N = 2 Supersymmetric Kinks and real algebraic curves

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Abstract

The kinks of the (1+1)-dimensional Wess-Zumino model with polynomial superpotential are investigated and shown to be related to real algebraic curves.

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1

The dimensional reduction of the (3+1)-dimensional Wess-Zumino model, produces an interesting (1+1)-dimensional Bose-Fermi system; this eld theory enjoys N = 2 extended supersymmetry provided that the interactions are introduced via a real harmonic superpotential, see [1]. In a recent paper [2], Gibbons and Townsend have shown the existence of domain-wall intersections in the (3+1)D WZ model, the authors relying on the supersymmetry algebra of the (2+1)D dimensional reduction of the system. Although the domain-wall junctions are two-dimensional structures, their properties are reminiscent of the one-dimensional kinks from which they are made. In this letter we shall thus describe the kinks of the underlying (1+1)-dimensional system.

The basic fields of the theory are:

Two real bosonic elds, \( \phi_a(x) \), \( a = 1; 2 \) that can be assembled in the complex eld: \( \chi(x) = \phi^0(x) + i \phi^1(x) \). \( M \) maps \((R^2; C)\), \( x = (x^0; x^1) \) are local coordinates in the \( R^{1; 2} \) Minkowski space, where we choose the metric \( g^{00} = 1; g^{11} = 1; g^{12} = g^{21} = 0 \).

Two Majorana spinor elds \( \psi^a(x) \), \( a = 1; 2 \). We work in a Majorana representation of the Clifford algebra \( \gamma_a \); \( g = 2 \gamma \).

where \( 1 \), \( 2 \), \( 3 \) are the Pauli matrices, such that \( \gamma^a = \gamma^a \). We also denote the adjoint spinors as \( \psi^a(x) = \psi^b(x) 0 \) and consider Majorana-Weyl spinors: \( \psi^a(x) = \frac{1}{\sqrt{2}} \psi^a(x) \), \( \psi^a(x) \psi^a(x) \) with only one non-zero component.

Interactions are introduced through the holomorphic superpotential: \( \mathcal{W}(\phi, \chi) = \frac{1}{2} W^1(\phi, \chi) + i W^2(\phi, \chi) \).

One could in principle start from the supercharges:

\[
Q^{BC} = \int dx^1 X^{BC} \psi^a (\theta_a \gamma^b \theta_b) \psi^a \psi^a (\theta_a \gamma^b \theta_b) \psi^a \# \]

where \( W^B, B = 1; 2 \), are respectively the real part if \( B = 1 \) and the imaginary part if \( B = 2 \), \( W(\phi, \chi) \) and \( \psi^a \) is either the identity or the complex structure endomorphism in \( R^2 \).

where

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}
\]
Nevertheless, the Cauchy-Riemann equations:

$$\frac{\partial W^1}{\partial 1} = \frac{\partial W^2}{\partial z} \quad \frac{\partial W^1}{\partial z} = \frac{\partial W^2}{\partial 1};$$

tell us that the theory is fully described by choosing either $W^1$ or $W^2$. We thus set $W^C = W^1$ and find the basic SUSY charges to be $Q^B = Q^B$:

$$Q^B = \int \frac{dx}{x} f^B_{ab} (\theta_0^a \theta_1^a) \frac{\partial W^1}{\partial b} a^b \frac{\partial W^1}{\partial a^b}$$

(2)

From the canonical quantization rules

$$[a(x_1), b(y_1)] = i^{ab} (x_1, y_1) = f^a(x_1); i^b(y_1) g$$

one checks that the $N = 2$ extended supersymmetric algebra

$$fQ^B; Q^C g = 2 \frac{b d^c}{P} \quad fQ^B; Q^C g = (1) (1) \frac{b d^c}{2} + \frac{b d^c}{2}$$

is closed by the four generators $Q^B$, described in [3]. Here

$$P = \frac{1}{2} \int dx^1 X (\theta_0^a \theta_1^a)(\theta_0^a \theta_1^a) Z a^a_1 + a^a_2 + \frac{1}{2} \int dx^1 X (\theta_0^a \theta_1^a)(\theta_0^a \theta_1^a) Z a^a_1 +$$

are the light-cone momenta and

$$T = \int dx^1 \frac{\partial W^1}{\partial \theta_1^1} + \frac{\partial W^1}{\partial \theta_2^1} \frac{\partial W^1}{\partial x^1} = \frac{\partial W^1}{\partial \theta_1^1} \frac{\partial W^1}{\partial \theta_2^1} \frac{\partial W^1}{\partial x^1}$$

the central extensions.

2

From the SUSY algebra one deduces,

$$2P_0 = 2 \frac{a}{\partial j^+} (Q^B, (1)^B Q^B) j^3 = 2 \frac{a}{\partial j^+} (Q^B, (1)^B Q^C) j^3;$$

see [4]. We thus define the charge operators on zero momentum states:

$$Q^1 = Q^1, \quad Q^1 = \int dx^1 \frac{\partial W^1}{\partial \theta_1^a + \theta_2^a \frac{\partial W^1}{\partial x^1}}$$

$$Q^2 = \frac{\partial W^1}{\partial \theta_1^a + \theta_2^a \frac{\partial W^1}{\partial x^1}}$$

Spatially extended coherent states built from the solutions of any of the two systems of first order equations, [4]:

$$\frac{d^1}{dx} = \frac{\partial W^1}{\partial 1} \quad \frac{d^2}{dx} = \frac{\partial W^1}{\partial 2} \quad \frac{d^3}{dx} = \frac{\partial W^1}{\partial 3} \quad \frac{d^4}{dx} = \frac{\partial W^1}{\partial 4}$$

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have minimum energy because they are respectively annihilated by $Q^1$ (system [3]) and $Q^2$ (system [4]).

The ow in $R^2 \setminus \mathbb{C}$ of the solutions of [3] is given by:

$$\frac{d^2}{dz} = \frac{\partial W}{\partial z} \frac{\partial W}{\partial z}^1 \frac{\partial W}{\partial z}^2 \frac{\partial W}{\partial z}^2 \frac{\partial W}{\partial z}^2 = dW^2 = 0$$

If $W(\cdot)$ is polynomic, the solutions of [3] live on the real algebraic curves determined by the equation:

$$W^2(1; 2) = \gamma$$

where $\gamma$ is a real constant. Similarly, the solutions of $\{3\}$ in $\mathbb{C}$

$$\frac{d^2}{dz} = \frac{\partial W}{\partial z} \frac{\partial W}{\partial z}^1 \frac{\partial W}{\partial z}^2 \frac{\partial W}{\partial z}^2 \frac{\partial W}{\partial z}^2 = dW^2 = 0$$

run on the real algebraic curves:

$$W^2(1; 2) = \gamma$$

where $\gamma$ is another real constant. There are two observations: (I) Solutions of system [3] live on curves for which $W^2 = \text{constant}$ and solutions of [3] have support on curves for which $W^1 = \text{constant}$. (II) The curves that support the solutions of [3] are orthogonal to the curves related to the solutions of [3].

A sume that $W(\cdot)$ has a discrete set of extremes, including the vacuum orbit of the system: $\frac{\partial W}{\partial z}^i = 0, i = 1; 2; \ldots; m$. Kinks are solutions of [3] and/or [3] such that they tend to $v_{i}^{(i)}$ when $x_i$ reaches $1\cdot v_{i}^{(i)}$ and $v_{i}^{(i)}$ thus exit either to curves [3] or [3], and this xes the values of $\gamma$ for which the real algebraic curves support kinks. In Reference [3], a general proof based in singularity theory of the existence of these solutions, that counts its number, is achieved. The energies of the states grown from kinks are $P_0 = \int \mathcal{J}_i = W^2(v_{i}^{(i)}) W^2(v_{i}^{(i)})$ for solutions of [3] and $P_0 = \int \mathcal{J}_i = W^2(v_{i}^{(i)}) W^2(v_{i}^{(i)})$ for solutions of [3]. The kink form factor is obtained from a quadrature: one replaces either [3] or [3] in the rst equation of [3] or [3] and integrates.

Therefore, the fermionic charges $Q^1$ and $Q^2$ are annihilated on coherent states $K^1$ and $K^2$ that correspond to the tensor product of the quantum antikink/kink, living respectively on curves $W^2 = \text{constant}$ and $W^1 = \text{constant}$, with its supersymmetric partners (the translational in time as a constant spinor). We end

$$Q^1 K^1 = \int_a^X dx_1 \frac{a_{K^2}}{4} \frac{a_{K^1}}{\theta} \frac{\partial W}{\partial z}^1 \frac{\partial W}{\partial z}^2 \frac{\theta}{\theta_{1}^a} \frac{1}{\theta_{1}^b} K^1 = 0$$

$$Q^2 K^2 = \int_a^X dx_1 \frac{a_{K^2}}{4} \frac{a_{K^1}}{\theta} \frac{\partial W}{\partial z}^1 \frac{\partial W}{\partial z}^2 \frac{\theta}{\theta_{1}^a} \frac{1}{\theta_{1}^b} K^2 = 0$$

$$Q^2 K^2 = \int_a^X dx_1 \frac{a_{K^2}}{4} \frac{a_{K^1}}{\theta} \frac{\partial W}{\partial z}^1 \frac{\partial W}{\partial z}^2 \frac{\theta}{\theta_{1}^a} \frac{1}{\theta_{1}^b} K^2 = 0$$

on solutions of (7) and/or (8); the SUSY kinks are thus $\frac{1}{2}$-BPS states. The energy of these states does not receive quantum corrections [3], because $N = 2$ supersymmetry forbids any anomaly in the central charges.

3

We focus on the case in which the potential is:

$$U(\gamma) = \frac{1}{2} \sum_{a} \frac{\partial W}{\partial z} \frac{\partial W}{\partial z}^1 \frac{\partial W}{\partial z} \frac{\partial W}{\partial z}^2 \frac{\theta}{\theta_{1}^a} \frac{1}{\theta_{1}^b} + 2(\frac{\gamma}{\gamma} + \frac{\gamma}{\gamma}) \cos (n \cdot \gamma) \arctan \frac{\gamma}{\gamma} \gamma + (\frac{\gamma}{\gamma} + \frac{\gamma}{\gamma})^2 \gamma$$

see [3] and [4]. In polar variables in the $R^2$ internal space,
\[
(x_1) = +\frac{p}{1} \left[ \frac{1}{2}(x_1)^2 + \frac{1}{2}(x_2)^2 \right];
(x_2) = \arctan \frac{2(x_2)}{1(x_1)}
\]

the potential reads:
\[
U(\mathbf{r}) = \frac{1}{2} n^2 \cos(n - 1) + 2(n - 1)
\]

There is symmetry under the \(D_{(n - 1)} Z_{2} Z_{n - 1}\) dihedral group:
\[
\begin{align*}
\phi &= 0, \quad \phi = \frac{2\pi}{n}; j = 0; 1; 2; \ldots; n - 2.
\end{align*}
\]

In Cartesian coordinates, these transformations form the \(D_{(n - 1)} Z_{2}\) sub-group of \(O(2)\) given by:
\[
\begin{align*}
(1) \quad \phi &= \frac{2\pi}{n}; j = 0; 1; 2.
\end{align*}
\]

The vacuum orbit is the set of \(n - 1\)-roots of unity:
\[
M = \frac{n}{\phi} = e^{i\frac{2\pi}{n}} = \frac{D_{(n - 1)} Z_{2}}{Z_{n - 1}} = Z_{n - 1};
\]

When the \(\phi^{(k)}\) vacuum is chosen to quantize the theory, the symmetry under the \(D_{(n - 1)} Z_{2}\) group is spontaneously broken to the \(Z_{2}\) sub-group generated by \(\phi = \frac{2\pi}{n}; 2\pi\); this transformation leaves a fixed point, \(\phi^{(k)}\), if \(n\) is even and two fixed points, \(\phi^{(k)}\) and \(\phi^{(k + \frac{1}{2})}\), if \(n\) is odd.

The \(Z_{n - 1}\) symmetry allows for the existence of \(n - 1\) harmonic superpotentials that are equivalent:
\[
W^{(j)}(\phi) = \frac{1}{n} \prod_{a=1}^{n-1} (\phi - \phi_{a})^{(1)}\phi_{1}^{(1)}; (j)^{0} = e^{i\frac{\phi}{n}}; \quad (j)^{1} = e^{i\frac{\phi}{n}}
\]

where \(\phi = \frac{2\pi}{n}\). There is room for closing the \(N = 2\) supersymmetry algebra \(\{\}\) in \(n - 1\) equivalent forms: denote the \(n - 1\) equivalent sets of SUSY charges:
\[
Q^{(j)^{0}} = \frac{1}{n^2} \prod_{a=1}^{n-1} f_{a}^{(1)} \phi_{a}^{(1)} = \frac{1}{n} \prod_{a=1}^{n-1} f_{a}^{(1)} \phi_{a}^{(1)}
\]

also in terms of the "rotated" fermionic fields \(\phi^{(j)^{0}}\), and the corresponding central charges \(T^{(j)^{0}}\) and \(T^{(j)^{1}}\).

Observe that the \(N = 2\) supersymmetry is unbroken, while the choice of vacuum that spontaneously breaks the \(Z_{n - 1}\) symmetry does not affect the physics, which is the same for different values of \(j\).

The \(j\) pairs of first-order systems of equations:
\[
\begin{align*}
\frac{d}{dx_{1}} = \sin (j)^{0}; n - 1 \sin n (j); 2 \frac{d}{dx_{1}} = \cos (j) n \cos n (j); (j)^{0} = \cos (j) n \cos n (j) \\
\frac{d}{dx_{1}} = \cos (j) n - 1 \cos n (j); 2 \frac{d}{dx_{1}} = \sin (j) + n \sin n (j); (j)^{1} = \sin (j) + n \sin n (j)
\end{align*}
\]

correspond to \(\{\}\) and \(\{\}\) for this particular case. The solutions lie respectively on the algebraic curves
\[
\frac{n}{n} \sin (j) = \sin (j)^{0}; n = \sin (j)^{1}; n = \cos (j) n \cos n (j)
\]

which form two families of orthogonal lines in \(R^{2}\). In the family of curves \(\{\}\) there are kinks joining the vacua \(\phi^{(k)}\) and \(\phi^{(k + \frac{1}{2})}\) if and only if:
\[
\begin{align*}
\sin \frac{2}{n} (k + \frac{1}{2}) n = \sin \frac{2}{n} (k + \frac{1}{2}) n; \quad \frac{1}{n} \sin \frac{2}{n} (k + \frac{1}{2}) n = \frac{1}{n} \sin \frac{2}{n} (k + \frac{1}{2}) n
\end{align*}
\]

\[
\frac{k}{n} = \frac{1}{n}
\]

\[
\begin{align*}
\frac{2}{n} (k + \frac{1}{2}) n = \frac{2}{n} (k + \frac{1}{2}) n; \quad \frac{1}{n} \sin \frac{2}{n} (k + \frac{1}{2}) n
\end{align*}
\]
This fixes the value of \( \gamma = \frac{K}{\beta} \) for which the algebraic curve supports a topological kink. Similarly, \( \cos \frac{2}{n} (k + j) \) is the value of the constant if the kink belong to the orthogonal family (14). Solutions of (15) and/or (16) exist, respectively, if and only if
\[
2(k + k_0 + 2j) = n \pmod{2(n + 1)}
\]
and/or
\[
k + k_0 + 2j = 0 \pmod{n + 1}
\]
Given the kink curves, the kink form factors are obtained in the following way: One solves for in (12) or (11),
\[
+ \frac{2}{n} j = h(\frac{K}{\beta};) \quad ; \quad + \frac{2}{n} j = h_\gamma(\frac{K}{\beta};)
\]
and plugs these expressions into the first equation of (12) or (11),
\[
\frac{d}{dx_1} = \sin h(\frac{K}{\beta};) \quad ; \quad \frac{d}{dx_1} = \cos h_\gamma(\frac{K}{\beta};)
\]
which are immediately integrated by quadratures: if \( a \) is an integration constant
\[
Z \quad \frac{d}{\sin h(\frac{K}{\beta};) \quad \sin h(\frac{K}{\beta};)} = (x_1 + a)
\]
\[
Z \quad \frac{d}{\cos h_\gamma(\frac{K}{\beta};) \quad \sin h_\gamma(\frac{K}{\beta};)} = (x_1 + a)
\]

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We first consider the lower odd cases, only for \( W (j = 0) \). The other kinks are obtained by application of a \( Z_{n+1} \) rotation.

\( n = 3 \):

\{Superpotential: \( W (\gamma) = \frac{1}{2} \)

\(W^1 = \frac{3}{3} + 1 \frac{2}{2} \) \quad \(W^2 = 2 \frac{2}{2} \frac{2}{2} + \frac{3}{3} \)

\{Potential: \( U (1; 2) = \frac{1}{2}[(1)2 + 4 \frac{2}{2}] \)
\{Vacuum orbit: \( M = \frac{a_1}{a_2} = f(y^0 = 1; y^1 = 1g) \)
\{Real algebraic curves:

\( 1 \frac{3}{3} + 1 \frac{2}{2} = \) \quad \(2 \frac{2}{2} + \frac{3}{3} = \)

\{Kink curve: \( \gamma = 0 \) (\( W^2 = 0 \)), tantam out to \( 2 = 0 \).
\{Kink form factor:

\( a) \) Solutions of \( K^1 = 0 \) \( (x_1) = \tanh(x_1 + a) \)
\{Kink energy: \( P_0 [K^1] = \gamma_j = W^1(y^0) \quad W^1(y^1) = 4 \)
\{Conserved SU SY charge: \( \xi^1 K^1 = 0 \)

5
n = 5:

\{ Superpotential: \( W(\cdot) = \frac{1}{2} \frac{1}{3} \)

\[ W^1 = 1 \ 1 \ \frac{4}{5} + 2 \frac{2}{1} \frac{2}{2} \frac{4}{2} \quad W^2 = 2 \ 1 \ \frac{4}{1} + 2 \frac{2}{1} \frac{2}{2} \frac{4}{2} \]

\{ Potential: \( U(\cdot; \cdot) = \frac{1}{2} (1 + 1)^2 (1 + 1)^2 (1 + 1)^2 \)

\{ Vacuum orbit: \( M = \frac{\partial}{\partial x^k} \) \( f_{\nu^0} = 1; \nu^1 = i; \nu^2 = 1; \nu^3 = i \g \)

\{ Real algebraic curves: \( \frac{1}{2} + 2 \frac{2}{1} \frac{2}{2} \frac{4}{2} \quad 2 \ 1 \ \frac{4}{1} + 2 \frac{2}{1} \frac{2}{2} \frac{4}{2} \quad \gamma \)

\{ Kink curves: a) \( \gamma = 0 \quad 2 = 0, b) \quad 0 \quad 1 = 0 \).

\{ Kink form factor: \n
a) Solutions of \( \frac{d}{dx^2} = 1 \quad \frac{4}{2} \) on \( 2 = 0: \) \( \arctan \frac{K^1}{1} + \arctanh \frac{K^1}{1} = 2(x_1 + a) \)

b) Solutions of \( \frac{d}{dx^2} = 1 \quad \frac{3}{2} \) on \( 1 = 0: \) \( \arctan \frac{K^2}{2} + \arctanh \frac{K^2}{2} = 2(x_1 + a) \)

\{ Kink energies: \( a) P_0[\cdot] = \mathcal{F} j = W^1(v^2) \quad W^1(v^2) = \frac{8}{9} \)

\( b) P_0[\cdot] = \mathcal{F} j = W^2(v^1) \quad W^2(v^1) = \frac{8}{9} \)

\{ Conserved SUSY charges: \( a) Q^1 K^1 = 0; b) Q^2 K^2 = 0 \)

n = 7:

\{ Superpotential: \( W(\cdot) = \frac{1}{2} \frac{7}{7} \)

\[ W^1 = 1 \ \frac{7}{7} + 3 \frac{5}{1} \frac{2}{2} 5 \frac{3}{1} \frac{4}{2} + 1 \frac{6}{2} \quad W^2 = 2 \ \frac{6}{1} + 5 \frac{4}{1} \frac{3}{2} 3 \frac{2}{1} \frac{5}{2} + \frac{7}{7} \]

\{ Potential: \( U(\cdot; \cdot) = \frac{1}{2} (1 \ 1)^6 2(\frac{7}{1} \frac{2}{2}) (1 \ 2)^2 16 \frac{2}{1} \frac{2}{2} + 1 \)

\{ Vacuum orbit: \( M = \frac{\partial}{\partial x^k} \) \( V^0 = 1; \nu^1 = \frac{1}{2} + \frac{P^P}{2}; V^2 = \frac{1}{2} + \frac{P^P}{2}; V^3 = 1; \nu^4 = \frac{1}{2} + \frac{P^P}{2}; V^5 = \frac{1}{2} + \frac{P^P}{2} \)

\{ Real algebraic curves: \( \frac{7}{7} + 3 \frac{5}{1} \frac{2}{2} 5 \frac{3}{1} \frac{4}{2} + 1 \frac{6}{2} \quad 2 \ \frac{6}{1} + 5 \frac{4}{1} \frac{3}{2} 3 \frac{2}{1} \frac{5}{2} + \frac{7}{7} \quad \gamma \)

\{ Kink curves: there are two choices of \( \gamma \) and three choices of \( P \) for which one ends kink curves. The other kinks associated with the other superpotentials can be obtained by \( Z_6 \) rotations.

a) \( \gamma = \frac{3P^P}{7} \): kink curve joining \( v^1 \) with \( v^2 \)

\( \gamma = \frac{P^P}{7} \): kink curve joining \( v^4 \) with \( v^5 \)

b) \( = \frac{1}{7} \): kink curve joining \( v^1 \) with \( v^3 \)

\( = \frac{1}{7} \): kink curve joining \( v^2 \) with \( v^4 \)

c) \( = 0 \): kink curve joining \( v^3 \) with \( v^5 \)

\{ Kink energies: \( a) P_0[\cdot] = \mathcal{F} j = W^1(v^k) \quad W^1(v^k+1)j = \frac{6}{7} \)

\( b) P_0[\cdot] = \mathcal{F} j = W^2(v^k+2)j = \frac{6}{7} \)

\( c) P_0[\cdot] = \mathcal{F} j = W^1(v^k+3)j = \frac{6}{7} \)

\{ Conserved SUSY charges: \( a) Q^1 K^1 = 0, b) \) and \( c) Q^2 K^2 = 0 \)
We now study two even cases.

The first and most interesting model occurs for \( n = 4 \). Here, we find that the kink curves are straight lines in \( W \)-space (true for any \( n \)) and curved in \(-W\)-space, in agreement with Reference [3]:

* Superpotential: \( W \) $\begin{array}{c}
\begin{array}{c}
W^1 = 1 \frac{6}{6} + \frac{5}{2} \frac{4}{2} + \frac{5}{2} \frac{4}{2} + \frac{5}{2} \frac{4}{2} = \frac{6}{6} \\
W^2 = 2 \frac{6}{6} + \frac{10}{1} \frac{3}{2} \frac{1}{2} + \frac{10}{1} \frac{3}{2} \frac{1}{2} = ?
\end{array}
\end{array}$

* Potential: \( U(1;2) = \frac{1}{2} \left( \frac{5}{2} \right)^2 2 \left( \frac{5}{2} \frac{4}{2} \frac{5}{2} \frac{4}{2} \frac{5}{2} \frac{4}{2} \right) + 1 

* Vacuum orbit: \( M = \frac{d}{dz} f v^0 = 1; v^1 = \frac{1}{2} + \frac{1}{2} \frac{P}{P} ; v^2 = \frac{1}{2} + \frac{1}{2} \frac{P}{P} 

* Real algebraic curves:

* Kink curve: $\frac{3}{1} + \frac{3}{1} + \frac{3}{1} + \frac{3}{1} = ?$

* Kink form factor: on the kink curve we nd $\frac{k}{1} = f^{-1} (x+2) \text{where}

\[
f(1) = \frac{2}{1 + 4 \frac{1}{1} + 8 \frac{1}{1} + 3 \frac{2}{1} + 2 \frac{1}{1} + 4 \frac{1}{1} + 8 \frac{1}{1}}
\]

* Kink energy: $P_0[K^2] = \frac{f}{f} j = W^2(v^k) W^2(v^{k+1}) j = \frac{P}{P}$

* Conserved \( SU(2) \) charge: $Q^2 K^2 = 0$

\( n = 6: \)

* Superpotential: \( W \) $\begin{array}{c}
\begin{array}{c}
W^1 = 1 \frac{6}{6} + \frac{5}{2} \frac{4}{2} + \frac{5}{2} \frac{4}{2} + \frac{5}{2} \frac{4}{2} + \frac{5}{2} \frac{4}{2} = \frac{6}{6} \\
W^2 = 2 \frac{6}{6} + \frac{10}{1} \frac{3}{2} \frac{1}{2} + \frac{10}{1} \frac{3}{2} \frac{1}{2} = ?
\end{array}
\end{array}$

* Potential: \( U(1;2) = \frac{1}{2} \left( \frac{5}{2} \right)^2 2 \left( \frac{5}{2} \frac{4}{2} + \frac{10}{1} \frac{3}{2} \frac{1}{2} \right) + 1 

* Vacuum orbit: \( M = \frac{d}{dz} f v^0 = 1; v^1 = \frac{1}{2} + \frac{1}{2} \frac{P}{P} ; v^2 = \frac{1}{2} + \frac{1}{2} \frac{P}{P} ; v^3 = \frac{1}{2} + \frac{1}{2} \frac{P}{P} 

* Real algebraic curves:

* Kink curves: there are two values of \( \frac{q}{24} (1 + \frac{P}{P}) \): kink curve joining \( v^2 \) with \( v^3 \), b) \( \frac{q}{24} (1 + \frac{P}{P}) \): kink curve joining \( v^4 \) with \( v^5 \). The other kink curves are obtained through \( Z_5 \) rotations.

* Kink energies: a) \( P_0[K^2] = \frac{f}{f} j = W^2(v^k) W^2(v^{k+1}) j = \frac{q}{24} \frac{q}{24} \frac{q}{24} \frac{q}{24} 

b) \( P_0[K^2] = \frac{f}{f} j = W^2(v^k) W^2(v^{k+2}) j = \frac{q}{24} \frac{q}{24} \frac{q}{24} \frac{q}{24} 

* Conserved \( SU(2) \) charges: (a) and (b) $Q^2 K^2 = 0$
References


Figure 1: Kink curves in the $n = 4$, $n = 5$, $n = 6$ and $n = 7$ models