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Advanced Control Strategies Based on Invariance Set Theory and Economic MPC: Application to WWTP

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CERTIFY

That the doctoral thesis entitled '**Advanced Control strategies Based on Invariance Set Theory and Economic MPC: Application to WWTP** " by Hicham El bahja , presented in partial fulfillment of the requirements for the degree of PhD, has been developed and written under their supervision.

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Preface

This thesis is submitted in partial fulfilment of the requirements for the degree of philosophiae doctor (PhD) at Salamanca University (USAL).

First of all, I am indebted to my supervisor professor Pastora Isabel Vega Cruz for offering me the opportunity to study for a PhD degree, for the excellent guidance during this work, for a good number of valuable discussions, for reading and correcting diverse reports, articles, and the drafts of this thesis. I am also thankful to you for your friendship and for so many enjoyable trips, café-visits, and parties.

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Summary

The increasing demand for environmental safety, energy efficiency and high product quality specifications dictate the development of advanced control algorithms in order to improve the operation and performance of industrial processes. Motivated by the fact that many industrially important processes are characterized by strong nonlinearities, model uncertainty and constraints, and the lack of general linear or nonlinear control methods for such systems, the first broad objective of this thesis is to develop a rigorous, but practical, unified framework for the design of control laws for nonlinear processes with input constraints, that integrates explicit constraint-handling capabilities in the controller designs and provide an explicit characterization of the stability and performance properties of the designed controllers. The second goal is to provide fundamental understanding and insight into the nature of the control problem for nonlinear processes as well as the limitations imposed by nonlinearities and constraints on our ability to steer the dynamics of such systems. The final aim is to evaluate the developed control strategies through simulation models, in a WWTP as case study.

The rest of the thesis is organized as follows. Chapter 2 contains a short description of WWTP and its different treatment steps. It also presents the details of three models of Activated Sludge Process (ASP) that will be used in subsequent chapters. Then, we describe the disturbances that affect the process, some dynamic indices of effluent quality and the operating conditions required for a suitable functioning. This chapter ends with the description of the control problem. Chapter 3 presents some theoretical notions and some tools that will be used later. After the chapter 3, the thesis is composed by two parts with different methodologies proposed. The first one is presents the control strategies based on positive invariance concept and the second one the control strategies based on dynamic real time optimization.

The first controller developed in the first part is a nonlinear feedback law that will cause the output to track a hight amplitude step input rapidly without experiencing large overshoot and without the adverse actuator saturation effects and ensuring stability and constraints fulfillment. The controller consists of a linear feedback law computed using the invariance positive concept and a nonlinear feedback law without any switching element. The linear

feedback part is designed to yield a closed-loop system with a small damping ratio for a quick response, while at the same time not exceeding the actuator limits for the desired command input levels. The nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part. The second controller is a closed loop MPC using polyhedral invariant sets that gives a simple solution to this type of control, ensuring stability and respecting constraints on control magnitude and moves in both modes of operation of the dual controller. The proposed solution can take into account symmetric and asymmetric constraints, and reduces significantly the computational burden associated with the constrained MPC problem in the presence of these constraints.

The second part is composed also by two controllers, the first one a new single layer Nonlinear Closed-Loop Generalized Predictive Control based on an economic nonlinear GPC, as an efficient advanced control technique for improving economics in the operation of nonlinear plants. The second controller focuses on integrating dynamic economic and nonlinear closed loop MPC. The architecture is composed by two-layer. The upper layer consisting of an economic MPC that receives state feedback and time dependent economic information, for computing economically optimal time-varying operating trajectories for the process, by optimizing a time-dependent economic cost function over a finite prediction horizon. The lower layer, utilizes a nonlinear closed loop MPC to compute a feedback control actions that force the outputs of the process to track the trajectories received from the upper layer. It is also proved that the deviation between the state of the closed loop system and the economically time varying trajectory is bounded.

Resumen

La creciente demanda de seguridad ambiental, eficiencia energética y altas especificaciones de calidad del producto promueven el desarrollo de algoritmos de control avanzado para mejorar el funcionamiento y el rendimiento de los procesos industriales. Motivado por el hecho de que muchos procesos industriales relevantes se caracterizan por fuertes no linealidades, incertidumbre y restricciones del modelo, y la falta de métodos generales de control lineal o no lineal para tales sistemas, el primer objetivo general de esta tesis es desarrollar una metodología rigurosa, práctica y unificada para el diseño de leyes de control para procesos no lineales con restricciones de entrada, que integra capacidades explícitas de manejo de restricciones en los diseños de controlador y proporciona una caracterización explícita de las propiedades de estabilidad y rendimiento de los controladores diseñados. El segundo objetivo es proporcionar una comprensión fundamental de la naturaleza del problema de control no lineal, así como de las limitaciones impuestas por las no linealidades y restricciones para dirigir la dinámica de tales sistemas. El objetivo final es evaluar el control y estrategias desarrolladas mediante modelos de simulación, utilizando EDAR como caso de estudio.

El resto de la tesis se organiza de la siguiente manera. El capítulo 2 contiene una breve descripción de la EDAR y sus diferentes etapas de tratamiento. También presenta los detalles de tres modelos del proceso de fangos activados (ASP) que se utilizarán en los capítulos siguientes. A continuación se describen las perturbaciones que afectan al proceso, algunos índices dinámicos de calidad del efluente y las condiciones operativas requeridas para un funcionamiento adecuado. Este capítulo termina con la descripción del problema de control. El capítulo 3 presenta algunas nociones teóricas y algunas herramientas que se utilizarán en las metodologías propuestas. A partir de ahí, la tesis se compone de dos partes con estrategias de control diferentes. La primera está formada por estrategias de control basadas en el concepto de invariancia positiva y la segunda fundamentalmente por estrategias de control basadas en optimización dinámica en tiempo real.

El primer controlador desarrollado en la primera parte es una ley de retroalimentación no lineal que hará que la salida siga rápidamente una entrada escalón de gran amplitud sin experimentar un sobreimpulso grande y

sin los efectos adversos de saturación del actuador y asegurando la estabilidad y el cumplimiento de las restricciones. El controlador consiste en una ley de realimentación lineal calculada usando el concepto de invariancia positiva y una ley de realimentación no lineal sin ningún elemento de conmutación. La parte de realimentación lineal está diseñada para producir un sistema en lazo cerrado con una pequeña relación de amortiguación para una respuesta rápida, mientras que al mismo tiempo no excede los límites del actuador para los niveles de entrada requeridos. La ley de realimentación no lineal se utiliza para aumentar la relación de amortiguación del sistema en lazo cerrado cuando la salida del sistema se aproxima a la referencia para reducir el sobreimpulso causado por la parte lineal. El segundo controlador es un MPC en lazo cerrado utilizando conjuntos invariantes poliédricos que proporciona una solución simple a este tipo de control, garantizando la estabilidad y respetando las restricciones sobre magnitud de control y sus incrementos en ambos modos de funcionamiento del controlador dual. La solución propuesta puede tener en cuenta restricciones simétricas y asimétricas, y reduce significativamente la carga computacional asociada con el problema del MPC con restricciones. La segunda parte presenta también dos metodologías de control, la primera de ellas consiste en un nuevo Control Predictivo Generalizado en lazo cerrado no lineal basado en un GPC no lineal económico, como una técnica de control avanzada eficiente para mejorar la economía en la operación de plantas no lineales. El segundo controlador se centra en la integración de optimización dinámica económica y un controlador MPC en lazo cerrado no lineal. La arquitectura está compuesta por dos capas. La capa superior, consistente en un MPC económico que recibe información de estado e información económica dependiente del tiempo, calcula trayectorias de operación variables en el tiempo económicamente óptimas para el proceso, optimizando una función de coste económico dependiente del tiempo sobre un horizonte de predicción finito. La capa inferior utiliza un MPC en lazo cerrado no lineal para calcular acciones de control que obligan a las salidas del proceso a seguir las trayectorias recibidas desde la capa superior. En esta última metodología, se demuestra que la desviación entre el estado del sistema en lazo cerrado y la trayectoria económica que debe seguir está acotada, asegurando la estabilidad

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Nomenclature

List of symbols

x	:	Biomass
s	:	Substrate
x_r	:	Recycled biomass
c	:	Dissolved oxygen
Y_s	:	Constant yield coefficient
K_c	:	Constant yield coefficient
K_s	:	Affinity constant
Y_c	:	Saturation constant
μ	:	Specific growth rate
μ_{max}	:	Maximum specific growth rate
D	:	Dilution rate
q_r	:	Ratio of recycled flow to influent flow
q_p	:	Ratio of waste flow to influent flow
K_{La}	:	Oxygen mass transfert coefficient
s_{in}	:	Influent substrate concentration
$X_{A,nit}$	:	Autotroph biomass in the aerated tank
$X_{H,nit}$	:	Heterotroph biomass in the aerated tank
$S_{S,nit}$	:	Substrate in the aerated tank
$S_{NH,nit}$	:	Ammonium in the aerated tank
$S_{NO,nit}$	:	Nitrate in the aerated tank
$S_{O,nit}$	:	Oxygen in the aerated tank
$X_{A,denit}$	:	Autotroph biomass in the anoxic tank
$X_{H,denit}$	:	Heterotroph biomass in the anoxic tank
$S_{S,denit}$	:	Substrate in the anoxic tank
$S_{NH,denit}$	:	Ammonium in the anoxic tank

$S_{NO,denit}$	:	Nitrate in the anoxic tank
$S_{O,denit}$	:	Oxygen in the anoxic tank
X_{rec}	:	Recycled biomass
V_{nit}	:	Volume of nitrification basin
V_{denit}	:	Volume of nitrification basin
V_{dec}	:	Volume of of settler
Q_{in}	:	Influent flow rate
Q_{r1}	:	Recycled flow rate
Q_{r2}	:	Intern recycled flow rate
Q_w	:	Waste flow rate
$X_{A,in}$	:	Autotrophs in the influent
$X_{H,in}$	:	Heterotrophs in the influent
$S_{S,in}$	:	Substrate in the influent
$S_{NH,in}$	:	Ammonium in the influent
$S_{NO,in}$	:	Nitrate in the influent
$S_{O,in}$	:	Oxygen in the influent
Y_A	:	Yield of autotroph mass
Y_H	:	Yield of heterotroph mass
K_S	:	Affinity constant
$K_{NH,A}$	:	Affinity constant
$K_{NH,H}$	:	Affinity constant
K_{NO}	:	Affinity constant
$K_{O,A}$	:	Affinity constant
$K_{O,H}$	:	Affinity constant
μ_{Amax}	:	Maximum specific growth rate
μ_{Hmax}	:	Maximum specific growth rate
b_A	:	Decay coefficient of autotrophs
b_H	:	Decay coefficient of heterotrophs
η_{NO}	:	Correction factor for anoxic growth
S_I	:	Soluble inert organic matter
X_I	:	Particulate inert organic matter
XS	:	Slowly biodegradable substrate
$X_{B,H}$	:	Active heterotrophic biomass
$X_{B,A}$	:	Active autotrophic biomass
X_P	:	Particulate products arising
S_{ND}	:	Soluble biodegradable organic nitrogen
X_{ND}	:	Particulate biodegradable organic nitrogen
S_{ALK}	:	Alkalinity
S_{NH}	:	$NH_4 + NH_3$concentration
S_{NO}	:	Nitrate and nitrite concentration
S_S	:	Readily biodegradable substrate concentration
S_O	:	Dissolved oxygen concentration
Q_{in}	:	Influent flow rate

$S_{S,in}$	:	Organic matter concentration
$S_{NH,in}$	:	Ammonium compounds concentration
Q_a	:	Internal recycle flow
V_1	:	Anoxic reactor volume
V_2	:	Aerobic reactor volume
i_{xb}	:	Nitrogen fraction in biomass
ρ_1	:	Aerobic growth of heterotrophic biomass
ρ_2	:	Anoxic growth of heterotrophic biomass
ρ_3	:	Aerobic growths of autotrophic biomass
$\mathbb{R}$	:	Set of real numbers
$\mathbb{R}_+^*$	:	Set of real numbers strictly positive
$\mathbb{R}^n$	:	Real space of dimension n
$\mathbb{R}^{n\times m}$	:	Set of real matrices of dimension $n \times m$
$\sigma(A)$	:	Spectrum of matrix A
$\lambda(A)$	:	The eigenvalues of $A \in \mathbb{R}^{n\times n}$
$\mathcal{R}e(\lambda(A))$	:	Real part of the eigenvalues of A
$\|\|v\|\|$	:	Euclidean norm of v
$\dot{v}$	:	Derivative of v

List of abbreviations

$WWTP$	:	Wastewater Treatment Plant
ASP	:	Activated Sludge Process
MPC	:	Model Predictive Control
RTO	:	Real Time Optimization
$D-RTO$	:	Dynamic Real Time Optimization
NLP	:	Nonlinear Programming
$ASM1$	:	Activated Sludge Model 1
$BSM1$	:	Benchmark Simulation Model 1
GPC	:	Generalized Predictive Control
$BIO-P$	:	Biological Phosphorous
$M1$	:	Mathematical model number 1
$M2$	:	Mathematical model number 2
$M3$	:	Mathematical model number 3
$IAWQ$	:	International Association on Water Quality
$COST$	:	European Cooperation in Science and Technology
IWA	:	International Water Association
ISE	:	Integral Square Error
EQ	:	Effluent Quality
AE	:	Aeration Energy
PE	:	Pumping Energy
COD	:	Chemical Oxygen demand
BOD	:	Biological Oxygen Demand
TSS	:	Total Production of Sludge
QP	:	Quadratic Programming
IQ	:	Influent Quality
DO	:	Dissolved Oxygen
$CLMPC$	:	Closed Loop Model Predictive Control
$NGPC$	:	Nonlinear Generalized Predictive Control
$NEGPC$	:	Nonlinear Economic Generalized Predictive Control
$NCLMPC$	:	Nonlinear Closed Loop Model Predictive Control
$NECLGPC$	:	Nonlinear Economic Closed Loop GPC
OCI	:	Overall Cost Index

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Introduction

1.1 Motivation

Nowadays, modern industrial processes have become highly integrated with respect to material and energy flows, constrained tightly by high quality product specifications, and subject to increasingly strict safety and environmental regulations. These more stringent operating conditions have placed new constraints on the operating flexibility of chemical processes and made the performance requirements for process plants increasingly difficult to satisfy. The increased emphasis placed on safe and efficient plant operation dictates the need for continuous monitoring of the operation of a chemical plant and effective external intervention (control) to guarantee satisfaction of the operational objectives. In this light, it is natural that the subject of process control has become increasingly important in both the academic and industrial communities. In fact, without process control it would not be possible to operate most modern processes safely and profitably, while satisfying plant quality standards.

The design of effective, advanced process control and monitoring systems that can meet these demands, however, can be quite a challenging undertaking given the multitude of fundamental and practical problems that arise in process control systems and transcend the boundaries of specific applications. Although they may vary from one application to another and have different levels of significance, these issues remain generic in their relationship to the control design objectives.

1.2 Previous work

Industrial plants usually involve constrained operation due to different types of constraints, such as limitations on the magnitude and/or increment of some variables, typically the control signals. For this reason, the design of control systems satisfying stability and performance conditions in the presence of constraints is a topic of ongoing interest. In the literature, there are several approaches to solve this type of problems, such as the invariant sets theory [1, 2] and Model Predictive Control (MPC) [3, 4].

The invariant sets theory is a useful instrument used in several branches of engineering systems, for reachability and stability analysis, as well as for the synthesis of constrained control laws. A positively invariant set is a subset of the state space of a system with the property that, if the system state is in this set at some initial condition, then the trajectories of the system will continue in this set in the future [5]. This motivates the development of effective constructive approaches to compute positively invariant sets for dynamical systems. One approach to obtain positively invariant sets can be obtained by noticing that the level surfaces of a Lyapunov function are also boundaries of positively invariant sets. For example, the terminal cost and constraints set approach in MPC [6] requires that the terminal set is positively invariant under some appropriate local feedback law. In [7], the authors studied the regulator problem for linear systems with constraints on both control and its increment in the state space representation using the invariant set technique, the controller is designed to yield a closed loop system with a small damping ratio for a quick response, while at the same time does not exceed the actuator limits for the desired command input and its rate levels.

Apart from control, the positive invariant set is also useful for examination and validation in fault detection cases. It can be used to study the safety properties of a system, that is, properties that specify that a system can never be in a pre-specified subset of ”risky” or ”hazardous” states, as well as stability issues [8].

The other approach to design control systems in the presence of constraints is MPC. MPC has become the accepted technique for complex constrained multivariable control problems in the process industries. MPC is an optimal control-based strategy that uses a plant model to predict the effect of an input profile on the state of the plant. At each sampling time, a constrained optimal control problem is solved over a finite horizon. The updated plant information is used in the next time step, to formulate and solve a new optimal problem, thereby providing feedback from the plant to the model, and the process is repeated. This technique yields a receding horizon control formulation. The solution relies on a linear dynamic model and satisfies all input and output constraints. The use of a quadratic performance index together with various constraints can be used to express real performance objectives, providing excellent MPC performance. Over the last decade, a solid theoretical foundation for MPC has emerged so that in real life, large-scale multi-input and multi-output applications controllers with non-conservative stability guarantees can be designed routinely [9]. The big drawback of the MPC is the large on-line computational effort needed, which limits its applicability to relatively fast systems. Also, the MPC can be used in closed loop control as it has been shown that this procedure is an effective strategy and has been exploited to decrease computational demand of solving optimization control problems. Traditionally, in this type of control two modes of operation are considered over an infinite prediction horizon at each sampling time, being a reformulation of a classical dual mode predictive control [10]. The predicted control moves are centered around a stabilizing control law, over the whole prediction horizon, but some additive degrees of freedom are added over a finite horizon to handle constraints and to guarantee feasibility improving performance. Therefore there is an implicit switching between one mode of operation and the other as the process converges to the desired state. Researchers in the MPC field due to its good properties have progressively adopted the closed loop MPC. For instance, it gives better numerical conditioning of the optimization [11, 12] and it makes robustness analysis more straightforward even for the constrained case [13, 14].

In addition, industry requires optimal operation procedures and advanced control systems to cope with the different factors that affect plant economics and process performance. The two-layer real time optimization strategy (RTO) has been successfully and widely applied in chemical processes for the economic optimization of plant operation. This process control architecture consists of the steady state real time optimization (RTO) of the trajectories for the regulated variables in terms of costs, in an upper level, followed by a model predictive controller (MPC) that executes the direct control actions on shorter time-scales, in a lower level [15, 16, 17].

Nevertheless, the steady state RTO approach may not be satisfactory in some cases leading to sub-optimal economic plant performance [18, 19, 20, 21, 22]. An important weakness of this approach is the inconsistency between the nonlinear steady-state models used in the RTO layer and the usual linear dynamic models used in the regulatory MPC layer. Another drawback is the delay in the optimization associated with the steady-state assumption in the RTO layer; moreover, it can produce an incorrect prediction of the operational point in the presence of frequent disturbances. The Dynamic Real Time Optimization (D-RTO) has been proposed to overcome the limitations of the stationary RTO accounting for the dynamic nonlinear behaviour of processes [23, 24].

The optimization of plant economic performance based on the integration of RTO and MPC has been addressed by in single level and two level strategies [21, 24]. In the single level strategies the economic optimization and control objectives are included in a single MPC algorithm in order to improve both economic and control performance in a cohesive manner. In [25], an optimizing MPC is defined to achieve both tasks by adding an economic objective term to the standard MPC objective function, observing that the one-layer procedure could react to frequent disturbances faster than the multilayer approach. However, a disadvantage of this procedure is that the incorporation of the economic objective turns the optimization problem, which is solved by an MPC algorithm, into a Nonlinear Programming (NLP) problem, where the objective function is nonlinear and there are nonlinear constraints corresponding to the steady-state model of the process system. Consequently, the expected computational effort required to compute the control sequence can be much higher than in the conventional MPC. As a solution, [26] propose a simplified version of the one-layer optimizing MPC. In their approach, the objective function of the MPC controller is also modified to include a term related to the economic objective, but the economic information is restricted to an estimation of the gradient of the economic objective. In [27] a stable MPC controller is presented that efficiently incorporate the stationary-control objectives into a single control formulation considering a velocity model in the input increment instead of the input.

1.3 Scope and outline of thesis

Motivated by the fact that many industrially important processes are characterized by strong nonlinearities, model uncertainty and constraints , and the lack of general linear or nonlinear control methods for such systems, the broad objectives of this thesis are:

- To develop a rigorous, but practical, unified framework for the design of control laws for nonlinear processes with input constraints, that integrates explicit constraint-handling capabilities in the controller designs and provide an explicit characterization of the stability and performance properties of the designed controllers.

- To provide fundamental understanding and insight into the nature of the control problem for nonlinear processes as well as the limitations imposed by nonlinearities and constraints on our ability to steer the dynamics of such systems.

- To evaluate the developed control strategies through simulation models, in a WWTP as case study.

The rest of the thesis is organized as follows. Chapter 2 contains a short description of WWTP and its different treatment steps. It also presents the details of three models of Activated Sludge Process (ASP) that will be used in subsequent chapters. In the first place, a plant with a reactor and a secondary settler is selected, and a mathematical model of the process is presented only with removal of organic matter, to further describe other nitrogen removal models. Then, we describe the disturbances that affect the process, some dynamic indices of effluent quality and the operating conditions required for a suitable functioning. This chapter ends with the description of the control problem. Chapter 3 presents some theoretical notions and some tools that will be used later. First, the problem of the stability of constrained linear systems, in particular the notion of Lyapunov function and the domain of attraction of the equilibrium is presented. Secondly, basic concepts of positive invariance theory are presented. Finally, this chapter is concluded by presenting some basic notion of predictive control.

The goal of chapter 4 is to design a nonlinear feedback law that will cause the output to track a hight amplitude step input rapidly without experiencing large overshoot and, without the adverse actuator saturation effects and ensuring stability and constraints fulfillment . The controller consists of a linear feedback law computed using the invariance positive concept and a nonlinear feedback law without any switching element. The linear feedback part is designed to yield a closed-loop system with a small damping ratio for a quick response, while at the same time not exceeding the actuator limits for the desired command input levels. The nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part. The state feedback control designs are subsequently combined with appropriate Luenberger observers to yield output feedback controllers.

Chapter 5 presents a novel methodology to design a closed loop MPC using polyhedral invariant sets that gives a simple solution to this type of control, ensuring stability and respecting constraints on control magnitude and moves in both modes of operation of the dual controller. It is well known that closed loop predictive control procedure is an effective strategy and has been exploited to decrease computational demand of solving optimization control problems. To achieve the objectives, the first step consists of the development of necessary and sufficient conditions for a linear system and a state feedback control law for the satisfaction of the constraints on both control and its increment over an infinite prediction horizon, proving also the asymptotic stability at the origin. Later on, these conditions are used to obtain the state feedback control law for the prediction computation which guarantees stability while fulfilling constraints and performance requirement of the closed loop system. The proposed solution can take into account symmetric and asymmetric constraints, and reduces significantly the computational burden associated with the constrained MPC problem in the presence of these constraints.

In the chapter 6, the principal scope is the proposal of a new single layer Non- linear Closed-Loop Generalized Predictive Control based on an economic nonlinear GPC, as an efficient advanced control technique for improving economics in the operation of nonlinear plants. The proposed approach, in contrast to classic closed loop MPC scheme, where the terminal control law is computed offline by solving a linear quadratic regulator problem, computes analytically the terminal control law online by solving an unconstrained Nonlinear Generalized Predictive Control minimizing a cost function constituted by tracking errors and economic costs. In order to be able to obtain an analytical solution of this non linear optimization problem two considerations have been made in the present work. Firstly, the prediction model consisting of a nonlinear phenomenological model of the plant is written in the extended linearization form or state dependent coefficient form, which actually allows having nonlinear model expressed with linear structure and state dependent matrices. Secondly, instead of including the nonlinear economic cost in the objective function, an approximation of the reduced gradient of the economic function is used. In this way the problem becomes a quadratic one, and can be solved analytically, at each sampling time, as in the linear case to obtain the terminal control law to be used within the closed loop MPC scheme.

Finally, chapter 7 focuses on integrating dynamic economic and nonlinear closed loop MPC. The architecture is composed by two-layer. The upper layer, consisting of an economic MPC that receives state feedback and time dependent economic information, for computing economically optimal time-varying operating trajectories for the process, by optimizing a time-dependent economic cost function over a finite prediction horizon. The lower layer, utilize a nonlinear closed loop MPC to compute a feedback control actions that force the outputs of the process to track the trajectories received from the upper layer. Also, proves that the deviation between the state of the closed loop system and the economically time varying trajectory is bounded. The proposed approach is based on the use of non-linear phenomenological models of the process to describe all relevant dynamics and to cover a wide operating range, providing accurate predictions and ensuring the performance of the control systems.

The proposed control methods are illustrated through their application to WWTP and compared with more traditional process control strategies.

1.4 Main contributions summary

The main contributions of this thesis are:

- The design of a nonlinear feedback law that will cause the output to track a hight amplitude step input rapidly without experiencing large overshoot and, without the adverse actuator saturation effects and ensuring stability and constraints.

- A novel methodology to design a closed loop MPC using polyhedral invariant sets that gives a simple solution to this type of control, ensuring stability and respecting constraints on control magnitude and moves in both modes of operation of the dual controller. Moreover, the proposed algorithm takes advantage of the design of a state feedback to increase the degrees of freedom in the design procedure.

- The proposal of a new single layer Nonlinear Closed-Loop Generalized Predictive Control based on an economic nonlinear GPC, as an efficient advanced control technique for improving economics in the operation of nonlinear plants.

- The proposal of a two layer strategy framework for integrating dynamic economic optimization and model predictive control for optimal operation of nonlinear process systems.

The thesis in particularly focussed on stability issues and constraint fulfillment in the design step of each control system.

2

Activated Sludge Processes for WWTP: Case Studies

2.1 Introduction

This chapter contains a short description of wastewater treatment process (WWTP) and its different treatment steps. Secondly, the activated sludge process (ASP) are briefly described. Third, three different processes of ASP and their models are detailed. In the first place, a plant with a reactor and a secondary settler is selected, and a mathematical model of the process is presented only with removal of organic matter, to further describe other nitrogen removal models. Then, we describe the disturbances that affect the process, some indices of effluent quality and the operating conditions required for a suitable functioning are described. The chapter ends with the description of the control problem.

2.1.1 WWTP description

WWTP is just one component in the urban water cycle; however, it is an important one since it ensures that the environmental impact of human usage of water is significantly reduced. It consists of several processes: biological, chemical and physical processes. That aim to reduce nitrogen, phosphorous, organic matter and suspended solids. To achieve that, WWTP have been designed with four treatment steps: a primarily mechanical pre-treatment step, a biological treatment step, a chemical treatment step and a sludge treatment step. (See Figure 2.1.)

2.1.1.1 The Primary Mechanical Pre-treatment

The purpose of the mechanical pre-treatment step is to remove various types of suspended solids from the incoming influent. Typically, it consists of grids that remove larger objects in the WWTP, an aerated sand filter that removes sand and a primary sedimentation unit that reduces the content of suspended solids in the wastewater by sedimentation. The primary sedimentation may also remove considerable amounts of organic matter in the particulate form and, hence, reduce the need for aeration later in the process.

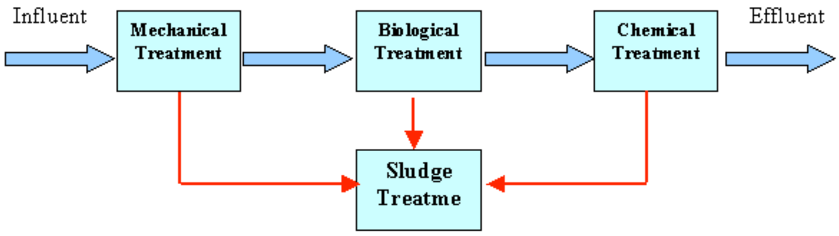


Figure 2.1: Layout of a typical wastewater treatment plant

2.1.1.2 The Biological Treatment Step

Traditionally, the aim of the biological treatment step has originally been solely to remove organic matter. However, nowday many WWTP are also designed for the biological removal of nitrogen and phosphorous. The most common type of biological treatment step is based on the activated sludge process. The simplest type of an activated sludge wastewater treatment system is illustrated in Figure 2.2. The biological reactor contains a mixture of microorganisms suspended in wastewater; called activated sludge. The microorganisms degrade the content of organic matter in the wastewater aerobically, i.e. when air is supplied to the biological reactor. To retain the sludge in the system, the biological reactor is followed by a sedimentation unit that separates the clean effluent wastewater from the sludge. The sludge is then recycled into the biological reactor. Due to the growth of the microorganisms, sludge has to be removed from the system continuously via the sludge outtake. In this simple system, the main control handles are: aeration, sludge outtake and sludge recirculation. These variables should be controlled to ensure a suitable treatment efficiency of the process, which includes maintaining a correct amount of sludge in the system.

The Organic matter elimination process can be summarized in the following chemical reactions:
Energy:

$$\text{Organic matter} + O_2 \xrightarrow{microorganisms} CO_2 + H_2O + \text{other products+energy}$$

$$\text{Synthesis: Organic matter} + O_2 + \text{energy} \xrightarrow{microorganisms} \text{ new cells}$$

$$\text{Decay: Microorganisms} + O_2 \rightarrow CO_2 + H_2O + \text{energy}$$

The nitrogen removal process is somehow more complicated, as the process requires both aerobic and anoxic conditions. A simplified process di-

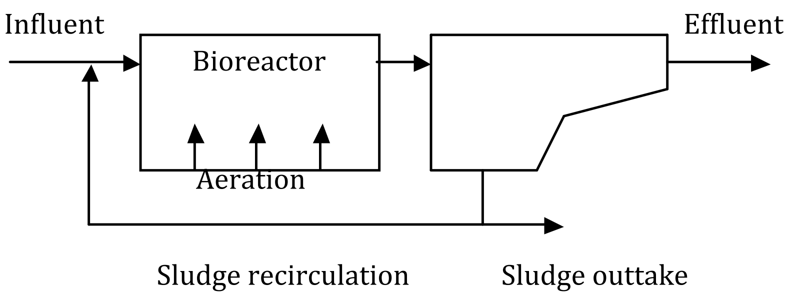


Figure 2.2: Simple activated sludge system

agram of the whole process can be seen in Figure 2.3. The first step is an aerobic nitrification process where nitrifies (i.e. microorganisms able to perform nitrification) convert ammonium to nitrate. This is followed by an anoxic process, known as denitrification, where nitrate is converted to free gaseous nitrogen, which leaves the water through the surface into the air. For this process, the denitrifying microorganisms use easily biodegradable organic matter.

Several types of wastewater treatment plants can perform the nitrogen removal processes; see an overview in e.g. [28]. One of the most widespread plant designs is the pre-denitrification system, which is depicted in Figure 2.4. To satisfy the need for easily degradable organic matter in the denitrification process the denitrifying biological reactor is located first so, that it can use the organic matter in the influent wastewater. To ensure the presence of nitrate in this reactor, the wastewater from the following nitrifying biological reactor is recycled via the internal recirculation.

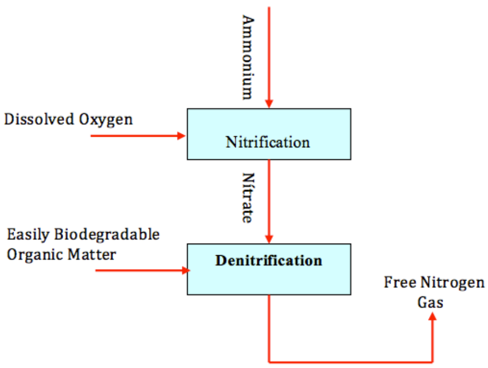


Figure 2.3: Nitrogen removal process

To maintain the sludge in the system there is also a sludge recirculation stream, which further supplements the internal recirculation by recycling more nitrates to the denitrification biological reactor.

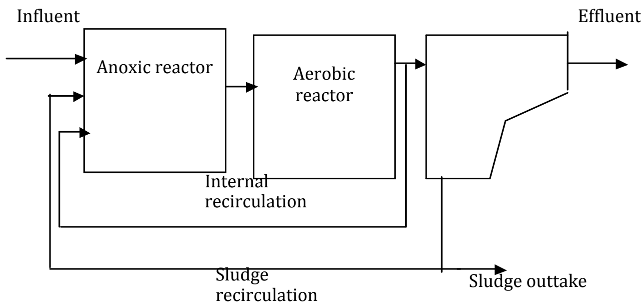


Figure 2.4: Pre-denitrification plant design

The main control handles in this process are: aeration, internal recirculation, sludge outtake and sludge recirculation. The Nitrogen elimination process can be summarized in the following chemical reactions, where the two first ones are for nitrification and the last one for denitrification:

$$NH_4^+ + \frac{3}{2}O_2 \xrightarrow{nitrosomonas} NO_2^- + H_2O + 2H^+ + New \quad biomass$$
$$NO_2^- + \frac{1}{2}O_2 \xrightarrow{nitrobacter} NO_3^-$$
$$NO_3^- + \frac{5}{6}CH_3OH \rightarrow \frac{1}{2}N_2 + \frac{5}{6}CO_2 + \frac{7}{6}H_2O + OH^-$$

Finally, phosphorous removal can be performed biologically or chemically by using advanced techniques [ref]. The chemical process is described later. The enhanced biological phosphorous removal is a fairly new process in the history of wastewater treatment. The process is performed by phosphorous accumulating organisms (PAOs). The PAOs release phosphate during anaerobic conditions (i.e. neither nitrate nor dissolved oxygen present) and take up phosphate during aerobic or anoxic conditions. As the uptake is larger than the release, it leads to a net uptake of phosphorous.

The process depends on the presence of volatile fatty acids (VFA), which are easily degradable organic matter. Wastewater treatment plant designs for biological phosphorous removal are typically similar to designs with nitrogen removal. Additionally, the plants are supplied with an anaerobic biological phosphorous release reactor (BIO-P reactor) preceding the nitrogen removal system, see Figure 2.5.

2.1.1.3 Sedimentation Process

In the biological treatment of wastewater, the sedimentation process enables to separate the treated wastewater from the biomass sludge and produces

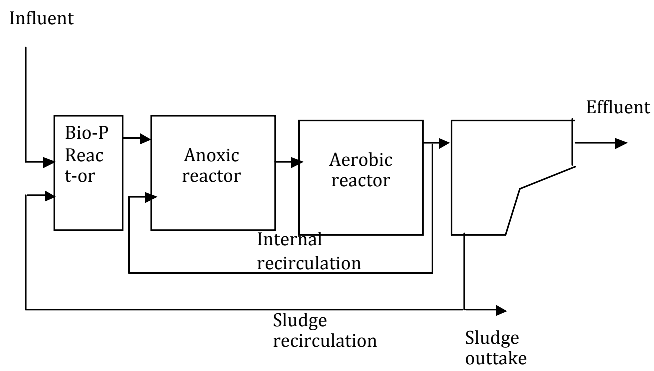


Figure 2.5: Activated sludge system designed for biological phosphorous removal

a clean treated effluent. In addition to clarification, secondary settler tanks or clarifiers have the function of thickening the activated sludge for returning it to the bioreactor and even the sludge. Settling problems are mainly associated to hydraulic loads or sludge characteristics (they arise from an unbalanced composition of the bacterial community and from the excessive proliferation of filamentous bacteria). Therefore, the identification and monitoring of these bacteria are critical to estimate the settling capacity of the activated sludge.

By all these reasons, secondary settling tanks have been considered essential and often they can be limiting factors for good removal efficiencies of the activated sludge system. *Tákacs* and assistants developed the first model of settling process that can consider or not possible biological reactions in the clarifier [29]. Simulation of sedimentation has enabled a better understanding of the settling tanks.

2.1.1.4 The Chemical Treatment

Before the biological phosphorous removal process was developed, the common procedure to remove phosphorous was by chemical precipitation. This is a well-proven technology that is still the dominating way of removing phosphorous. The purpose of the chemical treatment step is the chemical removal of phosphorous. The process consists of dosing of a chemical substance (typically an iron or aluminum salt) that binds phosphate molecules and forms flocs that can be removed by sedimentation. Hence, the phosphorous is removed via a chemical sludge. The process is depicted in Figure 2.6.

For the chemical precipitation process to function, two reactors are needed: a flocculation chamber where the chemicals are added and the flocs are formed and a sedimentation unit, which separates the flocs from the water. The precipitation process may take place at several locations in the wastewater treatment plant. In pre-precipitation plants, the process is carried out in the mechanical pre-treatment step. In simultaneous precipitation the precipitation is performed in the biological step and in post-precipitation plants the process is carried out in a separate chemical step following the biological step. These are the basic options, but others exist. Often a combination of two of these structures is used.

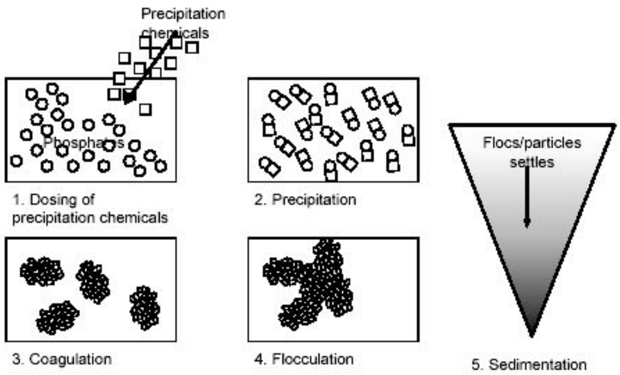


Figure 2.6: The process of phosphorous precipitation

2.1.1.5 The Sludge Treatment

The purpose of the sludge treatment step is to prepare the sludge for end disposal. Anaerobic digestion is probably one of the most used processes for reducing the amount of sludge. At the same time, the digestion process produces gas, providing a significant source of energy, which is usually used at the WWTP. Sludge treatment also includes various dewatering processes, which reduce weight and volume of the sludge. Sludge treatment is gaining in importance as it becomes increasingly difficult to dispose of the sludge. Sludge disposal is in many countries becoming one of the largest costs of wastewater treatment.

2.1.2 Management of Environmental Processes

Environmental processes are complex systems, involving many interactions between physical, chemical and biological processes, e.g. chemical or biological reactions, kinetics, catalysis, transport phenomena, separations, etc. The successful management of these systems requires multi-disciplinary approaches and expertise from different social and scientific fields. Some of the problematic and special features of environmental processes are described below [30].

- Intrinsic unsteadiness: most of the chemical and physical properties as well as the population of microorganisms (both in total quantity and number of species) involved in environmental processes do not remain constant over time. For example, in the WWTP, the characteristics of the influent can be highly variable both in quantity (flow rate) and in quality (concentrations) and there are different changing interrelations between substrates and microorganisms. This means that the often-assumed stability of these systems is wrong in many cases [31, 32].

- Ill-structured domain: environmental systems are poor or ill structured domain, that is, they involve knowledge difficult to clearly formulate due to its high complexity.

- Uncertainty and imprecision of data: many environmental systems are stochastic. The parameters used in models defining such processes are usually uncertain, and their operational ranges are only known approximately. Most of the information available in environmental processes involving biological reactions is qualitative and difficult to translate into numerical values. In the case of WWTP, there is a complex population community that evolves with time since it adapts to the process and influent characteristics.

- Huge quantity of data/information: the application of current computer technology to the control and supervisory elements for these environmental systems has led to a significant increase of amount of data acquired (improved SCADA equipment enables to acquire large quantities of data). However, such an increase in frequency, quality, quantity or diversification (quantitative as well as qualitative) of data of the same process not always corresponds to a similar increase in the process understanding and improvement.

- Heterogeneity and scale: because of the media in which environmental processes take place are not homogeneous and cannot easily be characterized by measurable parameters, data is often heterogeneous. For environmental real world problem, data comes from numerous sources, in different format, frequency and quality. In addition to the special features of environmental systems already explained, WWTP, especially biological processes, have additional particular features from of control that viewpoint make them even more difficult to control and supervise using a single conventional strategy, with respect to any other environmental process.

2.2 Models of activated sludge processes

In this section the mathematical models of the activated sludge process that will be considered in this thesis are described. The models will be used for control design as well as for testing the different methodologies. This section starts with a reduced model for organic matter removal, followed by two models for nitrogen and substrate removal.

2.2.1 Mathematical model for organic matter removal ($M1$)

The plant layout for $M1$ constituted by a bioreactor and a settler. The aerator is a well-stirred composed where suspended micro-organisms biochemically degrade the dissolved substrate. The suspended micro-organisms are separated completely in the secondary settler. A portion of the concentrated biomass is recycled to the bioreactor (see figure 2.7).

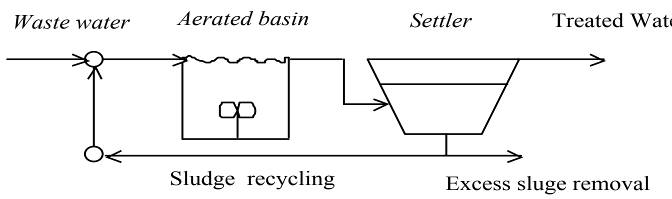


Figure 2.7: Schematic view of an activated sludge process.

The first nonlinear mathematical model considered is described here, and it will be denominated as $M1$.

The main characteristics and assumptions are the following:

- Assumptions:
  - A dissolved oxygen concentration denoted as (c)
  - A single homogeneous population of micro-organisms (x).
  - One soluble organic substrate (s).
  - A single biological process: bacterial growth and organic degradation. The biological kinetic is modeled by $\phi_a(s, c, x) = \mu(.)$ where $\mu(.)$ is the specific growth rate, that may depend on s, c, x and other external factors.
  - The aeration mechanism affects the dynamics of the bioreactor.
  - The mechanisms of sedimentation of sludge under the influence of gravity are negligible.

  - The concentration of soluble substances in the compression zone of the sludge is negligible.
  - The volumes of aeration tanks and sedimentation are assumed constant.
  - The concentration of micro-organisms in the influent is negligible.
  - The concentration of oxygen dissolved in the influent is negligible.
  - The soluble concentrations in the recycling flow and the concentrations of suspended particles in the effluent are supposed to be negligible

- Equations

The dynamic equations of the model $M1$ can be described as follow:

$$\begin{aligned}
\dot{x} &= \mu(.)x - (1 + q_r(t))D(t)x + q_r D(t)x_r \\
\dot{s} &= -\mu(.)x - (1 + q_r(t))D(t)s + D(t)s_{in}(t) \\
\dot{c} &= -\frac{\mu(.)x}{Y_c} - (1 + q_r(t))D(t)c + K_{La}(c_s - c) \\
\dot{x}_r &= \omega_{e_2}(1 + q_r(t))D(t)x - \omega_{e_2}(q_p(t) + q_r(t))D(t)x_r
\end{aligned} \tag{2.1}$$

Where:

- The term $K_{La}(c_s - c)$ is related to aeration of the bioreactor and defines the transfer of oxygen between the gas and liquid phases.
- c_s is the maximum dissolved oxygen concentration.
- The state variables are defined and summarized in table 2.1.
- $\mu(t)$ corresponds to the biomass specific growth rate. It assumed to follow the Olsson model [33]:

$$\mu(t) = \mu_{max}\frac{s(t)}{K_s + s(t)}\frac{c(t)}{K_c + c(t)} \tag{2.2}$$

  where μ_{max} is the maximum specific growth rate, K_s is the affinity constant and K_c is the saturation constant.
- $D(t)$ is the dilution rate.
- q_r and q_p represent the ratio of recycled flow to influent flow and the ratio of waste flow to influent flow.
- K_{La} represents the oxygen mass transfer coefficient.
- s_{in} corresponds to the influent substrate concentration.

Table 2.1: State variables for $M1$

State Variable	State Symbol	Units
Biomass	x	mg/l
Substrate	s	mg/l
recycled biomass	x_r	mg/l
Dissolved oxygen	c	mg/l

- Y_s and Y_c are constants yields coefficients.

- Value of plant parameters.

For the simulation of the equations of the model ($M1$), typical values of the model parameters and initial conditions are based on data available in literature [34, 35, 36] and summarized in the table 2.2.

Table 2.2: Typical values of process and kinetic parameters

Parameters/Functions	Values/Expression	Units
Specific growth rate	$\mu(t) = \mu_{max} \frac{s}{K_s+s} \frac{c}{K_c+c}$	
Kinetics parameters	$\mu_{max} = 0.2$	h^{-1}
	$K_s = 75$	mg/l
	$K_c = 2$	mg/l
Stoichiometric parameters	$Y_s = 1.8$	
	$Y_s = 0.8$	
Operational parameters	$\omega_{e_2} = 2$	
	$q_r = 1$	
	$q_p = 0.05$	
Process inputs	$D = 0.4$	h^{-1}
	$K_{La} = 1.8$	h^{-1}
Initial conditions	$s(0) = 20$	mg/l
	$x(0) = 1225$	mg/l
	$x_r(0) = 2333$	mg/l
	$c(0) = 6$	mg/l

2.2.2 The mathematical model $M2$ obtained by simplification of ASM1

A simplification of the ASM1 (Activated Sludge Model 1) is considered here for the removal of carbonaceous and nitrogen materials and it will be denoted $M2$. Consists of an anoxic basin followed by an aerated one, and a settler ($Fig.$2.8). In the presence of dissolved oxygen, the wastewater, is biodegraded in the reactor. Treated effluent is separated from the sludge in the secondary settler and fraction is returned to anoxic reactor to maintain the appropriate substrate-to-biomass ratio [37].

In this model, we consider six basic state variables in each reactor, together with the concentration of biomass in the settler (Table 2.3) .

In the formulation of the model $M2$ the following assumptions are considered:

- the physical properties of fluid are constant;
- there is no concentration gradient across the reactors and settler;
- substrates and dissolved oxygen are considered as a rate-limiting with a Monod-type kinetic;
- no bio-reaction takes place in the settler.

Based on the above description and assumptions, the full set of ordinary differential equations (mass balance equations), can be formulated.

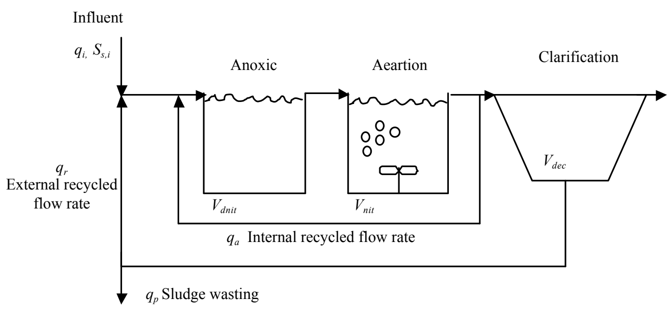


Figure 2.8: General layout for $M2$ oplant.

The mathematical model consists of the following equations obtained from mass balances, where the numerical values of the parameters, dimensions and characteristics of the influent are defined in Tables 2.4 and 2.5.

- Modeling of the aerated basin:

Table 2.3: State variables of $M2$

Description	Units	Variables
$X_{A,nit}$	mg/l	autotroph biomass in the aerated tank
$X_{H,nit}$	mg/l	heterotroph biomass in the aerated tank
$S_{S,nit}$	mg/l	substrate in the aerated tank
$S_{NH,nit}$	mg/l	ammonium in the aerated tank
$S_{NO,nit}$	mg/l	nitrate in the aerated tank
$S_{O,nit}$	mg/l	oxygen in the aerated tank
$X_{A,denit}$	mg/l	autotroph biomass in the anoxic tank
$X_{H,denit}$	mg/l	heterotroph biomass in the anoxic tank
$S_{S,denit}$	mg/l	substrate in the anoxic tank
$S_{NH,denit}$	mg/l	ammonium in the anoxic tank
$S_{NO,denit}$	mg/l	nitrate in the anoxic tank
$S_{O,denit}$	mg/l	oxygen in the anoxic tank
X_{rec}	mg/l	recycled biomass

Table 2.4: Process characteristics

Variables	Value	Description
V_{nit}	1000 m^3	volume of nitrification basin
V_{denit}	250 m^3	volume of nitrification basin
V_{dec}	1250 m^3	volume of of settler
Q_{in}	3000 m^3/j	influent flow rate
Q_{r1}	2955 m^3/j	recycled flow rate
Q_{r2}	1500 m^3/j	internal recycled flow rate
Q_w	45 m^3/j	waste flow rate
$X_{A,in}$	0 mg/l	autotrophs in the influent
$X_{H,in}$	30 mg/l	heterotrophs in the influent
$S_{S,in}$	200 mg/l	substrate in the influent
$S_{NH,in}$	30 mg/l	ammonium in the influent
$S_{NO,in}$	2 mg/l	nitrate in the influent
$S_{O,in}$	0 mg/l	oxygen in the influent

Table 2.5: Kinetic parameters and stoichiometric coefficients of $M2$.

Parameter	Value	Description
Y_A	0.24	yield of autotroph mass
Y_H	0.67	yield of heterotroph mass
i_{xb}	0.086	
K_S	20 mg/l	affinity constant
$K_{NH,A}$	1 mg/l	affinity constant
$K_{NH,H}$	0.05 mg/l	affinity constant
K_{NO}	0.5 mg/l	affinity constant
$K_{O,A}$	0.4 mg/l	affinity constant
$K_{O,H}$	0.2 mg/l	affinity constant
μ_{Amax}	0.8 l/j	maximum specific growth rate
μ_{Hmax}	0.6 l/j	maximum specific growth rate
b_A	0.2 l/j	decay coefficient of autotrophs
b_H	0.68 l/j	decay coefficient of heterotrophs
η_{NO}	0.8 l/j	correction factor for anoxic growth

$$
\begin{aligned}
\dot{X}_{A,nit}(t) &= (1+r_1+r_2)\,D_{nit}\,(X_{A,denit}-X_{A,nit}) + (\mu_{A,nit}-b_A)\,X_{A,nit}\\
\dot{X}_{H,nit}(t) &= (1+r_1+r_2)\,D_{nit}\,(X_{H,denit}-X_{H,nit}) + (\mu_{H,nit}-b_H)\,X_{H,nit}\\
\dot{S}_{S,nit}(t) &= (1+r_1+r_2)\,D_{nit}\,(S_{S,denit}-S_{S,nit}) + (\mu_{H,nit}+\mu_{Ha,nit})\,X_{H,nit}/Y_H\\
\dot{S}_{NH,nit}(t) &= (1+r_1+r_2)\,D_{nit}\,(S_{NH,denit}-S_{NH,nit}) + \left(i_{xb}+\frac{1}{Y_A}\right)\mu_{A,nit}X_{A,nit}\\
&\quad - (\mu_{H,nit}+\mu_{Ha,nit})\,i_{xb}X_{H,nit}\\
\dot{S}_{NO,nit}(t) &= (1+r_1+r_2)\,D_{nit}\,(S_{NO,denit}-S_{NO,nit}) + \mu_{A,nit}\frac{X_{A,nit}}{Y_A}\\
&\quad - \frac{4.57-Y_A}{2.86Y_A}\mu_{Ha,nit}X_{H,nit}\\
\dot{S}_{O,nit}(t) &= (1+r_1+r_2)\,D_{nit}\,(S_{O,denit}-S_{O,nit}) + a_0Q_{air}(C_S-S_{O,nit})\\
&\quad - \frac{4.57-Y_A}{Y_A}\mu_{Ha,nit}X_{H,nit} - \frac{1-Y_H}{Y_H}\mu Ha,nitX_{H,nit}
\end{aligned}
\tag{2.3}
$$

- Modeling of the anoxic basin:

$$
\begin{aligned}
\dot{X}_{A,denit}(t) &= D_{denit}(X_{A,in} + r_1 X_{A,nit}) - (1 + r_1 + r_2)\, D_{denit} X_{A,denit} \\
&+ \alpha r_2 D_{denit} X_{rec} + (\mu_{A,denit} - b_A) X_{A,denit} \\
\dot{X}_{H,denit}(t) &= D_{denit}(X_{H,in} + r_1 X_{H,nit}) - (1 + r_1 + r_2)\, D_{denit} X_{H,denit} \\
&+ (1 - \alpha) r_2 D_{denit} X_{rec} + (\mu_{H,denit} - b_H) X_{H,denit} \\
\dot{S}_{S,denit}(t) &= -(\mu_{H,denit - \mu_{Ha,denit}}) \frac{X_{H,denit}}{Y_H} - (1 + r_1 + r_2)\, D_{denit} S_{S,denit} \\
&+ D_{denit}(S_{S,in} - r_1 S_{S,nit}) \\
\dot{S}_{NH,denit}(t) &= D_{denit}(S_{NH,in} - r_1 S_{NH,nit}) - (1 + r_1 + r_2)\, D_{denit} S_{NH,denit} \\
&- (i_{xb} + 1/Y_A)\, \mu_{A,denit} X_{A,denit} - (\mu_{H,denit} + \mu_{Ha,denit})\, i_{xb} X_{H,denit} \\
\dot{S}_{NO,denit}(t) &= D_{denit}(S_{NO,in} - r_1 S_{NO,nit}) - (1 + r_1 + r_2)\, D_{denit} S_{NO,denit} \\
&+ \frac{\mu_{A,denit} X_{A,denit}}{Y_A} - \frac{1 - Y_H}{2.86 Y_H} \mu_{Ha,denit} X_{H,denit}
\end{aligned}
\tag{2.4}
$$

- Modeling of the settler

$$
\dot{X}_{rec} = (1 + r_2) D_{dec}(X_{A,nit} + X_{H,nit}) - (r_2 + w) D_{dec} X_{rec} \tag{2.5}
$$

Where the growth rates of autotrophs, heterotrophs and in both aerobic and anoxic conditions are defined as follow:

$$
\begin{cases}
\mu_{A,nit} & = \mu_{A,max} \cdot \frac{S_{NH,nit}}{(K_{NH,A} + S_{NH,nit})} \cdot \frac{S_{O,nit}}{(K_{O,A} + S_{O,nit})} \\
\mu_{H,nit} & = \mu_{H,max} \cdot \frac{S_{S,nit}}{(K_S + S_{S,nit})} \cdot \frac{S_{NH,nit}}{(K_{NH,H} + S_{NH,nit})} \cdot \frac{S_{O,nit}}{(K_{O,H} + S_{O,nit})} \\
\mu_{Ha,nit} & = \mu_{H,max} \cdot \frac{S_{S,nit}}{(K_S + S_{S,nit})} \cdot \frac{S_{NH,nit}}{(K_{NH,H} + S_{NH,nit})} \cdot \frac{K_{O,H}}{(K_{O,H} + S_{O,nit})} \cdot \\
& \quad \frac{S_{NO,nit}}{(K_{NO} + S_{NO,nit})} \cdot \eta_{NO} \\
\mu_{A,denit} & = \mu_{A,max} \cdot \frac{S_{NH,denit}}{(K_{NH,A} + S_{NH,denit})} \\
\mu_{H,denit} & = \mu_{H,max} \cdot \frac{S_{S,denit}}{(K_S + S_{S,denit})} \cdot \frac{S_{NH,denit}}{(K_{NH,H} + S_{NH,denit})} \\
\mu_{Ha,denit} & = \mu_{H,max} \cdot \frac{S_{S,denit}}{(K_S + S_{S,denit})} \cdot \frac{S_{NH,denit}}{(K_{NH,H} + S_{NH,denit})} \cdot \\
& \quad \frac{S_{NO,denit}}{(K_{NO} + S_{NO,denit})} \cdot \eta_{NO}
\end{cases}
$$

Other variables r_1,r_2 and w, that represent respectively, the ratio of the internal recycled flow Q_{r1} to influent flow Q_{in}, the ratio of the recycled flow Q_{r2} to the influent flow and the ratio of purge flow Q_p, C_S is the maximum dissolved oxygen concentration. D_{nit}, D_{denit} and D_{dec} are the dilution rates of nitrification, denitrification basins and settler tank respectively, X_{rec} is the concentration of the recycled biomass.

2.2.3 The mathematical model $M3$ obtained by simplification of BSM1

- The standard simulation platform BSM1

The idea to produce a standardized 'simulation benchmark' was first devised and developed by the first IAWQ Task Group on Respirometry-Based Control of the Activated Sludge Process [38]. This original benchmark was subsequently modified by the European Cooperation in the field of Scientific and Technical Research (COST) 682/624 Actions in co-operation with the second IWA Respirometry Task Group [39, 40, 41]. In an attempt to standardize the simulation procedure and the evaluation of all types of control strategies, the two groups have jointly developed a consistent simulation protocol.

The 'simulation benchmark' plant design is comprised of five reactors in series with a 10-layer secondary settling tank. Figure 2.9 shows a schematic representation of the layout. The tanks 1 and 2 are $1000m^3$ each, unaerated and fully mixed. The tanks 4, 5 and 6 are $1333m^3$ each and aerated.

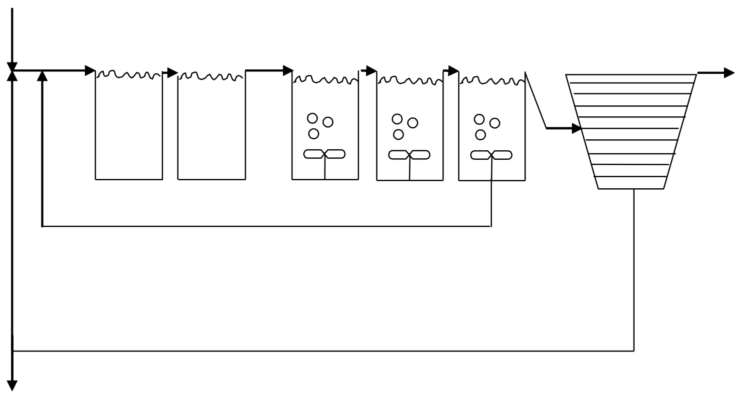


Figure 2.9: Schematic representation of the BSM1.

Other layout characteristic are:

- DO saturation in tanks 3, 4 and 5 of $8gO_2/m^3$
- a non-reactive secondary settler with a volume of $6000m^3$ (area of $1500m^2$ and a depth of $4m$) subdivided into 10 layers
- a feed point to the settler at $2.2m$ from the bottom (i.e. feed enters the settler in the middle of the sixth layer)
- two recycles:
  - nitrate internal recycle from the 5^{th} to the 1^{st} tank
  - Activated sludge recycle from the underflow of the secondary settler to the front end of the plant
- Activated sludge wastage is pumped continuously from the secondary settler underflow

The physical attributes of the biological reactors and the settler are listed in Table 2.6 and a selection of system variables are listed in Table 2.7.

Table 2.6: Physical attributes of the biological reactors and settling tank for the COST 'simulation benchmark' process configuration.

	Physical configuration	Units
Volume-Tnak 1	1000	m^3
Volume-Tnak 2	1000	m^3
Volume-Tnak 3	1333	m^3
Volume-Tnak 4	1333	m^3
Volume-Tnak 5	1333	m^3
Depth-Settler	4	m
Area-Settler	1500	m^2
Volume-Settler	6000	m^3

Table 2.7: A selection of system variables.

	Default System Flow Rates	Units
Influent flow rate	18446	$m^3 day^{-1}$
Recycle flow rate	18446	$m^3 day^{-1}$
Internal recycle flow rate	55338	$m^3 day^{-1}$
Wastage flow rate	385	$m^3 day^{-1}$
K_{La}-Tank 1	n/a	$-$
K_{La}-Tank 2	n/a	$-$
K_{La}-Tank 3	10	h^{-1}
K_{La}-Tank 4	10	h^{-1}
K_{La}-Tank 5	3.5	h^{-1}

The 'simulation benchmark' has 13 states variables and 8 processes. This model representation is included here only as a reference only. A complete description of the model and its development are available elsewhere [42]. Table 2.8 lists the $BSM1$ state variables, the associated symbols and the state variable units.

- Simplification of BSM1

A simplified version of BSM1 in [43, 44] is described here, where only significant variables are taken into account on an average time scale. Slower processes, generally related to insoluble compounds such as hydrolysis and

Table 2.8: State variables for $BSM1$

State Variable Description	State Symbol	Units
Soluble inert organic matter	S_I	$gCOD/m^3$
Readily biodegradable substrate	S_S	$gCOD/m^3$
Particulate inert organic matter	X_I	$gCOD/m^3$
Slowly biodegradable substrate	XS	$gCOD/m^3$
Active heterotrophic biomass	$X_{B,H}$	$gCOD/m^3$
Active autotrophic biomass	$X_{B,A}$	$gCOD/m^3$
Particulate products arising from biomass decay	X_P	$gCOD/m^3$
Oxygen	S_O	$gCOD/m^3$
Nitrate and nitrite nitrogen	S_{NO}	gN/m^3
$NH_4^+ + NH_3$ nitrogen	S_{NH}	gN/m^3
Soluble biodegradable organic nitrogen	S_{ND}	gN/m^3
Particulate biodegradable organic nitrogen	X_{ND}	gN/m^3
Alkalinity	S_{ALK}	mol/L

ammonification of organic nitrogen, are not considered and it is assumed that heterotrophic and autotrophic biomass does not vary, so that the number of states in each reactor is reduced from 13 to 4.

In this simplification of the BSM1, the complete system is reduced to one anoxic and one aerobic reactor, with a volume equivalent to the total volume of the Benchmark's anoxic and aerobic compartments (figure 2.10). it is assumed that the denitrification process only take place in the anoxic tank and the denitrification process occurs only in the aerated tank. The settler is approximated by a mixing point and the external reflux is assumed to be constant.

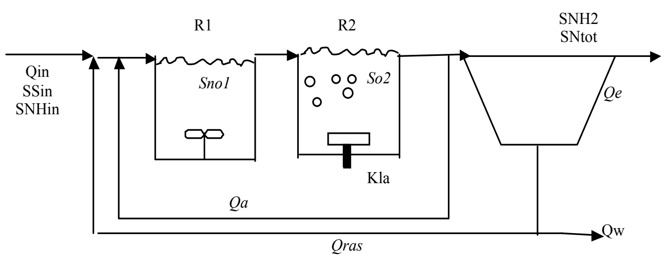


Figure 2.10: Schematic representation of the plant $M3$.

By simplifying the equations representing the mass balances, reaction rates and biological processes involved in BSM1, we obtain the set of differential equations for the simplified model $M3$, where the index 1 is referred to the anoxic tank and the index 2 is referred to the aerobic tank.

- Anoxic reactor:

$$\begin{aligned} \dot{S}_{NH1} &= \frac{1}{V_1}\left[Q_{in}S_{NHin} + Q_a S_{NH2} - (Q_{in}+Q_a)S_{NH1}\right] - i_{xb}\rho_{11} - i_{xb}\rho_{21} \\ &\quad - (i_{xb} + \frac{1}{Y_A})\rho_{31} \\ \dot{S}_{NO1} &= \frac{1}{V_1}\left[Q_a S_{NO2} - (Q_{in}+Q_a)S_{NO1}\right] - \frac{1-Y_H}{2.86Y_H}\rho_{21} + \frac{1}{Y_A}\rho_{31} \\ \dot{S}_{S1} &= \frac{1}{V_1}\left[Q_{in}S_{Sin} + Q_a S_{S2} - (Q_{in}+Q_a)S_{S1}\right] - \frac{1}{Y_H}\rho_{11} - \frac{1}{Y_H}\rho_{21} \\ \dot{S}_{O1} &= \frac{1}{V_1}\left[Q_a S_{O2} - (Q_{in}+Q_a)S_{O1}\right] - \frac{1-Y_H}{Y_H}\rho_{11} - (\frac{4.57}{Y_A}+1)\rho_{31} \end{aligned} \tag{2.6}$$

- Aerobic reactor:

$$\begin{aligned} \dot{S}_{NH2} &= \frac{1}{V_2}\left[(Q_{in}+Q_a)(S_{NH1} - S_{NH2})\right] - i_{xb}\rho_{12} - (i_{xb} + \frac{1}{Y_A})\rho_{32} \\ \dot{S}_{NO2} &= \frac{1}{V_2}\left[(Q_{in}+Q_a)(S_{NO1} - S_{NO2})\right] - \frac{1-Y_H}{2.86Y_A}\rho_{22} + \frac{1}{Y_A}\rho_{32} \\ \dot{S}_{S2} &= \frac{1}{V_2}\left[(Q_{in}+Q_a)(S_{S1} - S_{S2})\right] - \frac{1}{Y_H}\rho_{12} - \frac{1}{Y_H}\rho_{22} \\ \dot{S}_{O2} &= \frac{1}{V_2}\left[(Q_{in}+Q_a)(S_{O1} - S_{O2})\right] - \frac{1-Y_H}{Y_H}\rho_{12} - \frac{4.57-Y_A}{Y_A}\rho_{32} \\ &\quad + KLa(S_{O,Sat} - S_{O2}) \end{aligned} \tag{2.7}$$

In the first reactor, the anoxic growth of heterotrophic biomass is the main biological process, related to denitrification:

$$\rho_{21} = \mu_H.(\frac{S_{S1}}{K_S + S_{S1}}).(\frac{K_{O,H}}{K_{O,H} + S_{O1}}).(\frac{S_{NO1}}{K_{NO} + S_{S1}})\eta_g X_{B,H} \tag{2.8}$$

In the second reactor, where there is a higher concentration of oxygen, the aerobic growths of heterotrophic and autotrophic biomass are considered, related to nitrification:

$$\begin{aligned} \rho_{12} &= \mu_H.(\frac{S_{S2}}{K_S + S_{S2}}).(\frac{S_{O2}}{K_{O,H} + S_{O2}}).X_{B,H} \\ \rho_3(2) &= \mu_A.(\frac{S_{NH2}}{K_{NH} + S_{NH2}}).(\frac{S_{O2}}{K_{O,A} + S_{O2}}).X_{B,A} \end{aligned} \tag{2.9}$$

The rest of processes ρ are assumed to be zero in equations (2.7) and (2.8).

The definitions of the state variables are given in table2.9. The definitions of kinetic and physical parameters are presented in tables 2.10 and 2.11, their values are the same as for BSM1 [41].

Table 2.9: List of state variables of $M3$.

Notation	Definition	Unit
S_{NH}	$NH_4 + NH_3$ concentration	grN/m^3
S_{NO}	Nitrate and nitrite concentration	grN/m^3
S_S	Readily biodegradable substrate concentration	$grCOD/m^3$
S_O	Dissolved oxygen concentration	gr/m^3

Table 2.10: Process characteristics.

Notation	Definition
Q_{in}	Influent flow rate
$S_{S,in}$	Influent organic matter concentration
$S_{NH,in}$	Influent ammonium compounds concentration
Q_a	Internal recycle flow
K_{La}	oxygen transfer coefficient
V_1	Anoxic reactor volume
V_2	Aerobic reactor volume

Table 2.11: Kinetic parameters and stoichiometric coefficient characteristics.

Notation	Definition
$S_{O,sat}$	Oxygen saturation concentration
μ_H	Heterotrophic max. specific growth rate
K_S	Half saturation coefficient for heterotrophs
$K_{O,H}$	Oxygen saturation coefficient for heterotrophs
K_{NH}	Ammonia saturation coefficient for heterotrophs
$K_{O,A}$	Oxygen saturation coefficient for autotrophs
Y_H	Heterotrophic yield
Y_A	Autotrophic yield
i_{xb}	Nitrogen fraction in biomass

2.3 Operating conditions

2.3.1 Influent load and disturbances

The activated sludge process is subjected to strong perturbations in the flow and the concentration of organic matter and other pollutants in the influent. As a result the plant is usually in a transient state, the quality of the effluent is deteriorated and it is difficult to achieve steady state operation.One of the main control objective is to satisfy the requirements on the quality of the effluent, therefore to reject the disturbances and to achieve an efficient operation at the lowest cost is very important.

In order to test the performance of control strategy in different situations, the$BSM1$ provides standardized influent data considering different weather situations. In this work, data for 336 hours, corresponding to 2 weeks are considered, with a sampling period of $0.25h(= 15min)$. Figures (2.11), (2.12) and (2.13) present the profiles for stormy weather.

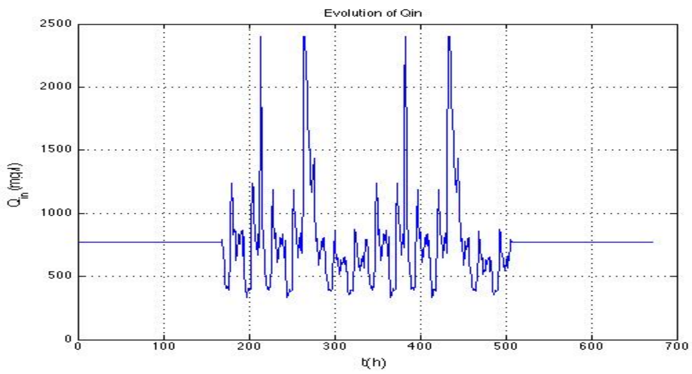


Figure 2.11: Influent flow Q_{in} for stormy weather.

2.3.2 Manipulated variables

For the model $M1$ the manipulated variables are the dilution rate D and the mass transfer coefficient K_{La} and the two manipulated variables for $M2$ and $M3$ are the internal recycle flow rate Q_a and the mass transfer coefficients K_{La}.

2.3.3 Outputs

For the model $M1$ the control objective is to make the system outputs that are the residual substrate (s) and the dissolved oxygen (c) to track the set point. For $M2$ and $M3$ the controlled variables are the S_O in the second bioreactor and nitrate S_{NO} levels in the first unit of the bioreactor. Five effluent variables - the ammonium concentration, the concentration of

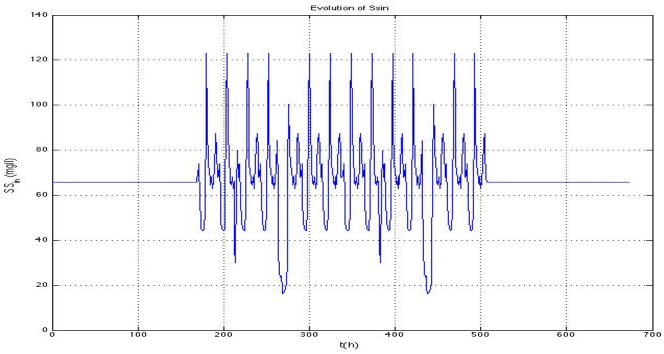


Figure 2.12: Concentration of organic matter in the influent $S_{s_{in}}$ for stormy weather.

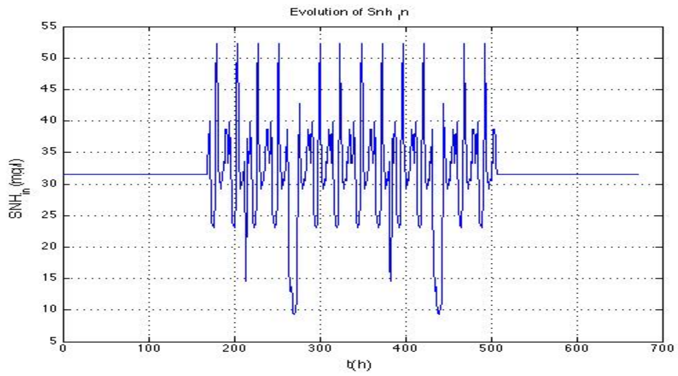


Figure 2.13: Concentration of ammonium compounds in the influent $S_{NH_{in}}$ for stormy weather.

suspended solids, the BOD_5, the COD and the total nitrogen are used to demonstrate the performance of the control system.

2.3.4 Bounds

The limits on the effluent for ammonium ($S_{NH,e}$) concentration, total nitrogen ($N_{tot,e}$) concentration, suspended solid ($S_{S,e}$) concentration, biological oxygen demand over a 5-day period ($BOD_{5,e}$) and (COD_e) are given Table 2.12.

2.3.5 Performance indices

The measures used to characterize the effluent quality and energy usage are the standard performance indices recommended in the BSM1 platform for the evaluation of control strategies applied to WWTPs. Therefore, the pa-

Table 2.12: Bounds of the effluent concentrations.

Effluent Qualities	Upper Bound	Unit
S_{NH}	4	mg/l
S_{NO}	10	mg/l
N_{tot}	18	mg/l
TSS	30	mg/l
COD	100	mg/l
K_{La}	200	d^{-1}
Q_a	3850	m^3/d

rameters related to the effluent quality and other characteristics of the process are expressed in terms of certain composite variables such as chemical oxygen demand (COD), biological oxygen demand (BOD), total nitrogen (N_{tot}) and total production of sludge (TSS) [43]:

$$\begin{aligned} COD &= (S_I + S_S + X_I + X_S + X_{B,A} + X_{B,H} + X_P)\ gCOD/m^3 \\ BOD &= 0.25((S_S + X_S) + (1 - 0.08)(X_{B,A} + X_{B,H}))\ gCOD/m^3 \\ N_{tot} &= S_{NH} + S_{ND} + X_{ND} + i_{xb}(X_{B,H} + X_{B,A}) + i_{xp}(X_P + X_I)\ gN/m^3 \\ TSS &= 0.75(X_S + X_I + X_P + X_{B,A} + X_{B,H})\ gSS/m^3 \end{aligned} \tag{2.10}$$

There are also some dynamic effluent quality indices such as the integral square error (ISE) and the effluent quality (EQ) defined in the $BSM1$ model for wastewater treatment.

First of all, EQ ($Kgpollution/d$) is considered as a direct and important indicator of the performance of the control systems as well as the entire wastewater treatment plant. For the $BSM1$, it is defined as a daily average of a weighted summation of the concentration of different compounds in the effluent over a certain time period T:

$$\begin{aligned} EQ = \frac{1}{1000T} \int_{t_0}^{t_0+T} & (2TSS_e(t) + COD_e(t) + 30N_{tot}(t) \\ & + 10S_{NO}(t) + 2BOD_e(t))Q_e(t)dt \end{aligned} \tag{2.11}$$

With the concentrations measured in the effluent (denoted with the subscript e), expressed in g/m^3 and the flow rate in m^3/d.

As for the energy consumption, the total average pumping energy expressed in KWh/day (PE) over a certain period of time, T, depends directly on the internal recirculation flaw rate Q_a and is calculated as [41]:

$$PE = \frac{0.04}{T} \int_{t_0}^{t_0+T} (Q_r(t) + Q_a(t) + Q_w(t))dt \tag{2.12}$$

Where Q_r denotes the return sludge flow rate and Q_w the excess sludge flow rate, both in units of m^3/day.

The aeration energy (AE) in KWh/day required to aerate the last three comportments can in turn be written as:

$$AE = \frac{24}{T} \int_{t_0}^{t_0+T} (0.4032K_{La}(t)^2 + 7.8408K_{La}(t))dt \tag{2.13}$$

Where K_{La_k} is the oxygen transfer function in the k^{th} aerated tank in units of h^{-1}.

2.4 Description of the control problem

WWTP are large nonlinear systems subject to significant perturbations in flow and load, together with variation in the composition of the incoming wastewater. Nevertheless, these plants have to be operated continuously, meeting strict regulations. The tight effluent requirements defined by the European Union (European Directive 91/271 "Urban wastewater") become effective in 2005 and this have increased both operational costs and economic penalties. Many control strategies have been proposed in the literature but their evaluation and comparison, either practical or based on simulation is difficult. This is partly due to the variability of the influent, to the complexity of the biological and biochemical phenomena and to the large range of time constants (from a few minutes to several days) but also to the lack of standard evaluation criteria (among other things, due to region specific effluent requirements and cost levels). In addition, the microorganisms that are involved in the process and their adaptive behavior coupled with nonlinear dynamics of the system make the WWTP to be really challenging from the control point of view [45, 46, 47].

The main control objective in the activated sludge process is to maintain the quality of the effluent within limits defined by the current legislation, which implies the design of a control system that guarantees a good rejection of the disturbances in the influent. The conditions of the influent vary depending on the hour or the day of the week as seen in the previous point, and the plant must try to keep the quality of the effluent constant despite these strong variations in the influent. Likewise, as in any industrial plant, another primary objective is the minimization of operating costs, which are mainly pumping and aeration costs. In addition, some other important reasons of requirement of advanced control strategies are:

- The increased public awareness as reflected in more stringent regulations is an efficient driving force,
- Economic motivation: There exists a lack of fundamental knowledge concerning benefits versus costs of automated treatment processes. In

addition, WWTP processes are not productive and automation can only contribute to a decrease of operating costs but does not directly lead to increased profit,

- Rejection of large process disturbances, e.g. large variations in the load to the plant requires monitoring and process control.

Some other specific features of this process that hinder its control, regardless of the strategy used, are the following [48]:

- Existence of a large amount of residual water to be treated daily.
- The interaction between the different variables is large due to internal recirculation.
- Existence of few manipulatable variables for the control, and the difficulty in measuring certain variables of interest.
- The need to maintain continuously the effluent specifications to minimize environmental impact.

For all this, traditionally the implementation of the control system is not implemented and simply operates in an open loop. This way of operating leads in the best case to the design of oversized plants and high costs of operation, without obtaining guarantees on the quality of the effluent. However, in the literature, numerous control strategies have been proposed for the activated sludge process [49, 50].

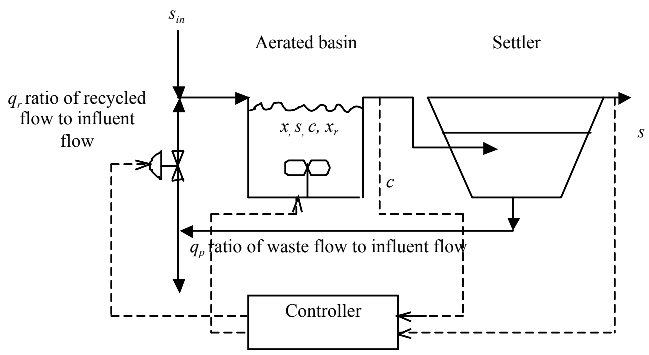


Figure 2.14: Schematic representation of the plant $M3$.

The control problems raised depend obviously on the model of the process considered. In the most simplified model corresponding to plant $M1$ (figure 2.7), the objective is to investigate the problem of regulating the process states (s, c) around a specific set points (s^*, c^*) under the following five assumptions:

- The dilution rate $D(t)$ and the oxygen transfer coefficient K_{La} are the two control variables, which are bounded.
- The biomass, substrate and recycled biomass concentrations $(x(t), s(t), x_r(t))$ are unavailable on-line.
- A noisy measure of the dissolved oxygen concentration c is available.

With regard to nitrogen control in plant $M2$ (Figure 2.8), the objective is to maintain the concentration of nitrites and nitrates in the anoxic tank (SNO_1) below certain legal limits and to regulate the dissolved oxygen concentration in the aerated tank (SO_2), despite of the incoming disturbances. The manipulated variables used are the internal recirculation flow (Q_a) and aeration in the aerated tank, namely through the oxygen transfer coefficient (K_{La}) ([41, 51]). The external recirculation flow rate remains constant in the chosen control configuration. In the plant $M3$ corresponding to the simplified $BSM1$, the control problem is analogous.

2.5 Conclusions

This chapter describes the process of activated sludge in a WWTP, detailing the simple mathematical model for removal of organic matter ($M1$) and two more complex models that include the nitrification and denitrification processes for nitrogen removal, such as the simplified model of $ASM1$ ($M2$) and the simplification of $BSM1$ which gives $M3$. It is a non-linear biological process with complicated dynamics, which makes its consideration very important for study and design of advanced control methodology.

The operation of the process begins with a description of the typical perturbations that affect the process, following with a description of some dynamic quality indexes associated with this particular process, defined in the $BSM1$ simulation platform, such as the effluent quality index (EQ), pumping and aeration energies and other generals such as ISE.

The control problem, which consists of maintaining the quality of the effluent despite of the disturbances and with a reasonable energy consumption in pumps and aeration turbines, has also been described. In particular, considering plant $M1$, the control objective is the regulation of the substrate and the oxygen around a specifics set points manipulating the dilution rate and aeration flow rate. For plants $M2$ and $M3$, the objective is to maintain the concentration of nitrites and nitrates in the anoxic zone below certain limit and regulate the dissolved oxygen concentration in the aerated zone. The manipulated variables in this case are the internal recirculation flow rate and the oxygen transfer coefficient.

3

PRELIMINARY CONCEPTS

3.1 Introduction

The purpose of this chapter is to present some theoretical notions and some tools that we will use later. First we detail the problem of stability of constrained linear systems. We define in particular the notion of Lyapunov function and the domain of attraction of the equilibrium. We then present the notion of positive invariance. We conclude this chapter by presenting some basics of predictive control.

3.2 Background on Analysis of linear Systems

3.2.1 Notation

For a scalar $a \in \mathcal{R}$ we define $a^+ = sup(a, 0)$, $a^- = sup(-a, 0)$. For a vector $x \in \mathcal{R}^n$ we define $x^+ = (x_j^+)$ and $x^- = (x_j^-)$ for $j = 1, ..., n$. Furthermore, for a matrix $A = (a)_{ij}, \quad i, j = 1, ..., n$, the tilde transform is defined by

$$\tilde{A} = \begin{pmatrix} A^+ & A^- \\ A^- & A^+ \end{pmatrix},$$

where $A^+ = (a^+)_{ij}$ and $A^- = (a^-)_{ij}$, $i, j = 1, ..., n$.

Moreover, $\sigma(A)$ denotes the spectrum of matrix A, $\mathbb{D}_s$ denotes the stability domain for eigenvalues and $\mathcal{R}_+^*$ denotes the set of real numbers strictly positive.

3.2.2 Dynamic system constrained in state space representation

Let the stationary dynamic system, in continuous time, be described by a vectorial differential equation of the type:

$$\dot{x}(t) = f(x(t), u(t)), \quad t \in [0, \infty) \tag{3.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}^n$ it is a continuous vector function, $x \in \mathcal{X}$ and $u \in \mathcal{U}$. If $\mathcal{X} \equiv \mathbb{R}^n$ and $\mathcal{U} \equiv \mathbb{R}^n$, the dynamic system is said to be unconstrained.

Considiring a control law $u : \mathbb{R}^n \to \mathbb{R}^m$ defined by:

$$u(t) = g(x(t)) \tag{3.2}$$

The closed loop system is represented by:

$$\begin{aligned} \dot{x}(t) &= f(x(t), g(x(t))), \\ &= h(x(t)), \quad t \in [0, \infty) \end{aligned} \tag{3.3}$$

Let $\phi(t, x(t_0), t_0)$ be the transition function or trajectory of the closed-loop system 3.3. It shows how the state $x(t_0)$ at time t_0, evolve in time to reach $x(t)$ at time $t \geqslant t_0$. With no loss of generality, suppose that $t_0 = 0$. Thus,

$$\begin{aligned} \phi(t, x(0), 0) &= \phi(t, x_0) \\ \text{where} \quad x(0) &= x_0 \end{aligned}$$

If $\mathcal{X} \subset \mathbb{R}^n$ is defined by a subset of of constraints applied to the state vector, $x \in \mathcal{X}$, and/or $\mathcal{U} \subset \mathbb{U}^m$ is defined by a subset of constraints applied to input vector $u \in \mathcal{U}$, thus the dynamic system (3.3) is a constrained system.

In a large majority of practical constrained control problems, these define convex polyhedral sets in the state and/or control space.

Definition 3.1 (Convex set) *A subset $\mathcal{C} \subset R^n$ is convex if and only if*

$$\forall x \in \mathcal{C}, \quad \forall y \in \mathcal{C} \qquad (1-\lambda)x + \lambda y \in \mathcal{C} \quad \forall \lambda \in]0, 1[\tag{3.4}$$

Properties:

- The intersection of an arbitrary convex collection of sets is a convex set.
- Given any system of linear equations and inequality in n variables, the set $\mathcal{C}$ of solutions is a convex set in $\mathbb{R}^n$.

Definition 3.2 *(Convex polyhedral) Any non-empty convex polyhedral of $\mathbb{R}^n$ can be characterized by a matrix $G \in \mathbb{R}^{q \times n}$ and a vector $w \in \mathbb{R}^q$. It is defined by:*

$$R[G, w] = \{x \in \mathbb{R}^n \quad \textit{such as} \quad Gx \leq w\}, \tag{3.5}$$

Consider then the constraints on the state vector of system (3.1). In general, the polyhedral of constraints is defined around an operating point of the dynamic system 3.1, and often it can be written in a symmetrical or asymmetrically form with respect to the origin ($\mathcal{X} \equiv \mathcal{L}(F, v)$ ou $\mathcal{X} \equiv \mathcal{L}(F, v_M, v_m)$).

Definition 3.3 *(Symmetric polyhedral) A symmetric polyhedron $\mathcal{L}(F, v)$ is defined by* $(Fig3.1a)$*:*

$$\mathcal{L}(F, v) = x \in \mathbb{R}^n \quad \textit{such as} \quad Fx \leqslant v \tag{3.6}$$

where $F \in \mathbb{R}^{q \times n}$, $v \in \mathbb{R}^q$ *with* $v \geqslant 0$

Definition 3.4 *(Asymmetric polyhedral) A asymmetric polyhedron $\mathcal{L}(F, v_M, v_m)$ os defined by* $(Fig3.1b)$*:*

$$\mathcal{L}(F, v_M, v_m) = x \in \mathbb{R}^n \quad \textit{such as} \quad -v_m \leqslant Fx \leqslant v_M \tag{3.7}$$

where $F \in \mathbb{R}^{q \times n}$, $v_M, v_m \in \mathbb{R}^q$ *with* $v_M, v_m \geqslant 0$

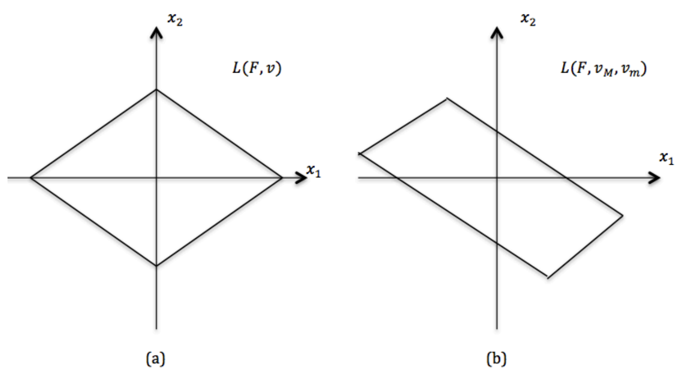


Figure 3.1: -(a) Symmetric polyhedral -(b) Asymmetric polyhedral.

3.2.3 Positive invariance and stability

The characterization of positive invariance of a subset of the state space of a dynamical system is very important for both the stability analysis and the design of constrained controllers [52, 53, 54]. For linear systems, positively invariant sets with 'quadratic' boundary can be obtained by the derivation of a quadratic Lyapunov function.

Indeed, the positive invariance of a closed set of states of a given dynamic system ensures that all the trajectories starting from this set remain in this set. Thus, this property can be used to calculate stabilizing control laws and such that the system (3.3) admits positively invariant sets inside the domain generated by constraints. So if the initial states of the system are constrained to belong within the invariant domain, then the trajectories emanating from these states respect the constraints.

3.2.3.1 Positively invariant set

Let $D \subset \mathbb{R}^n$ be a non-empty set of the state space. The set of trajectories of (3.3) starting from D is denoted by:

$$\phi(t, D) = \phi(t, x_0) \quad \text{such as} \quad x_0 \in D \tag{3.8}$$

We then present the definition of positively invariant set, which will be used later.

Definition 3.5 *Positively invariant set*
The closed nonempty set D is a positively invariant set of system (3.3) *if for any initial condition belonging to the set D, the corresponding trajectories remain in D. In other words, the dynamic system* (3.3) *has the property:*

$$\phi(t, D) \subseteq D, \quad \forall t \geqslant 0. \tag{3.9}$$

3.2.3.2 Stability of dynamic systems

The stability of a system is the ability of the system to return to its equilibrium point when it is slightly perturbed. For all control systems, stability is the primary requirement. One of the most widely used stability concepts in control theory is that of Lyapunov stability, which we employ throughout the manuscript. In this section we briefly review basic facts from Lyapunov's stability theory. To begin with, we note that Lyapunov stability and asymptotic stability are properties not of a dynamical system as a whole, but rather of its individual solutions.

Definition 3.6 *(Equilibrium point)*
A vector $x_e \in \mathbb{R}^n$ is called a point or state of equilibrium of (3.3) *if and only if :*

$$f(x_e) = 0$$

Remark 3.1 *Any point of equilibrium can be reduced to the origin by a simple change of variable $x - x_e \to x$.*

For convenience, we state all definitions and theorems for the case when the equilibrium point is at the origin of $\mathbb{R}^n$; that is, $x_e = 0$. There is no loss of generality in doing so since any equilibrium point under investigation can be translated to the origin via a change of variables.

Definition 3.7 *The equilibrium state of system* (3.3) *is:*

- *Stable if for every $\epsilon > 0$ there exists a $r = r(\epsilon)$ such as:*

$$||x(t = 0)|| < r \quad \Rightarrow ||x(t)|| < \epsilon, \quad \forall t > 0.$$

- *Unstable, if it is not stable.*
- *Asymptotically stable if it is stable and r can be chosen so that:*

$$||x(t=0)|| < r \quad \Rightarrow \lim_{t\to\infty} x(t) = 0.$$

- *Marginally stable, if it is stable without being asymptotically stable*
- *Exponentially stable if there exist positive real constants r, c, and λ such that all solutions of Eq.3.3 with $||x(0)|| < r$ satisfy the inequality:*

$$||x(t)|| \leqslant c||x(0)||e^{-\lambda t} \quad \forall t \geqslant 0.$$

  If this exponential decay estimate holds for all r, the system is said to be globally exponentially stable.

When the origin is asymptotically stable, we are often interested in determining how far from the origin the trajectory can still converge to the origin as t approaches ∞. This gives rise to the definition of the region of attraction (also called region of asymptotic stability, domain of attraction, and basin).

Definition 3.8 *(globally attractive)*
A system is globally asymptotically stable if its solutions converge to the origin from all initial conditions.

3.2.3.3 Stability of continuous systems

Consider the following autonomous continuous linear system:

$$\dot{x} = Ax \tag{3.10}$$

Theorem 3.1 *The equilibrium $x_e = 0$ of the system (3.10) is said*

1. *Asymptotically stable if and only if $\mathcal{R}e(\lambda_i(A)) < 0$, $\forall i = 1, ..., n$.*
2. *Unstable if there exists at least one eigenvalue of A such as $\mathcal{R}e(\lambda_i(A)) > 0$.*
3. *However, when there are certain eigenvalues $\lambda(A)$, Whose real part is zero: $\mathcal{R}e(\lambda_i(A)) = 0$, with $\mathcal{R}e(\lambda_j(A)) < 0$, for $i \neq j$, the critical case is obtained, it this case the equilibrium $x_e = 0$ is said critically stable if and only if the following conditions are satisfied*
   (a) *All the real part of the eigenvalues of A are negative.*
   (b) *To each eigenvalue $\lambda_i(A)$ with null real part and multiplicity s corresponds exactly s eigenvectors.*

4. *If in 3 the condition (b) is not satisfied then the equilibrium $x_e = 0$ is unstable.*

Having defined stability and asymptotic stability of equilibrium points, the next task is to find ways to determine stability. For practical interest, stability conditions must not require that we explicitly solve *Eq*.3.1. The direct method of Lyapunov aims at determining the stability properties of an equilibrium point from the properties of $f(x)$ and its relationship with a positive-definite function $V(x)$.

Definition 3.9 *A function $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is*

- *Definite positive on a set D if there exists a scalar function*

$$\Phi : \mathbb{R}^+ \to \mathbb{R}^+,$$

  Such as

$$0 < \Phi(||x||) \leqslant f(x,t), \quad \forall x \in int(D), \quad \forall t \in \mathbb{R}^+$$

- *Semi-definite positive on D if there exists a neighborhood G of the origin such as*

$$f(x,t) = 0, \quad \forall x \in G, \quad \forall t \in \mathbb{R}^+.$$

  and $f(x,t)$ is definite positive for all $x \in D \backslash G$ for all $t \in \mathbb{R}$.

Definition 3.10 *The function $V : W \subset \mathbb{R}^n \to \mathbb{R}^+$ is a Lyapunov function if it satisfy the two following conditions:*

- *$V(x)$ is continuous and its partial derivatives $\frac{\partial V(x)}{\partial x_i}$ exist and are continuous, for all $i = 1, ..., n$,*
- *$V(x)$ is definite positive.*

From this definition, the stability of continuous systems of type (3.10) can be characterized by the existence of a contractive Lyapunov function along the trajectories.

Theorem 3.2 *In the neighborhood $W \subset \mathbb{R}^n$, the equilibrium $x_e = 0$ is:*

- *Locally stable if there exists a candidate Lyapunov function $V : W \to \mathbb{R}^+$ such as:*

$$\dot{V}(x) \leqslant 0, \quad \forall x \in W.$$

- *Locally asymptotically stable if there exists a candidate Lyapunov function $V : W \to \mathbb{R}^+$ such as*

$$\dot{V}(x) < 0, \quad \forall x \in W$$

  $\dot{V}(x)$ refers to the derivative of $V(x)$ with respect to time along the trajectories of the system.

In what follows, we speak of stability of the system instead of talking about stability of the point of equilibrium.

The theorem proposed above provides sufficient stability conditions but without giving a guide to the user for the choice of the Lyapunov candidate function nor to conclude if is possible to find such a function.

A Lyapunov candidate function is a positive definite function whose decay is tested around the equilibrium point. A class of Lyapunov functions that plays an important role is the class of quadratic functions of the form:

$$V(x) = x^T P x.$$

This function is a positive definite if P is positive-definite symmetric matrix, i.e., all eigenvalues of P are positive. So, we write $P = P^T$.

In the case of the continuous linear autonomous system (3.10), a necessary and sufficient condition for the derivative

$$\dot{V}(x) = x^T (A^T P + PA) x$$

is negative-definite consists of finding a matrix P such as the following inequality matrix hold:

$$A^T P + PA < 0$$

Theorem 3.3 *The equilibrium* $x_e = 0$ *of system* (3.10) *is asymptotically stable if and only if for any positive-definite symmetric matrix* Q*, there exist a unique positive-definite symmetric matrix* P *that satisfies the Lyapunov equation:*

$$A^T P + PA = -Q \tag{3.11}$$

and the function

$$V(x) = x^T P x$$

is a Lyapunov function.

A continuously differentiable positive-definite function $V(x)$ satisfying $Eq.$3.11 is called a Lyapunov function. The surface $V(x) = c$, for some $c > 0$, is called a Lyapunov surface or a level surface. The condition $\dot{V}(0)$ implies that when a trajectory crosses a Lyapunov surface $V(x) = c$, it moves inside the set

$$\Omega_c = \{x \in \mathbb{R}^n : \quad V(x) \leqslant c\}$$

and can never come out again. When $\dot{V}(x) < 0$, the trajectory moves from one Lyapunov surface to an inner Lyapunov surface with smaller c. As c decreases, the Lyapunov surface $V(x) = c$ shrinks to the origin, showing that the trajectory approaches the origin as time progresses. If we only know that $\dot{V}(x) \leqslant 0$, we cannot be sure that the trajectory will approach the origin, but we can conclude that the origin is stable since the trajectory can be contained inside any ball, B_ϵ, by requiring that the initial state x_0 to lie inside a Lyapunov surface contained in that ball.

3.2.3.4 Stability of discrete linear systems

Consider the discrete system described by:

$$x(k+1) = Ax(k) \tag{3.12}$$

The study of the eigenvalues of A allows us to determine the stability of these systems.

Theorem 3.4

- *The system* (3.12) *is asymptotically stable if and only if* $\rho(A) < 1$.
- *if* $\rho(A) = 1$, *the system converges to an equilibrium if and only if* $\rho(A) = 1$ *is eigenvalue of the unit circle.*
- *the system*(3.12) *is unstable if* $\rho(A) > 1$

The stability of the system (3.12) can also studied by Lyapunov function.

Theorem 3.5 *Let* $x_e = 0$ *be an equilibrium point of the discreet system* (3.12) *and*

$$V : W \to \mathbb{R}^+$$

a continuous function defined in a neighborhood $W \subset \mathbb{R}^n$ *of* $x_e = 0$ *such as*

$$V(0) = 0, \quad V(x) > 0 \quad \textit{for} \quad x \neq 0.$$

Let $\Delta V(x(k)) = V(x(k+1)) - V(x(k))$.

- *If* $\Delta V(x(k)) \leqslant 0$, $\forall x(k) \in W$, *then the origin is stable.*
- *If* $\Delta V(x(k)) < 0$, $\forall x(k) \in W\backslash\{0\}$, *then the origin is asymptotically stable.*

Theorem 3.6 *The system* (3.12) *is asymptotically stable at the origin if and only if, for all* $Q = Q^T \in \mathbb{R}^{n\times n}$, Q *positive-definite, there exist a unique matrix* $P = P^T \in \mathbb{R}^{n\times n}$, *positive-definite, such as*

$$A^T PA - A = Q \tag{3.13}$$

and the function $v(x) = x^T Px$ *is a Lyapunov function of the system* (3.12).

3.3 Model predictive control

Model Predictive Control (MPC) is an optimal control strategy based on numerical optimization. Future control inputs and future plant responses are predicted using a system model and optimized at regular intervals with respect to a performance index. From its origins as a computational technique for improving control performance in applications within the process and petrochemical industries, predictive control has become arguably the most widespread advanced control methodology currently in use in industry. MPC has solid theoretical basis and its stability, optimality, and robustness properties are well understood.

This section covers the basic principles of model predictive control.

3.3.1 Predictive control strategy

A model predictive control law contains the basic components of prediction, optimization and receding horizon implementation. A summary of each of these ingredients is given below.

3.3.1.1 Prediction

The future response of the controlled plant is predicted using a dynamic model. This course is concerned mainly with the case of discrete-time linear systems with state-space representation

$$x(k+1) = Ax(k) + Bu(k) \tag{3.14}$$

where $x(k)$ and $u(k)$ are the model state and input vector at the k^{th} sampling instant. Given a predicted input sequence, the corresponding sequence of state predictions is generated by simulating the model forward over the prediction horizon, of say N sampling intervals. For notational convenience, these predicted sequences are often stacked into vectors U, X defined by

$$U(k) = \begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+N-1|k) \end{bmatrix}, \quad X(k) = \begin{bmatrix} x(k|k) \\ x(k+1|k) \\ \vdots \\ x(k+N|k) \end{bmatrix}$$

Here $u(k+i|k)$ and $x(k+i|k)$ denote input and state vectors at time $k+i$ that are predicted at time k, and $x(k+i|k)$ therefore evolves according to the prediction model:

$$x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k), \quad i = 0, 1, \cdots \tag{3.15}$$

with initial condition (at the beginning of the prediction horizon) defined

$$x(k|k) = x(k)$$

3.3.1.2 Optimization

The predictive control feedback law is computed by minimizing a predicted performance cost, which is defined in terms of the predicted sequences U, X. This thesis is mainly concerned with the case of quadratic cost, for which the predicted cost has the general form:

$$J(k) = \sum_{i=0}^{N}(||x(k+i|k)||_Q^2 + ||u(k+i|k)||_R^2) \tag{3.16}$$

Where Q and R are weight matrices with appropriate dimensions. Clearly $J(k)$ is a function of $u(k)$, and the optimal input sequence for the problem of minimizing $J(k)$ is denoted $u^*(k)$.

3.3.1.3 Reciding horizon implementation

Only the first element of the optimal predicted input sequence $u^*(k)$ is input to the plant:

$$u(k) = u^*(k|k).$$

The process of computing $u^*(k)$ by minimizing the predicted cost and implementing the first element of u^* is then repeated at each sampling instant $k = 0, 1, ...$ For this reason the optimization defining u^* is known as an online optimization. The prediction horizon remains the same length despite the repetition of the optimization at future time instants ($Fig.$3.2), and the approach is therefore known as a receding horizon strategy.

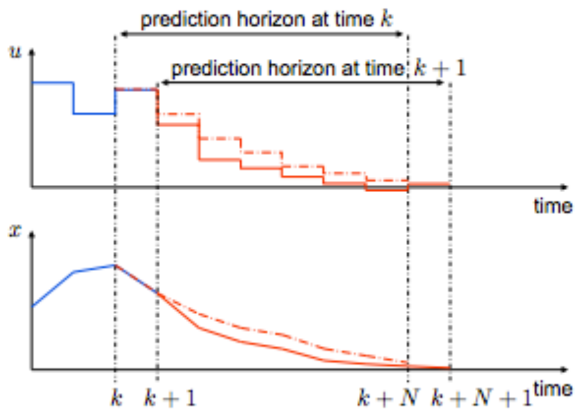


Figure 3.2: The receding horizon strategy

3.3.2 Prediction model

A very wide class of plant model can be incorporated in a predictive control strategy. This includes linear, nonlinear, discrete and continuous-time models. Prediction models may be deterministic, stochastic, or fuzzy.

3.3.2.1 Linear plant model

For linear systems, the dependence of predictions x(k) on u(k) is linear. A quadratic predicted cost such as (3.16) is therefore a quadratic function of the input sequence $u(k)$. Thus $J(k)$ can be expressed as a function of U in the form

$$J(k) = U^T(k)HU(k) + 2f^T u(k) + g \tag{3.17}$$

where H is a constant positive definite (or possibly positive semidefinite) matrix, and f, g are respectively a vector and scalar which depend on $x(k)$. Linear input and state constraints likewise imply linear constraints on $u(k)$ which can be expressed

$$A_c U(k) \leqslant b_c$$

where A_c is a constant matrix and, depending on the form of the constraints, the vector be may be a function of $X(k)$.

3.3.2.2 Nonlinear plant model

If a nonlinear prediction model is employed, then due to the nonlinear dependence of the state predictions $x(k)$ on $u(k)$, the MPC optimization problem is significantly harder than for the linear model case. This is because the cost in equation (3.16), which can be written as $J(U(k), x(k))$, and the constraints, $g(u(k), x(k)) \leqslant 0$, are in general non convex of $U(k)$, so that the optimization:

$$\begin{aligned} \min_{u} \quad & J(U, x(k)) \\ \text{subject to} \quad & g(U, x(k)) \leqslant 0 \end{aligned} \tag{3.18}$$

becomes a non convex nonlinear programming (NLP) problem. As a result there will in general be no guarantee that a solver will converge to a global minimum of (3.18), and the times required to find even a local solution are typically orders of magnitude greater than for QP problems of similar size.

3.3.3 Constraint handling

In addition to the obvious equality constraints, which are the model dynamics fulfillment of states and inputs, inequality constraints on input and state variables are found in every control problem. While the equality constraints are usually handled implicitly (i.e. the plant model is used to write predicted state trajectories as functions of initial conditions and input trajectories), the inequality constraints are imposed as explicit constraints within the online optimization problem.

3.4 Conclusion

In this chapter we have briefly reviewed the tools on which we will base our work, namely, the concept of stability and positive invariance for dynamic systems in continuous and discrete time, and the notions of stability of Lyapunov. Finally, a summary of model predictive control fundamentals are given.

Part I

CONTROL STRATEGIES BASED ON POSITIVE INVARIANCE CONCEPT

4

Nonlinear State Feedback Control Based on Positive Invariance

4.1 Introduction

The goal of this chapter is to present the design a nonlinear feedback law that will cause the output of the system to track a hight amplitude step input rapidly without experiencing large overshoot and, without the adverse actuator saturation effects and ensuring stability and constraints.

The controller consists of a linear feedback law computed using the invariance positive concept and a nonlinear feedback law without any switching element. The linear feedback part is designed to yield a closed-loop system with a small damping ratio for a quick response, while at the same time not exceeding the actuator limits for the desired command input levels. The nonlinear feedback law is used to increase the damping ratio of the closed-loop system as the system output approaches the target reference to reduce the overshoot caused by the linear part.

The obtained control scheme combines the problems of non availability of the state to measure with the limitations of some variables. The control is achieved by an observer based controller that can take into account constraints on the control and on observation error. The control strategy is worked out to meet all design required conditions. The efficiency of the controller is showed via simulations of a WWTP.

4.2 Problem statment

Consider a linear time-invariant system represented in the state space by:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{4.1}$$

Where $x(t) \in \mathbb{R}^n$ is a vector of state variables, $y(t) \in \mathbb{R}^p$ a vector of outputs or measured variables and $u(t) \in \mathbb{R}^m$ is the input vector, which is constrained to evolve in the following domain:

$$D_u = \{u(.) \in \mathbb{R}^m, \quad -u_{min} \leqslant u(.) \leqslant u_{max}, \quad u_{min}, u_{max} \in \mathbb{R}^m_+\}. \tag{4.2}$$

The controller proposed in this chapter is a control law that is composed by combining a linear and nonlinear state feedback laws such as:

$$u(t) = Fx(t) + Gr + g(x(t), r). \tag{4.3}$$

where

- $Fx(t)$ is a linear stabilizing state feedback control law which is computed using the positive invariance concept such that the closed loop system remains asymptotically stable and respecting constraints.
- Gr is a step command input, to lead the system output to set point r.
- $g(x(t), r)$ is a nonlinear state feedback control law, which used to change the system closed loop damping ratio as the output approaches the step command input reducing the overshoot, and at the same time, ensuring stability and respecting constraints.

The design of the proposed controller is sequential. First a linear feedback law is designed and then, based on this linear state feedback law, a nonlinear state feedback law is constructed.

The implementation of the proposed control strategy is depends on the availability of on-line information about the current state of the process. But due to lack or prohibitive cost, in many instances, of on-line sensors for these components and due to expense and duration (several days or hours) of laboratory analyses, sometimes there is a need to develop and implement algorithms which are capable of reconstructing the time evolution of the unmeasured state variables on the base of the available on-line data. To overcame this problem a Luenberger observer is used in this work.

4.3 Controller design

As mentioned before the design of the controller is sequential, so first we begin with the design of the linear state feedback law ,u_L, and then, using this controller to construct the nonlinear part $g(x(t), r)$.

4.3.1 The Linear State Feedback Control

In this section, we present the linear control scheme that it is designed for a linearized obtained system. The control strategy is based on the positive invariance concept that has shown efficiency in handling constrained control systems. Moreover, as the system state is composed by non measurable quantities, observers as software sensors are introduced.

Let us first recall the observer based regulator with constraints. To this end, the matrices A and B of the system (4.1) are constant of appropriate dimension and (A, B) is supposed to be controllable and it assumed that A possesses at least $(n - m)$ stable eigenvalues.

The control u_L is constrained in the set Ω defined as follow:

$$\Omega = \{u_L \in \mathbb{R}^m \backslash - u_{Lmin} \leqslant u_L \leqslant u_{Lmax}, u_{Lmin}, u_{Lmax} \in \mathbb{R}^m_+\} \tag{4.4}$$

First, we will use a feedback control given by:

$$u_L = sat(Fx(t)) = \begin{cases} u_{Lmax} & \text{if} \quad Fx \geqslant u_{Lmax} \\ u_L & \text{if} \quad -u_{Lmin} < Fx < u_{Lmax} \\ -u_{Lmin} & \text{if} \quad Fx \leqslant -u_{Lmin} \end{cases} \tag{4.5}$$

that leads to a domain of linear behavior for the closed loop system that is given by

$$D(F, u_{Lmin}, u_{Lmax}) = \{x \in \mathbb{R}^n \backslash - u_{Lmin} \leqslant Fx \leqslant u_{Lmax}\} \tag{4.6}$$

and the closed loop system in this case is defined by

$$\dot{x}(t) = (A + BF)x(t) \tag{4.7}$$

Hence, if the domain (4.6) is positively invariant, in the sense of the definition given below, one guarantees the respect of the control constraints for all $t \geqslant 0$.

Definition 4.1 *A subset D of $\mathbb{R}^m$ is said to be positively invariant with respect to system* (4.7) *if the condition $x(t_0) \in D$ implies that $x(t) \in D$ $\forall t \geqslant t_0$.*

At this level, one may introduce the observer for this class of systems. Note that the proposed observer is a reduced order one as the measurable part of the output is a linear combination of the states. Hence, only a part of the state is needed to be reconstructed via the reduced order observer [55].

Let this part be noted as

$$z(.) = Tx(t) \quad z \in \mathbb{R}^{n-p} \tag{4.8}$$

where matrix $T \in \mathbb{R}^{n-p \times n}$ is chosen in such a way that the matrix $\begin{pmatrix} C \\ T \end{pmatrix}$ is invertible, $z(.)$ is the state of the observer dynamics that may be generated from an auxiliary dynamical system as follows:

$$\dot{z}(.) = Dz(.) + Ey(.) + Gu_L(.) \tag{4.9}$$

At this stage, our problem can be stated as finding matrices F, D, E and G such that the closed loop system (4.7) is asymptotically stable and the input constraints are respected.

The observer state is given by:

$$\hat{x} = \begin{pmatrix} C \\ T \end{pmatrix}^{-1} \begin{pmatrix} y(.) \\ z(.) \end{pmatrix} = \begin{pmatrix} V & P_o \end{pmatrix} \begin{pmatrix} y(.) \\ z(.) \end{pmatrix} \tag{4.10}$$

Where the matrices V, C, T, P_o, satisfy

$$VC + P_oT = \mathbb{I}. \tag{4.11}$$

Recall that the matrices of the observer of minimal order are given in [56] and can be expressed as:

$$D = TAP_o, \quad E = TAV, \quad G = TB \tag{4.12}$$

or equivalently, the matrices are calculated to satisfy the following relation

$$TA - EC = DT \tag{4.13}$$

where the matrix P_o is chosen to ensure asymptotic stability of the matrix D. In fact, matrix D defines the dynamic of the observation error and it guarantees a vanishing error [57].

Note that

$$\begin{aligned} \dot{\epsilon}(t) &= \dot{z}(t) - T\dot{x}(t) \\ &= Dz(t) + Ey(t) + Gu_L(t) - T(Ax(t) + Bu_L(t)) \\ &= Dz(t) + ECx(t) - TAx(t) \\ &= Dz(t) - DTx(t) \\ &= D\epsilon(t) \end{aligned}$$

For the observation error, we define the field $D(\mathbb{I}, \epsilon_{max}, \epsilon_{min})$ that give us the limits within which we allow change of the error $\epsilon(.)$.

The reconstruction error is always given by

$$e(t) = \hat{x}(t) - x(t) \tag{4.14}$$

and it is related to the observation error in the following way:

$$\begin{aligned} e(t) &= Vy(t) + P_o z(t) - x(t) \\ &= VCx(t) + P_o z(t) - (VC + P_o T)x(t) \\ &= P_o(z(t) - Tx(t)) \\ &= P_o \epsilon(t) \end{aligned}$$

The evolution of the control $u_L(t)$ can be written using previous relationship (4.11), (4.12), (4.13):

$$\begin{aligned} \dot{u_L}(t) &= F\dot{\hat{x}} \\ &= FP_o\dot{z}(t) + FVC\dot{x}(t) \\ &= FP_o(Dz(t) + Ey(t) + Gu(t)) + FVC(Ax(t) + Bu_L(t)) \\ &= FP_o(TAP_oz(t) + TAVy(t)) + (FPTB + FVCB)u_L(t) + FVCAx(t) \\ &= FP_oTA(Pz(t) + Vy(t)) + F(PT + VC)Bu_L(t) + FVCAx(t) \\ &= FP_oTA\hat{x}(t) + FBu_L(t) + FVCA(\hat{x}(t) - e(t)) \\ &= (FA + FBF)\hat{x}(t) - FVCAe(t) \\ &= H_oF\hat{x}(t) - FVCAP_o\epsilon(t) \\ &= H_ou_L(t) + L_r\epsilon(t) \end{aligned}$$

Therefore the system formed by the control $u_L(t)$ and the error $\epsilon(t)$, can be expressed as:

$$\begin{pmatrix} \dot{u_L}(t) \\ \dot{\epsilon}(t) \end{pmatrix} = \begin{pmatrix} H_o & L_r \\ 0 & D \end{pmatrix} \begin{pmatrix} u_L(t) \\ \epsilon(t) \end{pmatrix}$$

This background enables one to recall the theorem [58] giving conditions for computing the controller that respects all the need requirements:

Theorem 4.1 *The field $D(\mathbb{I}, u_{max}, u_{min}) \times D(\mathbb{I}, \epsilon_{max}, \epsilon_{min})$ is positively invariant with respect to the system trajectory $\begin{pmatrix} u_L(t) \\ \epsilon(t) \end{pmatrix}$ if and only if, there exists a matrix H_o in $\mathbb{R}^{m \times m}$ such that:*

$$\begin{cases} H_oF = FA + FBF \\ \widetilde{M}q_\epsilon \leq 0 \end{cases} \tag{4.15}$$

where

$$M = \begin{pmatrix} H_o & L_r \\ 0 & D \end{pmatrix}; \quad q_\epsilon = \begin{pmatrix} u_{max} \\ \epsilon_{max} \\ u_{min} \\ \epsilon_{min} \end{pmatrix}; \quad L_r = -FVCAP_o$$

for every pair: $(u(0), \epsilon(0)) \in D(\mathbb{I}, u_{max}, u_{min}) \times D(\mathbb{I}, \epsilon_{max}, \epsilon_{min})$

To compute the feedback gain, the inverse procedure is used [58]. Hence, matrix H_o satisfying all required conditions is chosen and the feedback F is obtained as a solution to the equation:

$$H_oF = FA + FBF \tag{4.16}$$

Without loss of generality, we can assume that matrix A has $(n-m)$ stable eigenvalues. Resolution of equation (4.16) gives a state feedback assigning spectrum of matrix H_o ($\sigma(H_o) \subset \mathcal{D}_s$) together with the stable part of spectrum of matrix A in closed loop.

For this equation to have a valid solution, matrix H_o must satisfy:

$$\begin{cases} \sigma(H_o) \cap \sigma(A) \\ B\zeta_i, \quad i = 1, ..., m \\ \zeta_i, i = 1, ..., m \quad \text{are linearly independent} \end{cases} \tag{4.17}$$

for ζ_i eigenvectors of matrix H_o.

Note that an unique solution to equation (4.16) exists if and only if

$\{\chi_1 \quad ... \quad \chi_n\}$ are linearly independent

where $\chi_i, i = m+1, ..., n$ are eigenvectors associated to stable eigenvalues of matrix A, and $\chi_i, i = 1, ..., m$ are computed by

$\chi_i = (\lambda_i I_n - A)^{-1} B\zeta_i, \quad i = 1, ..., m$

The solution for the controller is given by:

$$F = [\zeta_1 \quad ... \quad \zeta_m 0 \quad ... \quad 0] [\chi_1 \quad ... \quad \chi_m \chi_{m+1} \quad ... \quad \chi_n]^{-1} \tag{4.18}$$

Remark 4.1 *The control law in* (4.18) *is computed under the assumption that the system presents* $(n-m)$ *stable eigenvalues. If the system matrix does not fulfill such requirement, it is always possible to augment the representation.*

Let $v \in \mathcal{R}$ be a vector of fictitious inputs such that $-v_{min} \leqslant v \leqslant v_{max}$, where v_{min} and v_{max} are freely chosen constraints.

In this case, vectors U and Δ become:

$$U_c = \begin{bmatrix} u_{Lmax} \\ v_{max} \\ u_{Lmin} \\ v_{min} \end{bmatrix}$$

The augmented system is then given by:

$$x(k+1) = Ax(k) + [B \quad 0] \begin{bmatrix} u_L(k) \\ v(k) \end{bmatrix} \tag{4.19}$$

It is easy to see that for the obtained square system the problem of $(n-m)$ stable eigenvalues is eliminated and controllability is not changed.

Remark 4.2 *Note here that all computation effort in the design procedure is handled off line. By choosing an adequate matrix H_o satisfying all required conditions in* [60]*, solution of equation* (4.16) *is detailed in [63].*

4.3.1.1 Step command input

The variables $(\hat{x}, u, y)$ in (4.1) are deviation variables around the origin, so we need to apply a step command input to return the system to their original absolute value, rewriting the control law obtained from (4.17) as

$$u_L = F\hat{x}(t) + Gr \tag{4.20}$$

where r is a step command input and G is a scalar given by:

The command input Gr is applied to the system to lead the output to r.

$$\lim_{t\to\infty} y(t) = r \tag{4.21}$$

Using the Eq. (4.20), the closed loop system is given by:

$$\dot{\hat{x}}(t) = (A + BF)\hat{x}(t) + BGr \tag{4.22}$$

The Laplace transform of (4.22) and (4.1) gives:

$$Y(s) = C\hat{X}(s) = C(s\mathbb{I} - A - BF)^{-1}BGr \tag{4.23}$$

And by the final value theorem we have :

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} Y(s) = r \tag{4.24}$$

Which gives

$$G = -[C(A + BF)^{-1}B]^{-1} \tag{4.25}$$

Here, we note that G is well defined because $A + BF$ is stable.

4.3.2 The Nonlinear Feedback Control

The objective here is to design a nonlinear feedback control law for the system (4.3) with the constraints (4.2) that will cause the output to track a step input rapidly without expressing large overshoot.

The following assumptions on the system matrices are required:

1) (A, B) is stabilizable.

2) (A, B, C) is invertible and has no zero at $s = 0$.

In this section, we follow the idea of the work presented in [30] to develop a nonlinear feedback control technique for the case where we have $(n-p)$ states of the plant (4.1) are measurable as mentioned before.

Then, the nonlinear feedback control law $g(\hat{x}(t), r)$ is given by:

$$g(\hat{x}(t), r) = \rho(r, y)B^T P(\hat{x} - x_e) \tag{4.26}$$

Where

- $\rho(r, y)$ is a nonpositive scalar function, locally Lipschitz in y, and is to be chosen to improve the performance of the closed loop system. The freedom to choose the function $\rho(r, y)$ is used to tune the control laws so as to improve the performance of the closed loop system as the controller output y approaches to set point, reducing time, by adding a significant value to the control input when the tracking error , $r-y$, is small.

- $P \in \mathbb{R}n \times n$ is a positive-definite matrix solution of the following Lyapunov equation:

  $$(A + BF)^T P + P(A + BF) = -W \tag{4.27}$$

  For a given $W \in \mathbb{R}^{n\times n}$

  Note that such a P exists since $A + BF$ is asymptotically stable.

- x_e is the steady state of x defined by

  $$\lim_{t\to\infty} \hat{x}(t) = x_e. \tag{4.28}$$

  And by using (4.23) and (4.25), we get

  $$x_e := G_e r := -(A + BF)^{-1} BGr \tag{4.29}$$

4.3.3 Stability

The linear and nonlinear feedback laws derived in the previous steps are now combined to form a composite nonlinear feedback controller:

$$\begin{aligned} u &= u_L + g(\hat{x}(t), r) \\ &= F\hat{x} + Gr + \rho(r, y)B^T P(\hat{x} - x_e). \end{aligned} \tag{4.30}$$

The following theorem shows that the closed-loop system comprising the given plant in (4.1) and the nonlinear feedback control law in (4.2) is asymptotically stable. It also determines the magnitude of r that can be tracked by such a control law without exceeding the control limit.

Theorem 4.2 *Consider the given system in* (4.1)*, the linear control law of* (4.12) *and the nonlinear feedback control law of* (4.28)*. For any* $\alpha \in (0, 1)$*, let* $c_\alpha > 0$ *be the largest positive scalar satisfying the following condition:*

$$|F\hat{x}| \le u_{max}(1-\alpha), \quad \forall \hat{x} \in X_{alpha} := \{\hat{x} : \hat{x}^T P \hat{x} \le c_\alpha\} \tag{4.31}$$

Then the linear control law of (4.12) *is capable of driving the system controller output* $y(t)$ *to track asymptotically a step command input* r*, provided that the initial state* x_0 *and* r *satisfy*

$$\tilde{x}_0 = (\hat{x}_0 - x_e) \in X_{alpha}, \quad |Hr| \le \alpha u_{max}. \tag{4.32}$$

Furthermore, for any nonpositive function $\rho(r, y)$*, locally Lipschitz in* y*, the composite nonlinear feedback law in* (4.32) *is capable of driving the system controller output* $y(t)$ *to track asymptotically the step command input of amplitude* r*, provided that the initial state* x_0 *and* r *satisfy* (4.26)*.*

Proof: Let $\tilde{x} = x - x_e$. It is simple to verify that the linear control law of (4.22) can be rewritten as

$$\begin{aligned} u_L &= F\tilde{x}(t) + [1 - F(A+BF)^{-1}B]Gr \\ &= F\tilde{x}(t) + Hr. \end{aligned}$$

Hence, for all $\tilde{x} \in X_{alpha}$ and, provided that $|Hr| \le \alpha u_{max}, |F\tilde{x} + Hr| \le u_{max}$ and the closed-loop system is linear and it is given by

$$\dot{\tilde{x}} = (A+BF)\tilde{x} + Ax_e + BHr. \tag{4.33}$$

Noting that

$$\begin{aligned} Ax_e + BHr &= B[1 - F(A+BF)^{-1}B]Gr - A(A+BF)^{-1}BGr \\ &= [I - BF(A+BF)^{-1}]BGr - A(A+BF)^{-1}BGr \\ &= 0. \end{aligned}$$

the closed-loop system in (4.35) can then be simplified as

$$\dot{\tilde{x}} = (A_o + B_o F)\tilde{x} \tag{4.34}$$

Similarly, the closed-loop system comprising the given plant in (4.1) and the nonlinear feedback control (4.32) can be expressed as

$$\dot{\tilde{x}} = (A+BF)\tilde{x} + B_o w \tag{4.35}$$

where

$$w = sat(F\tilde{x} + Hr + g(\hat{x}(t), r)) - F\tilde{x} - Hr. \tag{4.36}$$

Clearly, for the given x_0 satisfying (4.26), we have $\tilde{x}_0 = (x_0 - x_e) \in X_\alpha$. We not that (4.37) is reduced to (4.22) if $\rho = 0$. Thus, we can prove the results, respectively, under the linear control and the non linear feedback control in one shot.

Next, we define a Lyapunov function $v = \tilde{x}^T P \tilde{x}$, and evaluate the derivation of v along the trajectories of the closed-loop system in (4.37), i.e.,

$$\begin{aligned} \dot{v} &= \dot{\tilde{x}}^T P \tilde{x} + \tilde{x}^T P \dot{\tilde{x}} \\ &= \tilde{x}^T (A + BF)^T P \tilde{x} + \tilde{x}^T P (A + BF) \tilde{x} + 2\tilde{x}^T P B w \\ &= -\tilde{x}^T W \tilde{x} + 2\tilde{x}^T P B w \end{aligned} \tag{4.37}$$

Note that for all

$$\tilde{x} \in X_\alpha = \{\tilde{x} : \tilde{x}^T P \tilde{x} \leq c_\alpha\} \Rightarrow |F\tilde{x}| \leq u_{max}(1 - \alpha) \tag{4.38}$$

We next study the $\dot{v}$ for the different case of the constraints on the input.

- *Case 1):* If $|F\tilde{x} + Hr + g(\hat{x}(t), r)| \leq u_{max}$, then $w = u_N = \rho B^T P \tilde{x}$ thus
$$\dot{v} = -\tilde{x}^T W \tilde{x} + 2\rho \tilde{x}^T P B B^T P \tilde{x} \leq -\tilde{x}^T W \tilde{x} \tag{4.39}$$

- *Case 2):* If $F\tilde{x} + Hr + g(\hat{x}(t), r) \succ u_{max}$, and by construction $|F\tilde{x} + Hr| \leq u_{max}$, we have
$$0 \prec w = u_{max} - F\tilde{x} - Hr \prec u_N = \rho B^T P \tilde{x} \tag{4.40}$$
which implies that $\tilde{x}^T P B \prec 0$ and hence
$$\dot{v} = -\tilde{x}^T W \tilde{x} + 2\tilde{x}^T P B w \leq -\tilde{x}^T W \tilde{x}. \tag{4.41}$$

- *Case 3):* Finally, if $F\tilde{x} + Hr + g(\hat{x}(t), r) \leq -u_{min}$, we have
$$\rho B^T P \tilde{x} = u_n \prec -u_{min} - F\tilde{x} - Hr \prec 0 \tag{4.42}$$
implying $\tilde{x}^T P B \succ 0$ and hence $\dot{v} \leq -\tilde{x}^T W \tilde{x}$

In conclusion, we have shown that

$$\dot{v} \leq -\tilde{x}^T W \tilde{x}, \quad \forall \tilde{x} \in X_{alpha} \tag{4.43}$$

which implies that X_{alpha} is an invariant set of the closed-loop system in (4.37). This, in turn, indicates that, for all initial states x_0 and the step command input of amplitude r that satisfy (4.34)

$$lim_{t \to \infty} \hat{x}(t) = x_e \Rightarrow lim_{t \to \infty} y(t) = r \tag{4.44}$$

This completes the proof.

4.4 Application to the WWTP

In this section, the above nonlinear feedback control law design method is applied to WWTP.

4.4.1 Process Model

A typical, conventional activated sludge plant $M2$ for the removal of carbonaceous and nitrogen materials consists of an anoxic basin followed by an aerated one, which is aerated by a submerged air bubble system or mechanical agitation at its surface and a settler ($fig.$4.1).

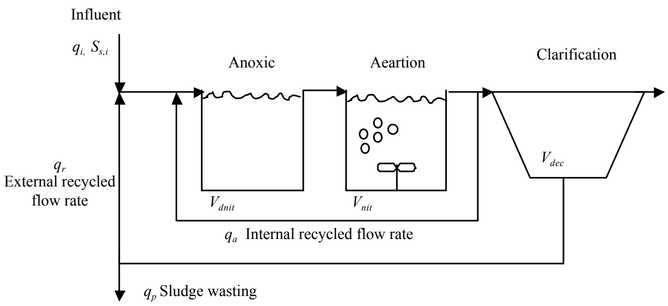


Figure 4.1: Schematic view of M2

The model $M2$ used in this application account for six basic components in the wastewater: autotrophic bacteria X_A, heterotrophic bacteria X_H, readily biodegradable carbonaceous substrates S_S, nitrogen substrates S_{NH}, S_{NO} and dissolved oxygen S_O, where X_A, X_H, S_S, S_{NH}, S_{NO}, and S_O represents the concentrations of these elements.

4.4.2 Linearization

To obtain a model in the state space of the model $M2$, the state vector is considered as

$$X(t) = [X_{A,nit}(t) \quad X_{H,nit}(t) \quad S_{S,nit}(t) \quad S_{NH,nit}(t) \quad S_{NO,nit}(t) \quad S_{O,nit}(t) \quad X_{A,denit}(t) \quad X_{H,denit}(t) \quad S_{S,denit}(t) \quad S_{NH,denit}(t) \quad S_{NO,denit}(t) \quad X_{rec}(t)]^T \tag{4.45}$$

Further, to complete the model, the following input and output vectors are used:

$$Y(t) = [S_{NH,nit}(t) \quad S_{NO,nit}(t) \quad S_{O,nit}(t)]^T \tag{4.46}$$

$$U(t) = [Q_{r1} \quad Q_{r2} \quad Q_{air}]^T \tag{4.47}$$

Linearizing the system around the equilibrium point computed from the nonlinear equations of $M2$ leads to the new variables (x, u, y) that are now deviation variables. That is, they are deviations from the point the model is linearized about, not their original absolute values. The equilibrium point is given by

$$\begin{aligned} \bar{x}(t) = [&69.6 \quad 623 \quad 13.5 \quad 3.2 \quad 10.4 \quad 2.4 \\ &68.9 \quad 624.6 \quad 20.9 \quad 8.9 \quad 5.3 \quad 1356.8]^T \end{aligned} \tag{4.48}$$

The matrices of the linearized model are:

$$A = \left[\begin{array}{cccccc} -29.07 & 0 & 0 & 2.65 & 0 & 2.17 \\ 0 & -29.48 & 6.04 & 0.642 & 0 & 4.40 \\ 0 & -0.34 & -38.99 & -1.02 & -0.05 & 5.01 \\ -2.22 & -0.02 & -0.55 & -40.74 & 0 & 12.23 \\ 2.18 & 0 & -0.06 & 11.05 & -29.41 & 9.64 \\ -9.45 & 0 & -0.18 & -47.90 & -0.02 & -167.01 \\ 2.30 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.30 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.30 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.30 & 0 \\ 6.14 & 6.14 & 0 & 0 & 0 & 0 \end{array} \right.$$

$$\left. \begin{array}{cccccc} 29.40 & 0 & 0 & 0 & 0 & 0 \\ 0 & 29.40 & 0 & 0 & 0 & 0 \\ 0 & 0 & 29.40 & 0 & 0 & 0 \\ 0 & 0 & 0 & 29.40 & 0 & 0 \\ 0 & 0 & 0 & 0 & 29.40 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -38.67 & 0 & 0 & 0.55 & 0 & 1.84 \\ 0 & -39.18 & 4.44 & 0.12 & 0 & 16.60 \\ 0 & -0.78 & -50.67 & -0.30 & -3.42 & 0 \\ -3.06 & -0.04 & -0.66 & -39.33 & -0.19 & 0 \\ 2.99 & -0.03 & -0.55 & 1.14 & -39.58 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.13 \end{array} \right] ;$$

$$B = 10^4 \begin{bmatrix} -0.0011 & -0.0011 & 0 \\ 0.0023 & 0.0023 & 0 \\ 0.0102 & 0.0102 & 0 \\ 0.0079 & 0.0079 & 0 \\ -0.0071 & -0.0071 & 0 \\ -0.0033 & -0.0033 & 0.0008 \\ 0.0014 & 0.1233 & 0 \\ -0.0031 & 1.1003 & 0 \\ -0.0386 & -0.0386 & 0 \\ -0.0105 & -0.0165 & 0 \\ 0.0094 & -0.0097 & 0 \\ 0 & -0.2042 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \end{bmatrix}$$

4.4.3 Decomposition

In order to apply the concept of positive-invariance, the studied system must be controllable and observable. However, our system does not completely satisfy the two later conditions. For this reason, we used the decomposition process that allow us to extract only the controllable and observable part.

Any representation in the state space can be transformed into the equivalent form by using the transformation $Z = T_o x$ [55]:

$$\begin{cases} \dot{Z} = \bar{A}Z + \bar{B}u \\ y(t) = \bar{C}Z \end{cases} \tag{4.49}$$

with:

$$\bar{A} = \begin{pmatrix} A_{no} & A_{12} \\ 0 & A_o \end{pmatrix} \quad ; \bar{B} = \begin{pmatrix} B_{no} \\ B_o \end{pmatrix}$$
$$\bar{C} = \begin{pmatrix} 0 & C_o \end{pmatrix}; \qquad Z = \begin{pmatrix} Z_{no} \\ Z_o \end{pmatrix}$$

So we obtain the following system of equations:

$$\begin{cases} \dot{Z}_{no} = A_{no}Z_{no} + A_{12}Z_o + B_{no}u \\ \dot{Z}_o = A_oZ_o + B_ou \\ y = C_oZ_o \end{cases} \tag{4.50}$$

where A_o and C_o are constant matrices of appropriate dimension and the pair (A_o, B_o) is controllable. Its assumed that A_o possesses at last $(n-m)$ stable eigenvalues.

The similarity matrix T used to compute the matrices A_o, B_o and C_o:

$$T = \left[\begin{array}{cccccccccccc}
0.0001 & 0.9975 & -0.0395 & 0 & 0 & 0 & 0 & -0.0561 & 0.0034 & -0.0001 & 0 & 0.0185 \\
-0.0014 & 0.0589 & 0.0680 & 0 & 0 & 0 & 0.0009 & 0.9943 & -0.0573 & 0.0012 & 0.0003 & -0.0003 \\
0 & -0.0184 & 0.0007 & 0 & 0 & 0 & 0 & 0.0013 & -0.0001 & 0 & 0 & 0.9998 \\
-0.0200 & 0.0355 & 0.9965 & 0 & 0 & 0 & 0.0064 & -0.0703 & 0.0001 & 0.0172 & 0.0037 & 0 \\
-0.0001 & 0 & 0.0041 & 0 & 0 & 0 & -0.0214 & 0.0572 & 0.9981 & 0.0001 & 0 & 0 \\
0.0001 & -0.0002 & -0.0064 & 0 & 0 & 0 & 0.9997 & 0.0008 & 0.0214 & -0.0001 & 0 & 0 \\
-0.9929 & -0.0007 & -0.0188 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0842 & 0.0821 & 0 \\
-0.0017 & 0.0005 & 0.0146 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6895 & -0.7242 & 0 \\
0.1173 & 0.0004 & 0.0122 & 0 & 0 & 0 & 0 & 0 & 0 & -0.7192 & 0.6847 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right];$$

Thereafter decomposition, the obtained system represented by (4.45), A_o, B_o, C_o are injected in closed loop and coupled with a minimal order Luenberger observer which has the role of estimating the non-measurable states from the measurable ones $(S_{NH,nit}, S_{NO,nit}, S_{O,nit})$.

$$Ao = \left[\begin{array}{ccccccccc}
-38.85 & 28.99 & -0.01 & 0.18 & 0.01 & -0.02 & -5.10 & 0.05 & 1.01 \\
2.33 & -50.29 & -0.27 & -0.25 & 2.72 & -2.09 & -0.02 & 0 & 0.01 \\
0.01 & -0.44 & -38.68 & -2.34 & -0.33 & -0.17 & 0.03 & 0 & -0.01 \\
0.01 & 0.07 & -28.69 & -29.23 & -0.04 & -1.26 & 2.26 & -0.19 & 2.81 \\
-0.01 & 1.29 & -0.08 & 0.11 & -38.99 & 0.81 & -0.07 & 1.67 & 1.60 \\
0.04 & 0.28 & 7.71 & -1.26 & -0.51 & -39.77 & -0.32 & -1.57 & 1.35 \\
0 & 0 & 0 & -9.39 & -0.01 & 1.11 & -167.01 & -0.02 & -47.91 \\
0 & 0 & 0 & -0.24 & 21.29 & -20.38 & 9.65 & -29.41 & 11.05 \\
0 & 0 & 0 & 0.25 & 20.27 & 21.41 & 12.24 & -0.01 & -40.75
\end{array}\right];$$

$$Bo = 10^3 \begin{bmatrix} 0.1040 & -0.6657 & 0 \\ -0.3872 & 0.2181 & 0 \\ 0.0052 & 1.2322 & 0 \\ 0.0252 & 0.0145 & 0 \\ 0.0057 & 0.1856 & 0 \\ 0.1403 & 0.0522 & 0 \\ 0.0332 & 0.0332 & -0.0076 \\ 0.0708 & 0.0708 & 0 \\ -0.0789 & -0.0789 & 0 \end{bmatrix}$$

$$Co = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}$$

The matrix T is chosen such that only the part $z(t) = TZ_{co}(t)$ is estimated. Further, matrix P_o is chosen to ensure asymptotic stability of matrix $D = TA_{co}P$. In fact, in this case matrix T and P_o are given by:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad P_o = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

According to Equation (4.10), the matrices D, G and E are computed.

$$D = \begin{bmatrix} -38.8580 & 28.9925 & -0.0070 & 0.1784 & 0.0119 & -0.0185 \\ 2.3306 & -50.2968 & -0.2666 & -0.2478 & 2.7231 & -2.0948 \\ 0.0147 & -0.4418 & -38.6780 & -2.3376 & -0.3296 & -0.1756 \\ 0.0119 & 0.0735 & -28.6943 & -29.2308 & -0.0443 & -1.2648 \\ -0.0105 & 1.2947 & -0.0824 & 0.1128 & -38.9935 & 0.8130 \\ 0.0442 & 0.2873 & 7.7115 & -1.2630 & -0.5140 & -39.7722 \end{bmatrix}$$

$$G = 10^3 \begin{bmatrix} 0.1040 & -0.6657 & 0 \\ -0.3872 & 0.2181 & 0 \\ 0.0052 & 1.2322 & 0 \\ 0.0252 & 0.0145 & 0 \\ 0.0057 & 0.1856 & 0 \\ 0.1403 & 0.0522 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} -1.0078 & -0.0489 & 5.1034 \\ -0.0042 & -0.0002 & 0.0205 \\ 0.0064 & 0.0003 & -0.0327 \\ -2.8104 & 0.1898 & -2.2577 \\ -1.6047 & -1.6664 & 0.0718 \\ -1.3549 & 1.5741 & 0.3185 \end{bmatrix}$$

We assume that in our case the control constraints are such as

$$u_{max} = \begin{bmatrix} 5000 \\ 18466 \\ 110 \end{bmatrix}, \quad u_{min} = \begin{bmatrix} 2000 \\ 18425 \\ 80 \end{bmatrix}$$

and reconstruction errors limits are such as:

$$\epsilon_{max} = [1 \quad 1 \quad 0.5 \quad 1 \quad 1 \quad 1]$$
$$\epsilon_{min} = [0.5 \quad 0.5 \quad 0.25 \quad 0.8250 \quad 0.5 \quad 0.5]$$

we choose the matrix H_o assigning spectrum $\{-170; -55; -51\}$ as follows:

$$H_o = \begin{bmatrix} -170 & 0 & 0 \\ 0 & -55 & 0 \\ 0 & 0 & -51 \end{bmatrix}$$

Hence, solving equation (4.13) leads to:

$$F = \left[\begin{matrix} 0.0010 & -0.0044 & 0.0319 & 0.0725 \\ 0.0001 & -0.0005 & 0.0000 & 0.0001 \\ 0.0148 & -0.0774 & 0.1421 & 0.0036 \end{matrix} \right.$$

$$\left. \begin{matrix} -0.0791 & -0.0944 & 1.0349 & -0.0000 & 0.4162 \\ 0.0000 & -0.0002 & 0.0017 & -0.0000 & 0.0007 \\ -0.1591 & -0.2023 & -0.0352 & -0.0024 & 0.0864 \end{matrix} \right]$$

4.4.4 Simulation Results

Figures below are devoted to present the evolution of the disturbances , outputs and inputs of the system. In fact, the nonlinear feedback controller, as defined in the sections above, is applied to the WWTP. The linear and non linear feedback controllers are compared by computing the indices IQ, EQ, PE and ISE as summarized in tables (4.1) and (4.2). The output variables evolution, that are $S_{NH,nit}$, $S_{NO,nit}$ and the dissolved oxygen concentrations, and their corresponding reference trajectories are 3.2, 10.4 and 2.4 , respectively.

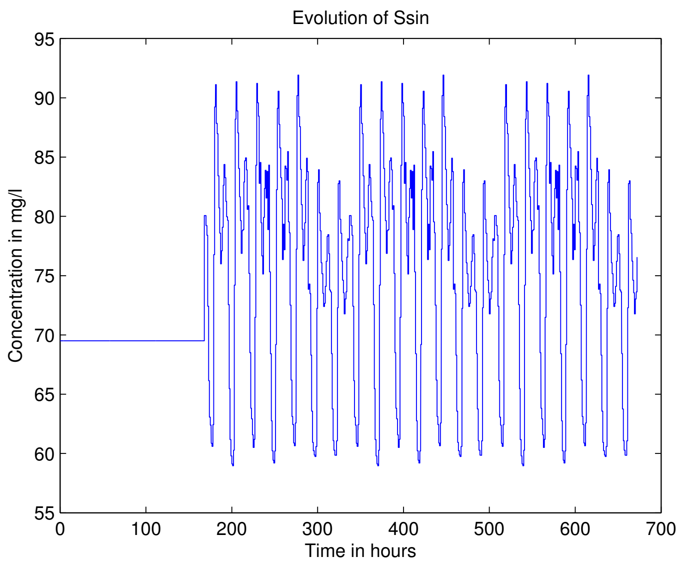
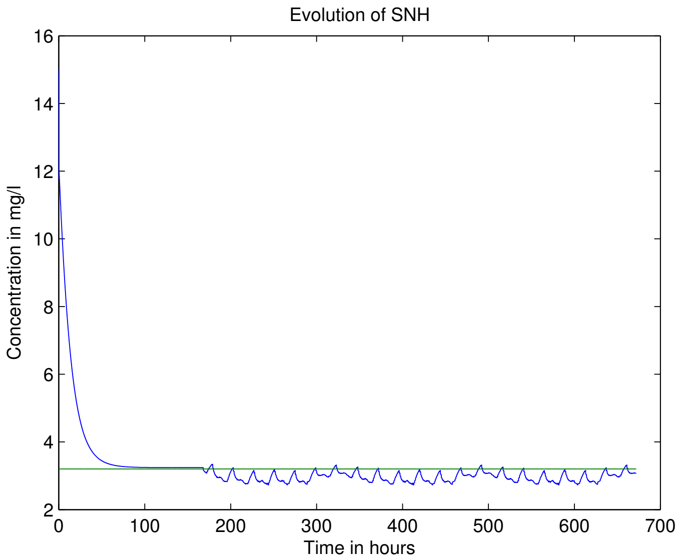
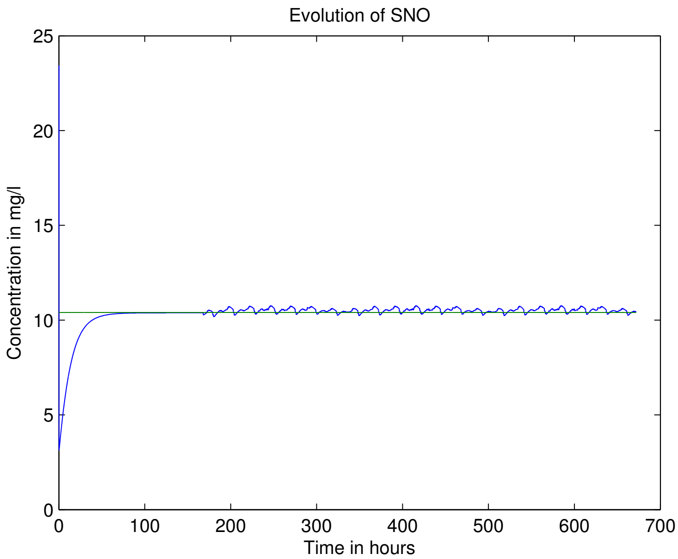


Figure 4.2: Evolution of the disturbance S_{in1}.

As general remarks asymptotic stability is obtained, all constraints are respected and the the amount of all non desired organic matter is reduced



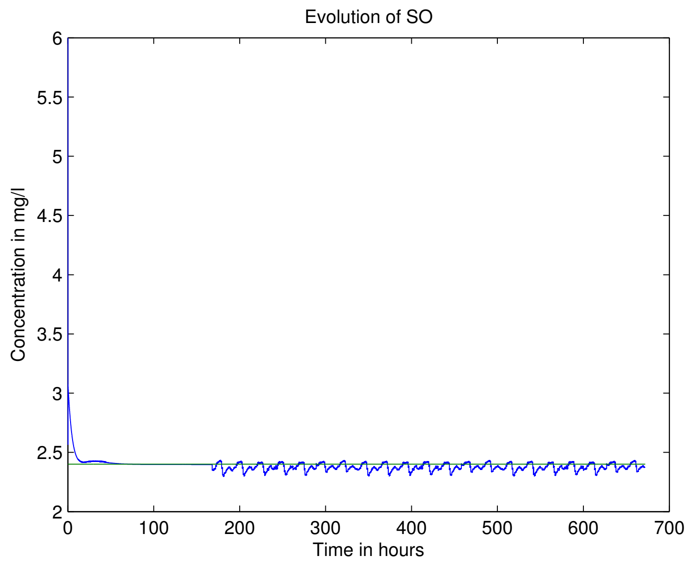
a):Evolution of $S_{NH,nit}$



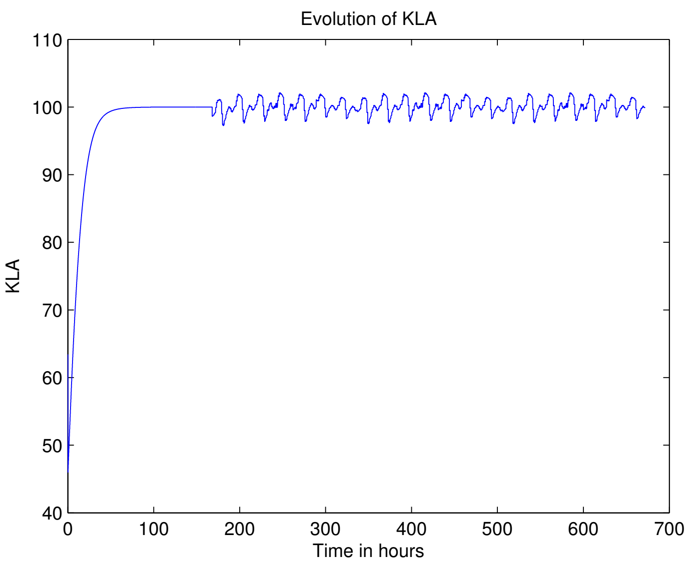
b):Evolution of $S_{NO,nit}$

Figure 4.3: Evolutions of the concentrations $S_{NH,nit}$ and $S_{NO,nit}$.

in the output to the desired values. The figures $(4.3 - 4.5)$ and $(4.7 - 4.9)$ shows the performance and the effectiveness of the regulator. In particular,



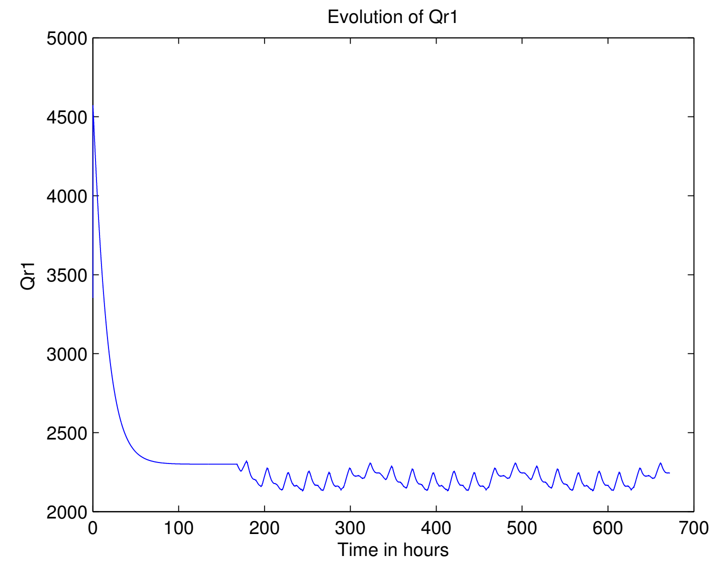
a):Evolution of S_O



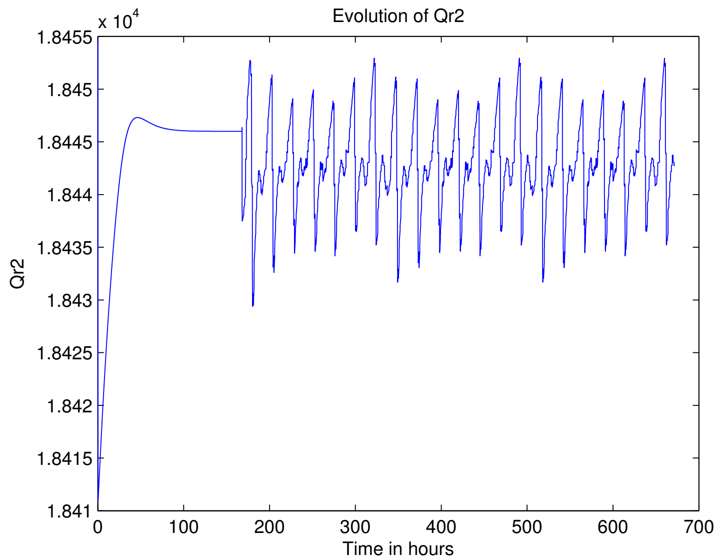
b):Evolution of K_{La}

Figure 4.4: Evolutions of the dissolved oxygen concentration S_O and the oxygen transfer coefficient K_{La}.

one can appreciate the ability of the controller to track the desired values of the controlled variables. Hence, and in practice, the nonlinear controller is



a):Evolution of Q_{r1}



b):Evolution of Q_{r2}

Figure 4.5: Evolutions of the recycled flows Q_{r1} and Q_{r2}.

capable to reduce the EQ, AE and ISE with an important percentage and this is clear from the tables (4.1) and (4.2). No change in IQ because the influent is constant.

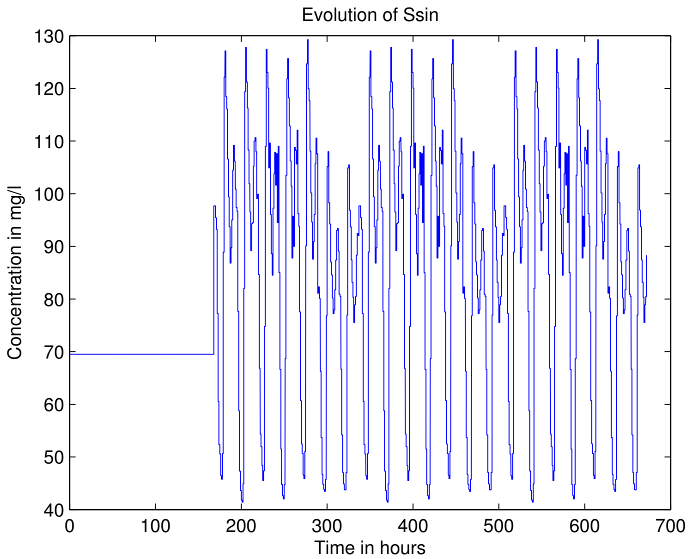


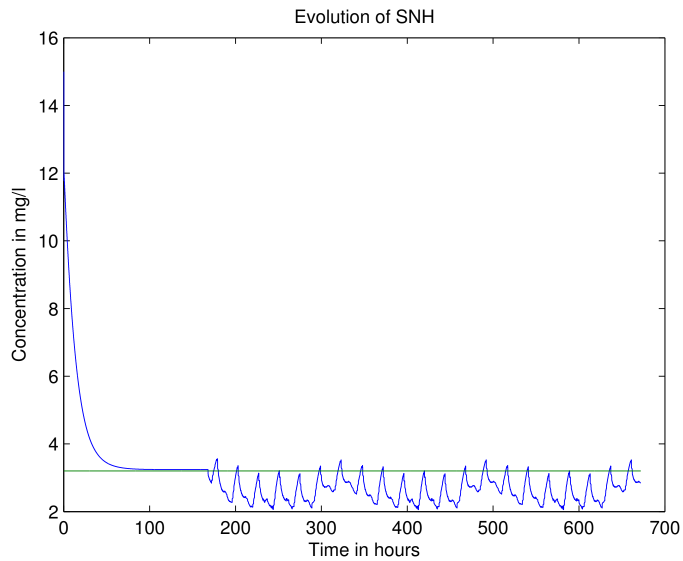
Figure 4.6: Evolution of the disturbance Sin2.

Table 4.1: Indices of the plant with the first disturbance

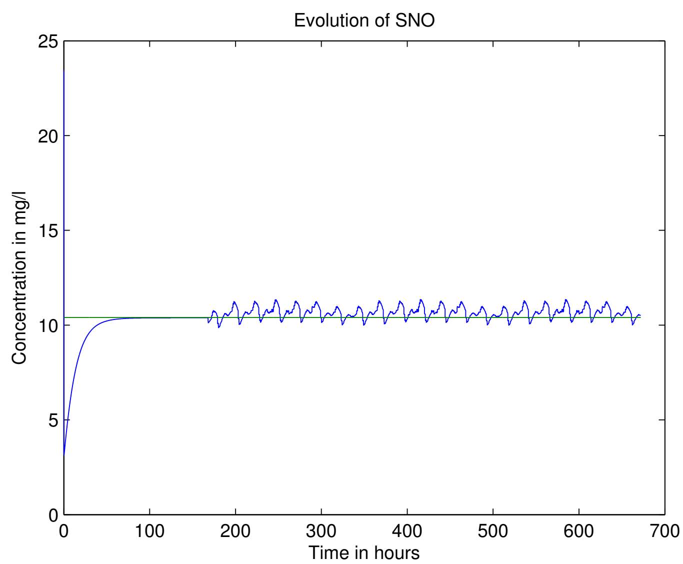
Indices	$u = u_L$	$u = u_L + u_N$	Units
IQ	5.4306e+007	5.4306e+007	$gPUd^{-1}$
EQ	8.9048e+007	8.9015e+007	$gPUd^{-1}$
PE	8.5923e+004	8.5923e+004	Whd^{-1}
AE	2.8433e+004	2.8173e+004	$Whd{-}1$
ISE	10.3677	1.9225	

4.5 Conclusion

In this chapter, we introduced the nonlinear feedback control of a non linear system with input constraints. In fact, positive invariance techniques together with minimal order observer (software sensor) are used to control the linearized model of a WWTP. For this process, modeled as a linear process, some state variables are unavailable to measure and more than that no adequate sensor exists. Hence, the introduction of the observer is of great interest. Further, in general case, linearizing a non linear process leads the variables (control in our case) to be limited within neighborhood of the steady point functioning. The positive invariance techniques that had emerged as very efficient to handle similar problems of constrained control is successfully used to control the nitrogen removal process. The observer based constrained control, as presented above may compete with approaches



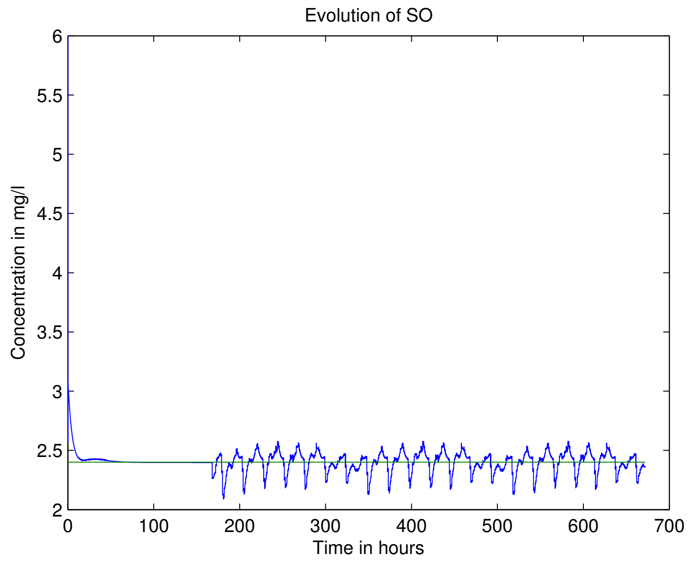
a):Evolution of $S_{NH,nit}$



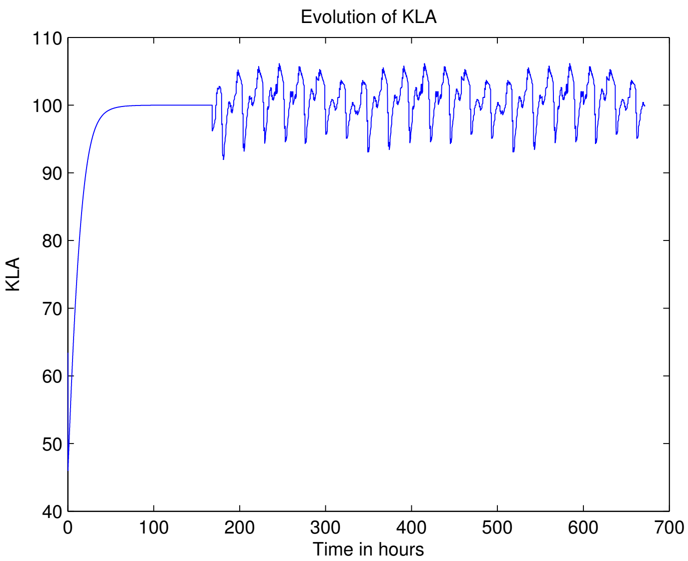
b):Evolution of $S_{NO,nit}$

Figure 4.7: Evolutions of the concentrations $S_{NH,nit}$ and $S_{NO,nit}$.

in easiness, applicability and computing effort.

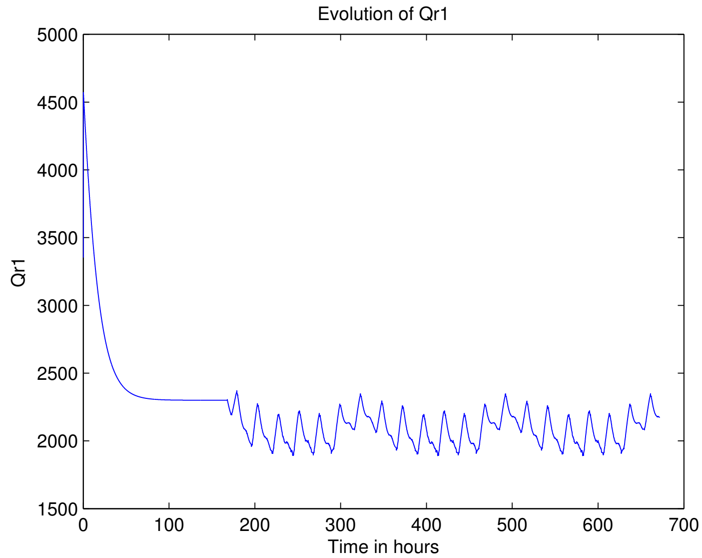


a):Evolution of S_O

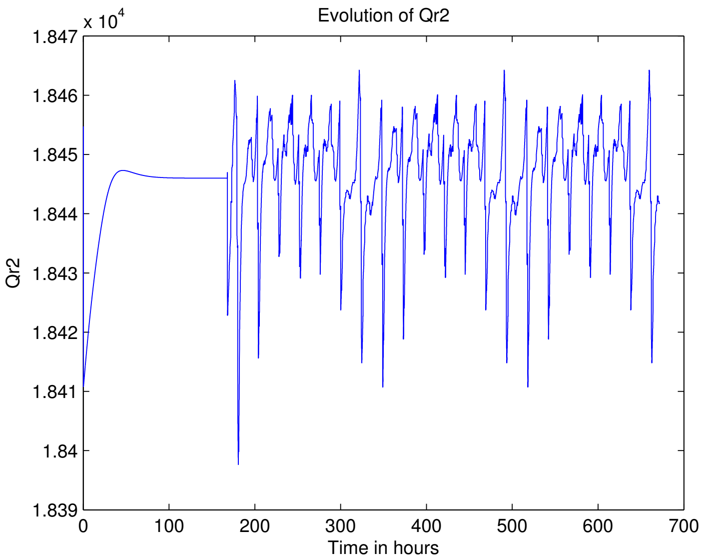


b):Evolution of K_{La}

Figure 4.8: Evolutions of the dissolved oxygen concentration S_O and the oxygen transfer coefficient K_{La}.



a):Evolution of Q_{r1}



b):Evolution of Q_{r2}

Figure 4.9: Evolutions of the recycled flows Q_{r1} and Q_{r2}.

Table 4.2: Indices of the plant with the second disturbance

Indices	$u = u_L$	$u = u_L + u_N$	Units
IQ	5.4678e+007	5.4678e+007	$gPUd^{-1}$
EQ	9.1354e+007	9.1306e+007	$gPUd^{-1}$
PE	8.5923e+004	8.5923e+004	Whd^{-1}
AE	2.8712e+004	2.8313e+004	Whd^{-1}
ISE	15.8029	5.2772	

5

A Constrained Closed Loop MPC Based on Positive Invariance Concept

5.1 Introduction

In this chapter we present a novel methodology to design a closed loop MPC using polyhedral invariant sets that gives a simple solution to this type of control, ensuring stability and respecting constraints on control magnitude and moves in both modes of operation of the dual controller. To achieve our objective, the first step consists of the development of necessary and sufficient conditions for a linear system and a state feedback control law for the satisfaction of the constraints on both control and its increment over an infinite prediction horizon, proving also the asymptotic stability at the origin. Later on, these conditions are used to obtain the state feedback control law for the prediction computation which guarantees stability while fulfilling constraints and performance requirement of the closed loop system. The proposed solution can take into account symmetric and asymmetric constraints, and reduces significantly the computational burden associated with the constrained MPC problem in the presence of these constraints. The objective to that is to transfer as much as possible the constraints handling to the off line computation.

The controller is applied for validation to an activated sludge process in a WWTP , where the bacteria and other microorganisms remove contaminants by degradation. The control problem is the regulation of the pollutant substrate and the dissolved oxygen (DO) concentrations around pre-specified levels.

5.2 Problem Statement

Consider the discrete-time linear system that is obtained by discretizing the continuous model of the process using the Euler integration method and rearranged into the state-space form as:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{5.1}$$

where $x(k) \in \mathcal{R}^n$ is a vector variables reflecting the systems state, called state variables, $y(k) \in \mathcal{R}^p$ a vector outputs or measured variables and $u(k) \in \mathcal{R}^m$ is the input vector, which is constrained to evolve in the following domain:

$$D_u = \{u(.) \in \mathbb{R}^m, \quad -u_{min} \leqslant u(.) \leqslant u_{max}, u_{min}, u_{max} \in \mathbb{R}_+^{m*}\} \tag{5.2}$$

and the control increment is constrained as follow:

$$-\Delta_{min} \leqslant u(k+1) - u(k) \leqslant \Delta_{max} \tag{5.3}$$

The controller proposed in this chapter is a CLMPC designed using the positive invariance concept. The predicted control moves are centered around a stabilizing feedback control law $u = Fx$ over the whole prediction horizon n_c, but adding some degrees of freedom c over a finite horizon n_c a controller to handle constraints fulfillment:

$$u(k) = Fx(k) + c(k) \tag{5.4}$$

Particularly, we want to obtain a stabilizing state feedback $Fx(k)$ computed using polyhedral invariance sets such as the CLMPC is asymptotically stable and the constraints on both the control signal magnitude and the control increment are respected.

5.3 Closed loop predictive control

Consider the following cost function:

$$\begin{aligned} J(k) = &\sum_{i=1}^{n_y}(x^T(k+i|k)Qx(k+i|k))+ \\ &\sum_{i=0}^{n_c-1}(u^T(k+i|k)Ru(k+i|k)) + x^T(k+n_c|k)Qx(k+n_c|k) \end{aligned}$$

with the constraints

$$\begin{aligned} -u_{min} \leq u(k+i|k) \leq u_{max} \\ -\Delta_{min} \leq u(k+i+1|k) - u(k+i|k) \leq \Delta_{max} \end{aligned} \tag{5.5}$$

for $i = 0, ..., n_c - 1$

where Q, R are positive definite matrices, and n_c, n_y are the control and prediction horizons respectively. Here $u(k+i|k)$ and $x(k+i|k)$ denotes input and state predictions vectors at time $k+i$ according to the prediction model:

$$x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k), \quad i = 0, 1, \cdots, n_y \tag{5.6}$$

with initial condition defined by:

$$x(k|k) = x(k)$$

In order to improve the numerical conditioning of the optimization and the controller performance, a closed loop MPC has been considered by defining the predicted input sequence in a dual mode fashion as:

$$\mathbf{u}(k) = \begin{bmatrix} Fx(k) + c(k) \\ Fx(k+1) + c(k+1) \\ \vdots \\ Fx(k+n_c-1) + c(k+n_c-1) \\ Fx(k+n_c) \\ \vdots \\ F\Phi^{n_y-1}x(k+n_c) \end{bmatrix} \tag{5.7}$$

with $\Phi = A + BF$.

With the new predictions model:

$$\begin{aligned} x(k+i+1) &= Ax(k+i|k) + Bu(k+i|k); \\ u(k+i|k) &= Fx(k+i|k) + c(k+i|k), i = 0, .., n_c - 1. \ (mode1) \\ u(k+i|k) &= Fx(k+i|k), i \geqslant n_c. \ (mode2) \end{aligned} \tag{5.8}$$

where: $F \in \mathcal{R}^{m \times n}$ is a stabilizing linear state feedback controller designed using positive invariance concept. In this thesis, we propose the design of F using positive invariance concepts as explained later.

The system model becomes

$$x(k) = (A + BF)x(k) + Bc(k). \tag{5.9}$$

where $c(k)$ is the new manipulated input.

That is, assume a finite number n_c of nonzero values for $\mathbf{c}$. Beyond n_c the perturbations are zero and the closed loop system is linear, equivalently to mode 2 of the dual mode predictions. Thus, the algorithm to obtain the control signal is , for each sampling time k:

- Solve the optimization problem

$$
\begin{aligned}
\mathbf{c}^* = \underset{\mathbf{c}(k)}{argmin} J(\mathbf{x}(k), \mathbf{c}(k))& \\
\text{subject to}& \\
-c_{min} \leqslant c(k+i|k) \leqslant c_{max}& \\
x(k+i+1|k) = \Phi x(k+i|k) + Bc(k+i|k),& \\
\text{for} \quad i = 0, .., n_c - 1& \\
x(k+i+1|k) = \Phi x(k+i|k),& \\
\text{for} \quad i \geqslant n_c&
\end{aligned}
\tag{5.10}
$$

- Implement $u(k) = Fx(k) + c^*(k)$

  where $\mathbf{c}(k) = \begin{bmatrix} c^T(k|k) & \cdots & c^T(k+n_c-1|k) \end{bmatrix}^T$ and $c^*(k)$ is the first element of $\mathbf{c}^*(k)$

In order to solve (5.10), J needs to be formulated in terms of the degree of freedom $\mathbf{c}(k)$.

The future response of the controlled plant is obtained using the prediction model (5.8).

Removing the dependent variable $u(k+i|k)$,

$$x(k+i|k) = (A+BF)x(k+i-1|k) + Bc(k+i|k); \tag{5.11}$$

Then the closed loop predictions with $\Phi = A + BF$ are;

$$\mathbf{x}(k) = P_x x(k) + H_x \mathbf{c}(k) \tag{5.12}$$

$$\mathbf{u}(k) = P_u x(k) + H_u \mathbf{c}(k) \tag{5.13}$$

Where

$$
P_x = \begin{bmatrix} \Phi \\ \Phi^2 \\ \Phi^3 \\ \vdots \\ \Phi^{n_c} \end{bmatrix}, \quad
H_x = \begin{bmatrix} B & 0 & 0 & \cdots & 0 \\ \Phi B & B & 0 & \cdots & 0 \\ \Phi^2 B & \Phi B & B & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \Phi^{n_c} B & \cdots & \cdots & \cdots & B \end{bmatrix},
$$

$$
P_u = \begin{bmatrix} F \\ F\Phi \\ F\Phi^2 \\ \vdots \\ F\Phi^{n_c-1} \end{bmatrix}, \quad
H_u = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ FB & I & 0 & \cdots & 0 \\ F\Phi B & FB & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ F\Phi^{n_c-1} B & \cdots & \cdots & \cdots & I \end{bmatrix},
$$

And

$$\mathbf{x}(k) = \begin{bmatrix} x^T(k|k) & \cdots & x^T(k+n_c|k) \end{bmatrix}^T;$$
$$\mathbf{c}(k) = \begin{bmatrix} c^T(k|k) & \cdots & c^T(k+n_c-1|k) \end{bmatrix}^T;$$
$$\mathbf{u}(k) = \begin{bmatrix} u^T(k+1) & \cdots & u^T(k+n_c-1) \end{bmatrix}^T;$$

The state after n_c steps will be denoted as:

$$x(k+n_c) = P_{nc}x(k) + H_{nc}\mathbf{c}(k) \tag{5.14}$$

Where P_{nc} and H_{nc} are the last block rows of prediction (5.12).

Then, for linear systems, the dependence of predictions $\mathbf{x(k)}$ on $\mathbf{c}(k)$ is linear, and therefore (5.5) is a quadratic function of the input sequence $\mathbf{c}(k)$. Assuming that $\mathbf{c}(k)$ comprises only a finite number of nonzero values, then the cost function can be set up as for dual mode predictions where mode 2 is the part where $\mathbf{c(k)} = \mathbf{0}$. Using the prediction equations (5.12), (5.13) and (5.14), $J(k)$ can be expressed as a function of $\mathbf{c}(k)$ in the form:

$$\underset{\mathbf{c}}{min} J(k) = \mathbf{c}^T(k)S_c\mathbf{c}(k) + 2\mathbf{c}^T(k)S_{cx}x(k) \tag{5.15}$$

where

$$\begin{aligned} S_c &= H_x^T diag(Q) H_x + H_u^T diag(R) H_u + H_{nc}^T P_o H_{nc} \\ S_{cx} &= H_x^T diag(Q) P_x + H_u^T diag(R) P_u + H_{nc}^T P_o P_{nc} \end{aligned}$$

The matrix P_o is the solution of the following Lyapunov equation:

$$\Phi^T P_o \Phi = P_o - \Phi^T Q \Phi - F^T R F \tag{5.16}$$

In this point, we present a novel algorithm to design a stable CLMPC using positive invariant sets. First, necessary and sufficient conditions are obtained for the satisfaction of the constraints on control magnitude and increments and also for the proof of asymptotic stability at the origin. Later, these conditions are used to obtain the state feedback control law F for the prediction computation, which guaranties stability respecting the constraints.

5.4 Controller Design

In this section, a method that gives a simple solution to stability to design a closed loop MPC with both constraints on control magnitude and its increment has been developed for stabilizing dynamical linear discrete system. The procedure starts with the development of necessary and sufficient conditions are given with respect to the autonomous system with disturbance for constraints satisfaction. Furthermore, a link between these conditions and a pole assignment procedure is used to find the stabilizing controller law.

In [58, 61], the authors solve the problem of stabilizing a linear system subject to constraints, using the positive invariance concept to find a stabilizing controller by state feedback. In this chapter, we follow a similar approach to extend this problem to a closed loop MPC based on positive invariance by the assumption that $\mathbf{c}(k)$ is a bounded additive disturbance.

5.4.1 Preliminary results

In this point, the concepts of positive invariance are presented and the conditions for respecting constraints for the autonomous system.

Consider the following linear discrete time invariant autonomous system

$$z(k+1) = Hz(k) + BN\mathbf{c}(k), z(k_0) = z_0 \tag{5.17}$$

where $z \in \mathcal{R}^m$ is the state constrained to evolve in

$$D_z = \{z \in \mathcal{R}^m, -z_{min} \leqslant z(k) \leqslant z_{max}, z_{min}, z_{max} \in \mathcal{R}_+^{m*}\} \tag{5.18}$$

and the perturbation $\mathbf{c}(k)$ is bounded in the domain

$$D_c = \{\mathbf{c}(k) \in \mathcal{R}^{n_c \times m}, -c_{min} \leqslant \mathbf{c}(k) \leqslant c_{max}, c_{min}, c_{max} \in \mathcal{R}_+^{n_c \times m*}\} \tag{5.19}$$

and $N = [I_m, 0, \cdots, 0]$, I denoting the identity matrix of adequate dimensions.

Consider also that the state increment is constrained as follows:

$$-\Delta^z_{min} \leqslant z(k+1) - z(k) \leqslant \Delta^z_{max} \tag{5.20}$$

First, recall the definition of D_c-positive invariance of domain D_z which will be used in the sequel.

Definition 5.1 *The domain D_z given by* (5.18) *is D_c-positive invariant w.r.t. motion of system* (5.17) *if for all initial condition $z_0 \in \mathcal{D}_z$, the trajectory of the system $z(k, k_0, z_0) \in \mathcal{D}_z$ for all $c(k) \in \mathcal{D}_c$, $k > k_0$.*

To design a closed loop MPC based on positive invariance that ensure constraint fulfillment for both control and increment, we begin by establishing conditions such that the state increment $\Delta z(k)$ constraints for the autonomous system (5.17) are respected. Further, for the proposed controller strategy presented later, control increment dynamics are separated from control dynamics in order to be studied sequentially. It will be easy then, to mix the conditions obtained separately for both control and increment constraints to obtain the controller that respects both of them. Hence, the following lemma studies the fulfillment of incremental state constraints for system (5.17).

Lemma 5.1 *The evolution of the autonomous system* (5.17) *respects incremental constraints* (5.20) *if and only if the matrix* H *satisfies:*

$$\widetilde{(H-I)}Z + \widetilde{BN}\begin{pmatrix} c_{max} \\ c_{min} \end{pmatrix} \leqslant \Delta \tag{5.21}$$

where $Z = \begin{pmatrix} z_{max} \\ z_{min} \end{pmatrix}$, $\Delta = \begin{pmatrix} \Delta^z_{max} \\ \Delta^z_{min} \end{pmatrix}$ *and* $N = [I_m, 0, \cdots, 0]$

Proof 5.1 ***If part*** *Assume that condition* (5.21) *is satisfied. it is known that*

$$-z_{min} \leqslant z(k) \leqslant z_{max} \tag{5.22}$$

hence, it is possible to write

$$\begin{aligned} z(k+1) - z(k) &= Hz(k) + BN\mathbf{c}(k) - z(k) \\ &= (H-I)z(k) + BN\mathbf{c}(k) \\ &= Gz(k) + BN\mathbf{c}(k) \end{aligned}$$

where we note $G = H - I$*,Next, decompose matrix* $G = G^+ - G^-$*, pre-multiplying* (5.24) *by* G^+ *and* $-G^-$*, gives*

$$-G^+ z_{min} \leqslant G^+ z(k) \leqslant G^+ z_{max} \tag{5.23}$$

$$-G^- z_{max} \leqslant -G^- z(k) \leqslant G^- z_{min} \tag{5.24}$$

and the addition of (5.23) *and* (5.24) *enables to write*

$$-G^+ z_{min} - G^- z_{max} \leqslant Gz(k) \leqslant G^+ z_{max} + G^- z_{min} \tag{5.25}$$

Further, the perturbation is bounded:

$$-c_{min} \leqslant \mathbf{c}(k) \leqslant c_{max} \tag{5.26}$$

and the same reasoning with matrix BN *and the* $c(k)$ *leads to the following inequalities:*

$$-(BN)^+ c_{min} \leqslant (BN)^+ \mathbf{c}(k) \leqslant (BN)^+ c_{max} \tag{5.27}$$

$$-(BN)^- c_{max} \leqslant -(BN)^- \mathbf{c}(k) \leqslant (BN)^- c_{min} \tag{5.28}$$

the addition of obtained inequalities gives the following:

$$\begin{gathered} -G^+ z_{min} - G^- z_{max} - (BN)^+ c_{min} - (BN)^- c_{max} \\ \leqslant Gz(k) + BN\mathbf{c}(k) \leqslant \\ G^+ z_{max} + G^- z_{min} + (BN)^+ c_{max} + (BN)^- c_{min} \end{gathered} \tag{5.29}$$

according to condition (5.21)

$$\begin{aligned}\Delta^z_{min} &\leqslant -G^+z_{min} - G^-z_{max} - (BN)^+c_{min} - (BN)^-c_{max} \\ &\leqslant Gz(k) + BN\mathbf{c}(k)\end{aligned} \tag{5.30}$$

and

$$\begin{aligned}Gz(k) + BN\mathbf{c}(k) &\leqslant G^+z_{max} + G^-z_{min} + (BN)^+c_{max} + (BN)^-c_{min} \\ &\leqslant \Delta^z_{max}\end{aligned} \tag{5.31}$$

this is equivalent to

$$\Delta^z_{min} \leqslant z(k+1) - z(k) \leqslant \Delta^z_{max}. \tag{5.32}$$

Only if part *Assume that* $\Delta z = z(k+1) - z(k)$ *respect the constraints and the condition* (5.23) *is not satisfied for an index* $1 \leqslant i \leqslant n$ *such that,*

$$\left[\widetilde{G}Z\right]_i + \left[\widetilde{BN}\begin{bmatrix} c_{max} \\ c_{min}\end{bmatrix}\right]_i > \Delta_i \tag{5.33}$$

$\left[G^+Z_{max} + G^-Z_{min}\right]_i > \Delta^i_{max}$ *Then, the following state vector for the system can be selected*

$$\phi(k) = \begin{cases} z^j_{max} & if \quad h_{ij} > 0 \\ 0 & if \quad h_{ij} = 0 \\ z^j_{min} & if \quad h_{ij} < 0 \end{cases} \quad j = 1, ..., n.$$

It is easy to check that $\phi(k)$ *is an admissible state for the system. Further the following admissible perturbation may occur:*

$$\omega(k) = \begin{cases} c^j_{max} & if \quad g_{ij} > 0 \\ 0 & if \quad g_{ij} = 0 \\ c^j_{min} & if \quad g_{ij} < 0 \end{cases} \quad j = 1, ..., n.$$

where g_{ij} *are the elements of matrix* BN

Calculation of the ith component of the increment of this state give

$$\begin{aligned}\left[\phi(k+1) - \phi(k)\right]_i &= \left[G\phi(k) + BN\mathbf{c}(k)\right]_i \\ &= \sum_{j=1}^n h_{ij}\phi_i(k) + \sum_{j=1}^n g_{ij}\omega_i(k) \\ &= \left[G^+z_{max} + G^-z_{min}\right]_i \\ &+ \left[(BN)^+c_{max} + (BN)^-c_{min}\right]_i\end{aligned}$$

taking into account inequality (5.33), *it is possible to write*

$$\left[\phi(k+1) - \phi(k)\right]_i > \Delta^i_{max}$$

which contradicts the assumption.

Evolution of the autonomous system (5.17) will respect both constraints on the state $z(k)$ and constraints on its increment if domain D_z, given by (5.18) is D_c-positively invariant and conditions given in the previous lemma are satisfied. But it was assumed that the state $z(k)$ does not leave domain D_z given by (5.18) which is not guaranteed in general case. For this we must also ensure the positive invariance of the domain D_c. Positive invariance conditions of polyhedral domains of type D_c have already been reported by [?, ?] and [?]. Combining these conditions to those proposed in Lemma 1 enables us to claim the following:

Lemma 5.2 *The domain D_z is D_c-positively invariant respect to motion of system* (5.18)*, and incremental constraints* (5.20) *are respected if and only if*

$$i) \quad \widetilde{(H-I)}Z + \widetilde{BN}\begin{pmatrix} c_{max} \\ c_{min} \end{pmatrix} \leqslant \Delta \tag{5.34}$$

$$ii) \quad \widetilde{H_0}W \leqslant W \tag{5.35}$$

where $H_0 = \begin{pmatrix} H & BN \\ 0 & M \end{pmatrix}$, $\quad M = \begin{bmatrix} 0 & I_m & 0 & \cdots & 0 \\ 0 & 0 & I_m & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I_m \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$, $\quad Z = \begin{pmatrix} z_{max} \\ z_{min} \end{pmatrix}$,

$\Delta = \begin{pmatrix} \Delta^z_{max} \\ \Delta^z_{min} \end{pmatrix}$, $\quad W = \begin{pmatrix} z_{max} \\ c_{max} \\ z_{min} \\ c_{min} \end{pmatrix}$, *and* $N = [I_m, 0, \cdots, 0]$

Proof 5.2 *First, note that the dynamics of the system state and those of the state increment are independent. Secondly, condition* (5.35) *implies that the magnitude bounds holds, and finally, condition* (5.34) *implies that the increment bounds also hold.*

Note also that M is a zero block matrix (whose partition is conformal with that of c) and whose i, $i+1$ block are all the identity matrix. The role of M is to shift all the vector elements of c by one position and to replace the last vector element by zero; ensuring that all but the first element of the c sequence are used again at the next time instant.

Relating the previous lemmas 5.1 and 5.2 to the so called inverse procedure a pole assignment procedure [60, 62] makes possible to solve the problem stated in the chapter, matrix H satisfying all required conditions exists and the feedback F is obtained as a solution to the equation :

$$HF = FA + FBF \tag{5.36}$$

Remark 5.1 *Note here that all computation effort is handled off line. Choice of an adequate matrix H with all required conditions is studied in* [63].

5.4.2 Main results

With this background, we are able to solve the problem stated in Section 5.3.1. Consider the discrete linear time invariant system (5.1) with constraints on both control magnitude and control increments (5.2) – (5.3). Using the feedback control law

$$u(k) = Fx(k) + c(k), \quad F \in \mathcal{R}^{m\times n} \quad \text{and} \quad c(k) \in \mathcal{R}^m, \\ \sigma(A + BF) \in \mathbb{D}_s \tag{5.37}$$

induces the following domain of linear behavior in the state space

$$D_u = \{x \in \mathbb{R}^n, \quad -u_{min} \leqslant Fx(k) + c(k) \leqslant u_{max}, u_{min}, u_{max} \in \mathcal{R}^m_+\} \tag{5.38}$$

Where $\mathbb{D}_s$ denotes the stability domain for eigenvalues (that is, the unit disk).

If the state does not leave the domain (5.38), the control signal does not violate the constraints. That is, the set D_u is $D_c$positively invariant w.r.t motion of system (5.1). This gives the following result:

Theorem 5.3 *System* (5.1) *with state feedback* (5.37) *is asymptotically stable at the origin with constraints on both the control and its increment if there exist a matrix $H \in \mathcal{R}^{m\times m}$ such that:*

$$\begin{array}{rc} i) & FA + FBF = HF \\ ii) & \widetilde{H_0}U \leqslant U \\ iii) & \widetilde{H_1}U \leqslant \Delta \end{array} \tag{5.39}$$

where

$$H_0 = \begin{pmatrix} H & FBN + NM - HN \\ 0 & M \end{pmatrix};$$

$$H_1 = \begin{pmatrix} H - I & FBN + NM - HN \end{pmatrix};$$

$$U = \begin{bmatrix} u_{max} \\ c_{max} \\ u_{min} \\ c_{min} \end{bmatrix} \qquad \textit{and} \qquad \Delta = \begin{bmatrix} \Delta_{max} \\ \Delta_{min} \end{bmatrix}$$

.

Proof 5.3 *Introduce the following change of coordinates $z = Fx + c$, it is possible to write*

$$\begin{array}{rcl} z(k+1) & = & Fx(k+1) + N\mathbf{c}(k+1) \\ & = & F(A+BF)x(k) + FBN\mathbf{c}(k) + NM\mathbf{c}(k) \\ & = & HFx(k) + (FBN + NM)\mathbf{c}(k) \\ & = & H(Fx(k) + N\mathbf{c}(k) - N\mathbf{c}(k)) \\ & + & (FBN + NM)\mathbf{c}(k) \\ & = & Hz(k) + (FBN + NM - HN)\mathbf{c}(k). \end{array} \tag{5.40}$$

With this transformation, domain D_u is transformed to domain D_z given by (5.18)*. Furthermore, with conditions* (5.39)*, it is easy to note that domain D_z is D_c-positive invariant w.r.t the system* (5.40) *while the constraints on the control increments are respected. Bearing in mind that $\sigma(A+BF) \in \mathbb{D}_s$ and that the linear behavior is guaranteed, it is possible to conclude to the asymptotic stability of the closed-loop system.*

Steps to follow for design of such controllers are summarized in the algorithm below:

Algorithm 5.1 *Closed loop predictive control design based on the positive invariance concept.*

> ***Step 1.*** *Check if the system is controllable and observable, if not extract the controllable and observable parts.*
>
> ***Step 2.*** *Check if matrix A has $(n-m)$ stable eigenvalues, if not augment the system (remark* 4.1*).*
>
> ***Step 3.*** *Choose the matrix $H \in \mathcal{R}^{m\times m}$ or $H \in \mathcal{R}^{n\times n}$ if the system is augmented satisfying conditions Eq*4.19 *and Eq.*4.20*.*
>
> ***Step 4.*** *Compute the gain matrix that is the solution of $HF = FA + FBF$ (section* 4.3*).*
>
> ***Step 5.*** *If condition* (5.39) *is satisfied go to* (5.17) *for the online implementation of the CLMPC, else return to step 3 and change the matrix H.*

5.4.3 Selection of the CLMPC parameters

In the proposed CLMPC scheme, a major obstacle for the designer is to get a balance between the maximal feasible invariant set (affected by F and n_c, c_{max}, c_{min}), the computational load (implied by n_c) and the performance (implied by n_c and F). A practical design objective is to enlarge the feasible invariant sets without sacrificing too much performance and an inexpensive computational optimization.

- Horizon of control n_c: The influence of n_c in the invariant set sizes is as follows. The size of the invariant sets increases when n_c increases until a maximum value is reached. When n_c is small, then mainly F, c_{max}, c_{min}, determine their size. In this case, then the volume of the maximal controlled admissible set may be dominated by the implied state feedback F. Note that a highly tuned F could give rise to small maximal controlled admissible set and loosely tuned F could provide much larger feasible regions. In contrast, for computational and sometimes for robustness reasons, n_c is chosen to be small.

- Limits c_{min} and c_{max}: Note first that these are fictitious limits that can be chosen with some freedom, since the real physical bounds of the control signal are specified through u_{max}, u_{min}. They must be chosen to be small in order not to affect too much the initial positive invariance set volumes. Note that large values of these limits the volume is significantly reduced as shown in the remainder of the paper.

- The matrix H: The selection of H can be done as in this article using an inverse procedure. It consists of starting with the choice of a matrix H that satisfies the conditions of the theorem 1, following with the calculation of F through the resolution of equation (5.36). The matrix H is a degree of freedom that can be exploited to improve the closed loop responses. When the goal is to enlarge the initial conditions set, the matrix H can be selected by linear programming as follows:

$$\begin{gathered} minimize \quad \epsilon \\ \text{subject to} \\ \begin{bmatrix} H^+ & H^- \\ H^- & H^+ \end{bmatrix} \begin{bmatrix} u_{max} \\ u_{min} \end{bmatrix} \leqslant \epsilon \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \\ 0 < \epsilon \leqslant 1,\ H^+ \geqslant 0 \text{ and } H^- \geqslant 0 \end{gathered}$$

  where $\omega_1 < u_{max}$ and $\omega_2 < u_{min}$ are tuning vectors. In fact, if the feasible solution H^+, H^- is such that $H = H^+ - H^-$ satisfies required conditions in theorem 1, use H to obtain F. Else, change vectors $\omega_1$and ω_2. A similar linear programming problem for finding simultaneously both matrices H and F can be found in [64].

5.5 Example

Consider the second order system described by the state-space equation:

$$x(k+1) = \begin{bmatrix} 1.7 & -3.3 \\ 1.3 & 0.3 \end{bmatrix} x(k) + \begin{bmatrix} 3.0 & 2.0 \\ -2.0 & 2.0 \end{bmatrix} u(k)$$

The control vector is subject to the following constraints

$$-\begin{bmatrix} 6 \\ 8 \end{bmatrix} \leqslant u(k) \leqslant \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

Assume that the control increment is constrained as follows:

$$-\begin{bmatrix} 3.5 \\ 4 \end{bmatrix} \leqslant u(k+1) - u(k) \leqslant \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

The design parameters and matrices of the CLMPC are the following. The parameter $c(k)$ is constrained as:

$$-\begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} \leqslant c(k) \leqslant \begin{bmatrix} 0.3 \\ 0.9 \end{bmatrix}$$

Other parameters selected are: $n_c = 2$, $Q = diag(0.5, 0.5)$, $R = diag(1, 1)$ and

$$P_o = \begin{bmatrix} 1.4206 & -0.9728 \\ -0.9728 & 1.7575 \end{bmatrix}$$

The unforced system is unstable. The eigenvalues of matrix A are: $\lambda_{1,2} = 1 \pm j1.95$. For the matrix H, we choose to assign the following closed loop eigenvalues $\{0.61, \quad 0.7\}$, which leads to the following choice of matrix H:

$$H = \begin{bmatrix} 0.61 & 0 \\ 0 & 0.7 \end{bmatrix}$$

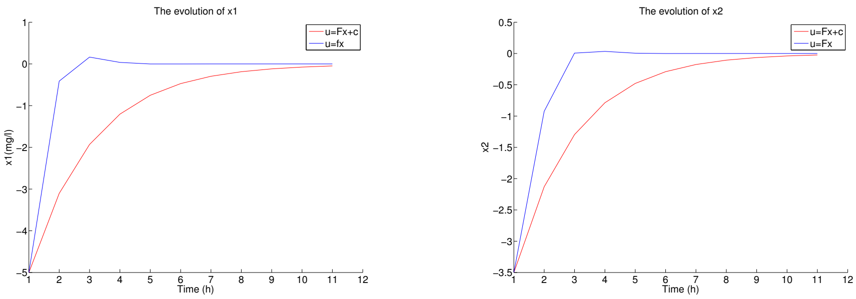
Solution of equation (5.36) leads to the following gain matrix F:

$$F = \begin{bmatrix} 0.0596 & 0.5762 \\ -0.5945 & 0.7363 \end{bmatrix}$$

At this step, one has to check that all required conditions (5.39) are fulfilled. In this case:

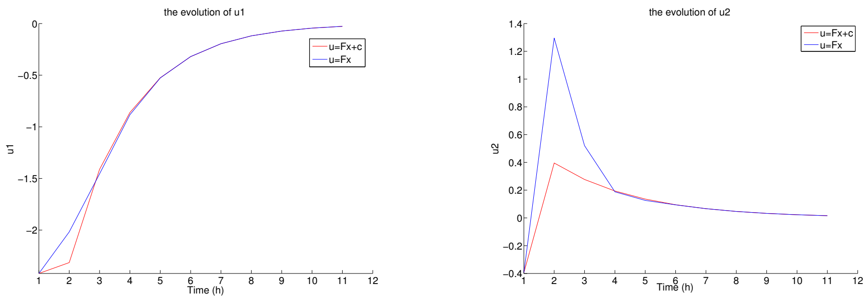
$$\widetilde{H_0}U \leqslant U$$

$$\begin{bmatrix} 5.2628 \\ 6.9089 \\ 0.3000 \\ 0.9000 \\ 0 \\ 0 \\ 4.4894 \\ 7.1516 \\ 0.1000 \\ 0.2000 \\ 0 \\ 0 \end{bmatrix} \leqslant \begin{bmatrix} 6.0000 \\ 8.0000 \\ 0.3000 \\ 0.9000 \\ 0.3000 \\ 0.9000 \\ 6.0000 \\ 8.0000 \\ 0.1000 \\ 0.2000 \\ 0.1000 \\ 0.2000 \end{bmatrix}$$



a):Evolution of x_1 b):Evolution of x_2

Figure 5.1: Evolutions of states x_1 and x_2 from the initial condition $x_0 = [-5 \quad -3.5]^T$



c):Evolution of u_1 c):Evolution of u_2

Figure 5.2: Evolutions of the control signals.

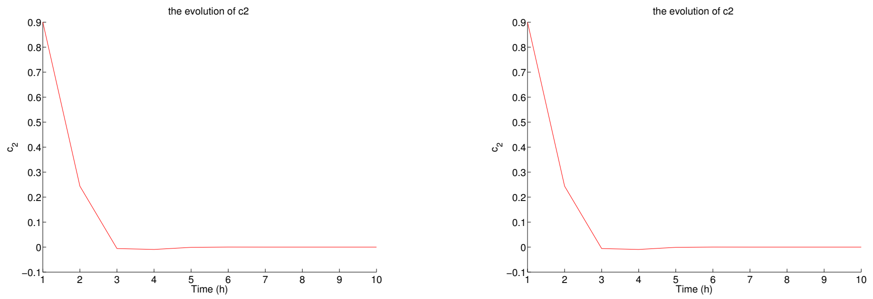
and

$$\widetilde{H_1} U \leqslant \Delta$$

$$\begin{bmatrix} 3.9428 \\ 3.7089 \\ 3.1694 \\ 3.9516 \end{bmatrix} \leqslant \begin{bmatrix} 4.0000 \\ 4.0000 \\ 3.5000 \\ 4.0000 \end{bmatrix}$$

Consequently, both conditions are satisfied and the asymptotic stability is guaranteed with the obtained control law.

Simulation results are summarized in figures $(5.1 - 5.4)$,comparing the proposed CLMPC with an existing state feedback control law based on positive invariance [58]. $Fig.$5.1 represents the motion of the system from the initial states condition $x_0 = [-5 \quad -3.5]^T$, while $Fig.$5.2 represents the evolutions in time of the inputs. The $Fig.$5.3 shows the evolution of the parameter **c**.



a):Evolution of c_1 b):Evolution of c_2

Figure 5.3: Evolution of parameter c.

Fig.5.5 presents the feasible set dependence on n_c, it can be shown that the size of $D_u(F, u_{max}, u_{min})$ increase when n_c increases until a maximum value is reached.

Fig.5.4 shows the influence of r_{max} and r_{min} on the size of the feasible set $D_u(F, u_{max}, u_{min})$ with $c_{1max} = [0.3 \quad 0.9]^T$, $r_{1min} = [0.1 \quad 0.2]^T$, $c_{2max} = [0.5 \quad 1.9]^T$, $r_{2min} = [0.3 \quad 0.5]^T$, $c_{3max} = [1.3 \quad 2]^T$, $r_{3min} = [1.5 \quad 2]^T$ and $c_{4max} = [2.5 \quad 4]^T$, $r_{4min} = [2.5 \quad 4]^T$. As it can be observed, the size of this set is inversely proportional to c_{max} and c_{min}, that is when c_{max} and/or c_{min} increase, the size of $D_u(F, u_{max}, u_{min})$ decreases. For this reason, they must be chosen to be small in order to not affect too much the size of this set.

5.6 Application to the WWTP.

In this section, the control scheme presented in the previous sections is adapted to the special case of the WWTP.

5.6.1 Process Modeling

The activated sludge process is usually constituted by a bioreactor and a settler. The aerator is taken to be a well-stirred tank in which suspended micro-organisms biochemically degrade the dissolved substrate. The suspended micro-organisms are separated completely in the settler. A portion of the concentrated biomass is recycled to the bioreactor. The remainder is wasted to maintain a limited micro-organism level in the system. The energy required is supplied by dissolved oxygen and carbon dioxide is in turn released. We assume that no bioreaction takes place in the settler and the aerator is considered to be perfectly mixed so that the concentration of each component is spatially homogeneous (Fig.5.5).

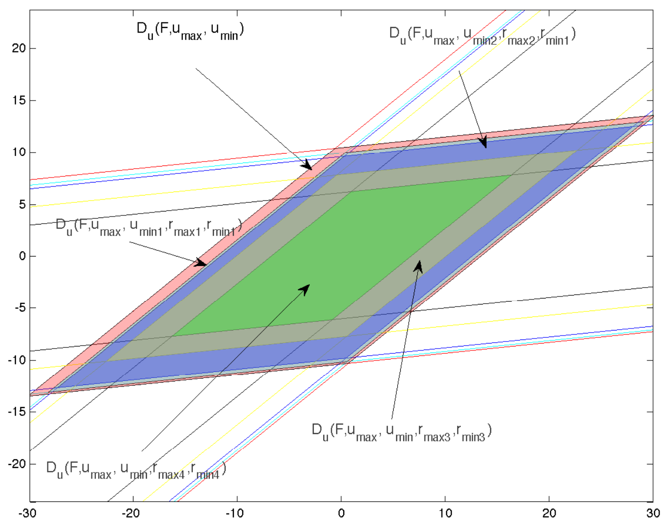


Figure 5.4: Feasible set $D_u(F, u_{max}, u_{min})$ dependence on c_{max} and c_{min}

The equations of the model $M1$are:

$$\begin{aligned}
\dot{X} &= \mu\left(.\right)X - \left(1 + q_r(t)D(t)\right)X + q_r(t)D(t)X_r \\
\dot{S} &= -\frac{\mu\left(.\right)}{Y_s}X - \left(1 + q_r(t)D(t)\right)S + D(t)S_{in}(t) \\
\dot{X}_r &= \left(1 + q_r(t)D(t)\right)X - \left(q_p + q_r(t)D(t)\right)X_r \\
\dot{C} &= -\frac{\mu\left(.\right)}{Y_C}X - \left(1 + q_r(t)D(t)\right)C + K_{La}(C_s - C)
\end{aligned}$$

Where:

- $X(t), S(t), X_r(t)$ and $C(t)$ are, respectively, the biomass, the substrate, the recycled biomass and the dissolved oxygen concentrations.
- $\mu(t)$ corresponds to the biomass specific growth rate, defined as:

$$\mu(t) = \mu_{max}\frac{S(t)}{K_S + S(t)}\frac{C(t)}{K_C + C(t)} \tag{5.41}$$

where μ_{max} is the maximum specific growth rate, K_S is the affinity constant and K_C is saturation constant.

- $D(t)$ is the dilution rate.

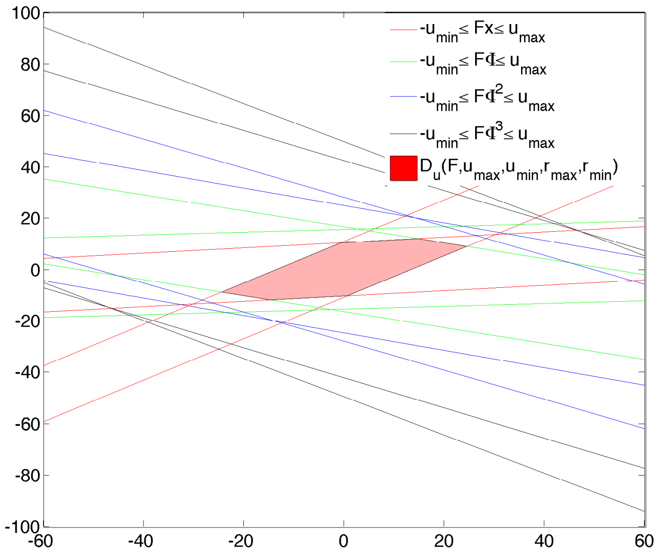


Figure 5.5: Feasible set $D_u(F, u_{max}, u_{min})$ dependence on n_c

- q_r and q_p represent the ratio of recycled flow to influent flow and the ratio of waste flow to influent flow.
- K_{La} represents the oxygen mass transfer coefficient.
- S_{in} corresponds to the influent substrate concentration.
- C_S is the maximum dissolved oxygen concentration.
- Y_S and Y_C are constants yields coefficients.

With the typical values of process and kinetic parameters $q_r = 0.6$, $q_p = 0.4$,$Y_s = 100 \quad mg.l^{-1}$, $Y_c = 2 \quad mg.l^{-1}$, $C_s = 10 \quad mg.l^{-1}$, $K_S = 100 \quad mg/l$, $K_C = 2 \quad mg/l$ $S_{in} = 200 \quad mg.l^{-1}$ and $\mu_{max} = 0.15 \quad h^{-1}$.

5.6.2 Controller design

To obtain a model in the state space, the state vector is considered as:

$$x(k) = [X(k) \quad S(k) \quad X_r(k) \quad C(k)]^T \tag{5.42}$$

Furthermore, to complete the model, the following input and output vectors are used

$$u(k) = [D(k) \quad K_{la}(k)]^T \tag{5.43}$$

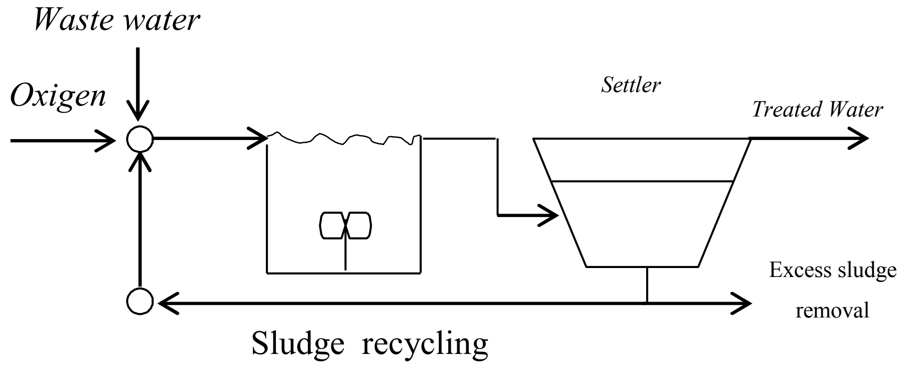


Figure 5.6: Schematic view of an activated sludge process.

$$y(k) = [S(k) \;\; C(k)]^T \tag{5.44}$$

The constraints on the control magnitude and on its increment are given by the following limitations :

$$\begin{cases} D_{min} \leq D \leq D_{max} \\ K_{lamin} \leq K_{la} \leq K_{lamax} \\ \Delta D_{min} \leq \Delta D \leq \Delta D_{max} \\ \Delta K_{lamin} \leq \Delta K_{la} \leq \Delta K_{lamax} \end{cases} \tag{5.45}$$

where $D_{min} = 0\ h^{-1}$, $D_{max} = 0.8\ h^{-1}$, $K_{lamin} = 0\ m^3.h^{-1}$, $K_{lamax} = 5\ m^3.h^{-1}$, $\Delta D_{min} = -0.5\ h^{-1}$, $\Delta D_{max} = 0.5\ h^{-1}$, $\Delta K_{lamin} = -2.5\ m^3.h^{-1}$ and $\Delta K_{lamax} = 2.5\ m^3.h^{-1}$

Linearizing and discretizing the system around the equilibrium point computed from the non linear equations leads to the new variables (x, u, y) that are now deviation variables. The equilibrium point is given by

$$\bar{X} = [122.7342 \quad 49.4714 \quad 196.3750 \quad 6.8300]^T \tag{5.46}$$

$$\bar{U} = [0.06 \quad 1.35] \tag{5.47}$$

The linearized system around this equilibrium point leads to the following discrete linear system :

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Cx(k) \end{cases} \tag{5.48}$$

Where,

$$A = \begin{bmatrix} 0.7685 & 0.1551 & 0.0576 & 0.1273 \\ -0.1438 & 0.4137 & -0.0859 & -0.0131 \\ -0.0109 & -0.0175 & 0.0026 & -0.0018 \\ 0.3396 & 0.0377 & 0.0253 & 0.8335 \end{bmatrix} \tag{5.49}$$

$$B = \begin{bmatrix} -250.0774 & 0.9268 \\ 398.0189 & -1.4129 \\ -13.8515 & 2.0454 \\ 102.6287 & 0.1800 \end{bmatrix} \tag{5.50}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{5.51}$$

with the constraints on the control and its increment are given by :

$$\begin{cases} u_{min} \leq u \leq_{max} \\ \Delta u_{min} \leq \Delta u \leq \Delta u_{max} \end{cases} \tag{5.52}$$

with u_{min}, u_{max}, Δu_{min} and Δu_{max} are computed from (5.42) and (5.43).

For the choice of matrix H, the selected closed loop eigenvalues are $\{0.65, \quad 0.67\}$, which leads to the following matrix H:

$$H = \begin{bmatrix} 0.65 & 0 \\ 0 & 0.67 \end{bmatrix} \tag{5.53}$$

It is worth noting here that the remaining closed loop eigenvalues are 2 $(n - m)$ stables ones coming from the open loop system. Hence, solving Equation (5.36) leads to:

$$F = \begin{bmatrix} 0.0085 & 0.0029 & 0.0067 & 0.0066 \\ 0.0301 & 0.0194 & 0.3502 & 0.0178 \end{bmatrix} \tag{5.54}$$

The design parameters and matrices of the CLMPC are selected by a trial and error procedure and following the guidelines of section 5.3.3: $n_c = 4$,

$$R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.005 & 0 & 0 & 0 \\ 0 & 0.005 & 0 & 0 \\ 0 & 0 & 0.005 & 0 \\ 0 & 0 & 0 & 0.005 \end{bmatrix}$$

and the resolution of equation (5.16) leads to:

$$P_o = \begin{bmatrix} 4.1984 & -8.3031 & -0.0910 & -2.7734 \\ -8.3031 & 16.8677 & 0.2910 & 5.7131 \\ -0.0910 & 0.2910 & 0.0463 & 0.1487 \\ -2.7734 & 5.7131 & 0.1487 & 2.0548 \end{bmatrix}$$

At this step, one has to check that all required conditions (5.39) are fulfilled in order to have a stable controller. In this case, both conditions are satisfied and the asymptotic stability is guaranteed with the obtained control law.

5.6.3 Simulation Results

Simulation results are given in $Figs.5.6 - 5.9$, comparing the proposed CLMPC ($u = Fx + c$) and an existing state feedback control law ($u = Fx$) computed in the some way as in [**?**].

$Fig.5.6$ illustrate the evolution of the output variables and their corresponding set points (equilibrium point). It can be seen the ability of the controller to track the desired values of the substrate and the dissolved oxygen after a short transient response. The evolution of the biomass and recycled biomass are shown in $Fig.5.7$. The evolution of the control variables which are the dilution rate and the air flow rate is shown in $Fig.5.8$. Finally, the $Fig.5.9$ shows the evolutions of the parameter **c**.

Table (5.1) presents the integral of the squared error (ISE) of the outputs, and it can be seen that the $CLMPC$ gives better ISE for both outputs.

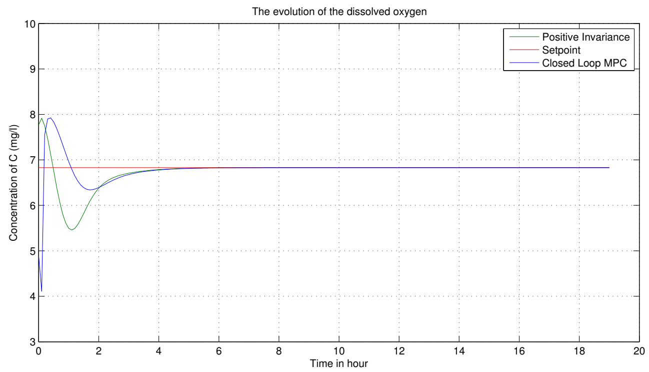
As general remarks asymptotic stability is obtained, all constraints are respected.

Table 5.1: Integral of the squared error (ISE)

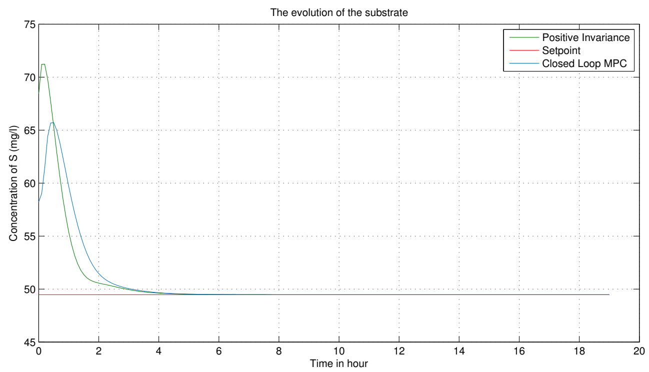
Outputs	$u(k) = Fx(k)$	$u(k) = Fx(k) + c(k)$
S (mg/l)	19.5055	16.2661
C (mg/l)	1.8419	1.7192

5.7 Conclusion

This chapter presents a new methodology to design a CLMPC, providing a simple solution that ensures stability and respects non-symmetrical constraints on control magnitudes and moves. The positive invariance theory has been used here, particularly polyhedral invariant sets, since it has been shown to be very efficient to handle constrained control. The proposed algorithm takes advantage of the design of matrix F to increase the degrees of freedom, in contrast to other $CLMPC$ approaches. For the controller design, necessary and sufficient conditions for asymptotic stability at the origin have been developed, for linear systems and a state feedback control law, while respecting constraints on both control magnitudes and its increments.



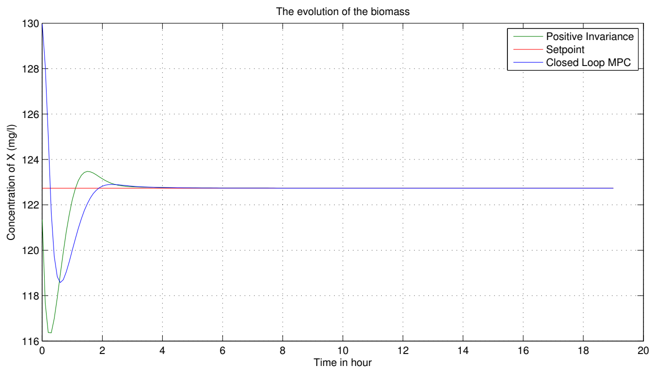
a):Evolution of C



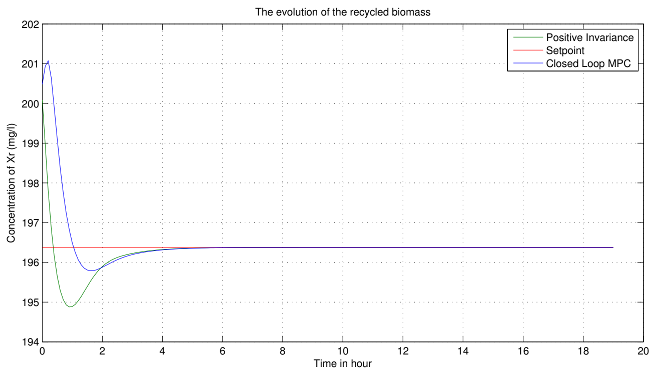
b):Evolution of S

Figure 5.7: The evolutions of the dissolved oxygen and the substrate concentrations.

The proposed methodology has been successfully applied to the activated sludge process in a wastewater treatment plant (WWTP), forcing the substrate concentration (organic matter) in the effluent and the dissolved oxygen concentration in the biological reactor to track a given set point. Simulation results show that integral squared error of the substrate in the effluent is reduced in 15% with respect to the other technique presented in the results, which is a significant improvement in effluent quality. The methodology of this work is general and can be easily extended to other applications.

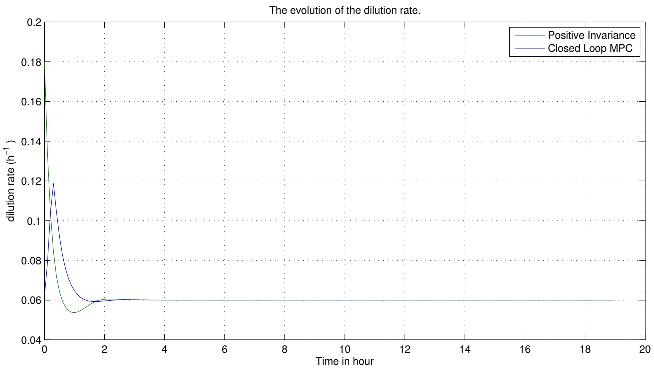


a):Evolution of X

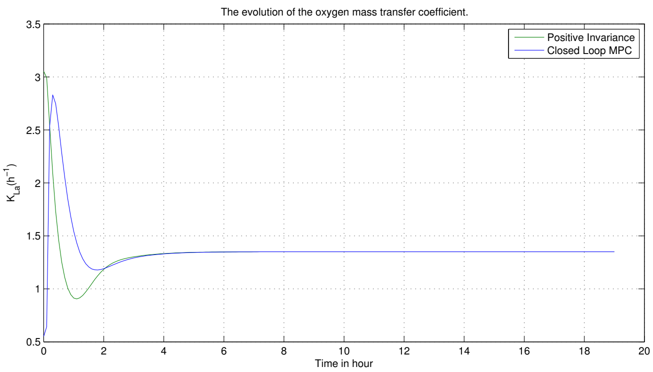


b):Evolution of X_r

Figure 5.8: The evolutions of the biomass and the recycled biomass concentrations.

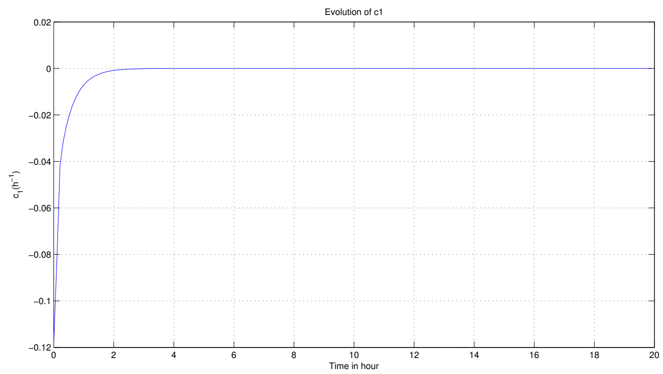


a):Evolution of D

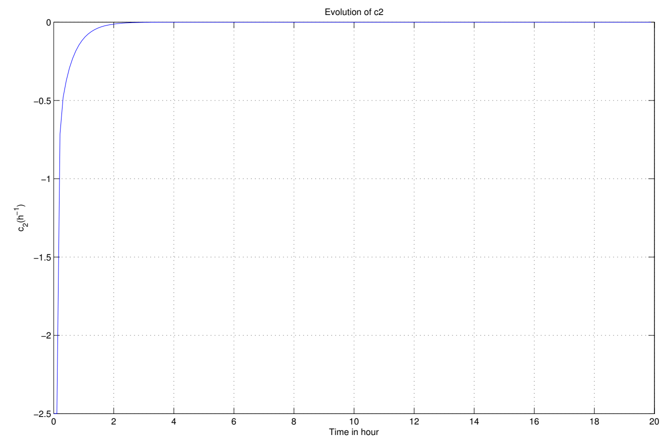


b):Evolution of K_{La}

Figure 5.9: The evolutions of the dilution rate and the oxygen mass transfer coefficient.



a):Evolution of c_1



b):Evolution of c_2

Figure 5.10: The evolutions of the parameter c.

Part II

CONTROL STRATEGIES BASED ON DYNAMIC REAL TIME OPTIMIZATION

6

One Layer Economic Nonlinear Closed-Loop Predictive Control

6.1 Introduction

The main scope of this chapter is the proposal of a new single layer Nonlinear Economic Closed-Loop Generalized Predictive Control ($NECLGPC$) based on an economic nonlinear GPC, as an efficient advanced control technique for improving economics in the operation of nonlinear plants. It is well known that closed loop predictive control procedure is an effective strategy and has been exploited to decrease computational demand of solving optimization MPC problems. The proposed approach, in contrast to classic closed loop MPC schemes, where the terminal control law is computed offline by solving a linear quadratic regulator problem [65, 66, 67], computes analytically the terminal control law online by solving an unconstrained Nonlinear Generalized Predictive Control ($NGPC$) minimizing a cost function constituted by tracking errors and economic costs. In order to obtain an analytical solution of this non linear optimization problem two considerations have been made here. Firstly, the prediction model consisting of a nonlinear phenomenological model of the plant is written in the extended linearization form or state dependent coefficient form, which actually allows having nonlinear model expressed with linear structure and state dependent matrices. Secondly, instead of including the nonlinear economic cost in the objective function, an approximation of the reduced gradient of the economic function is used. In this way the problem becomes a quadratic one, and can be solved analytically at each sampling time as in the linear case, to obtain the terminal control law to be used within the closed loop MPC scheme.

In the present work the $NECLGPC$ is used as an efficient advanced control technique for improving economics in the operation of the Nitrogen Removal of WWTP. As it is well known, this is an interesting case study because these types of plants need to operate efficiently in order to meet strict environmental regulations with minimum costs.

6.2 Problem statment

Consider the time varying system that is obtained by discretizing the continuous model of the process using the Euler integration method and rearranged into the state-dependent coefficient form [68] as:

$$\begin{cases} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= C(k)x(k) \end{cases} \tag{6.1}$$

Where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^p$ and $u(k) \in \mathbb{R}^m$ are the state, the output and the input vectors respectively at the k^{th} sampling instant.

The general formulation of the problem ($Eq.6.2 - 6.5$) consists of the optimization of a cost function that represents the control and economic objectives, subject to a set of constraints. The objective function includes the penalization of control error, the penalization of control efforts and a term (f_{eco}) that accounts for the economic objectives:

$$\begin{aligned} \min_{u(k)} = \min[&\sum_{i=1}^{n_y} ||w_1(r(k+i|k)) - y(k+i|k)||_2^2 \\ &+ \sum_{i=0}^{n_u} ||w_2 \Delta u(k+i|k)||_2^2 \\ &+ w_3 f_{eco}(u(k+n_u-1|k), \cdots, u(k|k), y(k+p|k))] \end{aligned} \tag{6.2}$$

subject to

$$u_{min} \leqslant u(k+i|k) \leqslant u_{max}, \quad i = 0, \cdots, n_u - 1 \tag{6.3}$$

$$y_{min} \leqslant y(k+i|k) \leqslant y_{max}, \quad i = 1, \cdots, n_y \tag{6.4}$$

$$\Delta u_{min} \leqslant \Delta u(k+i|k) \leqslant \Delta u_{max}, \quad i = 0, \cdots, n_u - 1 \tag{6.5}$$

Where n_y and n_u are the output and input horizon, respectively; $u(k+i|k)$ is the control input computed at time k to be applied at time step $k+i$; $y(k+i|k)$ is the output prediction at time step $k+i$; r is the desired value of the output; $\Delta u(k+i|k) = u(k+i|k) - u(k+i-1|k)$; w_1, w_2 and w_3 are positive definite matrices. Note that the different terms of the cost function must be weighted such that the economic criterion and the dynamic compensation of the output error have a similar influence on the values of the overall cost.

The control strategy proposed in this chapter is described schematically in $Fig.6.1$, This controller is achieved by using a new closed loop nonlinear predictive control paradigm that combines an unconstrained economic nonlinear Generalized Predictive Control law $F(k)$ with the parameterization $c(k)$, associated with the closed loop paradigm that allows taking into account the process constraints and improving the performance of the controller.

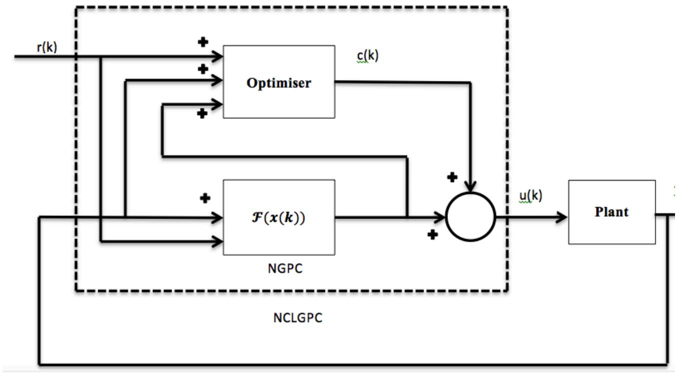


Figure 6.1: Control loop with the closed-loop paradigm

Some specific characteristics of this control strategy are:

- The optimizer shown in the control scheme ($Fig.$6.1) constitutes the economic nonlinear closed loop paradigm. The predicted control moves are centered around an unconstrained stabilizing control law, $u(k) = F(x(k))$, over the whole prediction horizon, and some additive degrees of freedom, $c(k)$, are added over a finite horizon to handle constraints. The resulting control $u(k) = F(x(k)) + c(k)$ is applied to the plant.
- In the objective function the nonlinear economic term is replaced by its gradient making this function a quadratic one.
- The prediction model, a nonlinear phenomenological model of the plant, is written as a state dependent coefficient model, also called extended linearization, which consists of factorizing the nonlinear system in a linear structure with state dependent matrices.
- The above assumptions allow us to design an economic unconstrained nonlinear GPC analytically, and its stabilizing control law, $u(k) = F(x(k))$, and to state the $NLCLGPC$ problem as a QP problem each sampling time.

6.3 Controller Design

In this section, the CLMPC that is the basis of the one layer economic controller proposed is presented, starting with the open loop MPC, the predictions using state dependent coefficient matrices and the optimization.

The predictions are obtained using a discrete time varying prediction model of the process along the prediction horizon n_y of the form:

$$x(k+i+1|k) = A(k+i|k)x(k+i|k)+B(k+i|k)u(k+i|k), \quad i = 0, 1, \cdots, n_y-1 \tag{6.6}$$

With initial condition established by:

$$x(k|k) = x(k) \tag{6.7}$$

The predicted input sequences are often stacked into the matrices $\underline{u}$ defined by:

$$\underline{u} = \begin{bmatrix} u(k|k) \\ u(k+1|k) \\ \vdots \\ u(k+n_u-1|k) \end{bmatrix} \tag{6.8}$$

Clearly $J(k)$ is a function of $u(k)$, and the optimal input sequence for the problem minimizing $J(k)$ is denoted $\underline{u}^*(k)$:

$$\begin{gathered} \underline{u}^* = arg \min_{\underline{u}(k)} J(k) \\ \text{Subject to(6.3)}, \text{(6.4)and(6.5)} \end{gathered} \tag{6.9}$$

To improve the numerical conditioning of the optimization and the plant performance, a closed loop MPC has been considered by defining the predicted input sequence as:

$$\underline{u} = \begin{bmatrix} F(k)x(k) + c(k) \\ F(k+1)x(k+1) + c(k+1) \\ \vdots \\ F(k+n_u-1)x(k+n_u-1) + c(k+n_u-1) \\ F(k+n_u)x(k+n_u) \\ \vdots \\ F(k+n_u+n_y)\Phi(k+n_u+n_y-1)\cdots\Phi(k+n_u)x(k+n_u) \end{bmatrix} \tag{6.10}$$

Where, $F(k) \in \mathbb{R}^{m\times n}$ is a nonlinear stabilizing state feedback and $\Phi(k) = A(k) + B(k)F(k)$.

With the new predictions model:

$$\begin{aligned} x(k+i|k) &= A(k+i-1|k)x(k+i-1|k) + B(k+i-1|k)u(k+i-1|k); \\ u(k+i|k) &= F(k+i|k)x(k+i|k) + c(k+i|k), \quad i = 0, \cdots, n_u - 1. \\ u(k+i|k) &= F(k+i|k)x(k+i|k), \quad i \geqslant n_u. \end{aligned}$$

The system model becomes

$$\begin{cases} x(k+1) &= (A(k)+B(k)F(k))x(k)+B(k)c(k) \\ y(k) &= C(k)x(k) \end{cases} \tag{6.11}$$

Where $c(k) \in \mathbb{R}^m$ is the new manipulated input.
Thus the problem to be minimized at each sampling time is:

$$\underline{c}^* = arg \min_{\underline{c}(k)} J(k)$$

subjet to

$$\begin{gathered} \text{Constraints}(6.3),(6.4)\text{and}(6.5) \\ x(k+i+1|k) = \Phi(k+i|k)x(k+i|k) + B(k+i|k)c(k+i|k) \\ \text{for} \quad i = 0, \cdots, n_{u-1}. \\ x(k+i+1|k) = \Phi(k+i|k)x(k+1|k), \quad \text{for} \quad i \geqslant n_u \end{gathered} \tag{6.12}$$

Where, $\underline{c}(k) = [c^T(k|k) \quad \cdots \quad c^T(k+n_u-1|k)]^T$ and $\underline{c}^*(k)$ is the first element of $\underline{c}(k)$.

The closed loop nonlinear *GPC* controller is implemented in a moving horizon framework. At current time step k, the plant state $x(k)$ is used as the initial condition and the economic optimization problem is solved on a horizon n_y, however, only the first calculated control action is implemented ($u(k) = F(k)x(k) + c^*(k)$). At the next time step $k+1$, we move the time frame one step ahead and the problem is solved with the new plant state $x(k+1)$ as the initial condition.

In the next section, the procedure for obtaining the controller is detailed. First, the analytical solution of the unconstrained economic *NLGPC* law is computed through the modification of the economic function, later, this law is used to predict the outputs over a prediction horizon with a Nonlinear Model Predictive Control.

With the aim to integrate RTO with NGPC in one single layer, the inclusion of the gradient of the economic function as an additional term in the cost function is proposed. This approach incorporates the economic objective into the NGPC controller such that the RTO and NGPC are solved in a single optimization routine.

6.3.1 The Nonlinear GPC with Economic Objective

The objective of this section is to design a nonlinear *GPC* controller that directly accounts for economic objectives. This is achieved designing a one-layer $RTO-GPC$ controller. The typical nonlinear *GPC* problem is the optimization of a nonlinear quadratic function calculated using a nonlinear steady state process model. In the proposed strategy, due to the presence of f_{eco}, the objective function of the economic ($Eq.6.2$) is not a quadratic function of the manipulated variables of the optimization problem that defines the controller. Thus, the control problem turns into an NLP, which may result difficult to resolve.

Then, assuming that the vector of the control action is changed to $u+\Delta u$, the first order approximation of the gradient of the economic function

$$F_{eco} = f_{eco}(u, \hat{y}) \tag{6.13}$$

can be represented as follows:

$$\xi_{u+\Delta u} = D + G\Delta\bar{u} \tag{6.14}$$

$$D = \frac{\partial F_{eco}}{\partial y} K_P + \frac{\partial F_{eco}}{\partial u} \tag{6.15}$$

$$G = K_P^T \frac{\partial^2 F_{eco}}{\partial y^2} K_P + \frac{\partial F_{eco}}{\partial y} (\frac{\partial^2 F_{eco}}{\partial y \partial u}) + \frac{\partial^2 F_{eco}}{\partial u^2} \tag{6.16}$$

Where $K_P = \frac{\partial y}{\partial u}$ corresponds to the process gain.

The computation of D and G involve the rigorous predicted steady-state model as shown in De Souza et al. (2010).

In the equation (6.14), $\Delta\bar{u} = u(k+m-1|k) - u(k-1|k)$ is the total move of the input vector, D is the gradient vector at the present time and G is the Hessian of the economic function with respect to the inputs. The gradient vector $\xi^T_{(u+\Delta u)}$ can be considered as a deviation vector, which is equivalent to considering that the gradient of the economic function is zero at the optimum. So, if bringing this vector to zero is considered as one of the controller objectives, the set of error equations represented in (6.14) can be included in the optimization problem. Thus, f_{eco} can be approximated by a quadratic function as $f_{eco} = \xi^T_{(u+\Delta u)}\xi_{(u+\Delta u)}$.

Remark 6.1 *In the unconstrained economic optimization, the operating point where the gradient ξ is equal to zero corresponds to a local maximum (when $G < 0$) or local minimum (when $G > 0$) of the economic function. However, when the constraints of the control problem are active, the optimum corresponds to the point where the reduced gradient of the economic function is equal to zero. The reduced gradient is obtained through the projection of the gradient on the tangent space of the active constraints.*

6.3.2 NECLGPC terminal control law $F(k)$

In this work, the terminal control law in the $NECLGPC$ is determined online by an unconstrained $NGPC$ Control with finite control and predictions horizons minimizing a cost function constituted by two important terms, the first one for set point tracking and the second for taking into account the economic cost that is approximated by means of its gradient.

The state dependent coefficient form of the model (6.1), in state space format, is stated as in the conventional *GPC* formulation, allowing for inherent integral action within the model, including the control increment as system input to the state space model. Consequently, an extra system state is incorporated.

$$\begin{cases} \chi(k+1) &= \tilde{A}(k)\chi(k) + \tilde{B}(k)\Delta u(k) \\ y(k) &= \tilde{C}(k)\chi(k) \end{cases} \tag{6.17}$$

Where:

$$\tilde{A}(k) = \begin{bmatrix} A(k) & B(k) \\ 0 & I \end{bmatrix}, \quad \tilde{B}(k) = \begin{bmatrix} B(k) \\ I \end{bmatrix},$$

$$\tilde{C} = \begin{bmatrix} C(k) & 0 \end{bmatrix}, \quad \chi(k) = \begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}$$

Considering that the future trajectory of the state of the system is known, the state-space model (6.11) matrices may be re-calculated for the future. The resulting state-space model may be seen as a time-varying linear model and the controller is designed using this model. The future trajectory of the system can be determined using this model.

The predictive control techniques address calculation of the vector of current and future controls by solving the following optimization problem:

$$\begin{aligned} \min_{u(k)} = \min(&\sum_{i=1}^{n_y} ||w_1(r(k+i|k) - y(k+i|k))||_2^2 \\ &+ \sum_{i=0}^{n_u-1} ||w_2\Delta u(k+i-1|k)||_2^2 \\ &+ ||w_3\xi_{u+\Delta u}^T||_2^2) \end{aligned} \tag{6.18}$$

Next the following vectors containing current and future values are introduced:

$$\begin{aligned} \underline{\chi}(k) &= \begin{bmatrix} \chi^T(k) & \cdots & \chi^T(k+n_y) \end{bmatrix}^T, \\ \Delta\underline{u}(k) &= \begin{bmatrix} \Delta u^T(k) & \cdots & \Delta u^T(k+n_{u-1}) \end{bmatrix}^T, \\ \underline{y}(k) &= \begin{bmatrix} y^T(k) & \cdots & y^T(k+n_y) \end{bmatrix}^T \\ \underline{r}(k) &= \begin{bmatrix} r^T(k) & \cdots & r^T(k+n_y) \end{bmatrix}^T \end{aligned} \tag{6.19}$$

Then, the cost function (6.18) may be written in the vector form:

$$\begin{aligned} J(k) = &(\underline{r}(k) - \underline{y}(k))^T w_1(\underline{r}(k) - \underline{y}(k)) + \Delta\underline{u}^T(k) w_2 \Delta\underline{u}(k) \\ &+ (D + G\Delta\underline{u}(k))^T w_3(D + G\Delta\underline{u}(k)) \end{aligned} \tag{6.20}$$

with $w_1 = diag(w_1^1, \cdots, w_1^p)$ and $w_2 = diag(w_2^1, \cdots, w_2^m)$.

Now, it is possible to determine the future state prediction:

$$\begin{aligned}\chi(k+i) = &\left[\tilde{A}(k+i-1)\tilde{A}(k+i-2)\cdots\tilde{A}(k)\right]\chi(k)\\ &+\left[\tilde{A}(k+i-1)\tilde{A}(k+i-2)\cdots\tilde{A}(k+1)\right]\tilde{B}(k)\Delta u(k)\\ &+\left[\tilde{A}(k+i-1)\tilde{A}(k+i-2)\cdots\tilde{A}(k+2)\right]\tilde{B}(k+1)\Delta u(k+1)\\ &+\cdots\\ &+\left[\tilde{A}(k+i-1)\tilde{A}(k+i-2)\cdots\tilde{A}(k+n_u)\right]\tilde{B}(k-1\\ &+\min(i,n_u))\Delta u(k-1+\min(i,n_u)) \quad For i = 1,\cdots,n_y\end{aligned} \tag{6.21}$$

Note that to obtain the state prediction at time instance $k+i$ the knowledge of matrix predictions $\tilde{A}(k)\cdots\tilde{A}(k+j-l)$ and $\tilde{B}(k)\cdots\tilde{B}(k-l+\min(i,n_u))$ is required. The control increments after the control horizon are assumed to be zero.

Next, the following notation has been introduced:

$$\left[\prod_{i=l}^{n_u}\tilde{A}(k+i)\right] = \begin{cases}\tilde{A}(k+n_u)\tilde{A}(k+n_u-1)\cdots\tilde{A}(k+l) & if \quad l \leqslant n_u\\ I & if \quad l \geqslant n_u\end{cases}$$

Where I denotes the identity matrix of appropriate size.

Then (6.21) may represented as:

$$\begin{aligned}\chi(k+i) = &[\prod_{i=0}^{i-1}\tilde{A}(k+i)]\chi(k) + [\prod_{i=1}^{i-1}\tilde{A}(k+i)]\tilde{B}(k)\Delta u(k)\\ &+[\prod_{i=2}^{i-1}\tilde{A}(k+i)]\tilde{B}(k+1)\Delta u(k+1)\\ &+\cdots\\ &+[\prod_{i=n_u}^{i-1}\tilde{A}(k+i)]\tilde{B}(k-1+\min(i,n_u)\Delta u(k-1+\min(i,n_u)))\end{aligned} \tag{6.22}$$

Now using (6.22) the following equation for the future state predictions vector $\underline{\chi}(k)$ is obtained:

$$\underline{\chi}(k) = \Omega(k)\tilde{A}(k)\chi(k) + \Psi(k)\Delta\underline{u}(k) \tag{6.23}$$

Where

$$\Omega(k) = \left[[\prod_{i=1}^{0}\tilde{A}(k+i)]^T\cdots[\prod_{i=1}^{n_{y-1}}\tilde{A}(k+i)]^T\right] \tag{6.24}$$

$$\Psi(k) =$$

$$\begin{bmatrix} [\prod_{i=1}^{0}\tilde{A}(k+i)]\tilde{B}(k)] & 0 & \cdots & 0 \\ [\prod_{i=1}^{1}\tilde{A}(k+i)]\tilde{B}(k)] & [\prod_{i=2}^{0}\tilde{A}(k+i)]\tilde{B}(k+1) & \ddots & \cdots \\ \vdots & \vdots & \ddots & \cdots \\ [\prod_{i=1}^{n_y-1}\tilde{A}(k+i)]\tilde{B}(k) & \cdots & \cdots & [\prod_{i=n_u}^{n_y-1}\tilde{A}(k+i)]\tilde{B}(k+n_u-1) \end{bmatrix}$$

From the output equation (6.17) it is clear that

$$y(k+i) = C(k+i)\chi(k+i) \tag{6.25}$$

Combining the outputs in (6.25) and (6.23) the following relationship between vectors $\underline{x}(k)$ and $\underline{y}(k)$ is obtained:

$$\underline{y}(k) = \theta(k)\underline{\chi}(k) \tag{6.26}$$

Where $\theta(k) = diag(C(k+1), C(k+2), \cdots, C(k+n_y))$

Finally substituting in (6.26) $\underline{\chi}(k)$ by (6.23) the following equation for output prediction is obtained:

$$\underline{y}(k) = \phi(k)\tilde{A}(k)\chi(k) + S(k)\Delta\underline{u}(k) \tag{6.27}$$

where

$$\phi(k) = \theta(k)\Omega(k), \quad S(k) = \theta(k)\Psi(k)$$

Substituting $\underline{y}(k)$ in the cost function (6.20) by the equation (6.27) and performing the analytical minimization, $\Delta\underline{u}$ is obtained by deriving the cost function:

$$\begin{aligned} \Delta\underline{u} = (S^T(k)w_1S(k) + w_2 + G^Tw_3G)^{-1}[S(k)w_1(\underline{r}(k) - \phi(k)\tilde{A}(k)\chi(k)) \\ - G^Tw_3D] \end{aligned} \tag{6.28}$$

By denoting:

$$\begin{aligned} F(k) &= (S^T(k)w_1S(k) + w_2 + G^Tw_3G)^{-1}S(k)w_1\Phi(k)\tilde{A}(k) \\ d(k) &= (S^T(k)w_1S(k) + w_2 + G^Tw_3G)^{-1}[S(k)w_1\underline{r}(k) - G^Tw_3D] \end{aligned} \tag{6.29}$$

The equation (6.28) becomes:

$$\Delta\underline{u}(k) = -F(k)\chi(k) + d(k) \tag{6.30}$$

6.3.3 Closed-Loop Paradigm

The dual mode controller proposed in this work differs from others proposed in the literature by three important points. First of all, usually in the classical dual mode MPC schemes, the terminal control law defined in the terminal region is obtained offline by solving a linear quadratic regulator problem, but in this paper the terminal control law is determined online by solving an unconstrained nonlinear GPC problem as presented in the previous paragraph. Secondly, the terminal controller takes into account the economic costs by including the gradient of the economic function as an additional term in the objective function of the $NGPC$. Finally, here, even though the parameters of $NGPC$ are tuned to assure a good performance and stability if there are not constraints, the dual mode approach is adopted in order to handle them when necessary and to ameliorate the performance of the closed loop system respecting them while maintaining stability.

A common choice is $u(k) = Fx(k)+c(k)$ as in [32] where F is a unchanging feedback gain computed offline and $c(k)$ is the new manipulated variable. From results of section 6.3.2 and particularly on equation (6.30), the control parameterization proposed is based on affine function disturbances as follows, making the controller less conservative.

$$\Delta u(k) = -F(k)\chi(k) + d(k) + c(k) \tag{6.31}$$

At each step time k, we assume that the feedback $F(k)$ and $d(k)$ are constant and $c(k)$ is the new decision variable.

The degrees of freedom are the disturbance $c(k)$ as is it described in Fig6.1. It is conventional to define these as:

$$\begin{aligned} \underline{c}(k) = [c^T(k) \quad \cdots c^T(k+n_u-1)]^T; \\ c(k+i+n_u) = 0 \quad \text{for} \quad i > 0 \end{aligned} \tag{6.32}$$

That is, suppose a limited number n_u of nonzero values for $\underline{c}(k)$. After n_u the disturbances are zero and the loop acts in a linear fashion and is equivalent to mode 2 of the dual mode predictions. Then the performance index in (6.2) and constraints $(6.3, 6.4 and 6.5)$, must be formulated in function of $c(k)$.

$$\begin{aligned} \min_{u(k)} = \min(\sum_{i=1}^{n_y} ||w_1(r(k+i|k)) - y(k+i|k)||_2^2 \\ + \sum_{i=0}^{n_u-1} ||w_2 \Delta u(k+i-1|k)||_2^2 \\ + ||w_3 \xi_{u+\Delta u}^T||_2^2) \\ \text{subject to} \quad (6.3), (6.4)\text{and} \quad (6.5) \end{aligned} \tag{6.33}$$

In order to obtain the prediction equations considering the control parameterization (6.31), those equations are rewritten here:

$$\begin{aligned}\Delta u(k) &= -F(k)\chi(k) + d(k) + c(k)\\ F(k) &= (S^T(k)w_1S(k) + w_2 + G^Tw_3G)^{-1}S(k)w_1\Phi(k)\tilde{A}(k)\chi(k)\\ d(k) &= (S^T(k)w_1S(k) + w_2 + G^Tw_3G)^{-1}[S(k)w_1\underline{r}(k) - G^Tw_3D]\end{aligned} \tag{6.34}$$

The predictions with the new control parameterization are:

$$\begin{aligned}\chi(k+i+1) &= \tilde{A}(k)\chi(k+i) + \tilde{B}(k)\Delta u(k+i);\\ \Delta u(k+i) &= -F(k)\chi(k) + c_t(k+i)\end{aligned} \tag{6.35}$$

With $i = 0, \cdots, n_u - 1$.

and $c_t(k+i) = d(k) + c(k+i)$.

Eliminating the dependent variable $\Delta u(k+i)$ one makes:

$$\chi(k+i+1) = (\tilde{A}(k) - \tilde{B}(k)F(k))\chi(k+i) + \tilde{B}(k)c_t(k+i) \tag{6.36}$$

Predicting onward in time with $\Phi(k) = \tilde{A}(k) - \tilde{B}(k)F(k)$ one gets;

$$\underline{\chi}(k) = \begin{bmatrix}\Phi\\ \Phi^2\\ \Phi^3\\ \vdots\end{bmatrix}\chi(k) + \begin{bmatrix}\tilde{B}(k) & 0 & 0 & \cdots\\ \Phi\tilde{B}(k) & \tilde{B}(k) & 0 & \cdots\\ \Phi^2\tilde{B}(k) & \Phi\tilde{B}(k) & \tilde{B}(k) & \cdots\\ \vdots & \vdots & \vdots & \vdots\end{bmatrix}c_t(k) \tag{6.37}$$

with

$$\begin{aligned}\underline{\chi}(k) &= [\chi^T(k+1) \quad \chi^T(k+2) \quad \cdots \quad \chi^T(k+n_y)]^T\\ \underline{c}_t(k) &= [c_t^T(k+1) \quad c_t^T(k+2) \quad \cdots \quad c_t^T(k+n_u)]^T\end{aligned} \tag{6.38}$$

Or in more compact structure we can redact the equation (6.37) as

$$\underline{\chi}(k) = P_{cl}\chi(k) + H_c\underline{c}_t(k) \tag{6.39}$$

The related input predictions can expressed as

$$\Delta\underline{u}(k) = \begin{bmatrix}-F(k)\\ -F(k)\Phi\\ -F(k)\Phi^2\\ \vdots\end{bmatrix}\chi(k) + \begin{bmatrix}I & 0 & 0 & \cdots\\ -F(k)\tilde{B}(k) & I & 0 & \cdots\\ -F(k)\Phi\tilde{B}(k) & -F(k)\tilde{B}(k) & I & \cdots\\ \vdots & \vdots & \vdots & \vdots\end{bmatrix}c_t(k) \tag{6.40}$$

with

$$\Delta\underline{u}(k) = [\Delta u^T(k+1) \quad \Delta u^T(k+2) \quad \cdots \quad \Delta u^T(k+n_u)]^T \tag{6.41}$$

or

$$\Delta\underline{u}(k) = P_{clu}\chi(k) + H_{cu}\underline{c}_t(k) \tag{6.42}$$

The state beyond n_u steps will be denoted as

$$\chi(k+mn_u) = P_{cl2}\chi(k) + H_{c2}\underline{c}_t(k) \tag{6.43}$$

Where P_{cl2} and H_{c2} are the n_y^{th} block rows of P_{cl} and H_c respectively.

6.4 Application to WWTP

In this point, the control methodology proposed is applied to a WWTP. The $WWTP$ have to be operated efficiently, minimizing the energy and recourses consumption while meeting the strict environmental regulations. Therefore, the advanced control strategies as the $NLGPC$ proposed in this paper are a promising alternative for improving their performance and economics.

6.4.1 Process Model

This application focuses specifically on the N-Removal process, which occurs in the biological treatment of the $WWTP$. The model $M4$ that represents the N-Removal process is taken from the Benchmark Simulation Protocol (BSM1) [41]. In order to represent the N-Removal process, the BSM1 is reduced to one anoxic and one aerated reactor, as shown in Figure 6.2. The volumes of the tanks are $2000m^3$ and $3999m^3$ respectively, to make them equivalent to total volumes of the anoxic and the aerobic compartments in the $BSM1$.

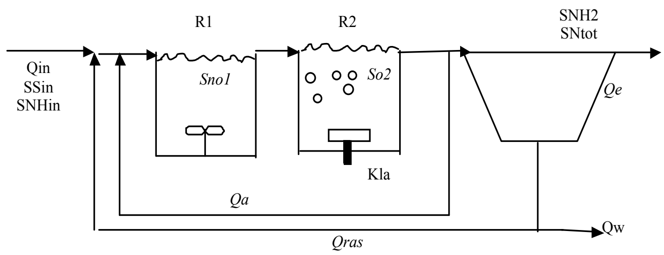


Figure 6.2: Schematic representation of the plant $M3$.

The equations of the model $M3$ representing the dynamic behavior of the plant are detailed in chapter 2.

6.4.2 Control problem

The basic control strategy proposed in the $BSM1$ is the feedback control of the dissolved oxygen S_{O2} level in the reactor by manipulation of the oxygen transfer coefficient K_{La} and the control of the nitrites and nitrates concentration in the last anoxic compartment by manipulation of the internal recycle flow rate Q_a.

In this work the $NECLGPC$ algorithm is applied for controlling the oxygen S_{O2} in the aerobic reactor and nitrate levels S_{NO1} in the anoxic reactor. A multivariable control strategy is used where the controlled variables are S_{NO1} and S_{O2}, the manipulated variables are oxygen transfer coefficient K_{La} and the internal recycle flow rate Q_a. The considered measurable disturbances are the influent flow (Q_{in}) (fig.2.14), the organic matter concentration (S_{sin}) (fig.2.15) and the ammonium concentration (S_{NHin}) (fig.2.16) in the influent.

6.4.3 Performance indices

The measures used to characterize the effluent quality and energy usage during the N-removal process are the standard performance indices recommended in the $BSM1$ platform for the evaluation of control strategies applied to WWTPs. The performance indices used in this section are the EQ, AE and PE as described in the section 2.3.

6.5 Simulations results

Different advanced control strategies based on nonlinear model predictive control are tested in the $WWTP$. The idea is to compare the proposed nonlinear GPC proposed in this chapter, including the economic term and considering the closed loop paradigm to account for restrictions with other GPC and $NMPC$ formulations.

6.5.1 Case studies

- Case 1 ($NGPC$): Unconstrained Nonlinear Generalized Predictive Control ($NGPC$) that minimize the following cost function that takes into account only the control objectives, without considering closed loop predictions:

$$\begin{aligned} J(k) = & \sum_{i=1}^{n_y} ||w_1(r(k+i|k) - y(k+i|k))||_2^2 \\ & + \sum_{i=0}^{n_u-1} ||w_2 \Delta u(k+i-1|k)||_2^2 \end{aligned} \tag{6.44}$$

- Case 2 ($NEGPC$): Unconstrained Nonlinear Economic Generalized predictive control ($NEGPC$) that minimize the following cost function which accounts for economics, but without considering closed loop predictions:

$$\begin{aligned} J(k) = & \sum_{i=1}^{n_y} ||w_1(r(k+i|k)) - y(k+i|k)||_2^2 \\ & + \sum_{i=0}^{n_u-1} ||w_2 \Delta u(k+i-1|k)||_2^2 \\ & + ||w_3 \xi_{u+\Delta u}^T||_2^2 \end{aligned} \tag{6.45}$$

- Case 3 ($NCLGPC$): The one layer optimization and control based on nonlinear closed-loop GPC presented in this work that minimize the following cost function without economics.

$$\begin{aligned} J(k) &= \sum_{i=1}^{n_y} ||w_1(r(k+i|k)) - y(k+i|k)||_2^2 \\ &+ \sum_{i=0}^{n_u-1} ||w_2 \Delta u(k+i-1|k)||_2^2 \end{aligned} \tag{6.46}$$

$$\text{subject to} \quad (6.3), (6.4) \text{and} \quad (6.5)$$

- Case 4 ($NECLGPC$): The one layer economic optimization and control based on nonlinear closed-loop GPC presented in this work that minimize the following cost function which accounts for economics.

$$\begin{aligned} J(k) &= \sum_{i=1}^{n_y} ||w_1(r(k+i|k)) - y(k+i|k)||_2^2 \\ &+ \sum_{i=0}^{n_u-1} ||w_2 \Delta u(k+i-1|k)||_2^2 \\ &+ ||w_3 \xi_{u+\Delta u}^T||_2^2 \end{aligned} \tag{6.47}$$

$$\text{subject to} \quad (6.3), (6.4) \text{and} \quad (6.5)$$

- Case 5 ($NEMPC$): Nonlinear Model predictive Control ($NMPC$) that minimizes the following cost function. Note that in this case the economics are considered including the full f_{eco} in the controller objective function

$$\begin{aligned} \min_{u(k)} = \min(& \sum_{i=1}^{n_y} ||w_1(r(k+i|k)) - y(k+i|k)||_2^2 \\ + & \sum_{i=0}^{n_u-1} ||w_2 \Delta u(k+i-1|k)||_2^2 \\ + & w_3 f_{eco}(u(k+n_u-1|k), \cdots, u(k-1|k), y(k+p|k)) \end{aligned} \tag{6.48}$$

subjet to

$$\begin{aligned} u_{min} \leqslant u(k+i|k) \leqslant u_{max}, &\quad i = 0, \cdots, n_u - 1 \\ y_{min} \leqslant u(k+i|k) \leqslant y_{max}, &\quad i = 1, \cdots, n_y \\ \Delta u_{min} \leqslant \Delta u(k+i|k) \leqslant \Delta u_{max}, &\quad i = 0, \cdots, n_u - 1 \end{aligned}$$

Those controllers are summarized in the following table 6.1:

Those control strategies are evaluated and compared by means of closed loop simulations of the process model ($M4$) implemented in Matlab. The simulations have been carried out considering the influent profile described in figures (2.11), (2.12) and (2.13) (storm weather scenario), and analogous influents for rain and dry weather described in the BSM1 specifications.

Table 6.1: Controllers characteristics.

Control	Economic function	Constraints	type of predictions	Predictions model
Case 1:	None	None	Open loop	State dependent coefficient
Case 2:	Quadratic Gradient based	None	open loop	State dependent coefficient
Case 3:	None	Yes	Closed loop	State dependent coefficient
Case 4:	Quadratic Gradient based	Yes	Closed loop	State dependent coefficient
Case 5:	Full nonlinear function	Yes	open loop	Nonlinear phenomenological model

6.5.2 Tuning parameters and operating conditions

The performance of the plant strongly depends on the selected controller set points due to the plant nonlinearities. The set point selected for the performance evaluation correspond to the economically optimal steady state condition found considering the average values of the inputs in one operating period. The variable DO (S_{O2}) in the second tank is controlled at a set point $2.09 g/m^3$ and the variable S_{NO} in the first compartment is controlled at a set point of $1.66 g/m^3$. The influent considered has been described in figures (2.11), (2.12) and (2.13). The plant responses and the corresponding performance indices for 678 (4 weeks) operating hours are compared.

The $NECLGPC$ weights, as well as the prediction and control horizons, affect the closed loop behavior of the plant, so a proper tuning is required. In this work, the tuning has been performed evaluating the plant behavior by means of simulations. The selected tuning parameters for the controllers described in cases 1, 2, 3 and 4 are: control horizon $n_u = 2$; prediction horizon $n_y = 4$; output weight $w_1 = diag(0.155, 0.01)$; input weight $w_2 = diag(0.01, 0.01)$, the weight of the economic term $w_3 = 0.01$ and sampling period of 15 minutes.

The control variables and its rates are bounded as shown in $Eqs.(6.3, 6.4)$ and therefore, the optimization problem (6.2) is a nonlinear and constrained. The bounds for the input variables and its rate are $Q_a < 3850 m^3/d$, $Kla < 200 day^{-1}$, $-100 < \Delta Q_a < 100$ and $-24 < \Delta K_{La} < 24$.

6.5.3 Results

The controller performance evaluation includes the analysis of the temporal responses and the corresponding performance indices. The first comparison is presented in figures 6.3 – 6.6, where the $NGPC$ (Case 1) is compared to the $NEGPC$ (Case 2), for stormy weather disturbances. For both controllers, the set point tracking is particularly good for the S_{O2}, and the S_{NH} concentration satisfies the legal constraint (table 2.12). The responses are very similar, and the only remarkable difference is that for S_{O2} tracking the $NEGPC$ shows a small offset due to the incorporation of the economic term. The OCI values (table 6.2) are smaller for $NEGPC$ as expected.

Secondly, in figures $6.7 - 6.12$ a comparison of the proposed $NECLGPC$ (Case 4) with a $NCLGPC$ (case 3) is presented, also for stormy weather. The responses are again very similar, only showing a small decrease of the manipulated variables for the Case 4 controller, due to the inclusion of the economic term. This is also seen in the OCI values of $table 6.2$. For these controllers the tracking for S_{NO} improves achieving a better balance between S_{O2} and S_{NO} tracking. The two manipulated variables are shown in $Figs.6.9$ and $Figs.6.10$ which indicate that suitable control signals Q_a and K_{La} drive the process to follow the set point, while satisfying the constraints ($Eqs.6.3-$ 6.5) imposed. The rest of constraints for the effluent (table 2.12) are also satisfied ($Fig.6.11$), ensuring a proper quality of the effluent. Figure 6.12 – 6.13 show the evolution of the parameters c_1 and c_2, where can be seen larger variations in c_1 due to the larger variations in Q_a.

Finally, a comparison of the $NECLGPC$ proposed with a $NEMPC$ where the economic term is the full economic function without any approximation is presented. Comparing the evolution of S_{O2} and S_{NO} concentrations, the $NEMPC$ controller (Case 5) presents better tracking for the S_{NO} than the $NECLGPC$ (Case 4) controller, but a slightly worse one for the S_{O2}, providing globally a similar performance. In spite of that, in the Case 5 controller, a main drawback is that the S_{NH} concentration does not fulfill legal regulations. Another advantage of the $NECLGPC$ is that the nonlinear internal model provides a good prediction with smaller computational effort due to its state dependent coefficient form.

In *table*6.2, a comparison of different performance indices is shown. Comparing the OCI for the different case studies, it is possible to observe that the introduction of the economic term in the $NECLGPC$ (case 2) and $NEGPC$ (case 4) improves the economics reducing the OCI index. For instance, for the storm weather influent it reduces the OCI from $1298.8 EUR/d$ (Case 1) to $1261.8 EUR/d$ (Case 2) and from $1299.9 EUR/d$ (Case 3) to $1245.8 EUR/d$ (Case 4). This is observed also for different scenarios, especially when dry weather influent profile is tested where a reduction of 16% of the OCI is achieved. Moreover, it can be seen that for the $NECLGPC$ (Case 4) the OCI values in all weather conditions are smaller than for cases 1, 2 and 3.

Table 6.2: Comparison of performance indices for case 1 to 4 controllers

Weather	Cases	Controller	AE Kwh/d	PE Kwh/d	OCI EUR/d	EQ Kg/d
-Storm:	Case 1	NGPC	889.65	409.1	1298.8	6412.5
	Case 2	NEGPC	849.23	412.57	1261.8	6422.3
	Case 3	NCLGPC	889.76	409.25	1299.9	6411.6
	Case 4	NECLGPC	833.42	412.39	1245.8	6426.8
-Rain:	Case 1	NGPC	870.05	411.93	1282	6401.2
	Case 2	NEGPC	830.13	415.13	1246.1	6410.9
	Case 3	NCLGPC	870.11	412.30	1282.4	6400.4
	Case 4	NECLGPC	816.39	414.75	1231.1	6415.7
-Dry:	Case 1	NGPC	927.57	405.76	1333.3	6566.9
	Case 2	NEGPC	885.15	408.9	1294.1	6577.6
	Case 3	NCLGPC	927.62	405.74	1333.4	6565.9
	Case 4	NECLGPC	868.7	408.9	1277.2	6583.1

In order to illustrate the efficiency of the $NECLGPC$ method proposed (case 4) for $WWTP$ control, a comparative study with a $NEMPC$ where the economic term is the full economic function without any approximation is presented, considering the same operating conditions. Comparing the evolution of S_{O2} and S_{NO} concentrations, and the corresponding manipulated variables (Figures 6.14, 6.15 and 6.16) the NEMPC controller (Case 5, Figure 6.14) presents better tracking for the S_{NO} than the $NECLGPC$ (Case 4, Figure 6.8) controller, but a slightly worse one for the S_{O2}, providing globally similar performance. However, in the $NEMPC$ (Case 5) controller, a main drawback is that the S_{NH} concentration does not fulfill legal environmental regulations (Figure 6.16), while the $NECLGPC$ (Case 4) satisfies that constraint ($S_{NH} < 4$) (Figure 6.11). Another important advantage of the $NECLGPC$ is that its nonlinear internal model provides good predictions with less computational effort due to the state dependent coefficient form. Computational times for the $NECLGPC$ are around 20 minutes, while for the NEMPC are more than 200 minutes, both for the simulations performed in this work.

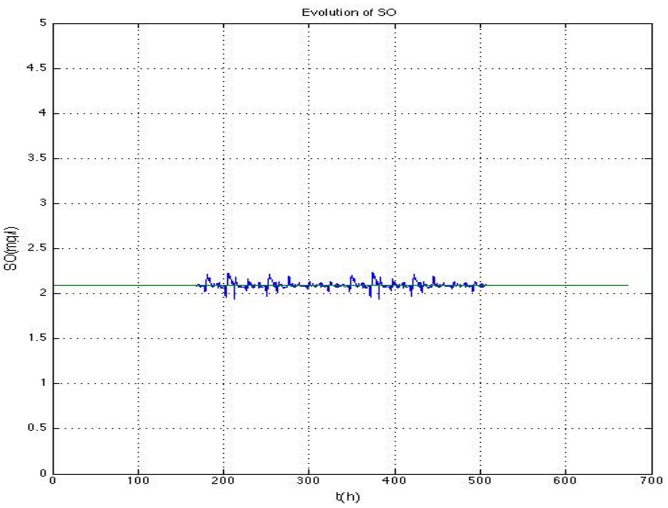
Finally, in table 6.3 a comparison of performance indices is shown, where can be seen that the $NEMPC$ controller (case 5) provides smaller operating costs than the $NECLGPC$ (case 4), as expected due to the use of the full nonlinear economic function instead of the gradient approximation, but through a much higher computational time as mentioned. As for the EQ index, for both controllers they are similar and dependent on the influent conditions.

Table 6.3: Comparison of performance indices for case 4 to 5 controllers

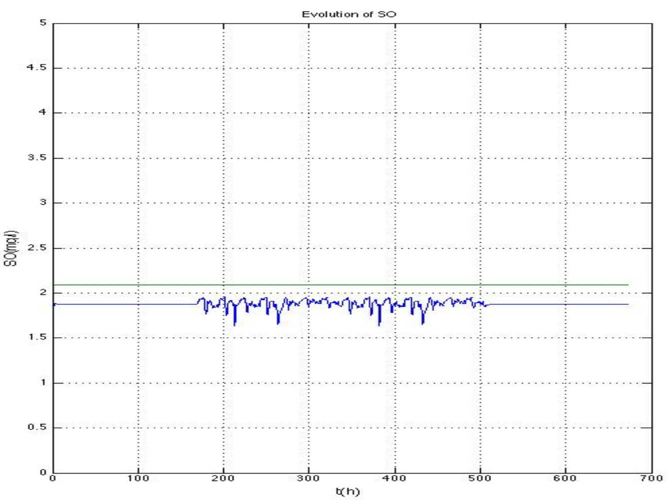
Weather	Cases	Controller	AE Kwh/d	PE Kwh/d	OCI EUR/d	EQ Kg/d
-Storm:	Case 4	NECLGPC	833.42	412.39	1245.8	6426.8
	Case 5	NEMPC	780.8	459.7	1240.5	6409.5
-Rain:	Case 4	NECLGPC	816.39	414.75	1231.1	6415.7
	Case 5	NEMPC	780.3	507.05	1287.4	6654.5
-Dry:	Case 4	NECLGPC	868.7	408.9	1277.2	6583.1
	Case 5	NEMPC	781.11	422.85	1204	6212.4

6.6 Conclusion

This study presents a nonlinear closed loop generalized predictive control scheme that uses a nonlinear model for predictions and includes the economic term in the controller cost function. The reduced gradient of the economic objective function is included as an additional term of the cost function of the nonlinear *GPC* controller. The proposed strategy allows the simultaneous optimization and control of the plant operation in one layer approach. The control law is based on direct use of the nonlinear model of the wastewater treatment processes.

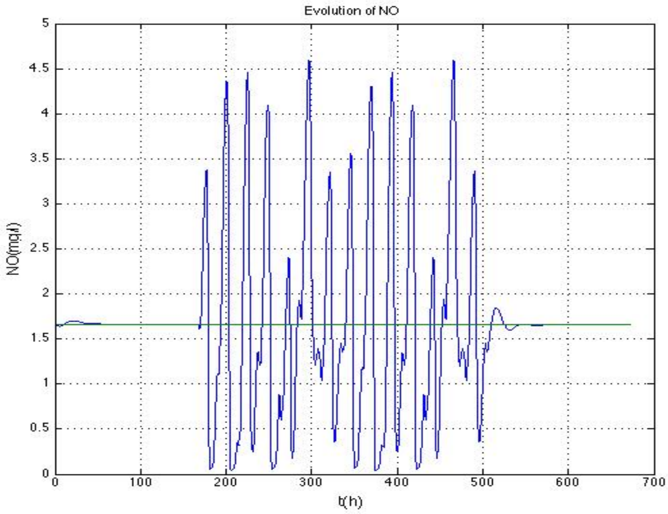


a):Evolution of S_{O2}

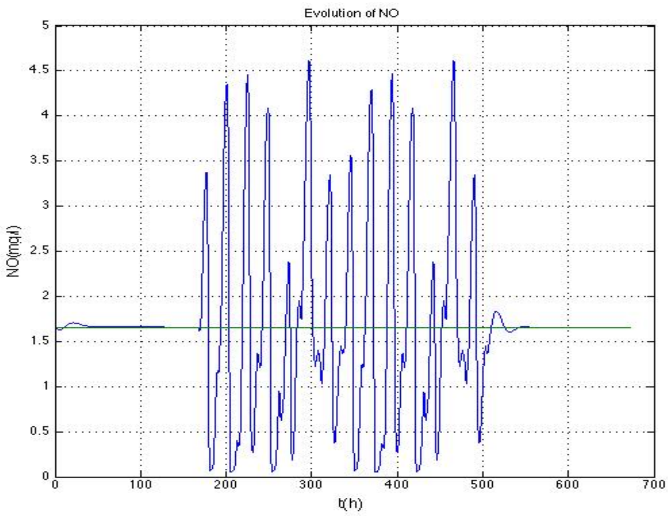


b):Evolution of S_{O2}

Figure 6.3: Responses of S_{O2} for the case 1 and case 2 controllers (from left to right).

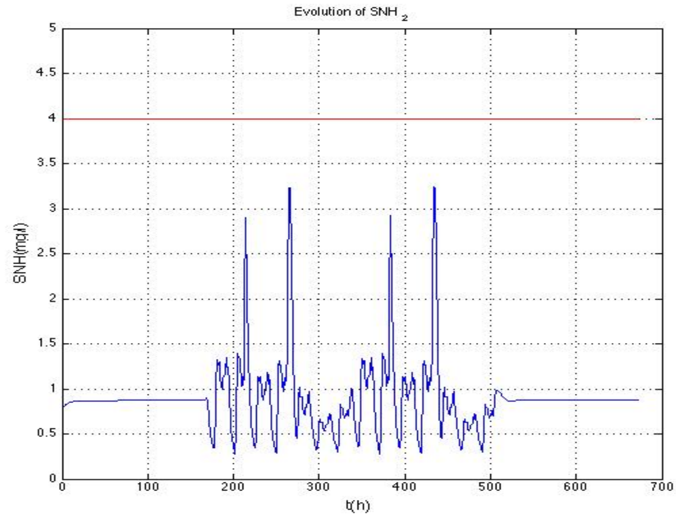


a):Evolution of S_{NO}

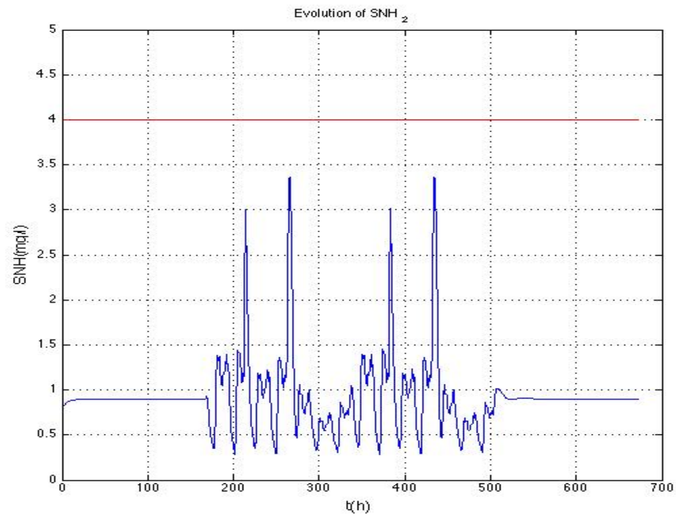


b):Evolution of S_{NO}

Figure 6.4: Responses of S_{NO} for the case 1 and case 2 controllers (from left to right)

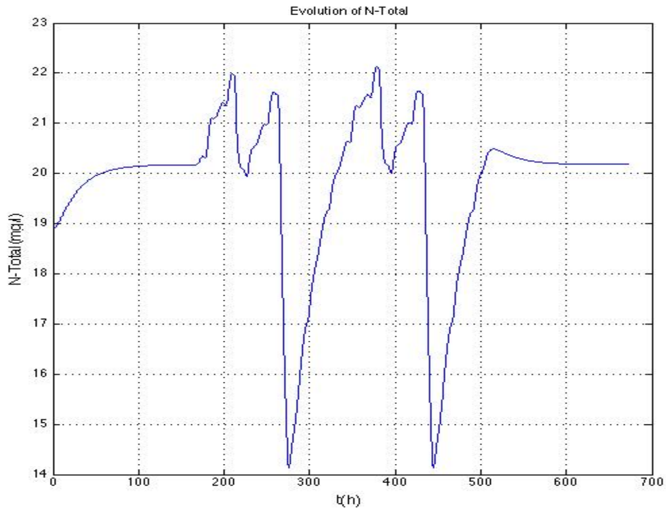


a):Evolution of S_{NH}

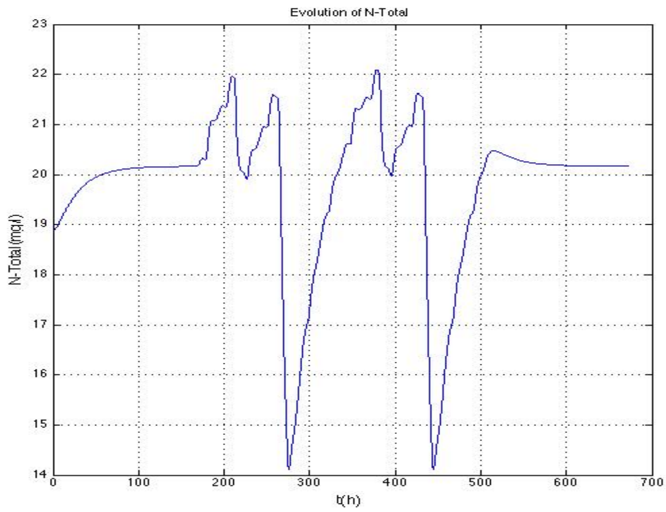


b):Evolution of S_{NH}

Figure 6.5: Responses of S_{NH} for the case 1 and case 2 controllers (from left to right)

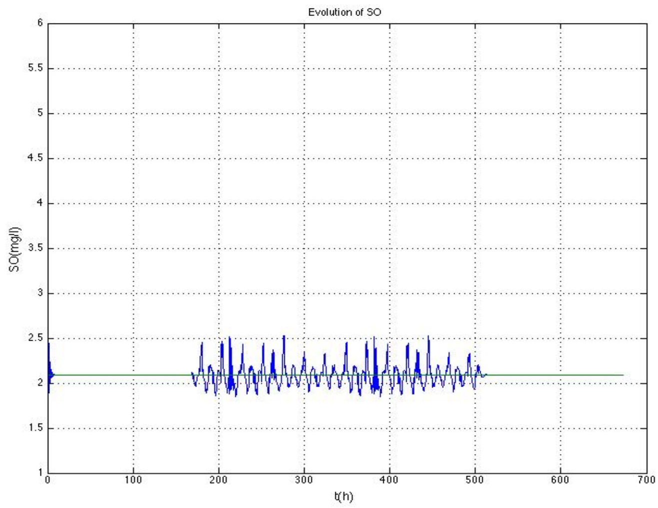


a):Evolution of N_{tot}

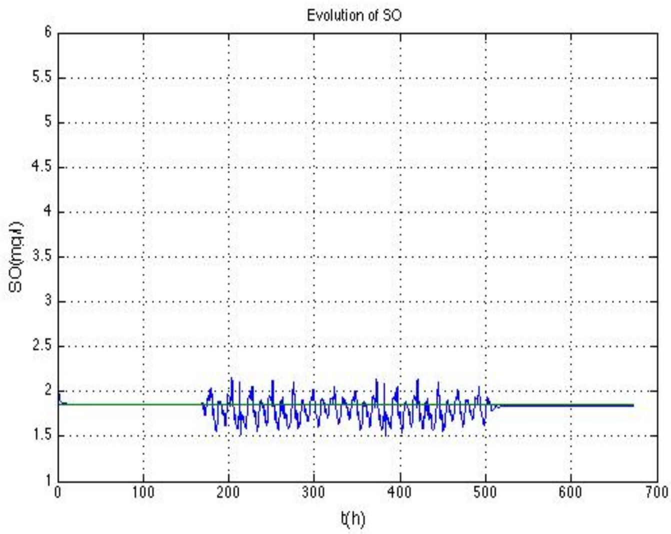


b):Evolution of N_{tot}

Figure 6.6: Responses of $N-total$ for the case 1 and case 2 controllers (from left to right)

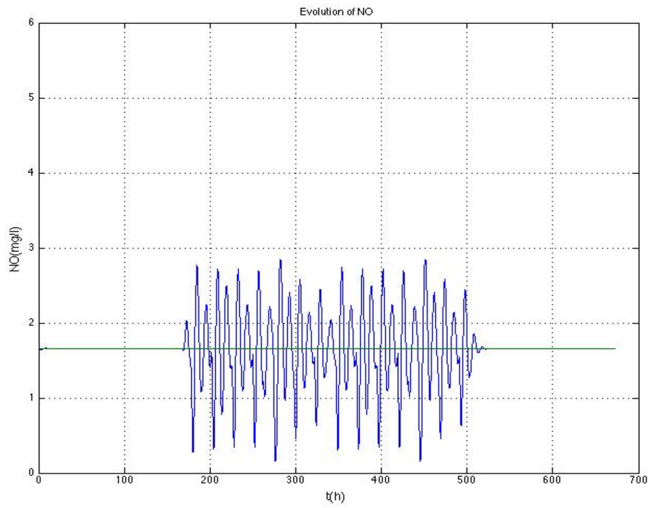


a):Evolution of S_{O2}

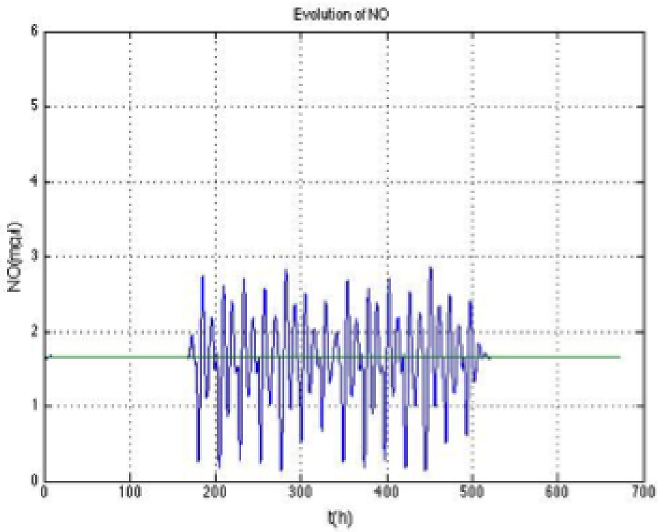


b):Evolution of S_{O2}

Figure 6.7: Responses of S_{O2} for the case 3 and case 4 controllers (from left to right).

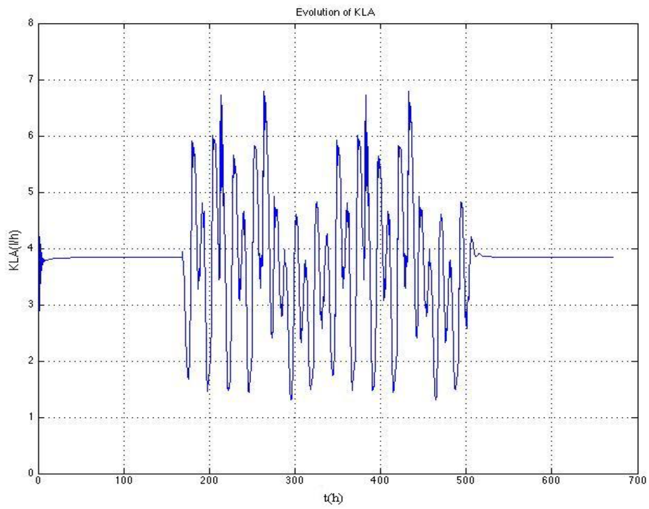


a):Evolution of S_{NO}

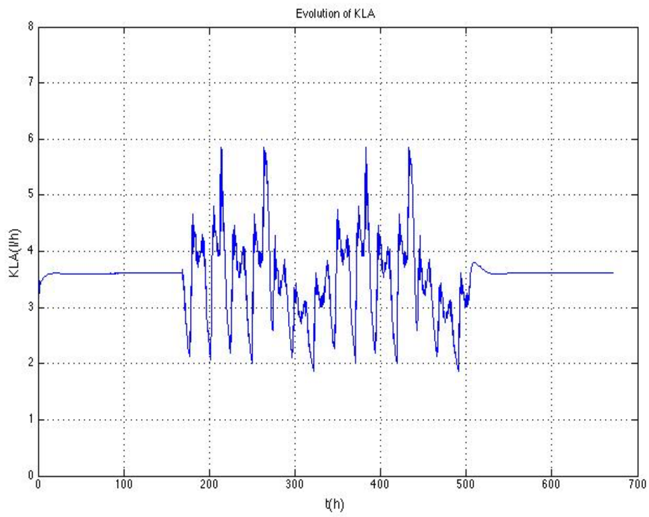


b):Evolution of S_{NO}

Figure 6.8: Responses of S_{NO} for the case 3 and case 4 controllers (from left to right)

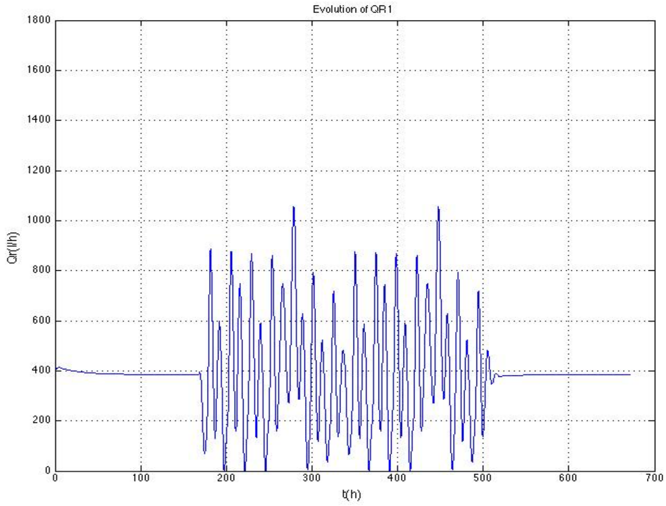


a):Evolution of K_{La}

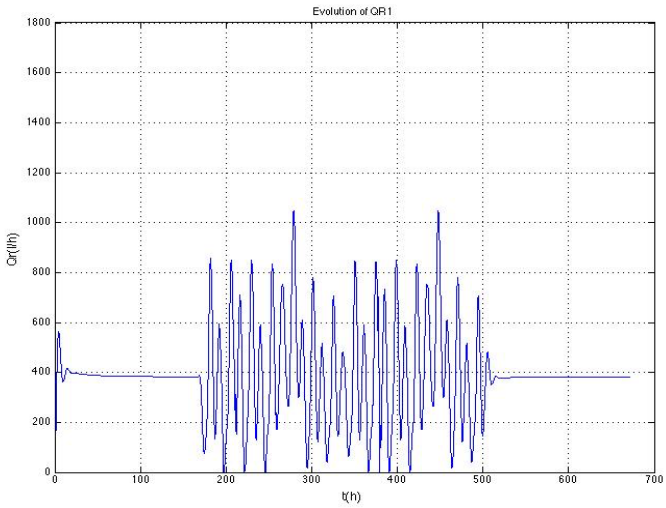


b):Evolution of K_{La}

Figure 6.9: Responses of K_{La} for the case 3 and case 4 controllers (from left to right)

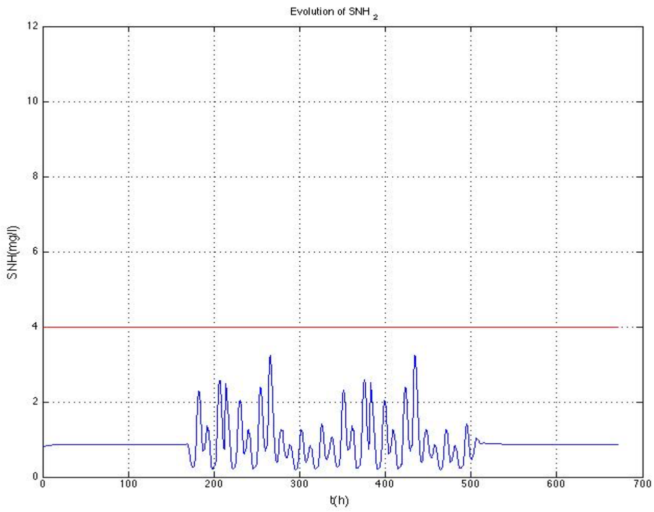


a):Evolution of Q_a

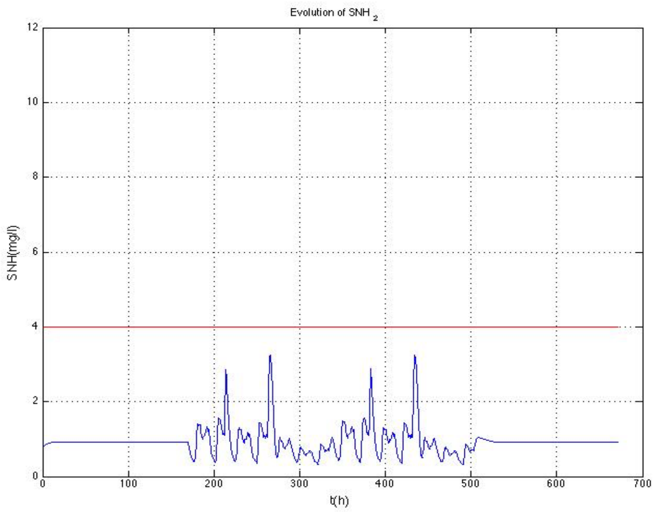


b):Evolution of Q_a

Figure 6.10: Responses of Q_a for the case 3 and case 4 controllers (from left to right)

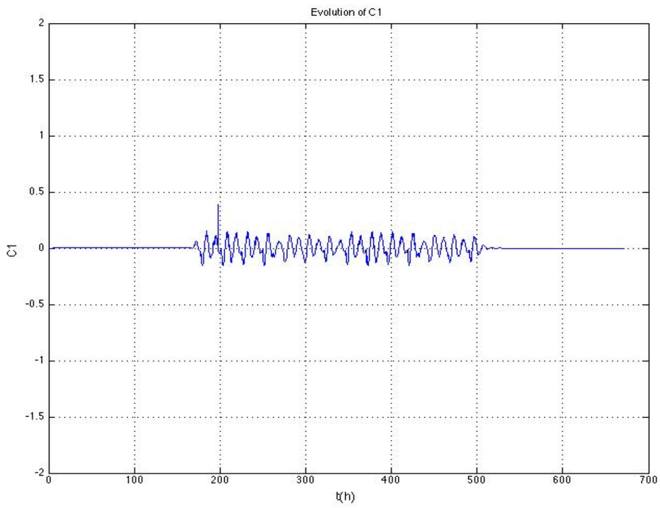


a):Evolution of S_{NH}

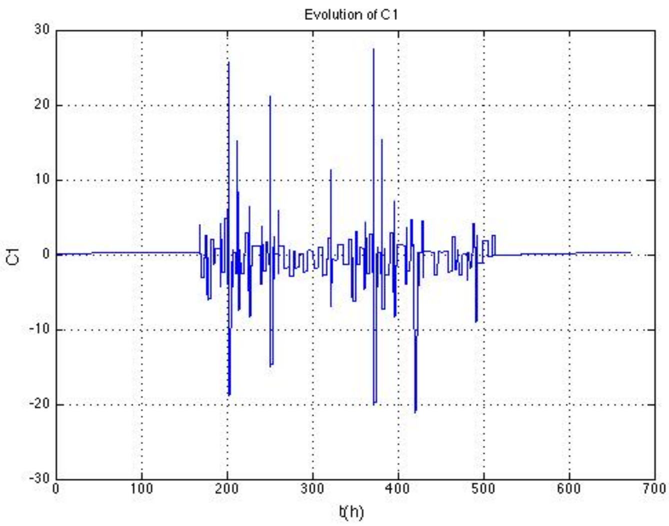


b):Evolution of S_{NH}

Figure 6.11: Responses of S_{NH} for the case 3 and case 4 controllers (from left to right).

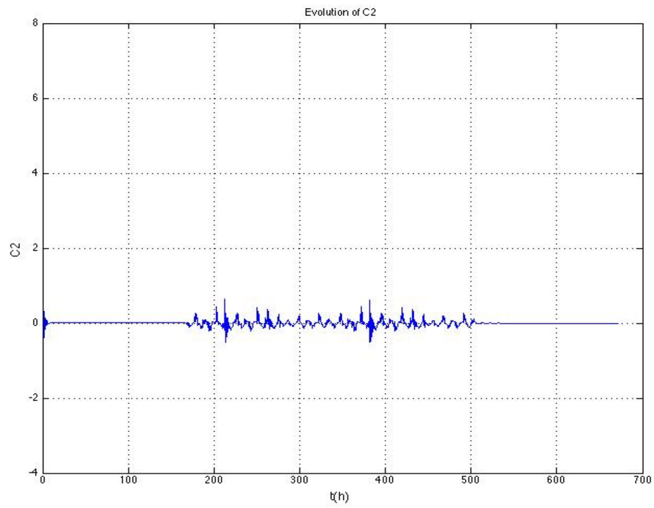


a):Evolution of c_1

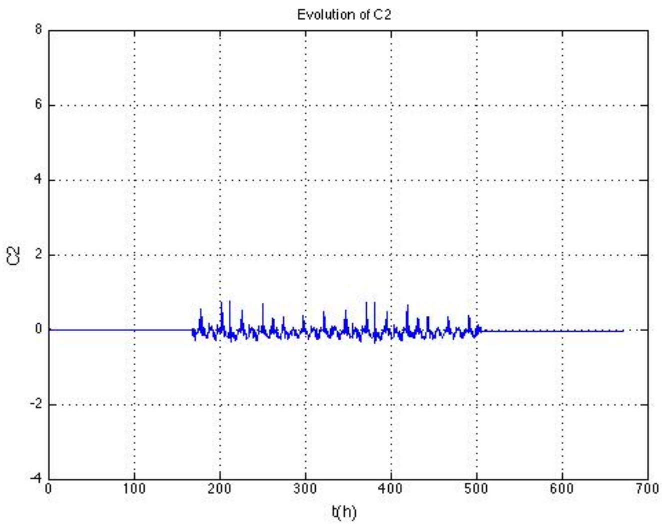


b):Evolution of c_1

Figure 6.12: Evolution of the degree of freedom c_1 for the case 3 and case 4 controllers (from left to right).

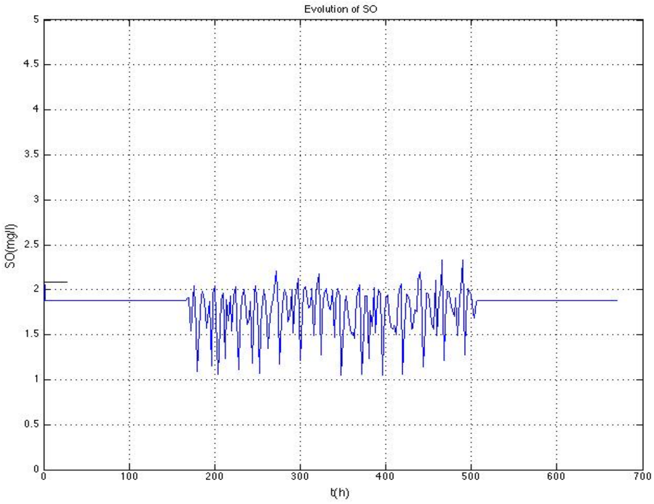


a):Evolution of c_2

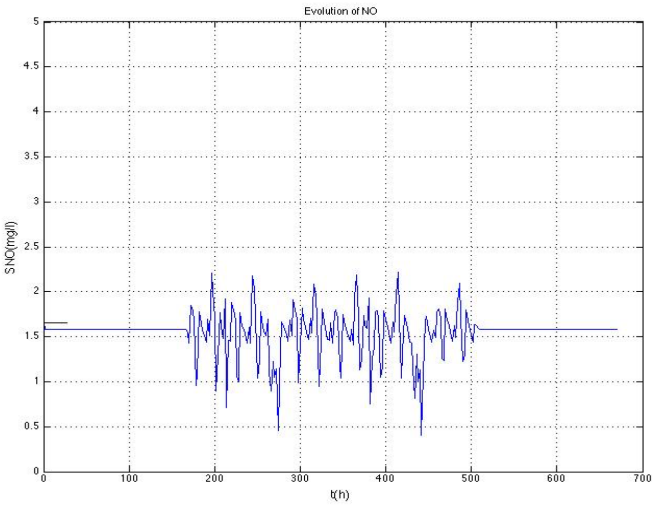


b):Evolution of c_2

Figure 6.13: Evolution of the degree of freedom c_2 for the case 3 and case 4 controllers (from left to right).

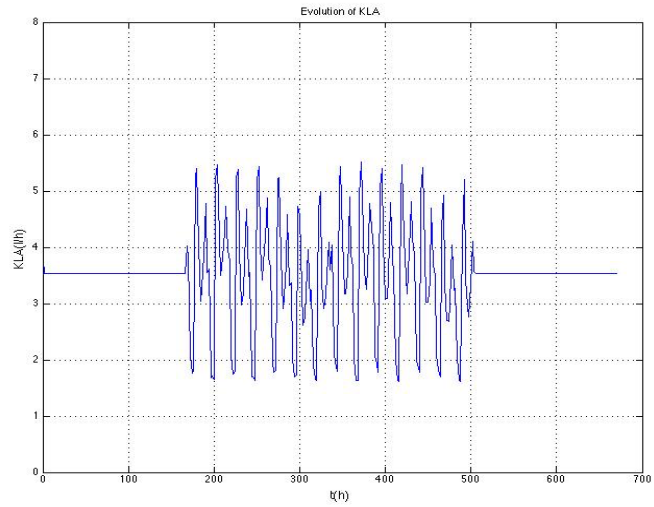


a):Evolution of S_{O2}

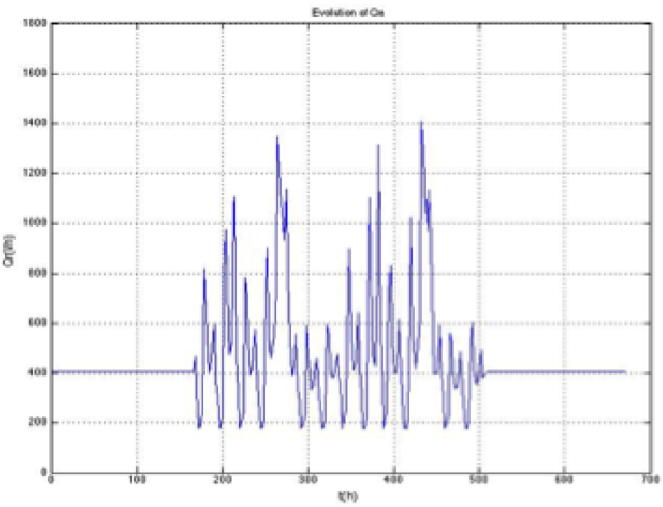


b):Evolution of S_{NO}

Figure 6.14: Responses of S_{O2} and S_{NO} for the case 5.



a):Evolution of K_{La}



b):Evolution of Q_a

Figure 6.15: Responses of K_{La} and Q_a for the case 5.

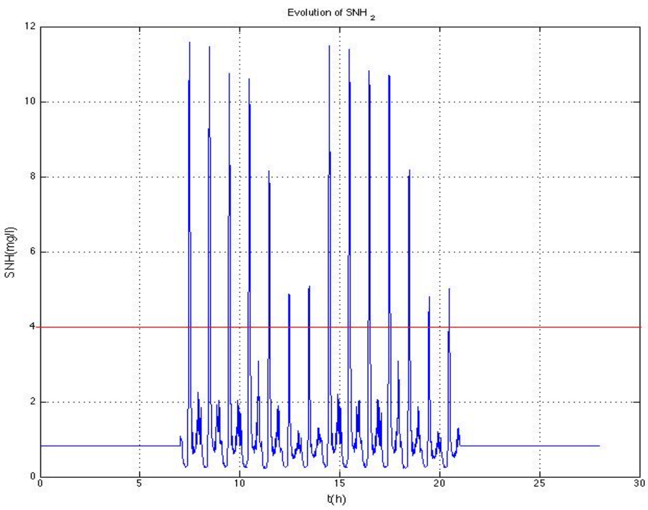


Figure 6.16: Response of S_{NH} for the case 5.

7

Integrating Dynamic Economic Optimization and Nonlinear Closed Loop MPC

7.1 Introduction

In this chapter a technique that integrates methods of dynamic economic optimization (DRTO) and real time control by including economic model predictive control and closed loop predictive control has been developed, using a two layer structure. The upper layer, which is constituted by an economical MPC, makes use of the updated state information to optimize some economic cost indices and calculate in real time the economically optimal trajectories for the process states. The lower layer uses a closed loop non-linear MPC to calculate the control actions that allow for the outputs of the process to follow the trajectories received from the upper layer. In the chapter it is also shown the theoretical demonstration proving that the deviation between the state of the closed loop system and the economically time varying trajectory provided by the upper layer is bounded, thus guaranteeing stability.

The proposed approach is based on the use of nonlinear phenomenological models to describe all the relevant process dynamics and cover a wide operating range, providing accurate predictions and guaranteeing the performance of the control systems.

7.2 Problem statment

Consider a nonlinear system represented in the state space by the following equations:

$$\dot{x}(t) = A(t)x(t) + \psi(x(t), u(t), w(t)) \tag{7.1}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in U \subset \Re^m$ is the manipulated input vector, $w(t) \in \Re^p$ is the disturbance vector.

It is also assumed that the inputs are restricted to a non-empty convex set defined as $U := \{u \in \Re^m || u(t)| \leqslant u_i^{max}, i = 1, \cdots, m\}$, $A(t) \in \mathbb{R}^{n \times n}$ is Hurwitz $\forall t \geqslant 0$, ψ is supposed to be locally Lipschitz on $\Re^n \times \Re^m \times \Re^p$ and the disturbance vector is bounded by the following inequality:

$$|w(t)| \leqslant \theta \tag{7.2}$$

where $\theta > 0$

The objective of the control methodology presented in this chapter is to solve a problem of economic dynamic optimization that provides a profile of optimal time-varying set points for a non-linear plant. More precisely, a hierarchical control in two layers is used, where in the upper layer the reference trajectories are generated by a non-linear economic MPC, satisfying with the restrictions imposed, and in the lower layer a CLMPC is used as described in the previous chapter of this thesis, in order to follow the marked references, also respecting the constraints and rejecting the existing disturbances.

The trajectory vector is denoted as $x_r(t) \in \Omega \subset \Re^n$ and the rate of change of $x_r(t)$ is bounded by

$$|\dot{x}_r(t)| \leqslant \gamma_r \tag{7.3}$$

The error is defined by the deviation between the state trajectory $x(t)$ and the reference trajectory $x_r(t)$ as

$$e(t) = x(t) - x_r(t) \tag{7.4}$$

The dynamics of the error can be studied from:

$$\begin{aligned} \dot{e}(t) &= \dot{x}(t) - \dot{x}_r(t) \\ &= A(t)e(t) + \psi(x(t), u(t), w(t)) - \psi(x_r(t), u_r(t), 0) \end{aligned} \tag{7.5}$$

In the sequel, it is shown that this error is bounded thus ensuring the stability of the control system.

7.3 Controller design

The control strategy proposed in this chapter is described schematically in $Fig.$7.1. In the upper layer, the control is achieved by using an economic MPC, while in the lower layer uses a closed loop nonlinear MPC that combines an unconstrained economic nonlinear feedback control law $F(k)$ with $c(k)$ the parameterization associated with the closed loop paradigm that allows taking into account the process constraints improving the performance of the controller.

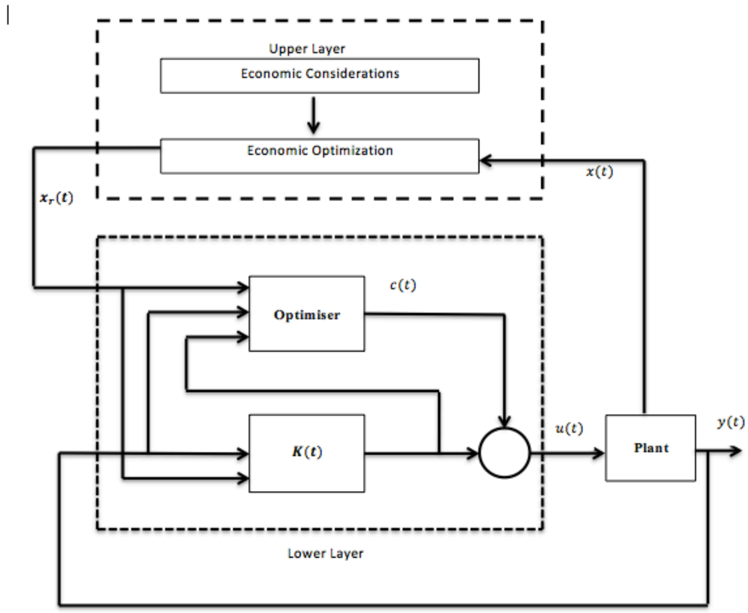


Figure 7.1: A block diagram of the proposed two layer framework

7.3.1 Upper layer problem formulation

This layer deals with the following nonlinear economic MPC optimization problem of the system of $Eq.(7.1)$:

$$\begin{aligned} &\underset{u_r}{min} \int_{t_k}^{t_{k+Ne}} f_{eco}(\tilde{x}_r(\tau), u_r(\tau), \tau) d\tau \\ &\quad \text{subject to :} \\ &\quad \dot{\tilde{x}}_r(t) = A(t)\tilde{x}_r(t) + \psi(\tilde{x}_r(t), u_r(t), 0), \\ &\quad \tilde{x}_r(t) = x(t), \\ &\quad u_r(t) \in U, \\ &\quad |\dot{\tilde{x}}_r(t)| \leqslant \gamma_r, \forall t \in [t_k, t_{k+Ne}[, \\ &\quad \tilde{x}_r(t) \in \Gamma \end{aligned} \tag{7.6}$$

where N_e is the prediction horizon of the economic MPC, $f_{eco}(\tilde{x}_r(\tau), u_r(\tau))$ is the time-dependent economic cost function, the state $\tilde{x}_r(t)$ is the predicted trajectory of the system with the manipulated input $u_r(t)$ computed by the economic MPC and $x_k(t)$ is the state measurement obtained at time t_k.

The first constraint is the nominal model of the system used to predict the future evolution of the process state. The second constraint defines the initial condition of the optimization which is the measurement of the process state at instant t_k. The third constraint presents the control limitation of all manipulated inputs. The fourth constraint limits the rate of change of the economically state trajectory. The fifth constraint ensure that the economically optimal state trajectory is maintained in the domain Γ.

In general, the function $f_{eco}(\tilde{x}_r(\tau), u_r(\tau), 0)$ is not a quadratic function of the decision variables of the stated optimization problem. Consequently, the problem becomes a nonlinear programming problem which may be difficult to solve. To overcome this problem The function f_{eco} is approximated by its gradient as in chapter 6. More precisely, the procedure is as follows. Consider a multivariable system with q outputs and m inputs, and any time step t_k, suppose that the stationary prediction of the controller output related to the present input u_r is $\hat{y}_r$. Also, consider that the economic function associated with the operation of the system to be concave function whose maximum has to be searched and this economic function can be represented as follows:

$$F = f_{eco}(u_r, \hat{y}_r) \tag{7.7}$$

Then, assuming that the vector of the control action is changed to $u_r + \delta u_r$, the first order approximation of the gradient of the economic function can be represented as follows:

$$\xi_{u_r+\delta u_r} = D + G\delta\bar{u}_r \tag{7.8}$$

The calculations of D and G are detailed in [69]. In the equation $Eq(7.8)$, $\delta\bar{u}_r = u_r(t_{k+N_e-1}) - u_r(t_{k-1})$ is the total move of the input vector. As a result of these assumptions f_{eco} can be approximated by a quadratic function as $f_{eco} = \xi^T_{u+\delta u}\xi_{u+\delta u}$ and the optimization problem 7.6 becomes:

$$\begin{aligned}
&\underset{u_r}{min} \int_{t_k}^{t_{k+Ne}} \xi^T_{u(\tau)+\delta u(\tau)}\xi_{u(\tau)+\delta u(\tau)} d\tau \\
&\quad \text{subject to :} \\
&\quad \dot{\tilde{x}}_r(t) = A(t)\tilde{x}_r(t) + \psi(\tilde{x}_r(t), u_r(t), 0), \\
&\quad \tilde{x}_r(t) = x(t), \\
&\quad u_r(t) \in U, \\
&\quad |\dot{\tilde{x}}_r(t)| \leqslant \gamma_r, \forall t \in [t_k, t_{k+Ne}[, \\
&\quad \tilde{x}_r(t) \in \Gamma
\end{aligned} \tag{7.9}$$

7.3.2 Lower layer problem formulation

This layer uses a nonlinear closed loop MPC to force the process state to track the economically optimal state trajectory $x^*_r(t)$ obtained by the controller of the upper layer solving the nominal model of $Eq.$(7.1) with manipulated input $u^*_r(t)$.

The nonlinear closed loop MPC at t_j is formulated as:

$$\begin{aligned} \min_{c} \int_{t_j}^{t_{j+N}} & (|\tilde{e}(\tau)|_Q + |u(\tau) - u_r^*(\tau)|_R) d\tau \\ & \text{subject to :} \\ & \dot{\tilde{e}}(t) = A(t)\tilde{e}(t) + \psi(x(t), u(t), w(t)) \\ & -\psi(x_r^*(t), u_r^*(t), 0), \\ & u(t) = K(t)x(t) + c(t), \\ & u(t) \in U, \\ & e(t) = x(t) - x_r^*(t) \end{aligned} \tag{7.10}$$

where N is the prediction horizon of CLMPC, $\tilde{e}(t)$ is the predicted deviation between the state trajectory predicted by the model $Eq.(7.1)$ with the manipulated input$u(t)$ and the economically optimal state trajectory $x_r^*(t)$. The optimal solution of the optimization $Eq.(7.10)$ is denoted by $u^*(t)$ defined for $t \in [t_j, t_{j+N}[$.

In the optimization problem of $Eq.(7.10)$, the first constraint is the nominal deviation system of $Eq.(7.5)$. The second and third constraint define the structure of the closed loop controller where $K(t)$ is terminal control law and the limitations of the manipulated variables $u(t)$ respectively. The last constraint presents the initial condition of the optimization of $Eq.(7.10)$.

The terminal region is obtained offline, here the terminal control law $K(t)$ is calculated each iteration as in the previous chapter and [3].

The implementation strategy of the proposed two layer dynamic optimization can be summarized as follows:

1. At time t_k the upper layer nonlinear MPC with a prediction horizon N_e receives the system state $x(t_k)$ from the process.

2. The controller of $Eq.(7.6)$ computes the economically optimal state trajectory $x_r^*(t)$.

3. The terminal control law $K(t)$ is computed.

4. The solution of the optimization of $Eq.(7.9)$ denoted by $u^*(t) = K(t)x(t)+ c^*(t)$ is calculated to track the economically state trajectory computed in step 2.

5. Go to step 1, $t_k = t_{k+t'}$

Remark 7.1 *t' is the operating period that can be chosen based on the frequency which the process economic information is updated.*

The control is applied in a sample-and-hold fashion for $t \in [t_k, t_k + t' + \Delta N[$, where t_k is the beginning of the operating period, Δ is the operating period, and ΔN is the prediction horizon of the closed loop MPC. We suppose that the manipulated inputs are recomputed synchronously every Δ.

7.4 Stability

7.4.1 Definitions and Assumptions

We need to make certain definitions and assumptions about the system of $Eq.(7.5)$ to guarantee that the time-varying trajectory $x_r(t)$ can be tracked. We assume that the nominal system of the $Eq.(7.1)$ is stabilizable at each fixed $x_r \in \Gamma$.

Scalar comparison functions, known as class $\mathcal{K}$, $\mathcal{K}_\infty$ and $\mathcal{KL}$, are important stability analysis tools that are frequently used to characterize the stability properties of a nonlinear system.

Definition 7.1 *A function $\alpha : [0, a[\rightarrow [0, \infty[$ is said to be of class $\mathcal{K}$ if it is continuous, strictly increasing, and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \rightarrow \infty$ as $r \rightarrow \infty$.*

Definition 7.2 *A function $\beta : [0, a[\times [0, \infty[\rightarrow [0, \infty[$ is said to be class $\mathcal{KL}$ if, for each fixed $t \geqslant 0$, the mapping $\beta(r, t)$ is of class $\mathcal{K}$ with respect to r and for each fixed r, the mapping $\beta(r, t)$ is decreasing with respect to t and $\beta(r, t) \rightarrow 0$ as $t \rightarrow \infty$.*

To be of practical interest, stability conditions should not require to solve the the system of $Eq.(7.1)$ explicitly. The direct method of Lyapunov allows to determine the stability properties of an equilibrium point from the $Eq.(7.5)$ and its relationship with a positive-definite function $V(e)$.

Definition 7.3 *Consider a $\mathcal{C}^1$ (i.e., Continuously differentiable) function $V : \Re^n \rightarrow \Re$. It is positive-definite if $V(0) = 0$ and $V(e) > 0$ for all $e \neq 0$. If $V(e) \rightarrow \infty$ as $||e|| \rightarrow \infty$, then V is said to be radially unbounded.*

7.4.2 Stability analysis

In this section, we present the stability properties of the proposed two-layer control framework presented in the systems $Eq.(7.9)$ and $Eq.(7.10)$. The following theorem provides sufficient conditions such that the closed loop MPC can track the economically time varying trajectory $x_r^*(t)$.

Theorem 7.1

The system of $Eq.(7.1)$ under the optimizations of $Eq.(7.9)$ and $Eq.(7.10)$ is exponentially stable if there exist $\beta_1 > 0$, $\beta_2 > 0$, $\gamma > 0$, $L_u > 0$, $L_e > 0$, $L_w > 0$ and there exist a matrices $P > 0$ and $Q \geqslant 0$ such that

$$\lambda_{min}(Q) > 2\beta_2(L_e + L_u\gamma) \tag{7.11}$$

Then

$$||e(t)|| \leqslant \frac{\sqrt{\beta_2}}{\sqrt{\beta_1}} exp(\frac{1}{2}\alpha_1 t)||e(0)|| - \frac{\alpha_2}{\alpha_1\sqrt{\beta_1}} \sum_{k=0}^{t-1} exp(\frac{1}{2}\alpha_1 k) \tag{7.12}$$

where λ_{min} stands for the smallest eigenvalue and α_1 and α_2 are:

$$\alpha_1 = \frac{2\beta_2(L_e + L_u\gamma) - \lambda_{min}(Q)}{\beta_1}$$
$$\alpha_2 = \frac{2\beta_2 L_w \theta}{\sqrt{\beta_1}}$$

The proof of this theorem requires the following Lemmas.

Lemma 7.2 [71]
Let u be a regularly persistent input. Then, $\forall P(0)$ a symmetric positive definite matrix, $\exists \beta_1 > 0$ and $\exists \beta_2 > 0$ such that

$$\forall t \geqslant 0, \qquad \beta_1 \mathcal{I} \leqslant P(t) \leqslant \beta_2 \mathcal{I} \tag{7.13}$$

where $P(0)$ is an arbitrary symmetric definite matrix and $\mathcal{I}$ is the identity matrix with the adequate dimension.

Lemma 7.3 [72]
let $v(t)$ be a positive differentiable function satisfying the inequality

$$\dot{v}(t) \leqslant f(t)v(t) + g(t)v^p(t), \quad t \in I = [a, b], \tag{7.14}$$

where the functions $f(t)$ and $g(t)$ are continuous in I, and $p \geqslant 0$, $p \neq 1$, is a constant. Then

$$\begin{aligned} v(t) \leqslant & exp(\int_a^t f(s)ds) \\ & \times \left[v^q(a) + q\int_a^t g(s)exp(-q\int_a^s f(\tau)d\tau)ds \right]^{\frac{1}{q}} \end{aligned} \tag{7.15}$$

where $q = 1 - p$.

Proof 7.1 *[Theorem 7.1]*
Let $\Phi(t, t_0)$ be the transition matrix of the system (1),

$$\begin{aligned} \frac{d}{dt}\Phi(t,t_0) &= A(t)\Phi(t,t_0) \\ \Phi(t_0,t_0) &= I \end{aligned} \tag{7.16}$$

Where I is the $(n \times n)$ identity matrix.

Let us define a Lyapunov function $V(t) = \Phi^T(t)P(t)\Phi(t)$, and its derivative is defined as follow:

$$\begin{aligned} \dot{V}(t) &= \dot{\Phi}^T(t,t_0)P(t)\Phi(t,t_0) \\ &+ \Phi^T(t,t_0)\dot{P}(t)\Phi(t,t_0) + \Phi^T(t,t_0)P(t)\dot{\Phi}(t,t_0) \\ &= \Phi^T(t,t_0)A^T(t)P(t)\Phi(t,t_0) + \Phi^T(t,t_0)P(t)\Phi(t,t_0) \\ &+ \Phi^T(t,t_0)P(t)A(t)\Phi(t,t_0) \\ &= \Phi^T(t,t_0)[A^T(t)P(t) + \dot{P}(t) + P(t)A(t)]\Phi(t,t_0) \end{aligned} \tag{7.17}$$

The derivative is negative if and only if there exist a positive-definite symmetric matrix Q such as

$$A^T(t)P(t) + \dot{P}(t) + P(t)A(t) = -Q(t) \tag{7.18}$$

we define a Lyapunov function that verifies the conditions of definition 7.3 *as:*

$$V(t) = e^T(t)P(t)e(t) \tag{7.19}$$

where the matrix $P(t)$ is the solution of the following Riccati equation:

$$A^T(t)P(t) + P(t)A(t) + \dot{P}(t) = -Q(t) \tag{7.20}$$

For $t \in [t_k, t_{k+1}]$

$$\begin{aligned} \dot{V}(t) &= 2e^T(t)P(t)\dot{e}(t) + e^T(t)\dot{P}e(t) \\ &= 2e^T(t)P(t)[A(t)e(t) + \psi(x(t), u^*(t), w(t)) \\ &- \psi(x_r^*(t), u_r^*(t), 0)] + e^T(t)\dot{P}(t)e(t) \end{aligned} \tag{7.21}$$

Using the Eq.(7.18) *gives*

$$\begin{aligned} \dot{V}(t) &= 2e^T(t)P(t)\dot{e}(t) + e^T(t)\dot{P}e(t) \\ &= 2e^T(t)P(t)[A(t)e(t) + \psi(x(t), u^*(t), w(t)) \\ &- \psi(x_r^*(t), u_r^*(t), 0)] \\ &+ e^T(t)[-A^T(t)P(t) - P(t)A(t) - Q(t)]e(t) \\ &= -e^T(t)Q(t)e(t) + \psi(x(t), u^*(t), w(t)) \\ &- \psi(x_r^*(t), u_r^*(t), 0) \\ &\leqslant -\lambda_{min}(Q(t))||e(t)||^2 + 2||e(t)||.||P(t)|| \\ &\times ||\psi(x(t), u^*(t), w(t)) - \psi(x_r^*(t), u_r^*(t), 0)|| \end{aligned} \tag{7.22}$$

By the Lipschitz property assumed for the vector ψ, there exist positive constants L_w, L_u and L_e such that

$$\begin{aligned} &||\psi(x(t), u^*(t), w(t)) - \psi(x_r^*(t), u_r^*(t), 0)|| \\ &\leqslant L_e|x(t) - x_r^*(t)| + L_u|u^*(t) - u_r^*(t)| + L_w|w(t)| \\ &\leqslant L_e|e(t)| + L_u\gamma|e(t)| + L_w\theta \end{aligned} \tag{7.23}$$

Substituting Eq.(7.23) *in Eq.*(7.22) *and using the Lemma.*7.2*, the derivative of the Lyapunov function in Eq.*(7.21) *becomes*

$$\begin{aligned} \dot{V}(t) \leqslant &[2\beta_2(L_e + L_u\gamma) - \lambda_{min}(Q(t))]||e(t)||^2 \\ &+ 2\beta_2 L_w\theta||e(t)|| \end{aligned} \tag{7.24}$$

And by the Lemma 7.2

$$\beta_1||e(t)||^2 \leqslant e^T(t)P(t)e(t) \leqslant \beta_2||e(t)||^2 \tag{7.25}$$

$$\begin{aligned} ||e(t)||^2 &\leqslant \frac{V(t)}{\beta_1} \leqslant \frac{\beta_2}{\beta_1}||e(t)||^2 \\ ||e(t)|| &\leqslant \frac{\sqrt{V(t)}}{\sqrt{\beta_1(P(t))}} \leqslant \frac{\sqrt{\beta_2}}{\sqrt{\beta_1}}||e(t)|| \end{aligned} \tag{7.26}$$

The Eq.(7.24) *becomes*

$$\dot{V}(t) \leqslant \alpha_1 V(t) + \alpha_2\sqrt{V(t)} \tag{7.27}$$

where

$$\begin{aligned} \alpha_1 &= \frac{2\lambda_2(L_e + L_u\gamma) - \lambda_{min}(Q(t))}{\beta_1} \\ \alpha_2 &= \frac{2\beta_2 L_w\theta}{\sqrt{\beta_1}} \end{aligned}$$

*By Lemma*2 *with*

$$\begin{aligned} f(t) = \alpha_1, &\qquad g(t) = \alpha_2 \\ p = \frac{1}{2}, &\qquad I = [t_k, t_{k+1}] \end{aligned}$$

Gives

$$\begin{aligned} \sqrt{V(t)} \leqslant & exp(\frac{1}{2}\alpha_1(t-t_k))[\sqrt{V(t_k)} \\ & - \frac{\alpha_2}{\alpha_1}(1 - exp(\frac{1}{2}\alpha_1(t-t_k)))] \end{aligned} \tag{7.28}$$

For $\forall t > t_{t_k}$ *and if there exist a semi definite symmetric matrix* $Q(t)$ *such* $\lambda_{min}(Q(t)) \geqslant 2\beta_2(L_e + L_u\gamma)$, *then*

$$0 \leqslant 1 - exp(\frac{1}{2}\alpha_1(t-t_k)) \leqslant 1 \tag{7.29}$$

The Eq.(7.28) *becomes*

$$\sqrt{V(t)} \leqslant exp(\frac{1}{2}\alpha_1(t-t_k))[\sqrt{V(t_k)} - \frac{\alpha_2}{\alpha_1}] \tag{7.30}$$

By iteration we have

$$\sqrt{V(t)} \leqslant exp(\frac{1}{2}\alpha_1 t)\sqrt{V(0)} - \frac{\alpha_2}{\alpha_1}\sum_{k=0}^{t-1} exp(\frac{1}{2}\alpha_1 k) \tag{7.31}$$

Using the second inequality of Eq.(7.26), *hence*

$$||e(t)|| \leqslant \frac{\sqrt{\beta_2}}{\sqrt{\beta_1}} exp(\frac{1}{2}\alpha_1 t)||e(0)|| - \frac{\alpha_2}{\alpha_1\sqrt{\beta_1}}\sum_{k=0}^{t-1} exp(\frac{1}{2}\alpha_1 k) \tag{7.32}$$

Consequently, the between the actual system trajectory and the economically optimal trajectory $e(t) = x(t) - x_r(t)$ is bounded by:

$$\lim_{t \to \infty} ||e(t)|| \leqslant \frac{\sqrt{\beta_2}}{\sqrt{\beta_1}} ||e(0)||. \tag{7.33}$$

7.5 Application to the WWTP

7.5.1 Process Model

The $WWTP$ process selected as a case study follows the specifications given in the $BSM1$ (model $M3$). The BSM1 representation is reduced to one anoxic and one aerated reactor as shown in Figure 7.2.

The volumes of the two tanks are $2000m^3$ and $3999m^3$, to make them equivalent to total volumes of the anoxic and the aerobic compartments in the BSM1.

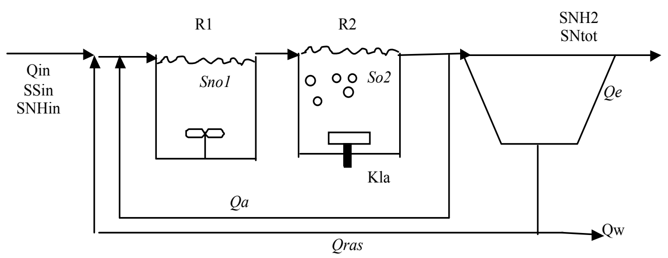


Figure 7.2: Schematic representation of the plant $M3$.

The values of the kinetic and physical parameters are assumed to be the same as for BSM1 [8]. Rewriting the equations of the model $M3$ as in $Eq.$1 gives:

$$\dot{x}(t) = A(t)x(t) + \psi(x(t), u(t), w(t)) \tag{7.34}$$

where

$$x(t) = \begin{bmatrix} S_{NH1}(t) \\ S_{NO1}(t) \\ S_{S1}(t) \\ S_{O1}(t) \\ S_{NH2}(t) \\ S_{NO2}(t) \\ S_{S2}(t) \\ S_{O2}(t) \end{bmatrix},$$

$$A(t) = \begin{bmatrix} \frac{-Q_{in}}{V_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-Q_{in}}{V_1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-Q_{in}}{V_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-Q_{in}}{V_1} & 0 & 0 & 0 & 0 \\ \frac{Q_{in}}{V_2} & 0 & 0 & 0 & \frac{-Q_{in}}{V_2} & 0 & 0 & 0 \\ 0 & \frac{Q_{in}}{V_2} & 0 & 0 & 0 & \frac{-Q_{in}}{V_2} & 0 & 0 \\ 0 & 0 & \frac{Q_{in}}{V_2} & 0 & 0 & 0 & \frac{-Q_{in}}{V_2} & 0 \\ 0 & 0 & 0 & \frac{Q_{in}}{V_2} & 0 & 0 & 0 & \frac{-Q_{in}}{V_2} \end{bmatrix}$$

$$\psi(x(t), u(t), w(t)) =$$
$$\begin{bmatrix} \frac{1}{V_1}\left[Q_{in}S_{NHin} + Q_aS_{NH2} - Q_aS_{NH1}\right] - i_{xb}\rho_{11} - i_{xb}\rho_{21} - (i_{xb} + \frac{1}{Y_A})\rho_{31} \\ \frac{1}{V_1}\left[Q_aS_{NO2} - Q_aS_{NO1}\right] - \frac{1-Y_H}{2.86Y_H}\rho_{21} + \frac{1}{Y_A}\rho_{31} \\ \frac{1}{V_1}\left[Q_{in}S_{Sin} + Q_aS_{S2} - Q_aS_{S1}\right] - \frac{1}{Y_H}\rho_{11} - \frac{1}{Y_H}\rho_{21} \\ \frac{1}{V_1}\left[Q_aS_{O2} - Q_aS_{O1}\right] - \frac{1-Y_H}{Y_H}\rho_{11} - (\frac{4.57}{Y_A} + 1)\rho_{31} \\ \frac{1}{V_2}\left[Q_a(S_{NH1} - S_{NH2})\right] - i_{xb}\rho_{12} - (i_{xb} + \frac{1}{Y_A})\rho_{32} \\ \frac{1}{V_2}\left[Q_a(S_{NO1} - S_{NO2})\right] - \frac{1-Y_H}{2.86Y_A}\rho_{22} + \frac{1}{Y_A}\rho_{32} \\ \frac{1}{V_2}\left[Q_a(S_{S1} - S_{S2})\right] - \frac{1}{Y_H}\rho_{12} - \frac{1}{Y_H}\rho_{22} \\ \frac{1}{V_2}\left[Q_a(S_{O1} - S_{O2})\right] - \frac{1-Y_H}{Y_H}\rho_{12} - \frac{4.57-Y_A}{Y_A}\rho_{32} + KLa(S_{O,Sat} - S_{O2}) \end{bmatrix}$$

7.5.2 Control Problem

For an efficient N-removal in the ASP, the typical controlled variables are the dissolved oxygen concentration in the aerated zone S_{O2} (DO) and the nitrate concentration in the anoxic zone S_{NO1}. The manipulated variables are: the internal recycle flow (Q_a) and the oxygen transfer coefficient (K_{La}). The disturbances are: the influent flow (Q_{in}) ($Fig.7.3a$), the organic matter concentration, (S_{Sin}) ($Fig.7.4$) and the ammonium concentration (S_{NHin}) ($Fig.7.3b$) in the influent.

The DO concentration in the aerobic zone should be sufficiently high to supply enough oxygen to the microorganisms in the sludge. However, high air flow rates can produce an excess in DO concentration in the aerobic zone that affect negatively the denitrification process through the internal recycle and increases unnecessarily the energy consumption. Hence, the control of the DO concentration is crucial for the satisfactory operation of the activated sludge process. In the denitrification process that takes place in the anoxic zone, the key variable is the nitrate concentration S_{NO1}.

7.5.3 Performance indices

The measures used to characterize the effluent quality and energy usage during the N-removal process are the standard performance indices recommended in the BSM1 platform for the evaluation of control strategies applied to WWTPs. The performance indices used in this section are the EQ, AE and PE as described in the section 2.3.

Now, we can define the economic objective f_{eco} as follow:

$$f_{eco} = w_3(AE + PE) \tag{7.35}$$

Where w_3 is a wight matrix.

The performance assessment is made at two levels. The first level concerns the control design by adding the economic term in the cost function. The second level measures the effect of the control strategy on plant performance.

7.6 Simulations Results

The selected values to tune the NMPC for the upper layer are: the control control horizon $m_e = 2$, prediction horizon $N_e = 4$ and the weight of the economic function is $w_3 = 1$ and the tuning parameters of the optimizing controller for the lower layer are: control horizon $m = 2$; prediction horizon $N = 4$; output weight $w_1 = diag(0.155, 0.01, 0.155, 0.01)$ and input weight $w_2 = diag(0.01, 0.01)$.

Figs. 7.2 and 7.3 present the different profiles of perturbations of $BSM1$ used in this study. In this influent we can observe a strong variations in the flow and concentrations during the dry weather.

In figures 7.5a and 7.5b present the two output that are respectively the dissolved oxygen in the aerobic reactor and the nitrate in the anoxic reactor, from the $Fig.$7, it can be appreciated the ability of the controller to track the desired set point and disturbances rejection.

The figures 7.6a and 7.6b present the evolution of the concentrations SNH_2 and $N - Total$.

The two manipulated variables are shown in $Figs.$7.7 which indicate that suitable control signals K_{la} and Q_r drive the process to follow the set point, while satisfying the constraints imposed. Finally, the figure 7.8 presents the evolution of the degree of freedom C.

In order to illustrate the efficiency of the two layer method proposed in this chapter, in table 7.1, a comparison of different performance indices is shown for the tow layer and one layer strategies. Comparing the two different case studies, it is possible to observe that the introduction of the two layer strategy improves the economics reducing the OCI index by 14.99%, AE by 17.87% and the PE by 4.72%.

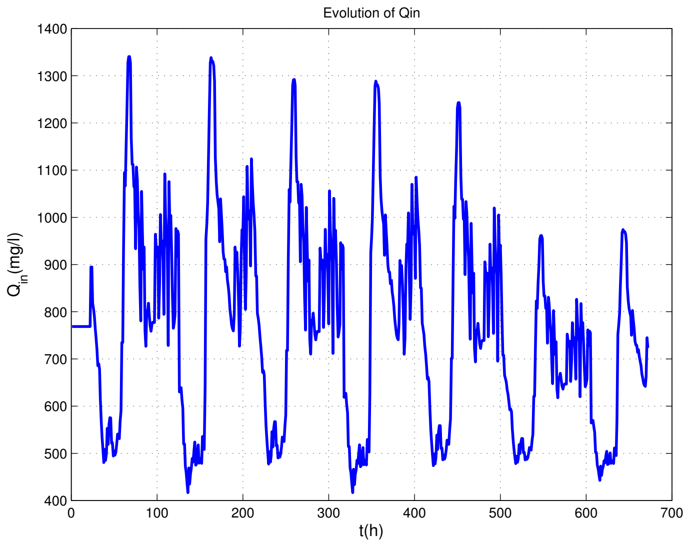
7.7 Conclusion

In this work, we proposed a two-layer strategy for integrating dynamic economic optimization and nonlinear closed loop MPC for nonlinear system. In

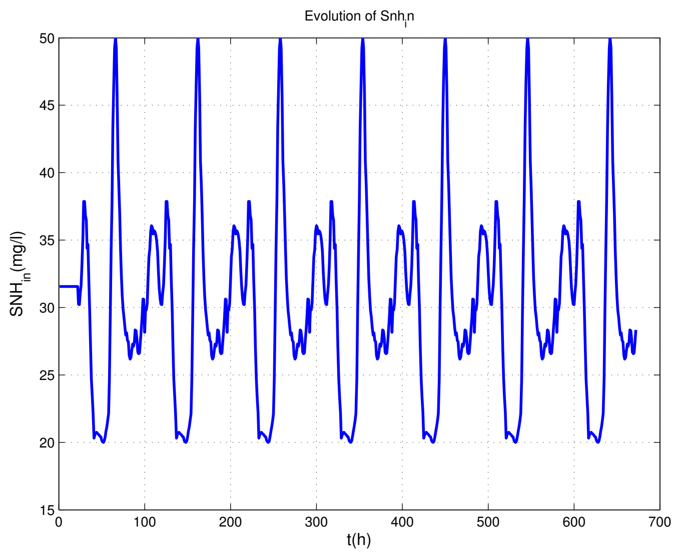
Table 7.1: Comparison of performance indices for one layer and two layers strategies

Strategy Index	One layer	Two layers	%
AE	1449.4	1190.4	-17.87%
PE	421.55	401.64	-4.72 %
EQ	5897	5845	-0.88%
OCI	1870.9	1592.1	-14.99 %

the upper layer the economic function is reduced by its gradient to compute economically optimal time varying operating trajectory. The lower layer is used to compute a feedback control actions that force the outputs of the process to track the trajectories received from the upper layer. We proved that the deviation between the actual closed loop system and the economically optimal closed loop trajectory is bounded. The control law is based on direct exploitation of the nonlinear model of the wastewater treatment processes.



a):Evolution of Q_{in}



b):Evolution of SNH_{in}

Figure 7.3: Responses of Q_{in} and SNH_{in}.

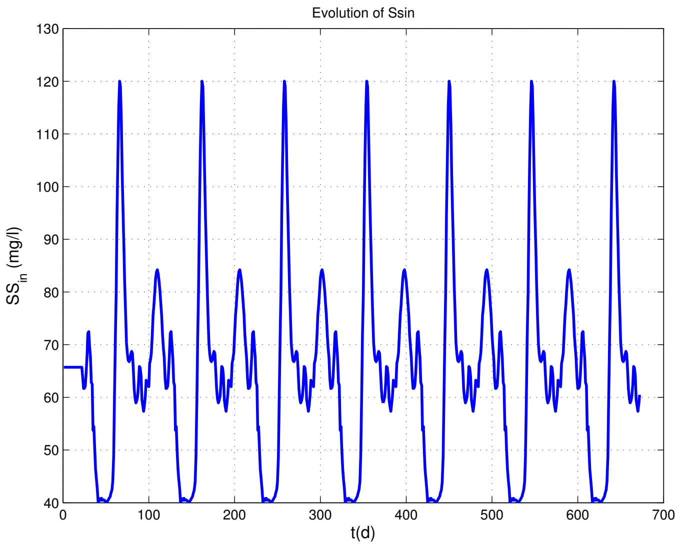
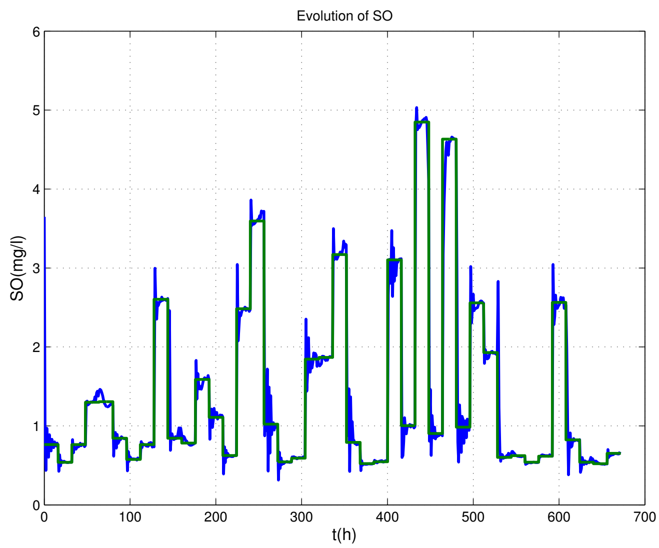
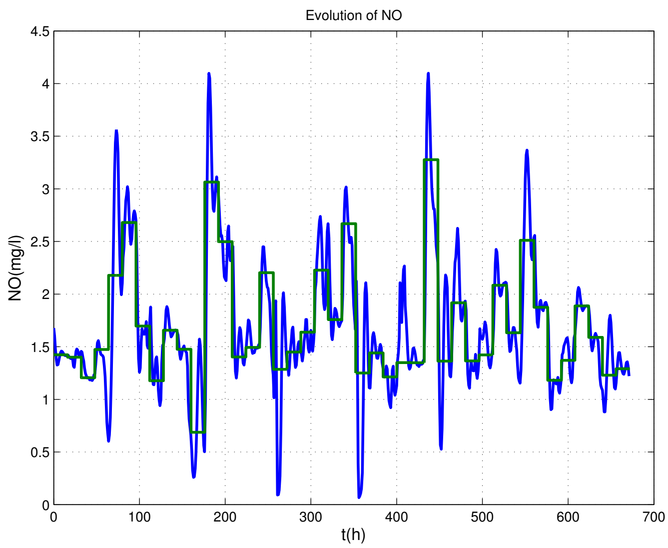


Figure 7.4: Responses of Ss_{in}.

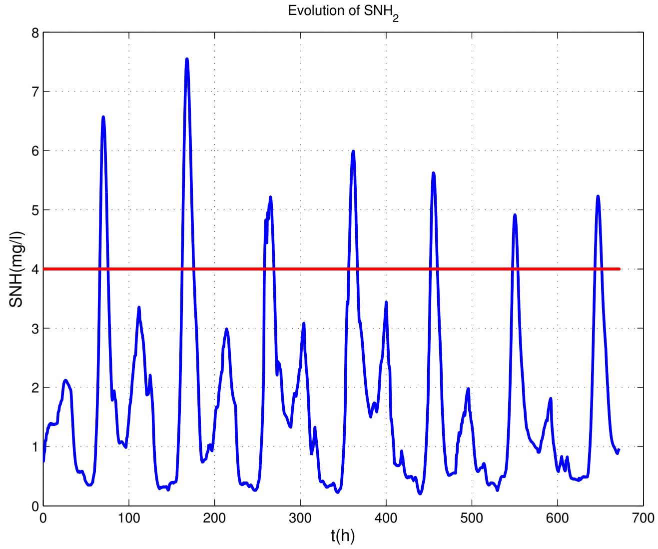


a):Evolution of S_{O2}

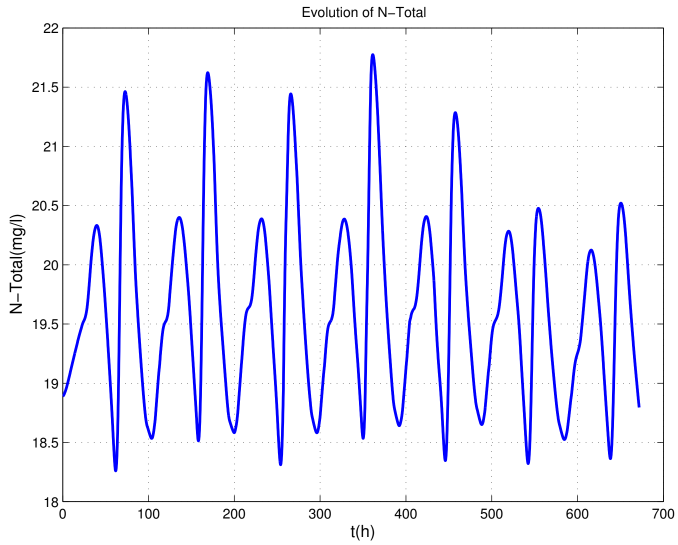


b):Evolution of S_{NO}

Figure 7.5: Responses of S_{O2} and S_{NO}.

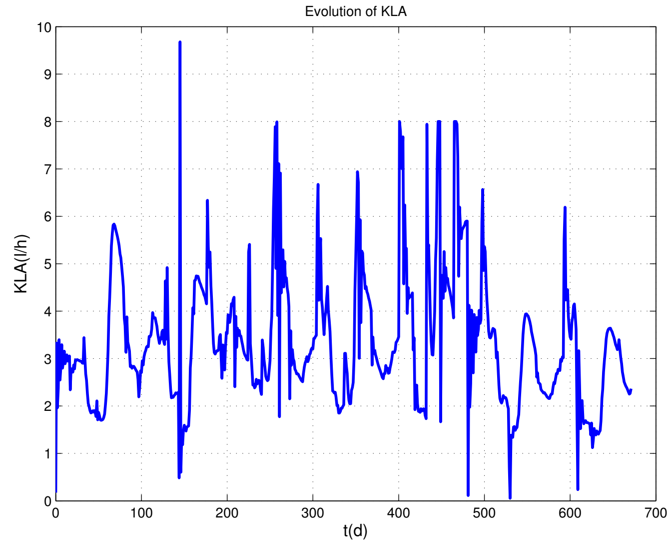


a):Evolution of NH_2

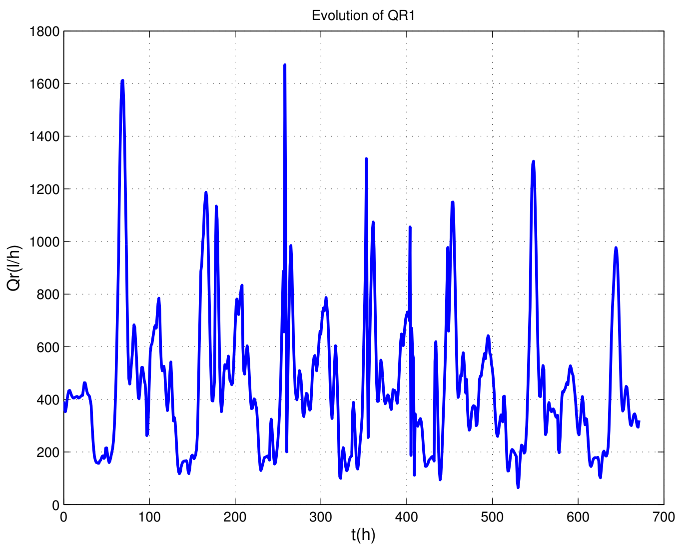


b):Evolution of $N - Total$

Figure 7.6: Responses of S_{NH2} and N_{tot}.

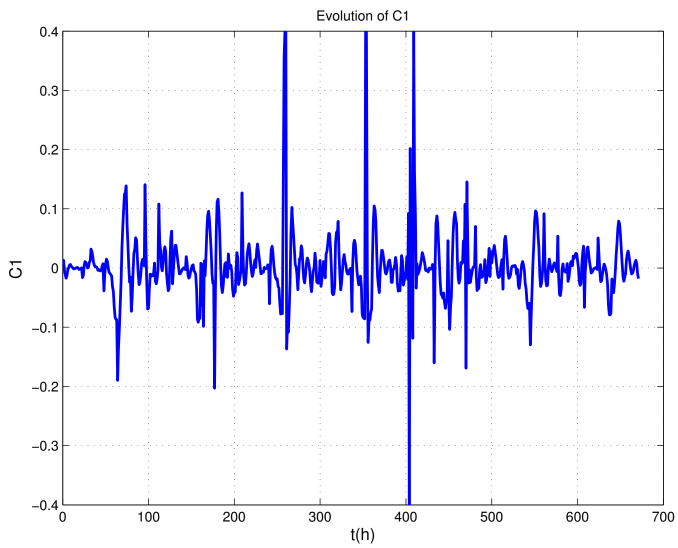


a):Evolution of K_{La}

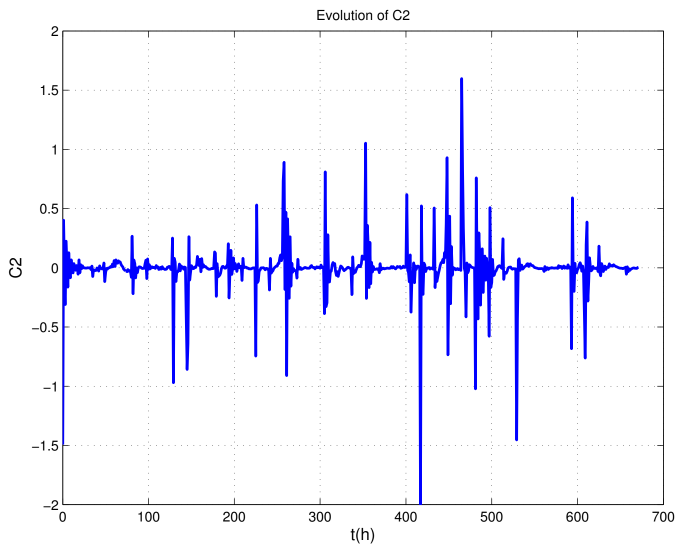


b):Evolution of Q_r

Figure 7.7: Responses of K_{La} and Q_r.



a):Evolution of C_1



b):Evolution of C_2

Figure 7.8: Evolution of the degree of freedom C.

8

Conclusions and future directions

8.1 Conclusions

This thesis proposes advanced methodologies of control systems, which provide a good rejection of disturbances and optimum operation in terms of of performance and economic costs. They have the advantage of reducing the computational load with respect to other alternatives techniques and they have been successfully applied to different simulation model of Wastewater treatment plants .

In this thesis, control techniques presented in this thesis are divided in two parts. The first one Part I, discuss the control strategies based on positive invariance theory and part II deals with control strategies based on dynamic real time optimization.

First, a nonlinear feedback control scheme based on a linearized model of the plant with input constraints is presented. Positive invariance techniques together with minimal order observer (software sensor) are used to control a nonlinear model of a WWTP. The positive invariance techniques that had emerged as very efficient to handle similar problems of constrained control is successfully used to control the nitrogen removal process. The observer based constrained control, as presented in this work compete with other approaches in easiness, applicability and computing effort.

Second, a new methodology to design a CLMPC, providing a simple solution that ensures stability and respects non-symmetrical constraints on control magnitudes and moves is developed. The positive invariance theory has also been used here, particularly polyhedral invariant sets. The proposed algorithm takes advantage of the design of a terminal control law to increase the degrees of freedom, of other CLMPC approaches guaranteeing stability and constraints fulfillment for both modes of the CLMPC. For the controller design, necessary and sufficient conditions for asymptotic stability at the origin have been developed, for linear systems and a state feedback control law, while respecting constraints on both control magnitudes and its increments. The proposed methodology has been successfully applied to the activated sludge process in a WWTP, forcing the substrate concentration (organic matter) in the effluent and the dissolved oxygen concentration in the biological reactor to track a given set point. Simulation results show that integral squared error of the substrate in the effluent is reduced in 15% with respect to the other technique presented in the results, which is a significant improvement in effluent quality. The methodology of this work is general and can be easily extended to other applications.

Third, an economic nonlinear closed loop generalized predictive control scheme is presented. The reduced gradient of the economic objective function is included as an additional term of the cost function of a nonlinear unconstrained GPC controller in order to obtain a quadratic function cost and an explicit terminal control law. The proposed strategy allows the simultaneous optimization and control of the plant operation in one layer approach. The control strategy is based on direct use of a state dependent coefficient representation based on the phenomenological model of a wastewater treatment processes.

Finally, a two-layer strategy for integrating dynamic economic optimization and nonlinear closed loop MPC for nonlinear system is developed. In the upper layer the economic function is reduced by its gradient to compute economically optimal time varying operating trajectories. The lower layer is used to compute a feedback control actions that force the outputs of the process to track the trajectories received from the upper layer. It has been proved that the deviations between the states of the actual closed loop system and the economically optimal closed loop trajectories are bounded. The control law is also based on direct exploitation of the nonlinear model of the WWTP.

The control results of the activated sludge process shown have been carried out under different conditions, varying the regions of uncertainty, index weights and optimization strategies. In all of them there is a compromise between costs, rejection of disturbances and control efforts that the designer must evaluate according to his needs.

8.2 Future directions

The future work will focus on large scale systems control within the plant wide control framework in order to obtain solutions according to their global requirements. Since the use of MPC strategies with distributed and hierarchical architectures has shown to be successful to tackle those problems and they are widespread in industry, solutions based on those techniques are the main future interest.

Particularly, the development of control strategies for integrated and networked systems based on hierarchical and distributed MPC structures will be investigated. Moreover, some properties such us economic and control performance optimality, stability and environmental quality will be guaranteed.

The validation of the proposals will be carried out on different types of complex systems: integrated urban water systems (sewer, wwtp and river basin) and water distribution networks.

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