject and the object, whereas we can see the quantities without problems with a compass and ruler. Precisely, a central point defended by Sánchez in his letter to Clavius is the

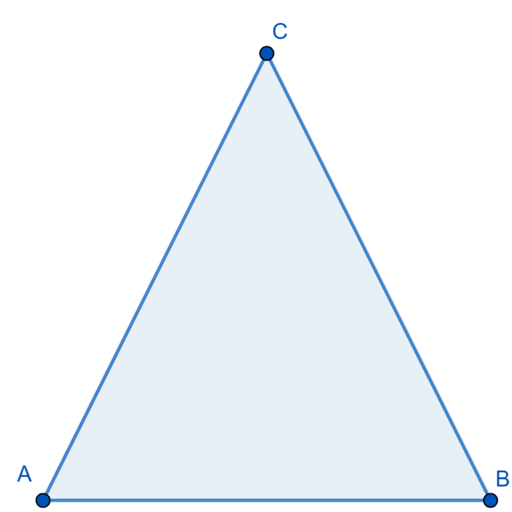

Fig. 3. Reconstruction of an equilateral triangle by Sánchez. Image drawn by the author with GeoGebra, 2022.

[16] Here, Sánchez refers to the Gregorian Calendar, whose commission was led by Clavius.

[17] Here, I would like to highlight that Piccolomini and other scholars of the *Quaestio* worked in mechanics. This means that although the debate focused on the logical structure of mathematics and its certainty because of its subordination to natural philosophy, there are a set of mechanical practices that could help us to understand the disciplinary transformations not only from the mathematization of natural philosophy but also from its physicalization.

incorporation of the senses in geometry. In other words, geometry is a scientific discipline because of the use of compass and ruler and the relation of these artifacts with senses.

Let us observe the case of Proclus's commentary to proposition 14 of the first book of Euclid mentioned by Sánchez. The Euclidean proposition is the following:

> If with any straight line, and at a point on it, two straight lines not lying on the same side make the sum of the adjacent angles equal to two right angles, then the two straight lines are in a straight line with one another (Euclid, 1956, p. 276; Fig. 4).

According to Proclus:

> In the construction he uses one postulate, the second (that a finite straight line can be extended in a straight line), just as in the proof he uses the preceding theorem and two axioms (things equal to the same thing are equal to one another, and if equals be subtracted from equals the remainders are equal); and for the reduction to impossibility he uses the axiom that the whole is greater than the part, for when the one common angle had been subtracted, the whole was equal to the part, which is impossible (Proclus, 1970, p. 231).

For Proclus, if we have a figure in which the straight line BE and BA touch the straight line CD in point B of the same side forming the angles CBA and CBE, then CBA and CBE add two right angles. Nevertheless, the straight lines BA and BE are not on the same straight line (Proclus, 1970, p. 231; Fig. 5).

Sánchez claims that Proclus was in error when he proposed that the angles CBA and CBE are equal to two right angles:

> However, all this would be unimportant if Proclus's proof were valid. But it turns out that it is not, and that it is even false—as I was making you see—when he says that the angles ACF and ACE are equal to two right angles. To which you answer me that these angles must be taken separately, and not insofar as the one is part of the other; and that, in that sense, both would add up to two right angles (qtd. in Mellizo & Cunningham, 1978, p. 401; Fig. 6).

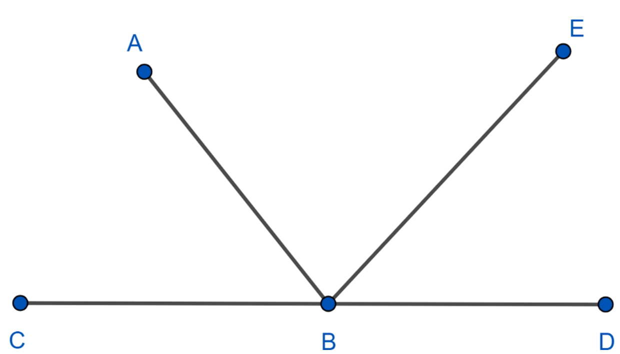

Fig. 4. Reconstruction of Euclid's Proposition 14. Image drawn by the author with GeoGebra, 2022.

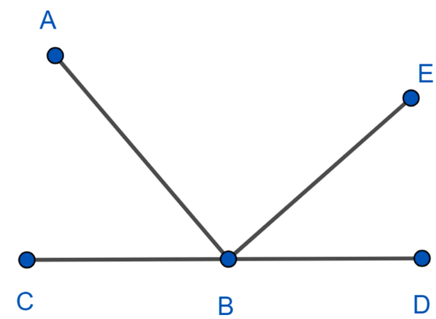

Fig. 5. Reconstruction of Proclus' interpretation of Euclid's Proposition 14. Image drawn by the author with GeoGebra, 2022.

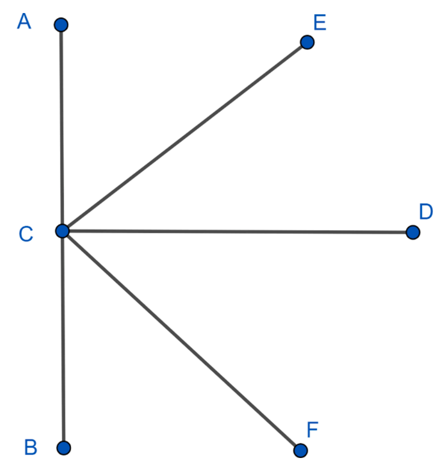

Fig. 6. Geometric reconstruction of Sánchez to refute Proclus in his reply to Clavius. Image drawn by the author with GeoGebra, 2022.

If we accepted the Proclus variant in which ACF is equivalent to the right angle and a half, then we should assume the entire amount of the angle which is included between AC and CF. According to Sánchez, for Proclus, the angles ACE, ECD, and DCF make up the angle ACF, and when he sums the angles ACF and ACE, he takes ACE separately as if they were not part of ACF (Mellizo & Cunningham, 1978).

Likewise, Sánchez presented Clavius's arguments based on everyday situations to refute Proclus[18]:

> if someone lends you twenty, and then you return fifteen—following Proclus's syllogism—arguing that the remaining five are included in the fifteen, you will not accept it. Likewise, if you buy a piece of cloth from a merchant to make you a toga (represented by segment A-B) and he gave you a quantity of cloth CD, adding to CD, CE to make you a cloth equal to AB, would not the toga be too short? (qtd. in Mellizo & Cunningham, 1978, p. 402; Fig. 7).

These arguments led Sánchez to criticise abstract demonstrations in

[18] This criticism of Proclus is probably related to the so-called "sceptical tropes." However, Sánchez did not use the tropes to avoid the judgments of authority he criticised (Bermúdez, 2019). Furthermore, the curricular policies in the universities incorporated the idea of obtaining knowledge through causally linked arguments, but this is the causal definition of knowledge that Sánchez seeks to criticise. However, this possibility is not closed because, according to Bermúdez (2014), there is a remarkable sceptical tendency in Sánchez; but he did not use the tropes explicitly because "it needed a long process of assimilation, as well as the appearance of a new generation of thinkers" (p. 7) who used the sceptical techniques.

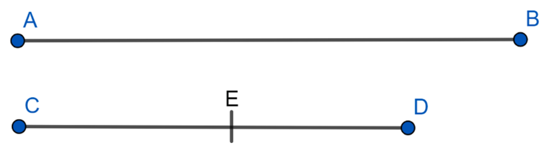

Fig. 7. Geometric segment of Sánchez in his refutation to Proclus. Image drawn by the author with GeoGebra, 2022.

mathematics and to instead proceed in geometry with an empirical approach. Thus, for Sánchez, geometry can achieve certainty because it is supported by the data provided by the senses, which have more evidence than reasoning, as Sánchez demonstrated through his critique of Proclus. Indeed, we may contrast this position with the arguments presented in *Quod nihil scitur*, where Sánchez argued that we can obtain knowledge from senses because they are related to immediate knowledge.

To conclude, Sánchez's correspondence with Clavius shows us a new way to analyse the problem of the certitude of mathematics through a sceptical approach. Sánchez's critique shows that scepticism raised doubts about not only knowledge in general but also about mathematics. As Sánchez claimed, we can avoid grandiloquent arguments in mathematics by considering our experience and artifacts, as in the case of Euclid's demonstrations of the equilateral triangle, where the use of the compass and ruler, according to Sánchez, is enough to know that it is indeed an equilateral triangle (qtd. in Mellizo & Cunningham, 1978, p. 400). Of course, Sánchez's position is complex because it involves arguing that the syllogism of of the first figure is unrelated to the certainty of mathematics.[19] Nevertheless, it is an approach that could help us to understand the different practices according to each context, in particular, the relationship between scepticism and the certainty of mathematics.

Conclusion

In this article, I have presented the central topics of the *Quaestio de certitudine mathematicarum* focused on the problems addressed by Christopher Clavius (1589) in his *Prolegomena,* which Sánchez discussed in his letter. I have introduced two approaches in this debate, namely, the critics and defenders of the causality of mathematics. Among the critics, Francisco Sánchez pointed out that mathematics does not achieve a perfect demonstration regarding causes and effects, which is a requirement to be a science, according to the participants in the debate. Nevertheless, like Piccolomini and Barozzi, Sánchez argued that mathematics could help us to improve our cognition and serve the purpose of understanding God. I have highlighted that Sánchez assigned non-absolute certainty to mathematics, and this certainty is derived from the uses of the senses and instruments like the compass and the ruler. Although the use of such artifacts is not an innovative aspect in early modern science, their appearance in Sánchez's sceptical arguments extends our knowledge about the history of mathematics and its certainty. In this light, scepticism is presented as a relevant source for suspending judgment about the certainty of mathematics. Sánchez presented this position in his critique of Proclus's interpretation of Proposition I. 14 of Euclid's *Elements* by considering the complexity of these demonstrations through abstraction. In this way, Sánchez defended the impossibility of causal demonstrations and connected mathematics with the senses. Thus, we have a sceptical position on the question of the certitude of mathematics. This analysis is one component within a larger comparative study of the relationship between scepticism and the *Quaestio* in the sixteenth and seventeenth centuries.[20]

[19] This position was previously defended by Piccolomini in his *Commentarium*. See, Piccolomini (1547, p. 69).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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[20] This research is part of my doctoral project, in progress, entitled "Science and colonialism: The mathematization of Natural Philosophy in the policies regarding the Spanish colonization of the New World.".

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