Reproduction in any form (including the internet) is prohibited without prior permission from the Society

829

British Journal of Educational Psychology (2007), 77, 829–848 © 2007 The British Psychological Society



www.bpsiournals.co.uk

# Influence of situational and conceptual rewording on word problem solving

Santiago Vicente<sup>1</sup>, Jose Orrantia<sup>1</sup>\* and Lieven Verschaffel<sup>2</sup>

<sup>1</sup>University of Salamanca, Spain

**Background.** Studies on rewording word problems can be grouped into two main groups: situational rewording, in which the situation denoted by the text is described more richly, and conceptual rewording, in which the underlying semantic relations are highlighted.

**Aims.** Our aims are to define and distinguish these two kinds of rewording and to test empirically their relative effectiveness in two different studies.

**Sample.** In the first study, 79 third graders, 64 fourth graders and 65 fifth graders took part; the sample for Study 2 was similar.

**Method.** In Study I, children were asked to solve both easy and difficult two-step change problems in three different versions: standard, situational and conceptual rewording. In Study 2, three different versions of the situational version were compared: one with only temporal elaborations, one with only causal elaborations and a 'complete' version combining both elaborations.

**Results.** In Study I, conceptually reworded problems elicited the best results, especially among younger children and for difficult two-step problems. Neither in Study I nor in Study 2 did the situationally reworded problems yield better performance than standard items.

**Conclusion.** Only conceptual rewording has proved to be useful for improving children's performance, especially among younger children and for difficult problems. The lack of impact of situational rewording cannot be explained in terms of the length of the resulting text.

Word problems have already, for a very long time, attracted the attention of researchers, both cognitive psychologists and mathematics educators. Throughout the 1980s and 1990s, there was concentrated research on how children learn to do one-step addition and subtraction problems involving small whole numbers or collections of discrete objects. In the early 1980s, a basic distinction emerged that guided much of the

<sup>&</sup>lt;sup>2</sup>Catholic University of Leuven, Belgium

<sup>\*</sup>Correspondence should be addressed to Jose Orrantia, Universidad de Salamanca, Facultad de Educacion, Pso. Canalejas 169, 37008, Salamanca, Spain (e-mail: orrantia@usal.es).

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

#### 830 Santiago Vicente et al.

subsequent research, distinguishing three classes of problem situations modelled by addition and subtraction. These are situations involving a change from an initial to a final state through the application of a transformation (change problems), the combination of two discrete sets or splitting of one set into two discrete sets (combine problems) and the quantified comparison of two discrete sets of objects (compare problems). Within each of these three major semantic categories, further distinctions were made resulting in 18 different types of one-step addition and subtraction problems (Riley, Greeno, & Heller, 1983; see also Fuson, 1992; Reed, 1999; Riley & Greeno, 1988; Verschaffel & De Corte, 1993, 1997).

Numerous empirical studies carried out during this period with children between the ages of 5 and 8 years demonstrated the psychological and educational significance of this classification scheme, especially that word problems that can be solved by the same arithmetic operation (i.e. a direct addition or a subtraction with the two given numbers in the problem) but that belong to different semantic problem types yield different degrees of difficulty, different ways of representing and solving these problems, and different error categories (for reviews of this research, see Fuson, 1992; Reed, 1999; Verschaffel & De Corte, 1997).

Computer simulations were developed concurrently with the empirical research (Briars & Larkin, 1984; Riley et al., 1983). Underlying these computer models was the general assumption that a skilful solution process of a word problem starts from a network representation of the basic semantic relationships between the main quantities in the problem, in terms of one of the three above-mentioned basic semantic structures. This network is considered the result of a complex interaction between bottom-up and top-down analysis; that is, the processing of the verbal input, as well as the activity of semantic schemata, contributes to the construction of this network representation. More difficult problem types require rerepresentations in terms of other schemata before a proper arithmetic action or operation can be selected and performed. For instance, according to the Riley et al. model, change problems with an unknown initial set or compare problems with an unknown compared or reference set can only be solved after the original problem representation in terms of, respectively, a change or a compare schema has been rerepresented in terms of a part-whole structure. The more competent the problem solver is, the more able (s)he is to process the text in a top-down way by relying on his(her) well-developed schemata.

A number of studies compared the performance, strategy and error data of students with the behaviour of these computer models (for reviews of these empirical studies, see Fuson, 1992; Reed, 1999; Verschaffel & De Corte, 1997). For example, empirical support for the assumed central role of part-whole knowledge in children's addition and subtraction word problem solving comes from the work by Sophian (Sophian & McCorgray, 1994; Sophian & Vong, 1995).

However, it was clear from an early stage that there were still several problematic issues and questions remaining. By no means all empirical findings were consistent with the (computer) models (for systematic comparisons of these empirical data with the predictions of the computer models, see Carpenter & Moser, 1984; De Corte & Verschaffel, 1988; Fuson, 1992). Many of these inconsistencies had to do with the fact that – because of the relatively weak elaboration of the initial text-processing stage of the word problem solving process and because of the neglect of the broader (instructional) environment wherein this problem-solving process occurs – these (computer) models were unable to account for the influence of textual and contextual variables on

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

Situational and conceptual rewording of word problems 831

Table 1. Eighteen problem types (taken from Riley & Greeno, 1988)

Combine I (combination unknown)

Joe has three marbles. Tom has five marbles.

How many marbles do they have

altogether?

Combine 2 (combination unknown) Joe and Tom have some marbles.

Joe has three marbles. Tom has five marbles.

How many marbles do they have

altogether?

Combine 3 (subset unknown)

Joe has three marbles. Tom has some marbles.

They have eight marbles altogether. How many marbles does Tom have?

Combine 4 (subset unknown)

Joe has some marbles. Tom has five marbles.

They have eight marbles altogether. How many marbles does Joe have?

Combine 5 (subset unknown)

Joe and Tom have eight marbles altogether.

Joe has three marbles.

How many marbles does Tom have?

Combine 6 (subset unknown)

Joe and Tom have eight marbles altogether.

Joe has some marbles. Tom has five marbles.

How many marbles does Joe have?

Change I (result unknown) Joe had three marbles.

Then Tom gave him five marbles.

How many marbles does Joe have now?

Change 2 (result unknown)

Joe had eight marbles.
Then he gave five marbles to Tom.
How many marbles does Joe have now?

Change 3 (change unknown)

Joe had three marbles.

Then Tom gave him some marbles.

Now Joe has eight marbles.

How many marbles did Tom give him?

Change 4 (change unknown)

Joe had eight marbles.

Then he gave same marbles to Tom.

Now Joe has three marbles.

How many marbles did he give to Tom?

Change 5 (start unknown) Joe had some marbles

Then Tom gave him five marbles.

Now Joe has eight marbles.

How many marbles did Joe have in the

beginning?

Change 6 (start unknown)

Joe had some marbles.
Then he gave five marbles to Tom.

Now Joe has three marbles.

How many marbles did Joe have in the beginning?

Compare I (difference unknown)

Joe has five marbles. Tom has eight marbles.

How many marbles does Tom have more

than Joe?

Compare 2 (difference unknown)

Joe has eight marbles.

Tom has three marbles.

How many marbles does Tom have less than Joe?

Compare 3 (compared quantity

unknown)

Joe has three marbles.

Tom has five more marbles than Joe. How many marbles does Tom have?

Compare 4 (compared quantity unknown)

Joe has eight marbles.

Tom has 5 marbles less than Joe. How many marbles does Tom have?

Compare 5 (referent unknown)

Joe has eight marbles.

He has 5 more marbles than Tom. How many marbles does Tom have?

Compare 6 (referent

unknown)

Joe has three marbles.

He has 5 marbles Jess than Tom. How many marbles does Tom have?

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

#### 832 Santiago Vicente et al.

children's problem representations and solutions (Cummins, Kintsch, Reusser, & Weimer, 1988; Kintsch & Greeno, 1985; Reusser, 1990).

Indeed, a number of empirical studies have shown how small changes in the wording of the problem texts may have a dramatic (positive) impact on children's solution processes and skills (Cummins, 1991; Cummins et al., 1988; Davis-Dorsey, Ross, & Morrison, 1991; De Corte, Verschaffel, & De Win, 1985; Hudson, 1983; Staub & Reusser, 1992; Stern & Lehrndorfer, 1992). Underlying all these studies is the idea that modifying the text of the problem, without changing its semantic structure, will lead to a higher success rate. A pioneering study on the rewording effect was carried out by Hudson. He presented nursery school, kindergarten and first-grade children, eight pictures of compare 1 situations (showing, e.g. five birds and two worms). Each time two questions were asked: first, the usual question in compare problems: 'Here are some birds and here are some worms. How many more birds than worms are there?' and second, an alternative question 'Here are some birds and here are some worms. Suppose the birds all race over and each one tries to get a worm. How many birds won't get a worm?' Hudson found that, as expected, the problem was significantly easier when the second question was asked.

Later investigations replicated Hudson's study and applied his method to other types of word problems as well. These studies on the impact of rewording word problems can be divided into two main groups. First, studies wherein the semantic relations between the sets implied in the problem are stated more explicitly and made more transparently (conceptual rewording), as in the studies by Cummins (1991), Davis-Dorsey *et al.* (1991) and De Corte *et al.* (1985). Second, investigations wherein the (real-world) situation to which the problem statement is referring to is presented in a more enriched and elaborated way (situational rewording); illustrative of this second approach are the studies by Cummins *et al.* (1988), Staub and Reusser (1992) and Stern and Lehrndorfer (1992).

In studies on conceptual rewording, reworded problems were formulated in such a way that the underlying semantic relations between the given and unknown sets were made more explicit than in the standard version, without affecting the underlying semantic/mathematical structure. De Corte *et al.* (1985) investigated the effect of conceptual rewording of change 5, combine 5 and compare 1 problems on first and second graders. Cummins (1991) tested the effect of conceptual rewording on first graders' solutions of combine 5 problems. Davis-Dorsey *et al.* (1991) reworded change 5, combine 5 and compare 1 problems in the same way as De Corte *et al.*, but manipulated conceptual rewording in combination with problem personalization (see Table 2). The results of De Corte *et al.*'s study showed that rewording had a positive effect for first and, to a lesser extent, for second graders. Cummins found a significant rewording effect. Davis-Dorsey *et al.* observed improved performance too, but in their study only the second grade students benefited from conceptual rewording and only when it was combined with personalization.

Problem personalization, as implemented by Davis-Dorsey *et al.* (1991), comes closely to the second kind of rewording mentioned above, namely situational rewording. Studies that belong to this second category share the idea of making the situational context in which the problem is embedded more explicitly, rather than by clarifying the underlying semantic relations between sets. However, there are some differences between this second type of studies. Cummins *et al.* (1988) compared second and third graders' comprehensions and solutions of combine 5, change 5,

Study	Problem type	Example(s) of reworded problem(s)
Hudson (1983)	CPI	'Here are some birds and here are some worms. Suppose the birds all race over and each one tries to get
De Corte et al. (1985)	CHS	'Joe had some marbles. He won three more marbles. Now he has five marbles. How many marbles did Joe have in the beginning?'
	CB5	Tom and Ann have nine nuts altogether. Three of these nuts belong to Tom. The rest belong to Ann. How many nuts does Ann have?
	CPI	'There are six children but there are only three chairs. How many children won't get a chair?'
Davis-Dorsey et al. (1991)	CH5 CB5	Same as De Corte et al. (1985) and in addition to personalizing the standard version of the problem with children's favourite movie, household pets' names, favourite food, and friends' names.
	CPI	(Best friend) walked 3/5 of a mile to see (favourite movie). Later he walked to (other friend's) house. (Best friend) walked 4/5 of a mile altogether. How far did (best friend) walk from the
	1	(Javourite movie) to (other friends) house!
Cummins (1991)	CBs	Same as De Corte et al. (1985), except for the first sentence, which is: 'I here are nine nuts'
Cummins et al. (1988)	CB5	Example for a Compare 5 problem. Similar rewording was applied to the other three kind of
	CHS	problems: Jane and Mimi play tennis together twice a week. They both always try hard to beat each other.
	CH6	Both of them decided to buy new tennis racquets. So far Jane has saved 13 dollars for her racquet.
	CP5	She saved five dollars more than Mimi. How many dollars has Mimi saved?
Stern and Lehrndorfer (1992)	CPI	Example for a Compare 2 problem. Similar rewording was applied to the other five kinds of compare
	CP2	problems: Peter is Laura's older brother. Because he is older, his bedroom is larger and his toys are more
	CP3	expensive than Laura's. Peter also gets more pocket money than Laura and he has a new bike whereas
	CP4	Laura has Peter's old bike. When Peter does his homework, Laura Doodles a little bit, Peter has
	CP5	six crayons. Laura has four crayons. How many crayons less does Laura have than Peter?
	CP6	
Staub and Reusser (1992)	CHI	Example for a change 6 problem. Similar rewording was applied to the other kind of problems:
	CH2	Peter has four marbles now. Today Peter gave Mary seven apples. How many apples did
	CHS	Peter pick yesterday?
	CH6	

Table 2. Rewordings used in previous studies

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

#### 834 Santiago Vicente et al.

change 6 and compare 6 standard word problems (in the authors' terminology: 'empoverished problems') with enriched problems embedded into little stories (see Table 2) 'showing plausible, realistic situations and setting up a motivation for the final arithmetic question that completed the story' (p. 427). Stern and Lehrndorfer (1992) presented first graders with (all six types of) standard compare problems and versions of these problems that were embedded in a concrete, enriched and familiar story context, where a competitive context dealing with qualitative comparison before presenting the arithmetic word problem was described. Children's performance on these 'enriched' problems was compared to the achievement on the standard problems in which the story had no qualitative comparison between the two people involved in the comparison. Finally, Staub and Reusser (1992) generated different versions of change 1 and 2, as well as of change 5 and 6, problems and asked first and third graders to solve them. Their rewording consisted of modifying the sequence of the events in the text, so that this sequence did no longer fit with the 'ordo naturalis' of the events as they occur in the real world (e.g. resulting state/transfer/initial state for change 5 and change 6 problems, see Table 2), which makes it more difficult for children to recover the intended situational structure. These studies on situational rewording yielded different results. Stern and Lehrndorfer (1992) and Staub and Reusser (1992) found results consistent with their predictions, namely, the former found that compare problems became easier when they were embedded in an 'enriched' situational context, whereas Staub and Reusser found that their reworded problems were indeed more difficult for children. However, Cummins et al. (1988) did not find any facilitation effect from an enriched story context.

The set of above-mentioned studies on rewording shows some limitations. Probably, the main limitation is that each study (except Davis-Dorsey *et al.*'s, 1991) has focused on only one kind of rewording, making mutual comparisons very difficult, if not impossible (see Table 2). Moreover, a detailed and formal description and account of the modified texts and their relation with the original standard versions are missing.

### STUDY I

Starting from an overview of the previous investigations, we set up two new empirical studies. Study 1 assessed the facilitating effect of the two types of rewording distinguished above and compared their relative effectiveness: first, a rewording aimed at facilitating the generation of a situation model (situational rewording) and second, a rewording whereby clues are added aimed at making more explicit the semantic relations between the given and unknown sets (conceptual rewording). Besides a comparison of the overall effect of both types of rewording on students' performance, we also wanted to investigate the impact of problem difficulty and children's age on the (relative) effectiveness of both kinds of rewording. In previous studies, impact of rewording was tested with first-, second- and third-grade children (except in Davis-Dorsey et al.'s, 1991, study, which also included fifth graders). Results of this latter study showed that conceptual rewording was not effective for fifth graders, because these more experienced learners' ability to construct an accurate problem representation even when confronted traditional, impoverished texts, as Davis-Dorsey argue. Like these authors, we were interested in analysing the impact of rewording on older elementary students too. In order to Reproduction in any form (including the internet) is prohibited without prior permission from the Society

Situational and conceptual rewording of word problems 835

adapt the difficulty level of the problems to the conceptual and mathematical level of the children, we chose for two-step instead of one-step problems. More concretely, we used two-step problems with a change structure.

The criteria for rewording the problems were based on some of the theoretical positions described earlier. For conceptual rewording, we followed Riley et al.'s (1983; Riley & Greeno, 1988) idea that understanding the semantic relations described by the text depends on understanding part-whole relations, especially for more difficult problems. Therefore, our conceptual reformulations of the two-step change problems make explicit the conceptual role of the total set shared by the two one-step change situations (i.e. the set that plays the role of result set in the first change problem and at the same time the role of start set of the second change problem).<sup>2</sup> An interesting feature of this type of conceptual rewording is that, contrary to previous studies wherein every kind of rewording was tied to a specific kind of problems, it can be applied to all problem types. After all, applying part-whole knowledge is needed for solving all (difficult) kinds of addition and subtraction problems (Riley & Greeno, 1988; Riley et al., 1983). Regarding situational rewording, we took Reusser's (1985, 1990) situation problem solving (SPS) model as theoretical frame, because it is the only one that involves the generation of an episodic situation model, implying 'the application of comprehension strategies to the text base, which generate an analysis of the temporal and functional structure of the situations and actions depicted in the problem text' (Staub & Reusser, 1995, p. 293). That is, the goal of this stage is to construct a cumulative representation of the events and actions depicted by the text of the problem. In order to generate the different kinds of situational information, we departed from two definitions from Reusser's theory, namely his definition of 'situational problems' and 'episodic situation model'. First, problems that the SPS model could solve were defined as situations 'organized around some protagonist or main actor with certain needs, motives, purposes and who is involved in some interactions with coactors, objects and instruments' (Staub & Reusser, 1995, p. 480). Departing from this definition, we distinguished three kinds of situational information that could be added to the problem text: depicting information (features of the protagonist), intentional information and information about actions. Second, Reusser defined the representation of the temporal and causal structure of the events described in the problem as the goal of the creation of the episodic situation model. Based on this second definition, two new categories of situational information were added to the situational problems, namely, temporal and causal information, so that we ended up with five types of extra situational information. We predicted that children will solve both situationally and conceptually reworded problems more accurately than standard problems. Moreover, we anticipated that the positive impact of rewording on children's performance will be higher for problems with a more difficult semantic structure. Finally, because two-step problems are cognitively (much) more demanding than one-step problems, we predicted that not only younger but also older elementary children will profit from these rewordings.

 $<sup>^{\</sup>rm I}$  Furthermore, the results from a pilot study had revealed that these two-step problems were suitable for the whole age range

of our interest.

We decided not to include explicit information about the sets as being the parts of this whole set for two reasons. First, it would complicate too much the problem text, and second, following Gilabert, Martínez, and Vidal-Abarca (2005), the most effective way for including inferences is not to insert into the text all needed inferences, but only those that will allow and help children to make new inferences themselves. Therefore, we provided children with an extra textual cue about the conceptual role of the whole set, but they had to infer themselves which are the parts of this whole set.

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

836 Santiago Vicente et al.

#### Method

### Subjects

The experimental task was administered to a sample of 208 grades 3-5 children (79 third graders, 64 fourth graders and 65 fifth graders) from two schools of the city of Salamanca, Spain. Ninety-four of them were males and 114 females, and ages varied between 9 and 11 years. The mean age was 8 years 10 months for third graders, 9 years 5 months for fourth graders and 10 years 11 months for fifth graders.

#### Tasks

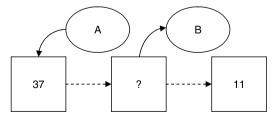
The experimental task included eight word problems: six experimental problems and two buffer items. The six experimental items were two-step change problems and involved two standard problems, two conceptually reworded problems (CRP) and two situationally reworded problems (SRP). We counterbalanced the order of the problems, so that the same problem did not appear in the same position across the different versions of the task. Moreover, we prevented that two problems with the same level of difficulty or with the same kind of rewording succeeded each other. The two buffer items were included to avoid stereotyped approaches and/or answers. These buffer items were also two-step problems, but they were compare instead of change problems, with the reference set unknown. There was no time limit, so that every student could spend the time he needed to solve them.

The standard problem text was as follows.

Peter had 37 metres of cable. He bought A metres of cable more. He used B metres of cable and he ended up with 11 metres of cable. How many metres of cable did he buy/use?

Figure 1 shows the structure of the above two-step change problem. It represents both the temporal structure of the problem and the actions and transformations specific of change problems. For this two-step problem, the first change situation begins with an initial state ('Peter had 37 metres of cable'), on which an action is executed ('he bought A metres of cable'), generating a resulting state represented in Figure 1 by the sign (?). This latter set is, at the same time, the initial set of the second change situation, on which a new action is executed ('he used B metres of cable') generating a final result set ('he ended up with 11 metres of cable').

Problems were considered to be more or less difficult depending on the set referred to in the question – the number of metres of cable being bought (set A, see Figure 1) or the number of metres of cable being used (set B) – because this determines, first, the



**Figure 1.** Problem structure of the two-step change problems used in the study. Squares represent static sets, that is start and/or result sets. Circles represent change sets. The normal arrows indicate the direction of the change: a transfer-in action is denoted by an arrow pointing to a start set and a transfer-out action by an arrow pointing to a change set. Dotted arrows indicate the temporal sequence of the problem.

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

Situational and conceptual rewording of word problems 837

level of difficulty of the two one-step change problems that constitute the two-step problem, and second, the possibility to follow the temporal sequence of the problem when solving it. We defined an easy problem as a problem that implies a combination of a change 1 and a change 4 situation (with the overall question referring to the unknown change set of the change 4 situation) and that can, therefore, be solved by simply following the temporal unfolding of the events described in the problem. Difficult problems were defined as problems that imply a combination of a change 3 and a change 6 situation (with the overall question referring to the unknown change set of the change 3 situation), and that have to be solved in a different than the actual sequence of the events denoted in the problem.

As already pointed out earlier, a conceptual reformulation clarifies the conceptual role of the total set shared by the two one-step change situations (i.e. the result set in the first change problem and at the same time the start set of the second change problem, represented in Figure 1 by a question mark). Following these ideas, the conceptual reworded problem (CRP) was as follows (additions to the standard form are put in italics):

Peter had 37 metres of cable. He bought A metres more *and joined them with those that he had. From the resulting total of metres of cable* he used B metres and he ended up with 11 metres of cable. How many metres of cable did he buy/use?

The second type of reformulation, namely situational rewording, was realized, as stated before, following Reusser's definitions of situational word problem and his episodic situation model, and, more specifically, by highlighting the intentional, causal and temporal structure of the situation described by the problem. The resulting situational reworded problem (SRP) version of the problems was as follows (again, additions to the standard form are put in italics):

Peter wants to renew his house's wiring. Peter still possesses 37 metres of cable from a previous renovation. As Peter realizes that these metres will not be enough cable for the whole installation, he bought A metres of cable more. After buying those metres of cable be began the renovation. While making the renovation he has used B metres of cable, and when he finishes he realizes that there remain 11 metres of cable. Peter wonders: How many metres of cable have I bought used?

To allow a rigorous comparative analysis of the three different wordings of the problem, we used the propositional analysis system developed by Graesser and Goodman (1985). This propositional analysis system assumes that the meaning that is embedded in a text can be 'represented as a network of labelled statement nodes that are interrelated by labelled, directed arcs' (p. 114). Applied to our topic, this implies that underlying every problem text there is a knowledge network consisting of two components: statement nodes, that is, the meaning units in which the text can be decomposed, and labelled arcs, which connect each statement node to another.<sup>3</sup> We carried out a propositional analysis to provide a formal account of the similarities and differences among the three kinds of problems. The standard version of the problem consisted of four statement nodes without any arcs linking these statements. Two extra action nodes and one arc were added to the standard version in the CRP. And, when compared with the standard problem, the SRP contained 11 new

<sup>&</sup>lt;sup>3</sup> The statement nodes are knowledge units corresponding to an event, state, process or action. Graesser and Goodman (1985) proposed six statement node categories: Physical State, Internal State, Physical Event, Internal Event, Goal and Style. Also, they categorized arcs into five categories: Reason, Initiate events, Consequence, Manner and Property.

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

#### 838 Santiago Vicente et al.

statement nodes and 11 new arcs. Finally, the propositional analysis revealed that, at the propositional level, there were no differences between the easy and the difficult problems. The internal-consistency reliability of the task was computed by means of Cronbach's alpha formula. The reliability of the task was 0.78.

#### **Procedure**

The task was administered during the normal school hours. The administration took place during two different moments of the same mathematics lesson, separated from each other by a break of 15 minutes. Task instructions stressed that in order to solve the problems comprehensive reading was required. There was no time limitation. All problems were distributed so that two problems with the same difficulty level or the same information did not appear consecutively.

#### Data coding

Children's responses were coded as correct (1 point) or incorrect (0 points). We considered as correct all solutions wherein the correct arithmetic operations with the appropriate numbers were chosen, without taking into account the computational exactness of the final result. We decided to code the data in this way because we wanted to assess children's problem comprehension rather than their calculation ability.

### Results and discussion

The results are shown in Table 3. The mean success rates were analysed in a 3 (grade: 3, 4 or 5) × 2 (difficulty: easy or difficult) × 3 (wording: standard, conceptual or situational) ANOVA with repeated measures on the last two factors. Effect size value is also included. All main effects proved significant. First, older children successfully solved more problems, F(2, 205) = 3.69, p < .03,  $\eta^2 = .03$ ; success rates were .51, .59 and .64 for grades 3, 4 and 5, respectively. Comparisons between means using the Tukey HSD procedure with p < .05 indicated that the differences were significant between grades 3 and 4, and between grades 3 and 5. Second, the mean success rates for easy problems (.81) were significantly higher than for difficult problems (.35), F(1, 205) = 257.64, p < .0001,  $\eta^2 = .53$ . Third, regarding wording type, the mean success rates were .55, .52 and .66 for standard, SRP and CRP, respectively, F(2, 410) = 18.98, p < .0001,  $\eta^2 = .15$ . The differences were significant between standard and CRP [F(1, 207) = 23.09, p < .0001,  $\eta^2 = .13$ ], and between SRP and CRP [F(1, 207) = 34.32, p < .0001,  $\eta^2 = .16$ ].

Table 3. Study 1, mean success rate per grade, difficulty level, and kind of rewording

		E	asy		Difficult		Total (grade)
Grade	Standar	Situational	Conceptual	Standar	Situational	Conceptual	Mean
3 (N = 79)	.74	.72	.73	.22	.17	.50	.51
4 (N = 64)	.84	.70	.85	.31	.35	.51	.59
5(N = 65)	.87	.89	.92	.33	.33	.44	.64
Mean	.82	.76	.83	.29	.28	.49	.58
Total (difficulty)			81			35	

Out of all two-way interactions, only the interaction between difficulty and wording was significant, F(2, 410) = 7.67, p < .0006,  $\eta^2 = .07$ . This interaction reflected the fact that the wording-type effect was only significant for difficult problems, F(2, 414) = 21.07, p < .0001,  $\eta^2 = .16$ , with success rates .29, .28 and .49 for standard, SRP and CRP, respectively. Significant differences were found between CRP and SRP, F(1, 207) = 30.61, p > .0001,  $\eta^2 = .13$ , and between CRP and standard, F(1, 207) = 27.63, p < .0001,  $\eta^2 = .13$ . For easy problems, the wording effect only approached significance, F(2, 414) = 2.54, p = .079, and the only significant effect was between CRP and SRP, F(1, 207) = 4.41, p < .04,  $\eta^2 = .29$ , with success rates of .83 vs. .76, respectively. Furthermore, difficulty × wording × grade interaction was significant, F(4, 410) = 2.78, p < .03,  $\eta^2 = .02$ .

Given our major interest, this three-way interaction effect was further explored by means of separate analyses for each grade level using a 2(difficulty) × 3(wording) repeated measures ANOVA. The data from the third graders mirror the earlier overall finding, F(2, 156) = 9.12, p < .0002,  $\eta^2 = .14$ , as, for this age group, the rewording effect was only significant for difficult problems, F(2, 156) = 16.80, p < .0001,  $\eta^2 = .28$ ; differences between standard and CRP were significant, F(1, 78) = 17.14, p < .0001,  $\eta^2 = .20$ , and also between SRP and CRP, F(1, 78) = 31.12, p < .0001,  $\eta^2 = .26$ . For the fourth graders, we also found a significant interaction effect,  $F(2, 126) = 6.135, p < .004, \eta^2 = .16$ . The wording effect was again significant for difficult problems, F(2, 126) = 6.26, p < .003,  $\eta^2 = .17$ . CRP were significantly easier than standard problems, F(1, 63) = 11.58, p < .002,  $\eta^2 = .17$ , and SRP, F(1, 63) = 5.98, p < .02,  $\eta^2 = .08$ . However, among these fourth graders, the rewording effect was also significant for easy problems, F(2, 126) = 5.75, p < .005,  $\eta^2 = .15$ , due to a negative influence of adding situational information. Therefore, easy SRP were significantly more difficult than standard,  $F(1, 63) = 8.19, p < .006, \eta^2 = .13$ , and CRP, F(1, 63) = 7.9, p < .007,  $\eta^2 = .12$ . Finally, for fifth graders, there was no significant interaction. Although the mean success rate was higher for difficult CRP (see Table 3) than for the other kinds of problem wordings (standard and SRP), planned comparisons showed that the differences did not reach significance, F(1, 64) = 2.38, p = .12.

In sum, results of Study 1 showed, on the one hand, that adding conceptual information resulted in all age groups in a significantly greater number of correct answers than the standard and situational forms for the difficult problems, but not for the easy ones. This facilitating effect of conceptual rewording was most evident among the youngest students, that is, grade 3 children, although it was still significant for fourth-grade students and marginally significant for fifth graders. Most probably, our finding that conceptual rewording was also helpful for older children was caused by the fact that we worked with two-step problems, whereas in previous investigations (wherein older children did not profit from this rewording), one-step problems were used (Davis-Dorsey et al., 1991; De Corte et al., 1985). Consequently, children from the older age groups kept profiting from reformulations aimed at clarifying the conceptual relationships underlying the difficult two-step problem. An unexpected result was that fifth graders reached a success rate that was not higher, but even slightly lower, than that of third and fourth graders, probably because some of these older children experienced an 'illusion of understanding' (Glenberg, Wilkinson, & Epstein, 1982; McNamara, Kintsch, Songer, & Kintsch, 1996) that prevented them from analysing carefully and mindfully the conceptual information present in these conceptually reworded difficult problems.

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

#### 840 Santiago Vicente et al.

On the other hand, our results also pointed out that adding markers for the temporal, intentional and functional structure of the problem did not result in any improvement in children's problem-solving performance. In retrospect, a plausible explanation for these outcomes is that the situational modifications inevitably led to problems that were much longer than the original standard ones, increasing the number of nodes and arcs, which might have made the problem text, may be, more accessible and comprehensible for (some) children, but, at the same time, inevitably resulted in a problem statement that was much longer and linguistically more complex, in terms of relations between nodes and of the resulting structure, than the original one. This explanation was tested in Study 2, wherein we tried to keep the extra text of the situationally reworded problems simple (when compared with the standard ones), by including only two kinds of situational additions, namely causal and temporal information, instead of the five information types used in our first study.

### STUDY 2

Based on the unexpected results of Study 1 for the situationally reworded problems, we decided to explore these results and their post boc interpretation in a second study that focused on situational rewording. Keeping in mind that including all five types of situational information that we derived from Reusser's theory inevitably results in a problematically extensive and linguistically complex problem text, we decided to drop the first three kinds of situational information that we derived from Reusser's theory (namely actions, descriptive, intentional information), and to keep only the last two types, namely, temporal and causal information. In other words, whereas in Study 1 we tried to develop a fully elaborated version of situational rewording strictly in line with Reusser's SPS theory, in Study 2 we took a more pragmatic position by restricting the situational enrichment to what we considered as absolutely crucial for providing a situationally enriched version and by erasing less important extra situational information. We decided not to include conceptual reformulations in Study 2, first because we were only interested in unravelling the puzzling negative impact of situational reformulations and including also conceptually reformulated problems would have made the experimental task more prone to learning and/or fatigue effects, and second, because the critical comparison was between these new types of situational reformulations of the word problems and the standard problems, and not with the conceptually reformulated ones, from Study 1.

As in Study 1, we predicted that children's performance would be higher for the situationally reworded problems than for the standard problems. These reworded problems would allow children to create a more elaborated episodic situation model (without leading to problem texts that were problematically long and linguistically complex), and thus would result in better performance than the standard problems.

### **Method**

### Subjects

The tasks were applied to 192 students (81 males and 111 females) from two different schools in the city of Salamanca, Spain. Out of these, 61 were third graders, 61 fourth graders and 70 fifth graders (mean age was 8 years 6 months for third graders, 9 years 4 months for fourth graders and 9 years 6 months for fifth graders).

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

Situational and conceptual rewording of word problems 841

### Task and procedure

The experimental task included 12 word problems: eight experimental problems and four buffer problems. These problems were split into two different tests, six problems each: four experimental problems and two filling problems. The application of the two tests was carried out in two different days, one for each application. Except that the two tests were administered on two different days, the tasks were administered under the same conditions as in Study 1. The procedure and data coding were the same as for Study 1.

Problems with temporal information. When compared with the standard problem, five new nodes were added in order to highlight the temporal structure of the problem situation. The resulting text was the following one (additions to the standard form are put in italics):

Peter had 37 metres of cable. Then he bought A metres of cable more. After buying those metres of cable be began a renovation. While making the renovation he has used B cable metres, and when he finishes there remain 11 metres of cable. How many meters of cable did he buy/use?

Problems with causal information. This version was designed to explicate reasons for the events that happened in the problem situation:

Peter had 37 metres of cable from a previous renovation. He bought A metres of cable more because he realized that he would need more cable. As he has made a wiring renovation, he has used B metres of cable, and there remain 11 metres of cable. Peter wonders: How many meters of cable did I buy/use?

Problems with temporal and causal information. This last type of problem reformulation is a combination of the two previous ones. Therefore, although it was longer and linguistically more complex than the versions with only causal or temporal information, it was still shorter and simpler than the situational version used in Study 1.

The internal-consistency reliability of the task, measured by means of Cronbach's alpha coefficient, was 0.82.

### Results and discussion

The results of Study 2 are shown in Table 4. The mean success rates were analysed with a 3 (grade: 3, 4 or 5)  $\times$  2 (difficulty: easy or difficult)  $\times$  4 (wording: standard, causal, temporal or complete) ANOVA with repeated measures on the last two factors. All main effects proved significant. First, as in Study 1, older children solved problems more successfully, F(2, 190) = 13.202, p < .001,  $\eta^2 = .12$ . Comparisons between means, using the Tukey HSD procedure (with p < .05), indicated that the differences between grades 3 and 4, between grades 3 and 5, and between grades 4 and 5 were all significant. As in Study 1, easy problems elicited a higher performance than difficult ones,  $F(1, 190) = 160.81, p < .0001, \eta^2 = .45,$  but, contrary to our expectations, no significant differences were found for rewording, F(3, 190) = 3.00, p > .14. Only one interaction proved significant: difficulty  $\times$  wording, F(1, 188) = 5.76, p = .001,  $\eta^2 = .08$ . For difficult problems, versions with extra causal information tended to be the easiest, with a significant difference with complete versions, F(1, 193) = 9.004,  $p < .005, \, \eta^2 = .04, \, \text{but not with standard problems} \, [F(1, 193) = 0.142, \, p > .70] \, \text{nor}$ with temporal versions [F(1, 193) = 2.71, p > .10]. For easy problems, the temporal versions were the easiest; they yielded a significantly higher performance than the

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

### 842 Santiago Vicente et al.

			Easy problems	\$			Diffic	Difficult problems	
	Standard	Causal	Temporal	Complete	Standard	Causal	Temporal	Complete	Mean (Grade)
Grade 3	09:	.49	.55	.52	41.	81.	Ξ:	0	.33
Grade 4	19:	.48	.74	99:	.26	.s.	.23	91.	.43
Grade 5	.72	.71	.79	.75	4.	.35	.35	.34	.55
Mean (rewording)	.64	.56	.70	.64	.27	.28	.23	71.	44.
Mean (Difficulty)			.63				.24		

 Table 4. Study 2, mean success rate per grade, difficulty level, and kind of rewording

Situational and conceptual rewording of word problems 843

causal versions, F(1, 190) = 12.93, p < .0001,  $\eta^2 = .06$ , but neither higher than the standard problems, F(1, 193) = 0.96, p > .32 nor the complete versions, F(1, 197) = 2.499, p > .110.

Regarding the influence of grade, planified comparisons on the interaction difficulty × wording × grade showed that for easy problems, temporal versions were solved significantly better than standard versions by fourth graders, F(1, 60) = 4.816, p < .04,  $\eta^2 = .07$ , whereas the better performance of the fifth graders did not reach significance, F(1, 67) = 1.683, p > .19. For difficult problems, causal versions tended to be solved more successfully than standard ones by third and fourth graders, but none of these differences was significant, F(1, 64) = 0.816, p > .35 and F(1, 60) = 1.000, p > .32, respectively.

The remarkable finding that the causal versions elicited most correct responses among the difficult problems but least correct answers among the easy ones was probably due to the way some children used the extra causal information. Some children may have used the joint cause for the existence of both the change set and the final set of the second change situation given in the causal versions as a superficial cue for arriving at the correct operations. More concretely, some children may have reasoned that as the causes of 'spending some meters of cable' and of 'the remaining of some cable' are the same (namely, the fact that 'he began a new renovation', see the example problem), these two quantities should be added, and afterwards they may have decided to take away the remaining data (namely the meters of cable he bought before the renovation) because it belongs to a different situation. However, for easy problems, this superficial cue in the problem statement towards the proper arithmetic operations is not available. As each set of the first change situation (of both easy and difficult problems) had its own cause (respectively 'because Peter did a previous renovation, he had some 47 cable meters left' and 'because he realized that he would need more, he brought some more') some children may have interpreted both sets from that first situation as being part of a different change situation, and because of this, they may have found it more difficult to add these two quantities as the first solution step of the 'easy' problems. Therefore, we claim that some children who correctly solved the causal versions of the difficult problems may have been using the extra causal information, not for improving their (deep) understanding of the complex situational model (as intended by the researchers), but merely as a superficial cue for (accidentally but correctly) adding quantities, without coming to a better or deeper understanding of the problem situation evoked by the problem text. Thus, there are good reasons to assume that improved comprehension of the causal structure was not responsible for pupils' slightly better achievement on the causal versions of the difficult problems when compared with the standard, temporal and complete ones.

In sum, the results obtained in Study 2 indicated that neither the temporal information, the causal information nor the combination of both the types of information yielded a significant improvement in children's solutions of two-step change problems when compared with standard versions. In addition, and in line with (our interpretation of) the results from Study 1, the complete version of the problems (involving both types of extra situational information) did not yield the expected highest success rate, but even resulted in a slightly lower success rate

<sup>&</sup>lt;sup>4</sup> But the temporal versions on easy problems for fourth graders.

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

#### 844 Santiago Vicente et al.

than the versions with only one type of added situational information. Therefore, even though the complete version in Study 2 contained already considerably fewer propositions than in Study 1, this complete version still did not yield the best results and even resulted in slightly worse performance than versions with fewer situational additions.

### **GENERAL DISCUSSION**

In the 1980s and 1990s, several studies on rewording arithmetic word problems have been realized. Generally speaking, in these studies, the problems were reworded in one of the following two ways. First, in a situational way, by enriching the problem statement with some extra pieces of information (such as motives, settings, time markers) with a view to allow children to generate a situational model containing the functional and/or the temporal structure of the problem. Second, in a conceptual way, by making more explicit the underlying semantic/mathematical relations between the given and the unknown sets than in the standard version, without affecting the underlying semantic/mathematical structure, and, in doing so, by helping the problem solver to build up the proper conceptual representation of the problem in terms of part-whole relationships. In previous empirical studies and theoretical analyses, these two kinds of rewordings were not explicitly distinguished and sometimes even confounded.

Starting from this categorization of different types of rewording, we set up two new empirical studies. In Study 1, we directly compared children's solutions of two different rewordings of a simple and a difficult two-step change problem: a situational reformulation, in which markers of the temporal, causal and intentional structure were added to the problem text, and a conceptual reformulation, in which the common whole set of the two simple word problems that constitute the two-step problem was explicitly marked. Results showed a facilitating effect of conceptual rewording, probably because it allowed mapping the problem text easily onto tacit knowledge concerning part-whole relations, while the situational reformulation did not result in any improvement in children's problem-solving performance. In our discussion of the results of Study 1, we pointed out that one explanation for these results might be that by having 'translated' Reusser's definitions of episodic situation model too strictly and too exhaustively, and by therefore having included an abundant amount of extra situational information, we may have turned the problem text into a problematically long and complex one. In this way, the cognitive load put on the (young) problem solvers may have become considerably bigger than for the standard and conceptually reworded versions of the problems. However, the results of Study 2 did not support this explanation. In this study, problems were reworded in such a way that only one kind of extra situational information (temporal or causal) or the combination of only these two situational additions was provided in the distinct conditions. This led to an important decrease in the number of nodes and linking arcs, and accordingly to linguistically considerably much shorter and less complex problem texts. However, results showed that this kind of 'textual economization' still did not result in a significant improvement of children's performance (except an isolated and restricted facilitative effect of temporal information on easy problems for fourth graders.)

After analysing the results of both studies, it is necessary to question why added situational information (as operationalized in both studies) did not help children to solve word problems more accurately. And, why some other studies did show positive results on situationally reworded problems. A possible explanation for why added situational

Situational and conceptual rewording of word problems 845

information was not helpful in our studies can be found in Sweller's (1999; Paas, Renkl, & Sweller, 2003) theory and is that there are three kinds of processing, competing for the resources of the learners' or problem solvers' working memory. First, intrinsic processing, which depends on the difficulty of the task and is fixed. Second, germane processing, allowing children to comprehend the information in a deeper way; this kind of processing must be controlled. And finally, extraneous processing, coming from irrelevant information, which must be eliminated from the task because it consumes resources from the working memory and is not useful at all. Relying on this theoretical framework, we tentatively conclude that, for word problem solving, extra conceptual information was directly relevant for the task (namely to find the semantic/mathematical structure 'hidden' in the word problem) and allowed children to develop germane processing, whereas added situational information was irrelevant and therefore caused an extraneous load. Furthermore, because easy problems were not so challenging and therefore required no additional information, all reworded versions did not yield better performance. However, our results concerning the effect of situationally reworded problems do not fit perfectly with Cognitive Load Theory (and also contrasts to recent results showed by Reed, 2006), because although situational rewording was not effective in our studies, it did not harm performance in some conditions through cognitive overload (as Cognitive Load Theory would predict).

It is interesting to note that recently Moreau and Coquin-Viennot (2003) found positive results on two-step problems that were reworded in a similar way as our situational problems. However, in their study, the children's task was not to solve the word problem, but (a) to select those pieces of information that make it easier to understand and (b) to make the problem as short as possible. Clearly, these two tasks come closer to text comprehension than to problem solving and therefore extra situational information was more relevant. Nevertheless, we hypothesize that when pupils would be confronted with situationally ambiguous, unclear or unfamiliar word problems, adding extra situational information should also become highly useful and lead to considerable positive effects of situational reformulation.

A closer look at the results of the previous studies that seem to be in contrast with ours, using our distinction between situational and conceptual rewordings as an analytical scalpel, suggests that all these findings are less contradictory than they seem at first sight. For example, Hudson (1983) circumvented the linguistic problems related to the complex 'how many more... than...' phrase by altering the question of his compare problems. But at the same time, in doing so, he also (deliberately or undeliberately) drastically changed the standard compare problem into a more dynamic and concrete situation which allowed children to use their (intuitive or informal) conceptual knowledge about comparison of and relationships between sets. Similarly, the rewording of Stern and Lehrndorfer (1992) provided children with an extra cue regarding which the bigger set is in a comparative situation; hence this kind of situational rewording also involves a kind of conceptual component (namely an extra cue as to which of the sets is the whole in the underlying part-whole structure). Therefore, in both the cases, the rewording was not a pure case of situational rewording, but rather a combination of situational and conceptual rewordings. These illustrative analyses show that our distinction between situational and conceptual reformulations helps us in characterizing the design of the different studies and in explaining their (seemingly) contrasting results.

In sum, conceptual rewording of two-steps change problems has shown to be useful for improving the achievement of third through fifth graders, while extra situational

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

#### 846 Santiago Vicente et al.

information did not result in better performance, neither in the full version (Study 1) nor in the shorter, causal and/or temporal versions (Study 2). Using Sweller's cognitive load theory, we have interpreted his latter failure as due to the irrelevance of this added situational information in relation to the specific difficulties and goals of the task and to the specific kind of goal-oriented processing required. Does this mean that Reusser's model (Reusser, 1985; Staub & Reusser, 1992) and the (theoretical and educational) implications that we have derived from it, are wrong? The answer is no. Rather, we claim that the negative results of our studies with respect to the facilitative effect of situational rewording were due to the fact that the children's major difficulty with these two-step problems was not to understand the situation in which the arithmetic problem was embedded; therefore, for these problems, getting additional situational information was not their major need and acted - in Sweller's (Paas et al., 2003; Sweller, 1999) terminology - even as an extra cognitive burden. That is, the difficulty of these problems was conceptual, not situational, and because of this, the only reworded problems that improved children's achievement were the conceptual versions, and not situational versions. However, for other word problems, where the construction of a proper episodic situational model becomes a major challenge, efforts to elaborate the problem text with situational enrichments may yield the expected positive impact. However, the present studies have revealed that such attempts to provide situationally richer problems necessarily result in texts that are linguistically more extensive and more complex (which might work against the positive effects of the situational enrichment), especially in younger children with poorer (technical) reading abilities. Providing this situational enrichment through non-textual means like pictures, animations, etc. might be a - both theoretically and educationally - promising alternative. The investigation of this latter suggestion is the topic of our current research.

### Acknowledgements

This research was supported by Grant BSO2003-05075 to J. Orrantia from the Ministerio de Ciencia y Tecnología in Spain and by Grant GOA 2006/01 from the Research Fund K.U. Leuven, Belgium. S. Vicente also gratefully acknowledges the support provided by the Junta de Castilla y León in Spain.

### References

- Briars, D. J., & Larkin, J. H. (1984). An integrated model of skill in solving elementary word problems. *Cognition and Instruction*, *1*, 245–296.
- Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, 15, 179–202.
- Cummins, D. D. (1991). Children's interpretations of arithmetic word-problems. *Cognition and Instruction*, *8*, 261–289.
- Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology*, 20, 405-438.
- Davis-Dorsey, J., Ross, S. M., & Morrison, G. R. (1991). The role of rewording and context personalization in the solving of mathematical word-problems. *Journal of Educational Psychology*, 83, 61-68.
- De Corte, E., & Verschaffel, L. (1988). Computer simulation as a tool in research on problem solving in subject-matter domains. *International Journal of Educational Research*, 12, 49-69.

#### Situational and conceptual rewording of word problems 847

- De Corte, E., Verschaffel, L., & De Win, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology*, 77, 460-470.
- Fuson, K. C. (1992). Research on learning and teaching addition and subtraction whole numbers. In G. Leinhardt, R. Putnam, & R. A. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 53–187). Hillsdale, NJ: Erlbaum.
- Gilabert, R., Martínez, G., & Vidal-Abarca, E. (2005). Some good texts are always better: Text revision to foster inferences of readers with high and low prior background knowledge. *Learning and Instruction*, *15*, 45–68.
- Glenberg, A. M., Wilkinson, A. A., & Epstein, W. (1982). The illusion of knowing: Failure in the assessment of comprehension. *Memory and Cognition*, 10, 597-602.
- Graesser, A. C., & Goodman, S. M. (1985). Implicit knowledge, question answering, and the representation of expository text. In B. K. Britton & J. B. Black (Eds.), *Understanding expository texts* (pp. 109–171). Hillsdale,NJ: Erlbaum.
- Hudson, T. (1983). Correspondences and numerical differences between disjoint sets. Child Development, 54, 84-90.
- Kintsch, W., & Greeno, J. (1985). Understanding and solving word arithmetic problem. Psychological Review, 92, 109-129.
- McNamara, D., Kintsch, E., Songer, N., & Kintsch, W. (1996). Are good texts always better? Interactions of text coherence, background knowledge, and levels of understanding in learning from text. *Cognition and Instruction*, 14, 1–43.
- Moreau, S., & Coquin-Viennot, D. (2003). Comprehension of arithmetic word problems by fifth-grade pupils: Representations and selection of information. *British Journal of Educational Psychology*, 73, 109–121.
- Paas, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational Psychologist*, 38, 1-4.
- Reed, S. K. (1999). Word problems: Research and curriculum reform. Mahwah, NJ: Erlbaum.
- Reed, S. K. (2006). Does unit analysis help students construct equations? *Cognition and Instruction*, 24(3), 341–366.
- Reusser, K. (1985). From situation to equation. On formulation, understanding and solving situation problems. Boulder, Colorado: Institute of Cognitive Science, Technical Report No. 143.
- Reusser, K. (1990). From text to situation to equation: Cognitive simulation of understanding and solving mathematical word problems. In H. Mandl, E. De Corte, N. Bennett, & H. F. Friedrich (Eds.), *Learning and instruction* (Vol. 2, pp. 477–498). Oxford: Pergamon.
- Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities of solving problems. *Cognition and Instruction*, *5*(1), 49-101.
- Riley, N. S., Greeno, J., & Heller, J. I. (1983). Development of children's problem solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153–196). New York: Academic Press.
- Sophian, C., & McCorgray, P. (1994). Part-whole knowledge and early arithmetic problem-solving. *Cognition and Instruction*, *12*, 3–33.
- Sophian, C., & Vong, K. I. (1995). The parts and wholes of arithmetic story problems: Developing knowledge in the preschool years. *Cognition and Instruction*, *13*, 469-477.
- Staub, F., & Reusser, K. (1992, April). *The role of presentational factors in understanding and solving mathematical word problems*. Paper presented at the meeting of the American Educational Research Association, San Francisco.
- Staub, F., & Reusser, K. (1995). The role of presentational structures in understanding and solving mathematical word problems. In C. A. Weaver, III, S. Mannes, & C. Fletcher (Eds.), *Discourse comprehension: Essays in bonor of Walter Kintsch* (pp. 285–305). Hillsdale, NJ: Erlbaum.
- Stern, E., & Lehrndorfer, A. (1992). The role of situational context in solving word-problems. *Cognitive Development*, 7, 259–268.

Reproduction in any form (including the internet) is prohibited without prior permission from the Society

### 848 Santiago Vicente et al.

- Sweller, J. (1999). *Instructional design in technical areas*. Camberwell, Victoria, Australia: Australian Council for Educational Research.
- Verschaffel, L., & De Corte, E. (1993). A decade of research on word-problem solving in Leuven: Theoretical, methodological and practical outcomes. *Educational Psychology Review*, 5, 239–256.
- Verschaffel, L., & De Corte, E. (1997). World problems: A vehicle for promoting authentic mathematical understanding and problem solving in the primary school. In T. Nunes & P. Bryant (Eds.), *Learning and teaching mathematics*. *An international perspective* (pp. 69–97). Hove: Psychology Press.

Received 28 October 2005; revised version received 12 December 2006