Modeling Solar Energy Through Mathematics

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Abstract. We live in a critical moment for humanity in which energy consumption is growing as the population grows, while fossil fuel resources are diminishing. It is time to bet on the search for renewable energy sources. Here arises, as researchers, the challenge of making these sources efficient and adapting them to the real demand. If we were able to harness all the solar energy that our planet receives, it would be enough to supply our current demand. And here we come into play as engineers, physicists and of course! Mathematicians. On the one hand, regarding the production of renewable electrical energy, one of the main lines of research that we develop at the Energy Optimization, Thermodynamics and Statistical Physics Group of the University of Salamanca is the simulation of Concentrated Solar thermal Power (CSP) such as central tower plants, analyzing possible aspects to improve the efficiency of the subsystems involved. And, on the other hand, to try to adapt the production to the demand, to study different aspects of thermal storage. All these processes are simulated by means of different programs such as Mathematica, Python, Matlab, etc. And, of course, mathematical tools such as analytical and numerical integrals, nonlinear equation solving, differential equation, interpolations, multiobjective optimization, etc., are used for this purpose. Let's see how mathematics becomes a magic wand that transforms this desire to harness the energy coming from the sun into a reality.

Keywords: Concentrated Solar Thermal Power · Thermal Storage · Applied Mathematics

1 Introduction

As evidenced Armstrong McKay [1], among many others, we are at a worrying time with respect to climate change. This is associated with the destruction of ecosystems and coupled with the reality in which energy consumption is growing as the population increases, while fossil fuel resources are diminishing. It is time to bet on the search for renewable energy sources. Here arises, as researchers, the challenge of making these sources efficient and adapting them to the real demand.

From these concerns, in Energy Optimization, Thermodynamics and Statistical Physics Group at the University of Salamanca [13], we propose:

- On the one hand, regarding the production of renewable electrical energy, one of the main lines of research that we develop is the simulation of concentrating solar thermal energy. Analysing possible aspects to improve the efficiency of the subsystems involved and their optimal integration [6] [9].
- On the other hand, to try to adapt the production to the demand, to study different technologies of thermal storage [12].

Today, the main types of concentrating power plants are shown in figure 1:

- In Solar Power Towers (SPT) flat rectangular mirrors (or heliostats) concentrate the radiation and direct it to the top of a tower, where the receiver is located.
- In the case of Parabolic Trough Collectors (PTC), these linear parabolic mirrors concentrate the light on a focal line, through which the working fluid, which is usually oil, circulates.
- The idea for parabolic dishes is similar, in this case a parabolic mirrow directs the radiation towards the focal point, where the receiver is located and also the associated gas turbine. In this case it is a micro gas turbine since the scale of energy produced is kW (in the case of central tower plants we speak of MW of power). These dishes are quite commercial with Stirling engines, but with Brayton cycles they are still under development. Generally, they are grouped to form big plants. But they can also be used for distributed power generation, which is the on-site generation of energy in places close to where it is consumed (without losses in transport), so that they can supply the demand of a farm or an isolated population, with or without access to the electricity grid.

Spain has been a pioneer in this type of technology, thanks to good solar conditions, especially in Andalusia, in the south of the country. There are already commercial plants of this type, others are prototypes and others are still in the process of research and development [10].

Figure 2 shows the basic scheme of the plants that we simulate, both for the case of a central tower plant or parabolic dish plant with hybrid gas turbine. They consist of three main subsystems: the solar collector, the combustion chamber and the heat engine. Initially, a working fluid, which can be air, is compressed in the compressor, increasing its temperature. Subsequently, heat from the solar collector is used to further increase the air temperature. If this temperature is suitable for the turbine, the working fluid expands in the turbine, generating mechanical energy, which is converted into electrical energy by a generator. If the temperature is not enough, the combustion chamber comes into play, which would increase the air temperature until it reaches the temperature required for turbine operation (this is known as hybridisation). The aim is to produce electrical energy with a constant power output regardless of weather conditions. At the end of the cycle, the air must return to its initial state of temperature, T_1, so the excess heat must be transferred by means of a heat exchanger. A regenerator can be placed at the outlet of the turbine to take advantage of part of this excess thermal energy, using it for an initial increase in the temperature

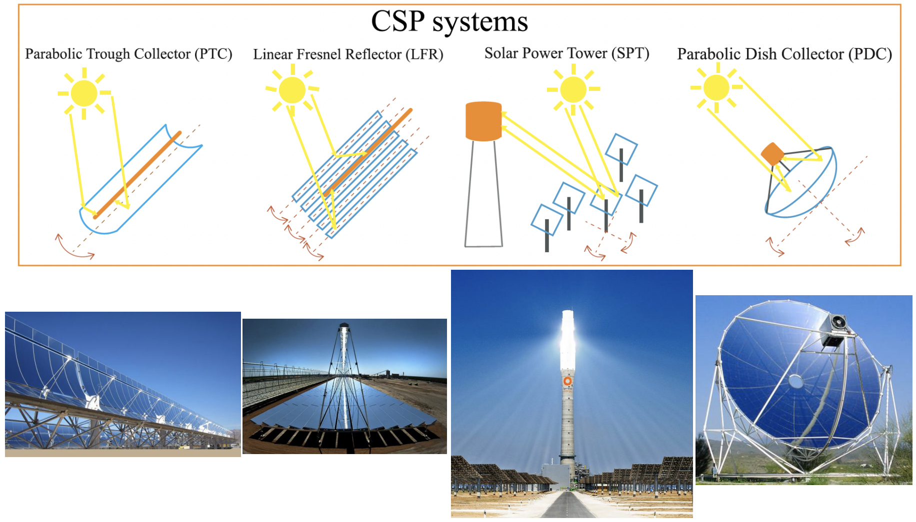


Fig. 1. Classification by reflector geometry of the commonly accepted CSP systems and some real examples of each (adapted from [10]).

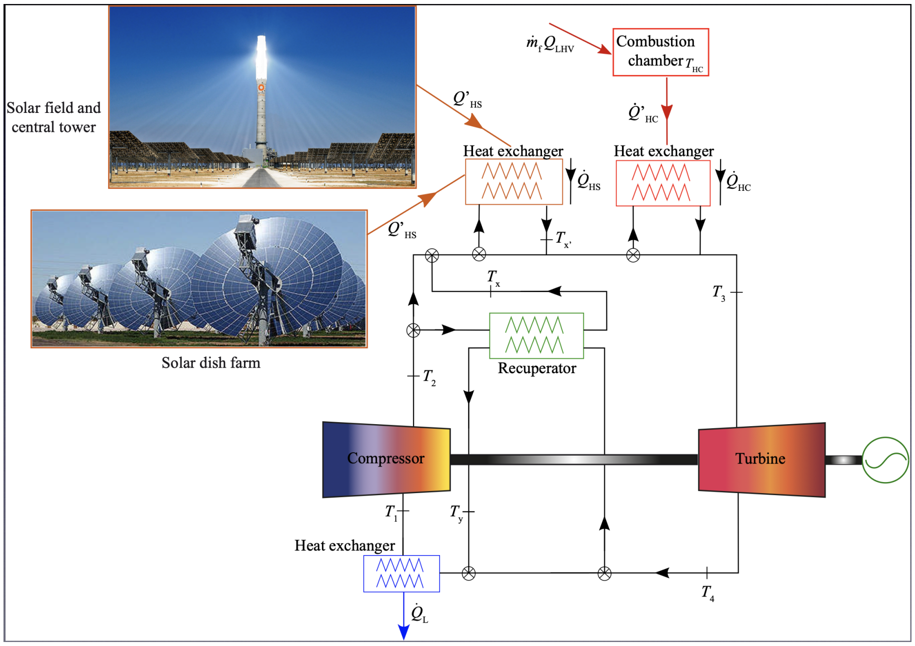


Fig. 2. Basic scheme of the hybrid solar gas-turbine plant considered, both for the case of a central tower plant or parabolic dish plant with hybrid gas turbine. The main heat transfers and temperatures are depicted [7].

of the gases after the compressor. After the regenerator, the temperature is T_y, to reach T_1, the remaining heat is transferred to the external medium through a heat exchanger. With these heat exchangers, which connect the working fluid with the external parts of the heat engine, what is achieved is that the Brayton cycle is closed and external combustion. All modeling takes into account the losses of the systems involved.

2 Methods and Results

2.1 Concentrated Solar Power

In the case of Solar Power Tower, we have modelled in Mathematica Software the heliostat field and the receiver [9]. Taking into account each of the subsystems (solar collector, combustion chamber and thermal machine), both the heat transfer equations and the losses of each of the devices are introduced into the model, which makes it possible to express the overall efficiency of the plant, as a function of the efficiency of each subsystem (solar collector, combustion chamber and heat engine) and the efficiency of the exchangers. The model considers all the subsystems using a reduced number of parameters, all with a clear physical meaning.

When distributing the heliostats in the field, several purely geometrical factors have to be taken into account, such as the size of the heliostats, the spacing distance between them and the distance between rows, and others that influence the efficiency, so that this efficiency is the maximum possible. Focusing all the heliostats in the best possible way pointing towards the tower is not a trivial task since there are several loss factors that cause the efficiency of the heliostat field to decrease. Overall, the efficiency of each heliostat depends on several loss factors. We take into account losses due to shading and blocking between heliostats, cosine effect, spillage, and attenuation. The optical efficiency of each heliostat is defined as a product of losses factors. We perform simulations by calculating the efficiency of each heliostat within the defined heliostat field [9]. The efficiency loss factors change with the location of the plant, with the particular weather conditions, with the season of the year and with the time of day, thus, the efficiency varies from time to time and from one heliostat to another.

Before obtaining results the model needs to be validated. Once we have programmed the model, we compare it with a real plant, if possible, and study whether the results of our model match those of the plant. This task is not always easy, because sometimes real data are not available. Some of these plants are under study, they are prototypes, and the companies do not publish all the data. Therefore, in some cases, we compare our results with other existing models that have been previously validated. There are many different patterns to place the heliostats in the field, we follow the Campo Code, developed by Francisco Collado of the University of Zaragoza [4]. Heliostats are placed in the field in the different ranks and taking into account all the space they can use during the follow-up along with a safety distance. Using this model for validation, it is seen that the deviation is small, below 7.1% in all cases.

Studies are carried out at the design point and once the model is validated, the conditions of input of solar irradiance and temperature throughout the day are varying, so that we can estimate the values of all the output parameters to the length of one day, of any day of the year. A seasonal study can also be done, for example for the plant similar to Gemasolar [3], with circular symmetry around an external receiver. The global efficiency maximum values in summer and the minimum in winter are obtained, with a relative increase from winter to summer of around 13%. Both spring and autumn present intermediate and very similar records between them [9].

Once we have designed the heliostat field and validated it, we will model the Brayton cycle. For this we use several mathematical tools as: interpolation functions, solve analytical and numerical equations or integrals. As a example, the working fluid performing the thermodynamic cycle can be air or another gas. To determine the heats involved in each phase, it is necessary to know the specific heat of the working fluid. This depends on the temperature, which, as explained above, changes throughout the process. Starting from the real behaviour of the specific heat of the gas under consideration with temperature, we look for an interpolation function that fits the real data. In other cases, to simplify the complexity of calculation, we determine the average of this function in the considered temperature range. We have also made a study of the possible differences using both methods and they are minimal [15]. In this case we use data from commercial programs, such as Thermoflex Software, which has a database from commercial components, to validate the simulation. The relative deviation of the results is below 2.95% [9].

Another example of mathematical tools used is to calculate the temperature of the solar collector, T_{hs}, we determine the heat involved in two different ways, as a function of T_{hs}, and equalise them. One could clear the temperature from this equation, but it has no analytical solution, so we use tools to solve the equation numerically.

Once we have the model designed and validated, we can study different configurations to determine in which intervals the behaviour is better. The model is flexible and allows us to modify the plant configuration, including a cycle with any number of multiple stages of compression and expansion with reheat. So we can analyse how the results are modified by introducing several compressors or several turbines and what the ideal configuration might be [9] [15].

We also perform sensitivity analyses, for example, with respect of the behaviour of the plant for different pressure ratios. Using mathematical techniques to optimise the design and operation of the plant. We look for optimal configurations from the point of view of various objectives, usually power and efficiency [11]. Generally, no configuration offers maximum values of efficiency and power simultaneously. Subsequently, engineers must decide which is more interesting, whether to work under maximum power or maximum efficiency conditions. This study can be carried out with other variables.

Following the same principle of looking for more efficient configurations, work fluids can be varied in a Brayton cycle. We have made four fluid simulations:

helium, nitrogen, air, and carbon dioxide. And this for multiple stages of compression and expansion with overheating [9]. With the objective of obtaining more efficient configurations, we can choose the relations of pressures that are associated with the maximum efficiencies in each case, in a process that we call pre-optimization.

So far we have a model that behaves like a real plant, which allows us to analyse various aspects of behaviour in an agile way. But the question is, can it be transferred to a commercial plant? To answer this question it is necessary to carry out economic studies of the solar plant, which are called thermoeconomic studies. From the economic point of view, through the LCOE (Levelised Cost of Electricity) indicator, various thermo-economic studies have been carried out, thus evaluating the profitability of this type of facilities. To calculate the LCOE, we take into account the typical costs of this type of plants: both capital costs (that is, the initial investment), those of decommissioning of the plant, as those of operation, maintenance and labour costs [5] [8] . We have conducted the study for different locations of the plant. While Seville in Spain is the typical place where this type of plants have been installed, or Ouarzazate, in Morocco; we wanted to check if other places, with less irradiance, but with less ambien temperature could be considered adequate. We also compare the LCOE of our simulation with other plant studies: both traditional, as renewable or with the solar concentration plants specifically [7].

To deepen in the model we wanted to understand the behaviour of heat transfer in the receiver, which is the main element of the solar part. Another challenge that we have raised is modelling heat losses in the receiver, and estimate its efficiency. The solar receiver can be considered as a special type of heat exchanger with the aim of converting direct solar radiation into heat. Many efforts have been dedicated to the design and optimisation of solar receiver, since they are the key component that connects the heliostat field and the energy conversion cycle. A volumetric receiver is made of a porous material that absorbs concentrated solar radiation. These receivers are usually closed with a quartz glass window that can reach temperatures of around 1200°C. Behind the glass, there is a cavity that contains the porous, the absorber, which is directly impacted by solar radiation. Gas flows through its pores heating at very high temperature. The porous means can be metallic or ceramic (Fig. 3).

Tonatiuh is an open-source Monte-Carlo Ray Tracer software for the optical simulation of solar concentrating systems. In this work, a parabolic dish and a target surface (receiver) placed at its focus will be simulated. The receiver will be a flat circle, representing the glass window aperture where the photons arrive after being reflected at the PDC surface. By this method we can obtain the optical efficiency of the parabolic dish efficiency, through Tonatiuh software, instead of taking a numerical value from experimental cases or other studies [6].

In García-Ferrero's work, a complete heat transfer analysis is carried out for a volumetric receiver of air coupled to a solar parabolic dish for distributed generation [6]. The model considers the main losses due to convection, conduction and radiation, as well as through the surrounding insulator. In this study there

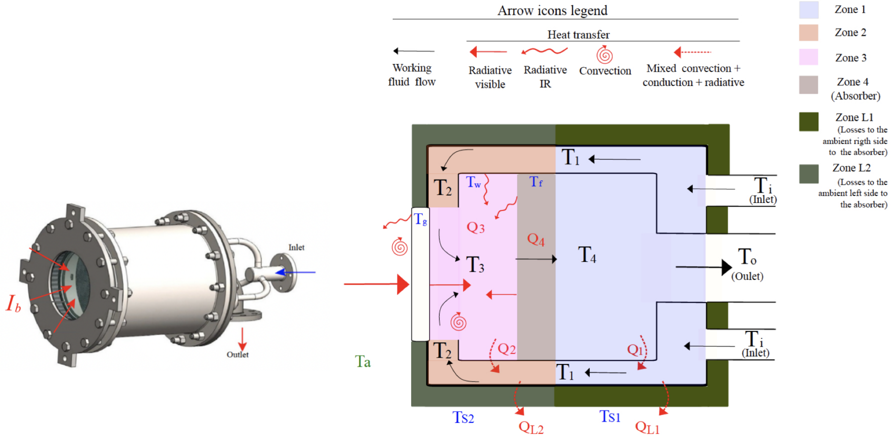


Fig. 3. 3D image of the receiver (left) and scheme of the receiver used in García-Ferrero work [6] (right). Heat Transfer Fluid temperatures are depicted in black. Surfaces temperatures related to glass (inner and outer surfaces), internal wall, absorber foam, front external insulator, and back external insulator in blue and ambient temperature is depicted in green [6].

are on the one hand equations defining the geometry, and on the other hand heat transfer by different modes, and temperatures varying in an interrelated way. For this, both analytical equations and multi-variable equations, including non-linear equations, have to be solved. Heat transfer involves the Nusselt number defined as a piecewise function. Every step is validated, as we discuss before. In this case, with Barreto and Canhoto's work [2] as a reference.

With our model we can also estimate the daily evolution of the main efficiencies for the four seasons. To obtain them, as input data, solar irradiance and temperature are introduced [14]. It can be seen that solar efficiency varies with the seasons due to the number of sun hours.

In the same way our model allows to simulate fuel consumption, in this case, of natural gas, along a day. This consumption can also be calculated. Associated with fuel consumption there are greenhouse gas emissions, which we can also calculate. One of the most important is associated with carbon dioxide. In general, it is observed that the differences between hybrid mode and pure combustion are higher in summer due to the greater solar irradiance, but also due to the largest number of solar hours [14].

2.2 Thermal Energy Storage

From the point of view of renewable energies, the challenge is to adapt the generation of electricity to demand. As it is known the generation of wind or solar energy does not correspond necessarily with demand. One possible solution to adapt electricity production to the demand is thermal storage. There are many storage techniques. The most widespread are the batteries, which are highly polluting and cannot be used for very large plants (scale of the order of MW). We will present below storage methods taking advantage of the thermodynamic properties of the materials, whether solid or liquid. While it is true that it is not easy to find an ideal method for energy storage.

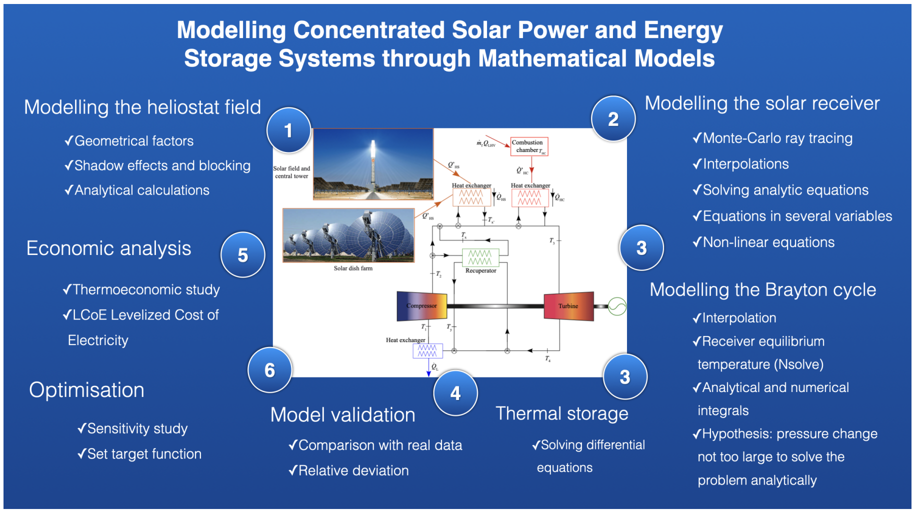
Nowadays there is a great interest in the so-called Pumped Thermal Energy Storage Systems (PTES) that are based in the subsequent use of thermal heat pumps and engines. As it is known, for instance, Brayton cycles can function as a heat pump and as a motor, generating power. When there is low demand for electricity or its price is cheap; the pump works to compress and heat the working fluid. Heat is stored in a reservoir that we call hot. To do this, working fluid is circulated through a tank that can contain, for example, rocks. In the discharge process, the Brayton cycle works as a motor, the fluid expands in the turbine and the heat accumulated in the tank is used to heat the fluid and finally generate electric current [12].

The so-called Packed Bed consist of tanks that contain: sand, rock pieces, remains of cement blocks, etc. They are suitable when working with high temperatures, as is the case of Brayton cycles. The great advantage they have is that they are cheap (compared to other technologies), since the material is easily accessible. The working fluid in Brayton cycles can be air. In the charging phase, hot air passes between the rocks and gives them heat and in the discharge, the cold air would pass and absorb the heat stored in the rocks, sand, etc. Here a field of study opens to the materials that can be used, its thermodynamic properties, the optimal size of the pieces, the characteristics of the storage tanks, temporary dynamic analysis of the temperature throughout the cylinder, among others. As we present before, different mathematical tools are used to simulate it, such as differential equations [12]. It must be taken into account that it is a dynamic system, the fluid moves in a three-dimensional system so there is a variation of time and a variation of space. In addition to the different forms of heat transfer, similar to what is shown in the model of the solar receiver.

3 Conclusions

Now we have an idea of how models can be designed to study concentrating solar thermal power and thermal energy storage through mathematical tools (Fig. 4). From the solar field, the receiver, the Brayton cycle, or the possibility of storing energy in the form of heat. These models have been validated so that they can be used to reproduce real behaviour. Subsequently, sensitivity and optimization analyses have been carried out. Finally, a thermo-economic study has been carried out to check their feasibility. Mathematics really is a magic wand that makes this whole study possible.

What role do we play in this world? Performing experiments with the complete, real plant is a huge investment. Then it is essential to make a previous detailed study, which allows a pre-desig, optimal, of the plant. We have developed the complete thermodynamic model for plants, both from the central tower



Modelling Concentrated Solar Power and Energy Storage Systems through Mathematical Models

1 Modelling the heliostat field
- ✓Geometrical factors
- ✓Shadow effects and blocking
- ✓Analytical calculations

2 Modelling the solar receiver
- ✓Monte-Carlo ray tracing
- ✓Interpolations
- ✓Solving analytic equations
- ✓Equations in several variables
- ✓Non-linear equations

3 Modelling the Brayton cycle
- ✓Interpolation
- ✓Receiver equilibrium temperature (Nsolve)
- ✓Analytical and numerical integrals
- ✓Hypothesis: pressure change not too large to solve the problem analytically

3 Thermal storage
- ✓Solving differential equations

4 Model validation
- ✓Comparison with real data
- ✓Relative deviation

5 Economic analysis
- ✓Thermoeconomic study
- ✓LCoE Levelized Cost of Electricity

6 Optimisation
- ✓Sensitivity study
- ✓Set target function

Fig. 4. Mathematics tools used to model concentrating solar thermal power and thermal storage.

and for parabolic dishes. It has the peculiarity of allowing rapid simulations and of sensitivity studies, evaluation of LCOE, losses and calculations all this in a short time to see how any change in the design of the global plant affects. On the other hand, we are working on storage issues in packed beds, a necessary step to adapt the demand for consumption renewable energy sources.

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