IMPROVING ENGINEERING THERMODYNAMICS LEARNING WITH MATHEMATICA

María Jesús Santos [1,3]*; Alejandro Medina [1,3]; José Miguel M. Roco[1,3]; Araceli Queiruga-Dios[2,3]

[1] Department of Applied Physics, Faculty of Science, University of Salamanca

[2] Department of Applied Mathematics, School of Industrial Engineering, University of Salamanca

[3] Research Institute on Fundamental Physics and Mathematics, University of Salamanca

ABSTRACT

Sophomore students from the Chemical Engineering Degree at the University of Salamanca, are involved in a Mathematics course during the first semester and in an Engineering Thermodynamics course during the second one. When they participate in the latter they are already familiar with mathematical software to solve numerical methods problems, including non-linear equations, interpolation or differential equations. On one hand, we present in this paper some of the materials elaborated in both courses with the Wolfram Mathematica package, and on the other, the didactic organization of the Engineering Thermodynamics course. The objective of the experience is to increase the interrelationship between different subjects, to promote transversal skills, and to make the subject closer to real working procedures the students will find in their future careers. The satisfactory results of the experience are exposed in this work.

1 Introduction

Chemical Engineers must be professionals capable of developing their work through innovation and continuous improvement of processes and products with analytical, creative and critical thinking, entrepreneurial spirit and the ability to lead highly productive teams. In their professional performance they will have to plan, analyze and interpret, design, implement, evaluate, investigate and put into practice possible solutions to needs that arise in society in their area of work or company [1]. All these skills are also essential for other types of engineering degrees. This work is focused on chemical engineering degree but the objectives, procedures and methodology could be easily applied to other specialities.

Traditionally, there has been a mismatch between the way in which universities have evaluated the results of their educational processes in Mathematics and Engineering Thermodynamics and the way in which society, in general, and companies, in particular, do so. We could say that until a few years ago, teachers at the university wanted to find out what knowledge the student had, but nowadays this has changed and now we focus on what skills and abilities do they have. Competency-based assessment seeks to change the educational process to guide it in this direction [2].

Competency-based learning is one remarkable change that the Bologna Process have brought. The European Higher Education Area implies a way of teaching and learning, where competencies represent the central axis of the new system. This means that it is no longer enough for a student to learn technical or specific knowledge. In this framework, students must also acquire a series of competencies that guarantee that they are capable of effectively and adequately exercise the work for which they are prepared. In any case, the contents are still necessary and essential, although they acquire a practical and applicable nature [3].

*Corresponding Author's E-mail: smjesus@usal.es

The definition of competencies arises from the need to understand an increasingly diverse and interconnected environment. Individuals need to master technologies and manage huge amounts of information. In these contexts, the competencies that individuals need to satisfy to achieve their goals have become more complex, requiring greater mastery of certain skills [4]. Based on the need for university students to acquire competencies, teachers must adapt the teaching-learning processes [5].

This is the reason why teachers at the University of Salamanca, who teach Mathematics III and Engineering Thermodynamics of second year of the Chemical Engineering Degree, in successive semesters, participate in this study. So, the methodology regarding the acquisition of competencies carried out in Mathematics, has explicit continuation in Engineering Thermodynamics, enriching the teaching-learning process of both subjects. Of course, coordination among all the teachers who tutor the same grade is essential, but also it is basic the coordination in subjects such as Mathematics and Physics. It is not necessary to justify this collaboration, since it is evident the need for mathematics as a language to express relationships in Physics. However, there are studies that analyze the elements that influence thermodynamic learning at university level, and they show the advantages of establishing collaborations between teachers of mathematics, physics and engineering, that is, collaborations between different disciplines [6, 7].

Furthermore, there are serious difficulties in teaching mathematics to engineering students. These students often face with difficulties in learning mathematical contents, in acquiring mathematical competencies and ultimately, in being proficient in mathematics. When we teach our engineering students subjects like Calculus, Linear Algebra, Numerical Methods, etc., one of our main concerns is usually how we can motivate them to learn mathematics. Engineering students often do not see the relationship between mathematics and other subjects, such as electricity, mechanics, mechanisms, automation, electronics or thermodynamics [2]. When students from engineering courses find the connection between topics from different subjects, they are more motivated [8]. With this proposal we try to make the students aware that the tools that are worked in Mathematics are useful to solve the problems of Engineering Thermodynamics.

In general, the use of the computer is of special relevance in Science and Engineering courses. Today, it is difficult to understand laboratory data processing without computational support. In the same way, programs with mathematical tools allow for solving Mathematics and Engineering Thermodynamics problems much more efficiently and similar to the real professional world.

Our objective is that students know how to use mathematical tools to formulate and solve problems that arise in other subjects, more specifically in Engineering Thermodynamics. It is generally accepted that computer-based (or computer-enhanced) problem solving is a very important application of the computer in engineering education and practice [9]. The *Wolfram Mathematica* package, without going any further, is a very suitable tool for solving mathematical problems. The use of symbolic software packages such as *Mathematica* has been steadily rising in academic instruction and specifically in Mathematics, and this trend is likely to gain strength in the upcoming years. There are numerous publications that reveal various applications of *Mathematica* for the training of engineers in general [8]. Moreover, for instance, interesting works were developed in the field of of chemical engineering [10, 11].

With all the arguments explained above, during the last two courses, we have organized a project in which students of Mathematics III (Numerical Methods) and Thermodynamic Engineering solve problems of these subjects with *Mathematica* program. In the first semester, in Mathematics III, as part of the training of the whole group, they carry out practices of numerical algorithms described in the theoretical classes using that software. The activity takes place in small groups in a computer room. In the second semester, the realization of some Engineering Thermodynamics problems with *Mathematica* software has been proposed to a test group of volunteer students. In five sessions, one per topic, work is done, together with the teacher, to solve problems with the help of the computer. This initiative is included in the university activities as an innovation and teaching improvement project "*Assessment mathematical competencies in science and engineering degrees ID2018/027*" [12]. The project has been carried out in coordination with all teachers (from physics and mathematics courses). On the other hand, it is part of the European project RULES_MATH [13], where we work on competencies-based learning.

This paper is organized as follows. Firstly, in Section 2 the context of the project carried out with students of the Chemical Engineering Degree will be exposed. This project is encompassed within the global methodology of the Thermodynamic Engineering course. Methodology is summarized in Sec. 3. Some highly relevant examples of the work done are introduced in Sec. 4. Finally, the evaluation methodology and the most significant conclusions are exposed (Secs. 5 and 6 respectively).

2 Context

The number of new students entering in the Chemical Engineering Degree at the University of Salamanca (USAL) each academic year is in the interval $60 - 70$. This Degree enables to exercise the regulated profession of Industrial Technical Engineer. The curriculum lasts 4 years (60 credits per year to complete 240). Adequate training in this scientific field involves the acquisition of the basic knowledge and skills that guarantee getting to know and being able to develop the design of processes and products characteristic of the chemical industry and the multiple sectors related to it (pharmaceutical, biotechnology, energy, food, environmental, etc.).

In this project we have focused on the following competencies established in the curriculum of the Chemical Engineering Degree, from the University of Salamanca [14], related with mathematics and engineering thermodynamics:

Transversal skills (the nomenclature corresponds to the official university program):

1. Computer knowledge in the field of study (TI5).
2. Problem resolution (TI8).
3. Critical thinking (TP8).
4. Ability to apply knowledge in practice (TS1).

Specific skills:

1. Ability to solve mathematical problems that may arise in Chemical Engineering, applying knowledge of algebra, geometry, calculus, numerical methods, statistics and optimization (DB1).
2. Basic knowledge on the use of computers, programming, operating systems, databases and programs with engineering applications (DB3).
3. Knowledge of the basic principles of thermodynamics and heat transmission and their application to the resolution of engineering problems (DR1).

2.1 Mathematics III

Mathematics III complements the basic mathematical training of the future Chemical Engineer, with elementary knowledge of Numerical Analysis, essential to translate an engineering problem into a mathematical problem. Moreover, to promote the capacity to solve the stated problems and to interpret the possible solutions are also objectives of the subject. Mathematics III is a course of 7.5 credits, developed in the first semester of the second year.

2.2 Engineering Thermodynamics

The main objective of Engineering Thermodynamics is the thermodynamic analysis of projected systems to carry out conversions among different energy sources. Among these, special attention is paid to cyclically operating devices (thermodynamic machines) designed for power generation and refrigeration. This subject covers 4.5 credits in the second semester of the second year.

2.3 *Wolfram Mathematica* Package

Information and Communications Technologies (ICTs) help to achieve higher levels of quality in teaching and, furthermore, allow to mimic the skills acquired by the students to those they will be required in their professional careers. The teaching of Mathematics and Engineering Thermodynamics cannot be an exception and should not be left out of the use of these methods. ICT provides students with the possibility of simulating experiences and posing very different situations, and comparing them. Sometimes, to do this manually can be difficult or at least tedious. For example, it allows the student to understand the true scope of a problem or the effectiveness of an algorithm by analyzing the results obtained by varying hypotheses, initial conditions, etc.

It is in these specialties where the use of specific software is really useful, such as symbolic calculation packages like Wolfram Mathematica package, among others. These programs, called CAS (*Computer Algebra System*) have an easy syntax to learn, since the syntax and commands resemble the mathematical operations they execute and, therefore, their learning is quick and intuitive. In addition, the help they offer is very complete and it is illustrated with numerous examples.

The University of Salamanca has a "*campus*" license for Mathematica, which gives legal coverage and allows its installation in the computer rooms and personal computers, so its utilization by the entire university community.

Particularly, it is installed in all the computers in the computer rooms in the Chemical Sciences Faculty. It is widely used in various undergraduate and master degrees. All these points makes this program an excellent software to carry out computer practices in different subjects.

3 Methodology

During a typical Thermodynamic Engineering course different methodological resources are used: master class, seminars, individual works, laboratory practices, tutoring and a final exam.

- Part of the training is given in the form of theory lectures. Videos and applets are used to enrich these sessions, which help to clarify the concepts and allow viewing experiences that would otherwise be difficult to carry out.
- An essential complement to theoretical classes are the problem resolutions in seminars. Facing problems and trying to solve them students can apply the knowledge acquired in theoretical classes and improve their skills.
- A third component are individual works of the students: not all the proposed problems are solved in class, therefore, students are asked to make two deliveries, throughout the course, with problems in which they have worked individually. These tasks contribute to the ongoing assesssment of the student.
- Because of bureaucratic issues in the developing of the official university program of the subject, no practical laboratory hours were established for Thermodynamic Engineering. However, we consider that it would be a really enriching complement for the in-depth understanding of the thermodynamic cycles that are addressed. For this reason, some seminars are dedicated to take students to the Thermodynamics laboratory, for example to observe the operation of a Stirling engine, for didactic purposes, as well as to experimentally measure cycle performance, maximum and minimum temperatures, and to analyze the pressure vs. volume diagram, among other aspects.
- Personal attention to students through face-to-face tutoring is essential to solve questions and doubts. In this way, we facilitate students to deepen their knowledge, while reinforcing direct and personal contact with teacher.
- To carry out the evaluation by competencies. This includes written tests with theoretical and numerical problems to solve. Students are allowed to use the class notes.

The use of the web-based *Studium* platform of the University of Salamanca (*Moodle*-based digital platform) is proposed for the subjects both to make available to students notes, presentations and figures used in class, proposed problems, and to enrich the teaching-learning process through forums, experience videos, etc.

An extra activity was planned the last two academic years: the realization of problems of the subject with the *Mathematica* tool. Briefly, the main objective was to allow the student a direct interaction with the topics developed in class. The display of the results "*in real time*" and using all possible graphic resources, is very effective in capturing the interest of the students. Moreover, computer practice familiarizes the students with a working method that, without doubt, will be essential in the development of their professional activity.

4 Using Mathematica software in Mathematics and Thermodynamic Engineering

Students from Chemical Engineering Degree take a Computer Science course in the first semester of the first year, so they are familiar with the Matlab programming environment. Thus they are already familiar with symbolics calculation packages.

Sophomore students must attend to Mathematics III course during the fall semester, this is a mandatory course and it includes several computer classes where the aim is to solve problems with Mathematica. These sessions are carried out in small groups (computer rooms usually have capacity for 15-20 students). In addition to the numerical methods described during theoretical lectures, several problems that connect students to their reality are also stated. They reveal the real usefulness of mathematics courses. As an example, we present the problem of the "Angry birds" (AB) game: to launch birds with parabolic trajectories. For the Thermodynamic Engineering course (during spring semester) we propose the students the problem of fugacity (F) as a bridge between both courses. Next, both problems are detailed.

"Angry birds" problem (AB)

The "Angry birds" game, developed by Rovio Entertainment Corporation, consists in destroying structures of different materials in order to elliminate the pigs inside or around them (green images in Fig. 1). With the help of a slingshot,

Figure 1: Image of the situation that raises the ‘Angry birds” problem.

the player launches an angry bird (in red) with the angle and strength necessary to achieve the proposed objective. To avoid a tower collision and not to reach its destination (Figure 1), the bird must pass as close to the first two towers as possible, but without touching them.

In a particular example, it is assumed, for instance, that the slingshot always shoots from a height of 1 cm. Tower 1, which is 1 cm from the slingshot measures 4.5 cm, Tower 2, at 2 cm, measures 6.5 cm and the desired point of impact, in the Tower 3, which is 4 cm from the slingshot, is exactly 5 cm. The question is set as: Draw the curve that the Angry Bird describes in the shot, knowing that we do not take much risk and go 0.5 cm above the towers.

Figure 2 shows the solution of the problem using Mathematica. In this way, a simple interpolation problem acquires meaning and a motivating application when connected to a popular videogame. On the other hand, problem solving

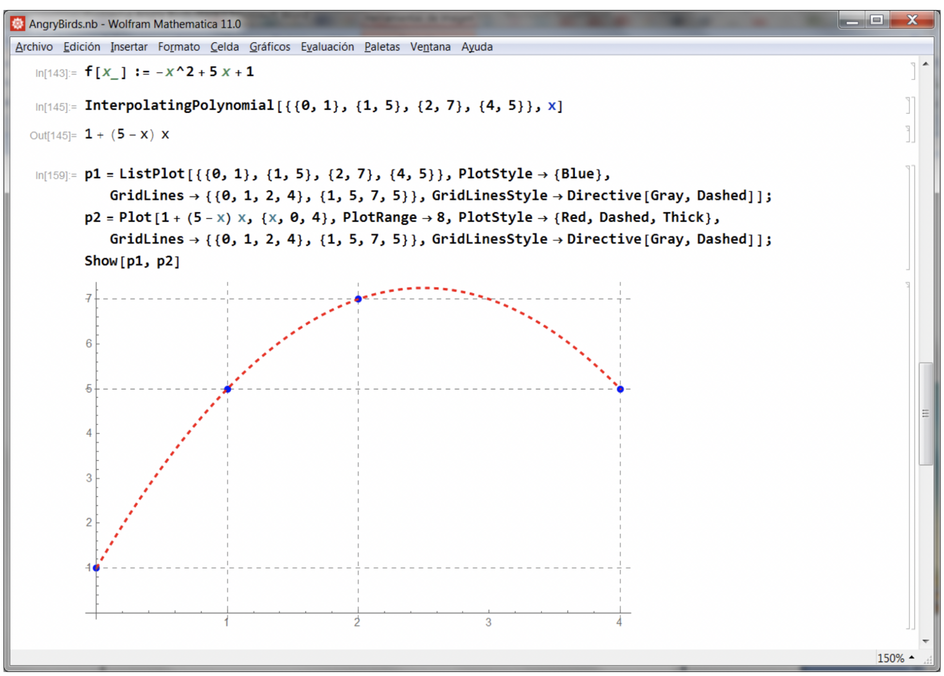


Figure 2: “Angry birds” problem solution.

skills (TI8), computer use (TI5 and DB3) are being acquired, as well as the ability to put knowledge into practice (TS1). Figure 3 shows a diagram where each proposed problem is related to the skills being worked on.

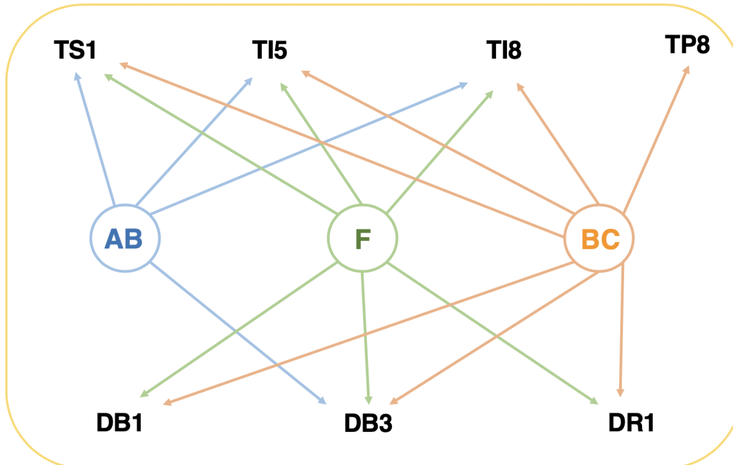


Figure 3: Relationship between the skills to be achieved (in black color) and the problems addressed (in different color).

Fugacity problem (F)

Use the following data to calculate N_2 fugacity at $0°C$ and 400 atm.

p (atm)	50	100	200	400
pV/RT	0.9846	0.9863	1.0365	1.2557

To help students to solve the problem using *Mathematica*, it is broken down into the following steps (the solution of this problem is shown in Fig. 4).

The fugacity f, can be calculated as: $\ln f = \ln p + \int_0^p \left(\frac{V}{RT} - \frac{1}{p}\right) dp$ or $\ln f = \ln p + \int_0^p \left(\frac{z-1}{p}\right) dp$ where $z = \frac{pV}{RT}$.

a) Determine the compressibility factor values $\frac{z-1}{p}$.

b) Make (using blue dots) the graphic representation of $\left(p, \frac{z-1}{p}\right)$.

c) Find the interpolation polynomial that best fits these value pairs $\left(p, \frac{z-1}{p}\right)$. It shall be denoted as $f(p)$.

d) Do represent $f(p)$ (in red color), along with the pairs of data (blue color).

e) Determine the area under the curve between $p = 0$ and $p = 400$ of the function $f(p)$. As it is known, the definite integral between $p = 0$ and $p = 400$ of the interpolation function, $f(p)$, corresponds to the area under the curve.

f) Clear the fugacity, f, of the expression: $\ln f = \ln p + \int_0^p \left(\frac{z-1}{p}\right) dp$ where $z = \frac{pV}{RT}$.

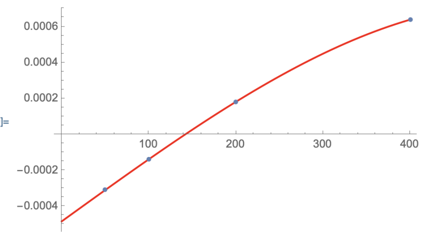
As it can be seen, students use the knowledge acquired in Mathematics to solve a problem of Thermodynamics, relying on different commands and functions of the *Mathematica* software. It is clear which competencies are developed: ability to solve problems (TI8), using Maths and Thermodynamic knowledge (DB1 and DR1), use of the computer (TI5 and DB3), and applying their knowledge practically (TS1) (see Fig. 3).

When students start the Thermodynamic Engineering course, they are already familiar with the Mathematica program and the necessary commands to solve the applied thermodynamics problems. One of the difficulties that this subject presents is that problems are long, and many operations are required. Usually, problems are solved in class, using a calculator. This is conditioned by the nature of the subject, since it is not about numerical methods, and because the exams will be done in a classroom without access to computers. However, when students tackle problems with computer, time is saved for resolution, allowing more time to be spent understanding the problems in depth. Since it is possible to vary different magnitudes and see how they affect the final solution. This is the reason that led us to offer students the opportunity to work on the same class problems, but with the Mathematica tool. It has been a voluntary

FugacityFugacity.nb

```
p = {50, 100, 200, 400};
z = {0.9846, 0.9863, 1.0365, 1.2557};
b = (z - 1) / p;
Datos = Transpose[{p, b}];
RepDatos = ListPlot[Datos];
f[p_] := InterpolatingPolynomial[Datos, p]
RepPol = Plot[f[p], {p, 0, 400}, PlotStyle → Red];
Show[RepPol, RepDatos]
Print["e) Defined Integral between p = 0 and p = 400 of the interpolation function f(p) :"]
Sol = NIntegrate[f[p], {p, 0, 400}]
"e) Fugacity (in atm), for p = 400 atm, is"
po = 400; (* atm *)
Solve[Log[x] == Log[po] + Sol, x]
```

Out[80]=




```
e) Defined Integral between p = 0 and p = 400 of the interpolation function f(p) :
Out[82]= 0.0591429
Out[83]= e) Fugacity (in atm), for p = 400 atm, is
Out[84]= {{x → 424.371}}
```

Figure 4: Fugacity problem solution.

activity, in 5 days outside the formally established hours we explain students how to solve some problems using a mathematical software. In that session, with the teacher, a problem was solved that had already been worked on in the problems class, but this time with the mathematical tool. This way, the emphasis was placed on the programming difficulties, not on those of Thermodynamics that were already explained in a previous case. Students were then asked to solve another new problem.

In the year 2018-19, 5 students from 34 participated in this activity and in the 2019-20 academic year, 30 of 48. This shows the inertness that this project has awakened.

Usually problems in Thermodynamic Engineering are long, with chained operations, and it is very useful to understand them to be able to visualize graphs of the situation. A typical example is presented below: a typical Brayton cycle problem (BC).

Brayton cicle problem (BC)

Consider a reversible air Brayton cycle (considered as ideal gas), with a mass flow of 1 kg/s and minimum and maximum temperatures of 290 K and 1430 K, respectively.

a) Calculate the different values of pressure and temperature at each vertex of the cycle and representation of the $p - T$ diagram for different compression ratios (rp): 5, 10 and 15.

b) Calculation for the three cases of the compression work, the one carried out by the turbine, the absorbed heat and the thermodynamic efficiency.

c) Repeat the exercise assuming an ideal regenerative Brayton cycle.

d) Compare and comment on the results of sections b) and c). In which cases is the use of the regenerator interesting?

As mentioned previously, one of the potentialities of Mathematica are graphs, as shown in the solution to the BC problem (Fig. 5). With a single image, three different situations can be identified, observing the consequences of modifying a variable in the problem, such as the particular case of the pressure relation (rp).

In the solutions of sections b) and c) of problem BC (Fig. 6 and Fig. 7), you can see how, through some orders, the problem is solved for 8 different cases (4 values of rp and 2 cycles: ideal and regenerative Brayton). Presenting the

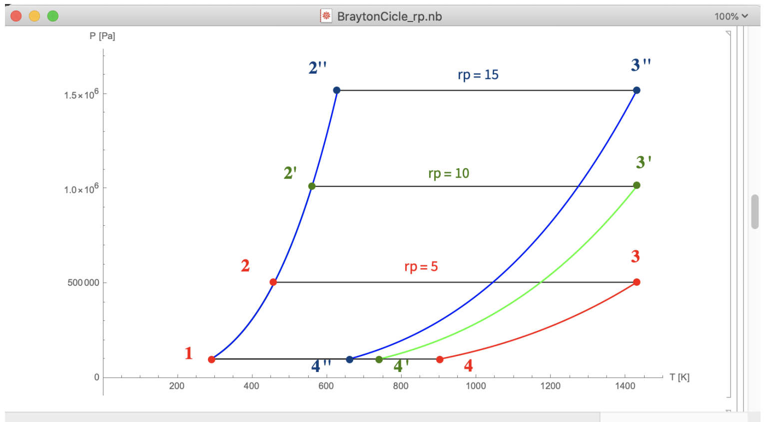


Figure 5: P-T diagram of an ideal Brayton cycle for different values of pressure ratio ($rp = 5$ $rp = 10$ and $rp = 15$). Results from section a) of Brayton cycle problem.

results in a visual and didactic way, through tables. This helps students to compare and deeply understand the root of the problem, leading them to achieve a better understanding of the physical phenomena involved [11].

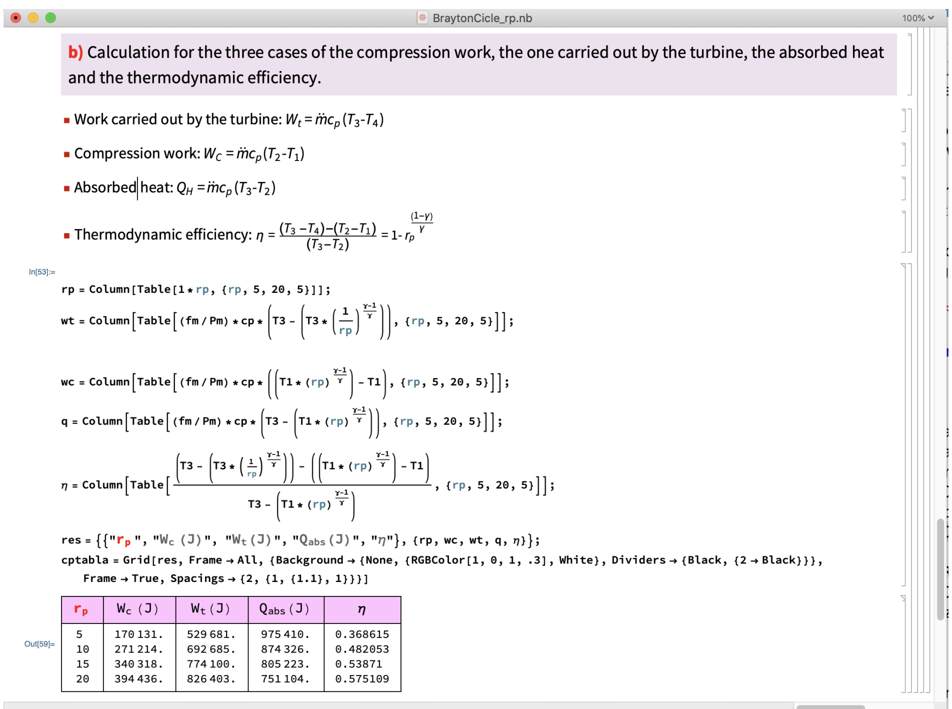


Figure 6: Results from section b) of Brayton cycle problem.

This is one of the examples where all the skills initially planned, including the critical capacity, are worked on, since they can tackle the same problem with multiple variants (Fig. 3).

Another reason for encourages our students to learn Mathematica software is that the professors responsible for Mathematics and Thermodynamic Engineering offer end-of-degree projects in which it is necessary to use such software.

5 Evaluation

With this methodology, the results in the last three academic years, $2017-2018$, $2018-2019$ and $2019-2020$, are shown in Table 1. Although the results of the 2019-2020 are really good, we consider that they should not be taken

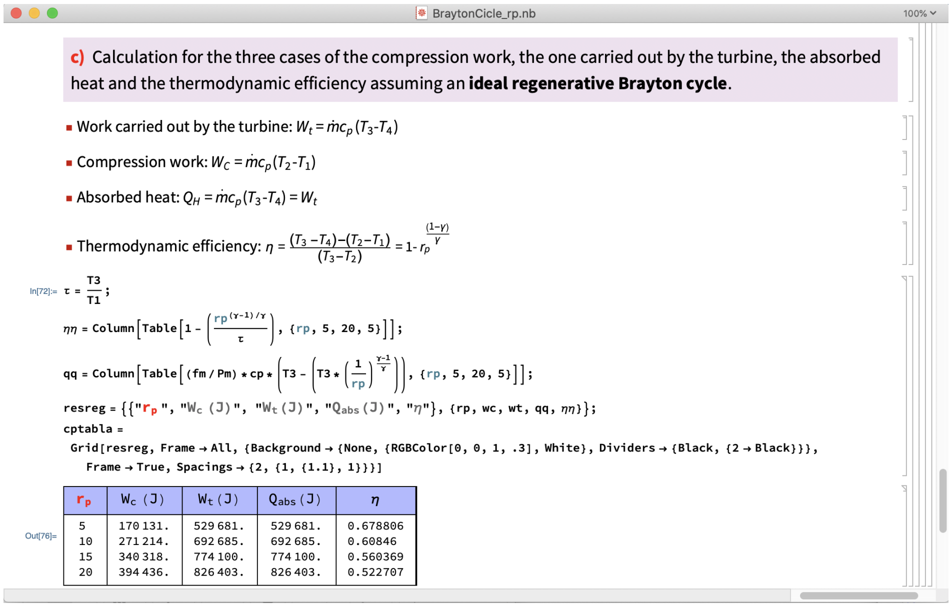


Figure 7: Results from section c) of Brayton cycle problem.

into account for this study, since half of the teaching and the entire evaluation (continuous and final) has been carried out on-line because of coronavirus pandemic. Both the success rate (percentage of students who passed the subject with respect to those presented) and the performance rate (percentage of students who pass the subject with respect to the total of enrolled students) are above 65%, which can be consider good results within an Engineering Degree.

Table 1: Results obtained in Thermodynamic Engineering in the last three academic years: $2017-2018$, $2018-2019$ and $2019-2020$.

Academic year	$2017-18$	$2018-19$	$2019-20$
Students involved in the project	–	5	30
Rate	–	14.7%	62.5%
Students that pass the course	–	100%	100%
Total number of students	41	34	48
Performance rate	65.8%	82.3%	95.8%
Success rate	75%	96.6%	100%

It is observed that the results have improved since 2018-2019, which is when we started with the project presented here. Although there are many variables to consider, the difference between 2017-2018 and 2018-2019 is the fact of starting to work through Mathematica problems in Thermodynamic Engineering. It is true that there are more hours of work both between teacher and student, as well as personal work of the student and closer student-teacher relationship.

Besides, the interest that the initiative has aroused has been verified: in academic year 2018-2019 it was carried out with 5 student volunteers, out of 34 enrolled (15%) in Thermodynamic Engineering, and in 2019-2020, 30 volunteer students participated out of 48 enrolled (63%), which is a considerable increase. It is remarkable the fact that all the students who have participated in the project have passed the course. It is true, that being a voluntary activity, those students who are most interested in learning generally sign up. We will think about the possibility of expanding it to the entire group of students enrolled in the subject, for future courses.

6 Conclusions

Our final goal is that the students of the Chemical Engineering Degree know how to use mathematical tools to formulate and solve problems that arise in Thermodynamic Engineering. So that the students understand that knowledge acquired in different subjects are interdependent. For reaching it, teachers at the University of Salamanca, who teach

Mathematics III and Engineering Thermodynamics of second year of the Chemical Engineering Degree, in successive semesters, have worked together.

First of all, in Mathematics III computer practices, it have been solved problems with Mathematica. Covering both classic problems and others that connect students with their close reality and motivate them, such as the case of the "Angy bird". In addition to Thermodynamic exercises where they discover the application of the knowledge acquired in Mathematics, as “fugacity case”. In second semester, the realisation of some Engineering Thermodynamics problems through Mathematica software has been proposed to a group of volunteers students. In five sessions, one per topic, work is done, together with the teacher, to solve problems raised with the help of this tool.

This work aims to achieve competency-based teaching, such as: problem resolution (Mathematics III and Engineering Thermodynamics); and basic knowledge of the use of computers, and operating systems, with engineering applications.

And, above all, to introduce in the teaching process, methods that are closer to those of the graduate’s later work and that involve an approach to the reality of engineering, such as solving problems with the program.

The interest that the initiative has aroused has been verified: in academic year 2018-2019 it was carried out with 5 student volunteers, out of 34 enrolled (15%) in Thermodynamic Engineering, and in academic year 2019-20, 30 volunteer students participated out of 48 enrolled (63%). Which is a considerable increase.

It is also notable that the degree of satisfaction of the teachers involved is high. On the one hand, perceiving that such a voluntary activity which students carry out outside the established schedule, arouses their interest. And on the other hand, we think that the fact that students perceive that teachers care about what they learn (not just their grade), establishes another type of perception, positive, towards the subject. This has been verified with the attendance to the usual classes, which has been very high, compared with that of other subjects of the same course.

References

[1] Mogens Niss. Mathematical competencies and the learning of mathematics: The danish kom project. In *3rd Mediterranean conference on mathematical education*, pages 115–124, 2003.

[2] Araceli Queiruga-Dios, M. Jesús Santos Sánchez, Juan José Bullón Pérez, Jesús Martín-Vaquero, Ascensión Hernández Encinas, Snezhana Gocheva-Ilieva, Marie Demlova, Deolinda Dias Rasteiro, Cristina Caridade, and Víctor Gayoso-Martínez. Evaluating engineering competencies: a new paradigm. In *2018 IEEE Global Engineering Education Conference (EDUCON)*, pages 2052–2055. IEEE, 2018.

[3] M.L. Montero Curiel. El proceso de bolonia y las nuevas competencias. 2010.

[4] D.S. Rychen and L.H. Salganik. *Key competencies for a successful life and well-functioning society*. Hogrefe Publishing, 2003.

[5] Deolinda Dias Rasteiro, Víctor Gayoso Martínez, Cristina Caridade, Jesús Martin-Vaquero, and Araceli Queiruga-Dios. Changing teaching: competencies versus contents. In *2018 IEEE Global Engineering Education Conference (EDUCON)*, pages 1761–1765. IEEE, 2018.

[6] Kinsey Bain, Alena Moon, Michael R Mack, and Marcy H Towns. A review of research on the teaching and learning of thermodynamics at the university level. *Chemistry Education Research and Practice*, 15(3):320–335, 2014.

[7] Jenna P Carpenter, Michael B Cutlip, Michael D Graham, Anton J Pintar, and Jan A Puszynski. Mathematics and chemical engineering education. *age*, 6:1, 2001.

[8] Araceli Queiruga Dios, Ascensión Hernández Encinas, Jesús Martín Vaquero, Ángel Martín del Rey, Juan José Bullón Pérez, and Gerardo Rodríguez Sánchez. How engineers deal with mathematics solving differential equation. *Procedia Computer Science*, 51:1977–1985, 2015.

[9] Mordechai Shacham. An introductory course of modeling and computation for chemical engineers. *Computer Applications in Engineering Education*, 13(2):137–145, 2005.

[10] Doraiswami Ramkrishna and Neal R. Amundson. Mathematics in chemical engineering: A 50 year introspection. *AIChe journal*, 50(1):7–23, 2004.

[11] Deolinda D. Rasteiro, Fernando P. Bernardo, and Pedro M. Saraiva. Using mathematica to teach process units: A distillation case study. *Chemical Engineering Education*, 39(2):n2, 2005.

[12] Araceli Queiruga Dios, Juan José Bullón Pérez, María Ascensión Hernández Encinas, Ángel María Martín del Rey, Jesús Martín Vaquero, Gerardo Rodríguez Sánchez, María Jesús Santos Sánchez, et al. Evaluación de competencias matemáticas en titulaciones de ciencias e ingeniería. 2019.

[13] Araceli Queiruga-Dios, Ascensión Hernández Encinas, Marie Demlova, Deolinda Dias Rasteiro, Gerardo Rodríguez Sánchez, and María Jesús Sánchez Santos. Rules_math: Establishing assessment standards. In *International Joint Conference: 12th International Conference on Computational Intelligence in Security for Information Systems (CISIS 2019) and 10th International Conference on EUropean Transnational Education (ICEUTE 2019)*, pages 235–244. Springer, 2019.

[14] Competencies of the degree in chemical engineering, 2020.