A Social Choice Approach to Graded Soft Sets: slides for FUZZ-IEEE 2017

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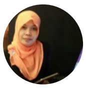

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A Social Choice Approach to Graded Soft Sets: slides for FUZZ-IEEE 2017

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Outline

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Motivation I

Standard soft sets produce a binary parameterized description of the universe of objects.

This representation replicates the description by binary evaluations (of the same universe) in social choice.

In the last decade, many works in social choice account for the case of ternary, quaternary, or even *m*-ary evaluations of the options.

Soft sets have been extended in various directions: bijective soft sets, fuzzy soft sets, probabilistic soft sets, or incomplete soft sets.

Does anyone produce *m*-ary evaluations of the options?

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Motivation II

Our precedent "*N*-soft sets and their decision making algorithms" pioneers *m*-ary parameterized descriptions of the universe of objects.

Here we present some parallelisms between extended notions of soft sets and very relevant concepts of Social Choice.
By doing so we introduce a particular class of *N*-soft sets: Graded soft set.

Driving ideas:

(1) Both soft sets and *N*-soft sets can be faithfully represented by well-established voting situations in Social Choice.

(2) Their standard decision making procedure by choice values coincide with approval voting (soft sets) and the Borda rule (graded soft sets).

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Theoretical background on soft sets

Henceforth, we assume that all sets are finite.

Soft set

Let U be a set of alternatives, E a set of parameters, $A \subseteq E$. (F, A) is a soft set over U when $F : A \to 2^U$.

Example. Let $U = \{h_1, h_2, h_3\}$ be options, $E_0 = \{e_1, e_2, e_3, e_4\}$ be attributes. A soft set (F_0, E_0) is defined by:

(a) $h_1 \in F_0(e_1) \cap F_0(e_3)$ and $h_1 \notin F_0(e_2) \cup F_0(e_4)$.

(b) $h_2 \in F_0(e_2)$ and $h_2 \notin F_0(e_1) \cup F_0(e_3) \cup F_0(e_4)$.

(c) $h_3 \in F_0(e_1) \cap F_0(e_4)$ and $h_3 \notin F_0(e_2) \cup F_0(e_3)$.

Tabular representation of the soft set (F_0, E_0):

(F_0, E_0)	e_1	e_2	e_3	e_4
h_1	1	0	1	0
h_2	0	1	0	0
h_3	1	0	0	1

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Theoretical background on social choice I

Many concepts represent rankings, evaluations, ... of the alternatives by the members of a group.

We are concerned with the following adapted definition:

Definition (Aleskerov, Chistyakov, Kalyagin, 2010)

Let U be a finite set (of alternatives) with cardinality $|U| \geqslant 2$, $E = \{a_1, a_2, ..., a_q\}$ be a set (of agents) with $q \geqslant 2$.

An *evaluation procedure* with a set of ordered grades $R = \{0, 1, 2, ..., n\}$ for U is a map $G : U \times E \to R$, which assigns to each alternative $u \in U$ and each agent $a_j \in E$ a grade $u_j = G(u, a_j) \in R$.

As in the case of soft sets, the information in an evaluation procedure can be captured in tabular form too:

Theoretical background on social choice II

Example. Let $U = \{x, y, z\}$ be alternatives, and $A = \{a_1, a_2, a_3\}$ be agents.
An evaluation procedure G_1 with a set of ordered grades $R = \{0, 1, 2\}$ on U is defined by:

$G_1(x, a_1) = 2$, $G_1(y, a_1) = 1$, $G_1(z, a_1) = 0$.

$G_1(x, a_2) = 2$, $G_1(y, a_2) = 0$, $G_1(z, a_2) = 1$.

$G_1(x, a_3) = 0$, $G_1(y, a_3) = 2$, $G_1(z, a_3) = 2$.

The information defining G_1 can be given in tabular form:

G_1	a_1	a_2	a_3
x	2	2	0
y	1	0	2
z	0	1	2

One can easily retrieve the original definition of the evaluation procedure G_1 from this table.

Outline

Motivation and background

Some motivation

Soft sets

Social choice

Graded soft sets

N-soft sets

N-soft sets and choice

Graded soft sets

Graded soft sets and choice

Conclusion

N-soft sets

Any soft set can be matched with an evaluation procedure with $n = 1$.

What corresponds to general evaluation procedures?

Definition (Fatimah et al., submitted)

Let U be a set of objects and E be attributes, $A \subseteq E$. Suppose $R = \{0, 1, \ldots, N-1\}$ are ordered grades, with $N \in \{2, 3, \ldots\}$.

The triple (F, A, N) is an *N-soft set* on U if $F : A \times R \to 2^U$ and $\{F(a, r) : r \in R, F(a, r) \neq \varnothing\}$ is a partition of U for each $a \in A$.

Proposition

Let U and A be fixed.
There is a one-to-one correspondence from the set of evaluation procedures with grades $R = \{0, 1, 2, ..., n\}$ on U and the set of N-soft sets (F, A, N) on U with $N = n + 1$.

N-soft sets and choice

In soft sets, each parameter induces a binary *evaluation* of the alternatives:

1 for alternatives that meet the parameter, 0 otherwise.

The basic decision making mechanism for soft sets ranks the alternatives by choice values (CVs): each alternative receives a score that equals the number of parameters for which it is positively evaluated.

Proposition

The CV decision mechanism for soft sets is equivalent to Approval Voting for the fictitious voting situation where each attribute becomes a voter which submits a ballot, and the attribute votes for all the options that verify it.

Approval Voting simply sums up the votes received.

Graded soft sets

What about groups of m-ary evaluations in Social Choice?

Then the Borda rule is a very popular decision making mechanism.

We need a new model to import it to our setting.

Definition

Let U be a set of objects and $A \subseteq E$ be a subset of the universal set of attributes E.
(G, A) is a *graded soft set* on U when $G : A \times R \to 2^U$ is an N-soft set with $R = \{0, 1, \ldots, |U| - 1\}$, and $G(a, r)$ is a singleton for each $a \in A$ and $r \in R$.

We have an N-soft set on U with $N = |U|$, such that for any attribute a, each alternative receives one unrepeated grade. the *unique* g_a s.t. $G(a, g_a) = \{u\}$.

Graded soft sets and DM I

It seems only natural to extend the CV criterion of soft sets to *N*-soft sets, hence to graded soft sets:

Definition

When (F, A, N) is an *N*-soft set, the *choice value* of $u \in U$ is

$$CV(u) = \sum_{j=1,\dots,q} \{g \in R : F(a_j, g) = u\}.$$

Definition

In this case we can associate a CV-ranking of the alternatives U with (F, A, N), namely, the complete preorder on U

$$u \succcurlyeq v \text{ if and only if } CV(u) \geqslant CV(v), \text{ each } u, v \in U.$$

Graded soft sets and DM II

Proposition

Every N-soft set (F, A, N) on U can be identified with a voting situation on U.

When (G, A) is a graded soft set, its CV-ranking $\succcurlyeq$ coincides with the Borda ranking associated with such voting situation.

In fact, choice values and Borda scores are the same throughout.

Lemma

Every graded soft set (G, A) induces a profile of $|A|$ linear orders on U: each fictitious voter is $a \in A$, and it is associated with the linear order on U:

$$uP_a v \text{ if and only if } G(a, r) = u,\ G(a, r') = v, \text{ and } r > r'.$$

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Motivation and background
- Some motivation
- Soft sets
- Social choice

Graded soft sets
- N-soft sets
- N-soft sets and choice
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- Graded soft sets and choice

Conclusion

Conclusion

We have shown that there is a correspondence between ideas from soft computing and social choice.

We have concentrated on extensions of the concept of soft set.

Notions from one field can be faithfully transferred to the other through very discernible identifications.

This intuitively appealing association extends to choice mechanisms as well.

Consequently, we hope that further ideas from social choice can provide interesting insights into soft computing in the future.

Thanks for your attention!

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