Ordering infinite utility streams: set-theoretical and topological issues

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Outline

Intergenerational aggregation: The framework

$\mathbf{X} \subseteq \mathbb{R}^{\mathbb{N}}$ is a domain of utility sequences or infinite-horizon utility streams.
Unless otherwise stated, $\mathbf{X} = [0,1]^{\mathbb{N}}$.
Sometimes we refer to $\mathbf{Z} = \{0,1\}^{\mathbb{N}} \subseteq \mathbb{R}^{\mathbb{N}}$.

Usual notation for utility streams: $\mathbf{x} = (x_1, ..., x_n, ...) \in \mathbf{X}$.

We use 'vector'-domination of various types:

- $\mathbf{x} \geqslant \mathbf{y}$ if $x_i \geqslant y_i$ for each $i = 1, 2, ...$
- $\mathbf{x} > \mathbf{y}$ if $\mathbf{x} \geqslant \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.
- $\mathbf{x} \gg \mathbf{y}$ if $x_i > y_i$ for each $i = 1, 2, ...$

Intergenerational aggregation: comparing streams

A *social welfare relation* (SWR) is a binary relation $\succcurlyeq$ on $\mathbf{X}$.

$\mathbf{x} \succcurlyeq \mathbf{y}$ means "$\mathbf{x}$ is (socially) at least as good as $\mathbf{y}$"

It is assumed that $\succcurlyeq$ is reflexive.

If $\succcurlyeq$ is an ordering (i.e., complete and transitive) then we call it a *social welfare ordering* (SWO).

▷ Its asymmetric factor is denoted by $\succ$ (i.e., $\mathbf{x} \succ \mathbf{y}$ iff $\mathbf{x} \succcurlyeq \mathbf{y}$ but not $\mathbf{y} \succcurlyeq \mathbf{x}$).

▷ Its symmetric factor is denoted by $\sim$ (i.e., $\mathbf{x} \sim \mathbf{y}$ iff $\mathbf{x} \succcurlyeq \mathbf{y}$ and $\mathbf{y} \succcurlyeq \mathbf{x}$).

Intergenerational aggregation: comparing streams

A *social welfare function* (SWF) is a function $\mathbf{W} : \mathbf{X} \longrightarrow \mathbb{R}$.

$\mathbf{W}(\mathbf{x}) \geqslant \mathbf{W}(\mathbf{y})$ means "$\mathbf{x}$ is (socially) at least as good as $\mathbf{y}$"

It induces a *representable* social welfare ordering according to the expression:

$$\mathbf{x} \succcurlyeq \mathbf{y} \text{ if and only if } \mathbf{W}(\mathbf{x}) \geqslant \mathbf{W}(\mathbf{y})$$

Intergenerational justice: comparing streams

We are concerned with combinations of axioms of different nature for SWRs / SWFs on $\mathbf{X}$.

- Axioms related to efficiency: Strong/Weak/Partial Pareto, Weak Dominance, or Monotonicity.
  Strong Pareto: If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\mathbf{x} > \mathbf{y}$ then $\mathbf{x} \succ \mathbf{y}$.
- Axioms related to equity: Pigou-Dalton transfer principle, variations on the Hammond Equity axiom, ...
  Anonymity: Any finite permutation of a utility stream produces a socially indifferent utility stream.

Ramsey, Economic Journal, 1928, expressed the conjecture that there exist incompatibilities among these properties.

Shortcomings of some classical representable criteria

The most popular criteria for evaluating infinite streams is the utilitarian criterion with positive discount rate $\beta \in (0,1)$:

$$\mathcal{F}_\beta(\mathbf{x}) = \sum_{i=1}^{+\infty} \beta^{i-1} x_i \quad \text{for all} \quad \mathbf{x} = (x_1, x_2, ...)$$

Criticism. Ramsey, Economic Journal, 1928, claimed that discounting is "ethically indefensible", something that "arises merely from the weakness of the imagination":

$$\mathcal{F}_\beta(1,0,0,...) > \mathcal{F}_\beta(0,1,0,0,...) > ... > \mathcal{F}_\beta(0,\overset{i}{\ldots},0,1,0,0,...) > ...$$

This yields ethically conflicting statements: e.g., it turns out that $(1,\overset{14}{\ldots},1,0,0,....)$ is socially better than $(0,\overset{14}{\ldots},0,1,1,...)$ when $\beta = 0.95$.

Shortcomings of some classical representable criteria

The Rawlsian criterion:

$$\mathcal{R}(\mathbf{x}) = \inf\{x_1, x_2, ..., x_n, ...\} \quad \text{for all} \quad \mathbf{x} = (x_1, x_2, ...)$$

Criticism. Very basic Paretian performance, although it verifies a large variety of equity postulates.

The next efficiency axioms are used

Statements for SWFs / SWRs:

- **Axiom SP** (*Strong Pareto*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\mathbf{x} > \mathbf{y}$ then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$ / $\mathbf{x} \succ \mathbf{y}$.

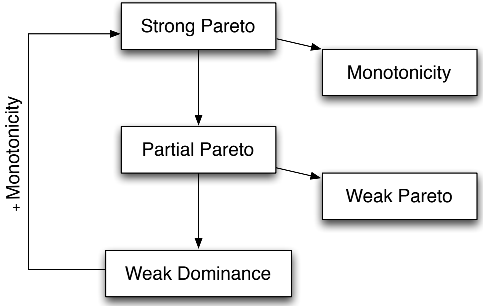
- **Axiom WD** (*Weak Dominance*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and there is $j \in \mathbb{N}$ such that $x_j > y_j$, and $x_i = y_i$ for all $i \neq j$, then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$ / $\mathbf{x} \succ \mathbf{y}$.

- **Axiom WP** (*Weak Pareto*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\mathbf{x} \gg \mathbf{y}$ then $\mathbf{W}(\mathbf{x}) > \mathbf{W}(\mathbf{y})$ / $\mathbf{x} \succ \mathbf{y}$.

- **Axiom PP** (*Partial Pareto*). The conjunction of WD and WP.

Some relationships

Axiom M (*Monotonicity*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and $\mathbf{x} > \mathbf{y}$ then $\mathbf{W}(\mathbf{x}) \geqslant \mathbf{W}(\mathbf{y})$ / $\mathbf{x} \succcurlyeq \mathbf{y}$.



The next *procedural equity* axiom is used

Anonymity is the usual "equal treatment of all generations" postulate à-la-Sidgwick and Diamond.

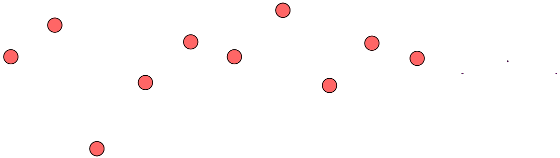
Axiom AN (*Anonymity*). Any finite permutation of a utility stream produces a socially indifferent utility stream. Formally:

For all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if there exist $i, j \in \mathbb{N}$ such that $x_i = y_j$ and $x_j = y_i$, and for $k \in \mathbb{N} - \{i, j\}$, $x_k = y_k$, then $\mathbf{W}(\mathbf{x}) = \mathbf{W}(\mathbf{y})$ / $\mathbf{x} \sim \mathbf{y}$.

The next *consequentialist equity* axioms are used

Axiom HE (*Hammond Equity*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $x_j > y_j > y_k > x_k$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{y} \succcurlyeq \mathbf{x}$.

If the consequence is $\mathbf{y} \succ \mathbf{x}$ then we obtain **Axiom SEP** (*Strong Equity Principle*).



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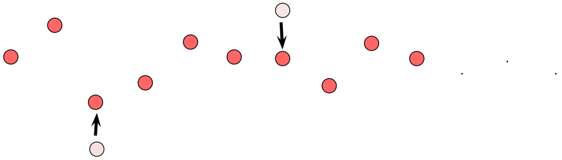

The next *consequentialist equity* axioms are used

Since $\mathbf{X} = [0,1]^{\mathbb{N}}$, in the presence of M the HE postulate implies the following very mild property:

Axiom HEF (*Hammond Equity for the Future*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ are such that $\mathbf{x} = (x_1, x, x, x, ...)$, $\mathbf{y} = (y_1, y, y, y, ...)$ and $x_1 > y_1 > y > x$, then $\mathbf{y} \succcurlyeq \mathbf{x}$.

Svensson's possibility theorem

Theorem (Svensson, Econometrica, 1980)

There are SWOs on $\mathbf{X} = [0,1]^{\mathbb{N}}$ that are Strongly Paretian and anonymous.

Idea: Extend the *incomplete* Suppes-Sen Grading Principle to a complete preorder using Szpilrajn's lemma:

Suppes-Sen Grading Principle

The preorder (reflexive, transitive) R on $\mathbf{X}$ defined as:
$\mathbf{x} \, R \, \mathbf{y}$ iff there is a finite permutation π such that $\mathbf{x} \geqslant \pi(\mathbf{y})$

Key: The extensions preserve Strong Pareto and Anonymity.

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There are SWOs on $\mathbf{X} = [0,1]^{\mathbb{N}}$ that are Strongly Paretian and anonymous.

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Lemma (Szpilrajn, Fundamenta Mathematicae, 1930)

For every preorder (reflexive, transitive) R on a set $\mathbf{A}$ there is a complete preorder R' on $\mathbf{A}$ that extends R $(R \subseteq R', P \subseteq P')$.

Key: The extensions preserve Strong Pareto and Anonymity.

The Basu-Mitra impossibility theorem

Theorem (Basu-Mitra, Econometrica, 2003)

No SWF on $\mathbf{Z} = \{0,1\}^{\mathbb{N}}$ is Strongly Paretian and anonymous (i.e., Strongly Paretian and anonymous SWOs on $\mathbf{X}$ *cannot* be represented by utilities).

Their argument relies on the following:

Lemma (Sierpiński, 1965)

There is an uncountable family of distinct nested sets $E(z)$, with each set containing an infinite number of positive integers.

Idea: Enumerate the rationals $\mathbb{Q} = \{q_1, q_2, ..., q_i, ...\}$, and for each $z \in \mathbb{R}$ define $E(z)$ as the set of indices of the rationals q_i such that $q_i < z$.

The Basu-Mitra impossibility theorem

Now their proof has an affinity with the demonstration that lexicographic preferences do not have real-valued representations: one "runs out of numbers" (*sic*).

More precisely: if a Strongly Paretian and anonymous SWF $\mathbf{W}$ exists on $\mathbf{Z}$, then for each $z \in (0,1)$ one can define an interval $I(z) = (A_z, B_z)$ in such way that the intervals associated with distinct values of $z \in (0,1)$ are nonoverlapping:

$$A_z = \mathbf{W}(\mathbf{a}(z)) \text{ such that } \mathbf{a}(z)_n = \begin{cases} 1 & \text{if } n \in E(z) \\ 0 & \text{otherwise} \end{cases}$$

$B_z = \mathbf{W}(\mathbf{b}(z))$ such that the first 0 in the stream $\mathbf{a}(z)$ is made a 1

$$z < t \Rightarrow B_z < A_t \text{ for all } z, t \in (0,1)$$

Furher incompatibilities with SWFs

An earlier use of the fact that lexicographic orders are not representable.

A SWO $\succcurlyeq$ is said to verify the **non-substitution condition** if $(y, v, v, ...) \succ (x, z, z, ...)$ whenever $x, y, z, v \in [0, 1]$ and $v > z$.

Proposition (Lauwers, Economic Theory, 1997)

No SWF on $\mathbf{X} = [0, 1]^{\mathbb{N}}$ is Strongly Paretian and verifies the non-substitution condition (i.e., if a Strongly Paretian SWO on $\mathbf{X} = [0, 1]^{\mathbb{N}}$ verifies the non-substitution condition then it *cannot* be represented by a utility function).

Reason: The restriction to the set of utility streams of the form $(u, v, v, ..., v, ...)$ induces a lexicographic order on $[0, 1]^2$.

A possibility result about SWFs

Proposition (Basu and Mitra, 2007)

There are SWFs on $\mathbf{X} = [0,1]^{\mathbb{N}}$ that are Weakly Dominant and Anonymous.

Argument: Consider the equivalence relation on $\mathbf{X} = [0,1]^{\mathbb{N}}$ defined as $\mathbf{x} \sim \mathbf{y}$ iff $\mathbf{x}, \mathbf{y}$ are eventually coincident.
From each equivalence class $[\mathbf{x}]_\sim$, choose a representative $\mathbf{x}_\sim$.

Solution: for each $\mathbf{x} \in \mathbf{X}$, $\mathbf{W}(\mathbf{x}) = \sum_{i=1}^{\infty} (\mathbf{x} - \mathbf{x}_\sim)_i$.

Handicap: Useless because $\mathbf{W}$ cannot be Monotonic, and it appeals to the Axiom of Choice.

A criticism to Svensson's theorem

- According to the Basu-Mitra theorem, only *non-representable* criteria are ensured by Svensson's theorem.

- Main criticism:
  - A class of criteria is provided: lack of uniqueness of the solution.
  - Non-constructive solution (hinges on Szpilrajn's lemma: it depends on the Axiom of Choice).

Fleurbaey and Michel, Journal of Mathematical Economics, 2003, conjectured that reliance on the Axiom of Choice is unavoidable.

Their conjecture has been confirmed:

Zame's theorem

Theorem (Zame, Theoretical Economics, 2007)

1. The existence of Weakly Paretian and anonymous SWOs on $\mathbf{X} = [0,1]^{\mathbb{N}}$ entails the existence of a *non-measurable set*.
2. The existence of Strongly Paretian and anonymous SWOs on $\mathbf{Z} = \{0,1\}^{\mathbb{N}}$ entails the existence of a *non-measurable set*.

Remark 1: Non-measurable sets are non-constructive: their existence does not follow from the Zermelo-Fraenkel axioms (without the Axiom of Choice).

Remark 2: When the existence of an object requires the Axiom of Choice, then it is said that the object "is non-constructive" or "does not have an explicit description".

Lauwers' theorem

An improvement of Zame's second statement:

Theorem (Lauwers, J. Mathematical Economics, 2010)

There is no explicit description of an 'Intermediate Paretian' and anonymous SWO $\succcurlyeq$ on $\mathbf{Z} = \{0, 1\}^{\mathbb{N}}$.

Remark: Strong Pareto is strictly stronger than 'Intermediate Pareto'.

Idea: The existence of such SWO $\succcurlyeq$ entails the existence of a *non-Ramsey set*. The existence of this object does not follow from the Zermelo-Fraenkel axioms without the Axiom of Choice.

Lauwers' theorem: sketch of the argument

Non-Ramsey set: $\mathcal{N} \subseteq [I]^\infty = \{A \subseteq I : A \text{ is countably infinite}\}$ with I infinite, such that for all infinite $J \subseteq I$, the class $[J]^\infty$ has elements from both $\mathcal{N}$ and $[I]^\infty - \mathcal{N}$.

Idea: Identify each $\mathbf{x} = (x_1, ..., x_n, ...) \in \mathbf{Z} = \{0, 1\}^{\mathbb{N}}$ that has an infinite number of $1's$, i.e.,

$$x_n = \begin{cases} 1 & \text{in coordinates } n_1 < n_2 < ... < n_k < ... \\ 0 & \text{otherwise} \end{cases}$$

with $S = \{n_1, n_2, ..., n_k, ...\} \in [\mathbb{N}]^\infty$.
SWOs on $\mathbf{Z}$ naturally induce complete preorders on $[\mathbb{N}]^\infty$.

A non-Ramsey set: $\mathcal{N} = \{S \in [\mathbb{N}]^\infty : S_2 \succ S_1\}$, where
$S_1 = [n_1, n_2[\cup [n_3, n_4[\cup ... \cup [n_{2k-1}, n_{2k}[\cup .....$ and
$S_2 = [n_2, n_3[\cup [n_4, n_5[\cup ... \cup [n_{2k}, n_{2k+1}[\cup .....$

Continuity ... with respect to what?

Quoting Shinotsuka: "It is desirable that social ranking of consumption plans is robust to 'small' perturbations .... the problem is that it is difficult to single out the 'right' notion of 'nearness' or topology".

- Lauwers, Social Choice and Welfare, 1997, analyses 5 metric topologies with relevance in the field.
  His motivation: there is no natural topology in the set of infinite utility streams.
  Continuity is manipulable.

- Quoting Svensson: "in the space $\mathbf{X}$ a continuity assumption of preferences is not only a mathematical assumption (...) but also reflects a value judgement".

The next *continuity* axioms are used

For $\succcurlyeq$, a *reflexive* binary relation on $\mathbf{X} \subseteq \mathbb{R}^{\mathbb{N}}$, the following definitions apply:

- **Axiom USC** (*Upper semicontinuity with respect to* τ). For each $\mathbf{x} \in \mathbf{X}$, $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \succcurlyeq \mathbf{x}\}$ is τ-closed.
- **Axiom LSC** (*Lower semicontinuity with respect to* τ). For each $\mathbf{x} \in \mathbf{X}$, $\{\mathbf{y} \in \mathbf{X} : \mathbf{x} \succcurlyeq \mathbf{y}\}$ is τ-closed.

Continuity w.r.t. τ is USC + LSC w.r.t. τ.

Restricted upper (or lower) semicontinuity w.r.t. τ, RUSC (or RLSC), is the restriction of the conclusion to *eventually constant* $\mathbf{x} \in \mathbf{X}$.

Relevant metric topologies

Lauwers studies the topologies induced by the following distances on l_∞, the set of bounded real-valued sequences:

- Product topology τ_p: $d_p(\mathbf{x},\mathbf{y}) = \sum_{k=1}^{\infty} 2^{-k}|x_k - y_k|$
- 'Strict'/Myopic τ_m: $d_m(\mathbf{x},\mathbf{y}) = sup\{|x_k - y_k|/k : k \in \mathbb{N}\}$
- Sup/Uniform topology τ_u: $d_u(\mathbf{x},\mathbf{y}) = sup\{|x_k - y_k| : k \in \mathbb{N}\}$
- Svensson topology τ_s: $d_s(\mathbf{x},\mathbf{y}) = \min\{1, \sum_{k=1}^{\infty} |x_k - y_k|\}$
- Campbell topology τ_c: $d_c(\mathbf{x},\mathbf{y}) = sup\{\delta(x_k, y_k)/k : k \in \mathbb{N}\}$, where $\delta(a,b) = 1$ if $a = b$, $\delta(a,b) = 0$ otherwise

It turns out that $\tau_p \subset \tau_m \subset \tau_u \subset \tau_s$ and $\tau_p \subset \tau_c$.

Myopia vs. Continuity

Intertemporal myopia refers to impatience of the ordering: present consumption is preferred to future consumption.

Brown and Lewis, Econometrica, 1981, motivate the use of a topology as a *behavioural assumption* reflecting the myopic behaviour of economic agents: “In the capital theory literature (...) continuity is now a behavioral assumption rather than a technical requirement”.

▷ Both the *product*, *Mackey*, and *Campbell* topologies on l_∞ have the property that every continuous SWO is impatient.

▷ The *sup* (thus the *Svensson*) topology does not share this property: there are sup-continuous SWOs that do not discount the future.

Anonymity vs. Continuity

Ramsey had expressed the conjecture that there is some incompatibility *among efficiency and equity* properties (1928).

Diamond, Econometrica, 1965, first formalized this fact *under continuity*:

Theorem (Diamond-Yaari impossibility)

There is no SWO on $\mathbf{X} = [0,1]^{\mathbb{N}}$ that verifies Strong Pareto, anonymity, and continuity w.r.t. the supremum topology.

There is no SWO on $\mathbf{X} = [0,1]^{\mathbb{N}}$ that verifies Monotonicity, Weak Pareto, anonymity, and continuity w.r.t. the product topology.

Remark. Its assumptions ensure representability.

Anonymity vs. Continuity

Svensson's topology permits to avoid Diamond's impossibility.

Theorem (Svensson, Econometrica, 1980)
There are SWOs on $\mathbf{X} = [0,1]^{\mathbb{N}}$ that verify Strong Pareto, anonymity, and continuity w.r.t. the topology induced by (the non-trivial) Svensson's metric.

But remember: all such SWOs are non-constructive objects.

Anonymity vs. Continuity

Diamond's impossibility theorem is *extended by the following contributions* on l_∞, in the absence of Paretian restrictions.:

- ▷ Campbell, Social Choice and Welfare, 1985: continuity w.r.t. the Campbell (metric) topology*. For asymmetric, negatively transitive relations: anonymity $\Leftrightarrow$ triviality.
- ▷ Shinotsuka, Social Choice and Welfare, 1998: continuity w.r.t. the Mackey topology* (hence w.r.t. the product topology). Asymmetry and equity ($\mathbf{x} \sim \mathbf{y} \Rightarrow \pi(\mathbf{x}) \sim \pi(\mathbf{y})$ for each π finite) imply triviality.
  He uses Conway's characterization of the Mackey topology (Trans. AMS, 1967).

*These are logically independent.

Inequality aversion vs. Continuity

Weak forms of sup-continuity conflict with mild rationality and *inequality aversion* even in the absence of Paretian restrictions.

Axiom AE (*Altruistic Equity*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, there are $\varepsilon > \delta > 0$ with $y_j = x_j - \delta \geqslant y_k = x_k + \varepsilon$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{y} \succ \mathbf{x}$.

Axiom PDT (*Pigou-Dalton transfer principle*). Same as above except $\varepsilon = \delta$.

- Hara et al., Social Choice and Welfare, 2008: under PDT or Lorenz domination principle, semicontinuity w.r.t. the sup topology is incompatible with acyclicity.
- Alcantud, Bull. SAET, 2012: same is true under AE.

SWFs as a tool for implementation

There is a general agreement that using SWFs imposes a too heavy burden to the analyst due to the large proportion of incompatibilities that it produces, *especially* when $\mathbf{X} = [0,1]^{\mathbb{N}}$ or $\mathbf{X} = l_\infty$.

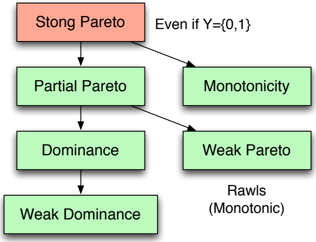
Nevertheless: "It seems fair to say that no criterion has achieved the analytical clarity of the discounted sum of utilities" (Chichilnisky, Social Choice and Welfare, 1996).

The same can be said about other *explicit* SWFs like the Rawlsian criterion.

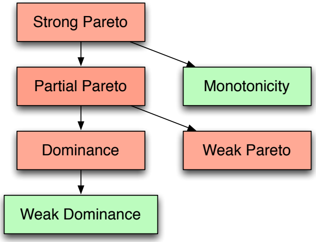
A different issue is what implications they have e.g., in optimal growth theory.

Influence of the structure of the set of streams

The next figures show what postulates of efficiency can/cannot be made compatible with *anonymous* SWFs, for two common choices of $\mathbf{X} = Y^{\mathbb{N}}$ (cf., Basu and Mitra):



The case $Y = \mathbb{N}$



The case $Y = [0,1]$

SWFs that verify the Hammond Equity principle

Past literature only gave trivial antecedents on representable criteria that verify the following equity principle:

Axiom HE (*Hammond Equity*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ verify $x_j > y_j > y_k > x_k$ some $j, k \in \mathbb{N}$, and $x_t = y_t$ if $j \neq t \neq k$, then $\mathbf{y} \succcurlyeq \mathbf{x}$.

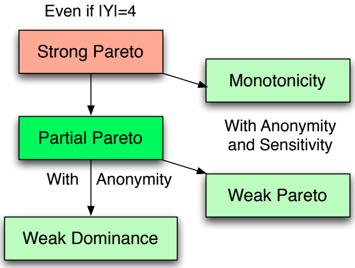
In the case of finite streams ($\mathbf{X} = [0,1]^n$ with $n \in \mathbb{N}$), in conjunction with Anonymity and Strong Pareto it characterizes the *Leximin ordering*.

- Rank-order the vectors and apply Lexicographic.

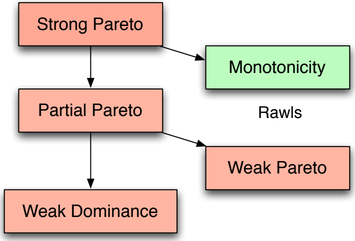
It is known that Leximin is non-representable even if $n = 2$.

SWFs that verify the Hammond Equity principle

Our contribution produces the following picture $(\mathbf{X} = Y^{\mathbb{N}})$:



The case $Y = \mathbb{N}$

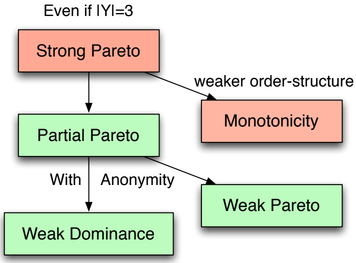


The case $Y = [0, 1]$

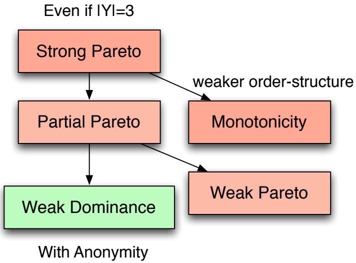
SWFs that verify Pigou-Dalton transfer principle

Little information on PDT, that we complement. Remember:

Axiom PDT (*Pigou-Dalton transfer principle*). If $\mathbf{x}, \mathbf{y} \in \mathbf{X} = Y^{\mathbb{N}}$, there are $\varepsilon > 0$ with $y_j = x_j - \varepsilon \geqslant y_k = x_k + \varepsilon$ for some $j, k \in \mathbb{N}$, and $x_t = y_t$ when $j \neq t \neq k$, then $\mathbf{y} \succ \mathbf{x}$.



The case $Y = \mathbb{N}$



The case $Y = [0, 1]$

SWFs that verify Pigou-Dalton transfer principle

The most striking fact is:

Theorem

Suppose there are $a, b, c, d \in Y \subseteq \mathbb{R}$ s.t. $c - d > b - c > a - b > 0$. Then there are not SWFs on $\mathbf{X} = Y^{\mathbb{N}}$ that verify M and AE (resp., PDT, or other axioms of *strict* aversion to inequality like the Strong Equity Principle).

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