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We calculate for the first time the phonon excitation rate in the outer crust of a neutron star due to scattering from light dark matter (LDM) particles gravitationally boosted into the star. We consider dark matter particles in the sub-GeV mass range scattering off a periodic array of nuclei through an effective scalar-vector interaction with nucleons. We find that LDM effects cause a modification of the net number of phonons in the lattice as compared to the standard thermal result. In addition, we estimate the contribution of LDM to the ion-ion thermal conductivity in the outer crust and find that it can be significantly enhanced at large densities. Our results imply that for magnetized neutron stars the LDM-enhanced global conductivity in the outer crust will tend to reduce the anisotropic heat conduction between perpendicular and parallel directions to the magnetic field.

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I. INTRODUCTION

Dark matter constitutes the most abundant type of matter in our Universe, and its density is now experimentally well determined $\Omega_{\text{CDM}}h^2 = 0.1199 \pm 0.0027$ [1]. Worldwide efforts to constrain its nature and interactions have led the community to a puzzling situation where null results coexist with direct detection experiments that find high significance excesses [2]. In particular, in the low mass region of dark matter (DM) candidates, i.e., $m_\chi < 1 \text{ GeV}/c^2$, cosmological, astrophysical and collider constraints seem to be the most important; see, for example, a discussion in Ref. [3]. Direct detection searches of thermalized galactic DM are mostly based on nuclear recoils on selected targets. In this scenario, light dark matter (LDM) particles with masses much smaller than that of the nucleon, $m_\chi \ll m_N$, can only provide energies $\sim \text{eV}$ which are below the $\sim \text{keV}$ threshold for conventional terrestrial searches. If one, instead, considers LDM scattering off bound electrons, energy transfer can cause excitation or even ionization and thus seems promising for exploring the phase space in a complementary way in the near future [4]. DM hitting terrestrial targets is expected to have low velocities $v_\chi \sim 10^{-3}$ (we use $c = \hbar = 1$ units) as the gravitational boost is small for the Earth, i.e., its Lorentz factor $\gamma_\oplus = 1/\sqrt{1 - \beta_\oplus^2} \sim 1$ with $\beta_\oplus = \sqrt{\frac{2GM_\oplus}{R_\oplus}}$. However, for compact objects, such as neutron stars (NSs) with typical

masses $M_{\text{NS}} \approx 1.5 M_\odot$ and radius $R_{\text{NS}} \approx 12 \text{ km}$, $\gamma_{\text{NS}} \sim 1.26$ providing $v_\chi \sim 0.7$ as a result of the gravitational velocity boost. Contrary to what happens around other less compact celestial bodies this mechanism allows to boost particles from tiny velocities to relativistic values or, accordingly, test the same length scales as in direct detection searches with smaller projectile masses. In particular, the outer crusts in NSs are formed by periodically arranged nuclei with typical densities ranging from $\rho \approx 210^6\text{--}410^{11} \text{ g/cm}^3$. In the single-nucleus description [5], a series of nuclei with increasing baryonic number, A , from Fe to Kr form a lattice before neutrons start to leak out of nuclei. At these high densities, electrons form a degenerate Fermi sea. At even larger densities and up to nuclear saturation density, around $\rho_0 \approx n_0 m_N \approx 2.410^{14} \text{ g/cm}^3$, a number of different nuclear structures called *pasta* phases appear [6].

In this work, we study the effect of LDM scattering in the production of quantized lattice vibrations (phonons) in the outer NS crust. Later, we will discuss how this result can impact subsequent quantities of interest, such as the ion thermal conductivity, that are relevant for computing the cooling behavior of NSs. Phonons are quantized vibrational modes characterized by a momentum \vec{k} and polarization vector \vec{e}_λ appearing in a nuclear periodic system [7]. They can have a number of different sources. They can be excited due to nonzero temperature T in the medium. The Debye temperature allows us to evaluate the importance of the ion motion quantization. For a body-centered cubic (bcc) lattice [8], for example, $T_D \approx 0.45T_p$, with $T_p = \omega_p/k_B = \sqrt{\frac{4\pi n_A Z^2 e^2}{k_B^2 m_A}}$ geing the plasma temperature

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associated to a medium of ions with number density n_A , baryonic number A , electric charge Ze and mass m_A . k_B is the Boltzmann constant. At low temperatures $T < T_D$, the quantization becomes increasingly important, and the thermal phonons produced are typically acoustic modes, following a linear dispersion relation $\omega_{k,\lambda} = c_{l,\lambda}|\vec{k}|$, where $c_l = \frac{\omega_p/3}{(6\pi^2 n_A)^{1/3}}$ is the sound speed. In addition, phonon production can be caused by an external scattering agent, for example, standard model neutrinos. In this respect, weak probes such as cosmological neutrinos with densities $n_\nu \sim 116 \text{ cm}^{-3}$ per flavor have been shown to provide small phonon production rates in a crystal target [9,10]. Due to the tiny mass of the neutrino, the experimental signature of this effect seems, however, hard to confirm. The main interest in the astrophysical context we discuss in this contribution follows as phonon excitation in a periodic system, such as the outer crust of a NS, can affect the thermal transport coefficients in the star. The potential modification of transport properties of heat/energy in the external layers in NSs is crucial to possibly identifying relevant distortions in the cooling behavior of these astrophysical objects in rich LDM environments.

The structure of this contribution is as follows. In Sec. II, we present the effective field theory Lagrangian model using dark matter-nucleon contact interactions via scalar and vector couplings in a relativistic framework and compute the single phonon excitation rate, discussing sources of uncertainty. Later, in Sec. III, we compute the thermal conductivity in the outer crust with LDM contributions comparing the results to the standard thermal value and discussing possible astrophysical consequences. Finally, in Sec. IV, we give our conclusions.

II. LIGHT DARK MATTER SCATTERING AND PHONON EXCITATION RATE

In this work, we consider the interaction of an incoming fermionic DM particle, χ , scattering quasielastically with a nucleus in the outer NS crust lattice via scalar and vector couplings [11,12] composed of Z protons (p) and $A - Z$ neutrons (n),

$$\mathcal{L}_{\mathcal{I}} = \sum_{N=n,p} g_{s,N} \bar{\chi} \chi N \bar{N} + g_{v,N} \bar{\chi} \gamma^\mu \chi N \gamma_\mu \bar{N}, \quad (1)$$

where $g_{s,N}$ ($g_{v,N}$) are the effective scalar (vector) couplings of the DM particle to the nucleon (N) field. We will focus on a weakly interacting candidate (WIMP) with mass in the sub-GeV range. This interaction is equivalent to considering a Fermi four-fermion interaction model, where the effective couplings of mass dimension (-2) for these operators are obtained by integrating out the propagator of a generic ϕ mediator with mass M_ϕ . Let us mention that, indeed, more effective operators for Dirac LDM candidates are possible; see, for example, Table I in Ref. [13] for leading coupling contributions to Standard Model fermions. However, in order to keep our description concise

for the sake of clarity, we will restrict here to the spin-independent interaction model used in previous works [12].

Motivated by the need to compare bounds from colliders to direct detection, we describe interactions of DM with quarks $q = u, d$, and averaging in terms of nucleon fields, we can write for the vector case $g_{v,N}/M_\phi^2 \sim 1/\Lambda_v^2$ and $g_{s,N}/M_\phi^2 \sim m_q/\Lambda_s^3$ where Λ_v (Λ_s) is the suppression mass scale for the vector (scalar) case, assuming the effective couplings are of order $O(1)$ and can be absorbed into $\Lambda_{s,v}$ [14]. Using bounds from CMS and ATLAS [15], we set $\Lambda_v \gtrsim 1 \text{ TeV}$ and $\Lambda_s \gtrsim 100 \text{ GeV}$.

We denote by $p_N^\mu = (E_N, \vec{p}_N)$ and $p_N^\mu = (E_N, \vec{p}_N)$ the four momentum for the outgoing and incoming nucleon and $p_\chi^\mu = (E_\chi, \vec{p}_\chi)$ and $p_\chi^\mu = (E_\chi, \vec{p}_\chi)$ as those analogous for the LDM particle, respectively. Momentum transfer is denoted by $q^\mu = p_\chi^\mu - p_\chi^\mu$.

Generically, given an interaction potential \mathcal{V} felt by an interacting DM particle when approaching a nucleus in the periodic lattice, the single phonon excitation rate *per mode* can be obtained using the Fermi golden rule, $R_{k,\lambda} = 2\pi\delta(E_f - E_i)|\langle f|\mathcal{V}|i\rangle|^2$, where i and f are the initial and final states considered and $\delta(E_f - E_i)$ assures energy conservation. Given the fact that incoming (outgoing) LDM particles suffer a very moderate perturbation from the plane wave state, we will describe its incoming (outgoing) quantum state as $\Psi_{\vec{p}_\chi}(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{p}_\chi \cdot \vec{r}}$ with V the volume of the system. The interaction potential felt by the LDM particle is the sum [9] over lattice sites $\mathcal{V}(\vec{r}) = \sum_j v(\vec{r} - \vec{r}_j)$ that we describe for the sake of simplicity as impenetrable pointlike spheres $v(\vec{r} - \vec{r}_j) = \delta^3(\vec{r} - \vec{r}_j)v_0$. We, nevertheless, comment on corrections to this picture later in the manuscript.

Using the Born approximation, the scattering amplitude for an incident χ particle can be written as

$$f(\vec{p}_\chi, \vec{p}'_\chi) \simeq -\frac{m_\chi}{2\pi} \int e^{i(\vec{p}_\chi - \vec{p}'_\chi) \cdot \vec{r}} v(\vec{r}) d^3 r', \quad (2)$$

and from its squared value, the differential cross section in the center-of-mass frame, $\frac{d\sigma}{d\Omega}|_{\text{CM}} = |f(\vec{p}_\chi, \vec{p}'_\chi)|^2$. The validity of the Born approximation is provided by the finite-range potential $\mathcal{V}(\vec{r})$ so the condition $|(\vec{p}_\chi - \vec{p}'_\chi) \cdot \vec{r}'| \ll 1$ is fulfilled, with $|\vec{r}'|$ being a typical target size. The effective interaction potential can be obtained from the squared interaction matrix element as calculated from the Lagrangian in Eq. (1) as $\frac{d\sigma}{d\Omega}|_{\text{CM}} = \frac{|\overline{\mathcal{M}}_{\chi N}|^2}{64\pi^2 s}$. First, we compute the scattering amplitude $|\overline{\mathcal{M}}_{\chi N}|^2$ with $s = (p_N + p_\chi)^2$ being the Mandelstam variable. Adding the contribution over proton and neutron amplitudes coherently, we can obtain the LDM particle-nucleus differential cross section and then integrate to find a relation between the total cross section $\sigma_{\chi A} \simeq 4\pi a^2$, or, equivalently, the effective potential

from Eq. (2), and the scattering length, a , at low incident energies. We obtain $v(\vec{r}) = \frac{2\pi a}{m_\chi} \delta(\vec{r})$. Besides, we have used a normalization of the delta function as $\int_{VT} \delta(x) d^4x = 1$. From the Lagrangian in Eq. (1), the spin-averaged scattering amplitude [12] reads

$$\begin{aligned} |\overline{\mathcal{M}}_{\chi N}|^2 &= 4g_{s,N}^2[(p_N p'_N + m_N^2)(p_\chi p'_\chi + m_\chi^2)] \\ &+ 8g_{v,N}^2[2m_N^2 m_\chi^2 - m_N^2 p'_\chi p_\chi - m_\chi^2 p_N p'_N] \\ &+ (p'_\chi p'_N)(p_N p_\chi) + (p'_N p'_\chi)(p_N p'_\chi) \\ &+ 8g_{s,N} g_{v,N} [m_N m_\chi (p_N + p'_N)(p_\chi + p'_\chi)]. \end{aligned} \quad (3)$$

Due to the mildly relativistic nature of nucleons inside the nucleus, energy and momentum will lie close to the Fermi surface values E_{FN} , $|\vec{p}_{\text{FN}}|$ and $|\vec{p}_N|^2 \sim |p'_N|^2 \sim |\vec{p}_{\text{FN}}|^2 \ll m_N^2$. We will approximate the product $p'_N p_N = E_N E'_N - |\vec{p}_N||\vec{p}'_N| \cos \theta_{\vec{p}_N, \vec{p}'_N} \simeq E_{\text{FN}}^2$. On the other hand, for the more relativistic DM particle products, $p_\chi p'_\chi = E_\chi E'_\chi - |\vec{p}_\chi||\vec{p}'_\chi| \cos \theta_{\vec{p}_\chi, \vec{p}'_\chi} = E_\chi^2 - |\vec{p}_\chi|^2 \cos \theta_\chi = m_\chi^2 + |\vec{p}_\chi|^2 (1 - \cos \theta_\chi)$, where we use $\theta_{\vec{p}_\chi, \vec{p}'_\chi} \equiv \theta_\chi$. The density dependence will be retained using a parametrization of the nuclear Fermi momentum $|\vec{p}_{\text{FN}}| \sim (3\pi^2 n_0 Y_N)^{1/3}$ and the nuclear fractions $Y_p = Z/A$, $Y_n = (A - Z)/A$. If we now average over angular variables,

$$\begin{aligned} &\int_{-1}^1 2\pi d(\cos \theta_\chi) |\overline{\mathcal{M}}_{\chi N}|^2 \\ &= 16\pi g_{s,N}^2 [(2m_N^2 + |\vec{p}_{\text{FN}}|^2)(2m_\chi^2 + |\vec{p}_\chi|^2)] \\ &+ 32\pi g_{v,N}^2 [2E_\chi^2 E_{\text{FN}}^2 - m_N^2 |\vec{p}_\chi|^2 - m_\chi^2 |\vec{p}_{\text{FN}}|^2] \\ &+ 128\pi g_{s,N} g_{v,N} [m_N m_\chi E_{\text{FN}} E_\chi]. \end{aligned} \quad (4)$$

In the nucleus, we can use the previous expression, Eq. (4), to find the coherent contribution of the A nucleons in a way similar to what is done in direct detection [16],

$$\begin{aligned} &\int_{-1}^1 2\pi d(\cos \theta_\chi) |\overline{\mathcal{M}}_{\chi A}|^2 \\ &\simeq m_A^2 \left(\frac{Z}{m_p} \sqrt{|\tilde{\mathcal{M}}_p|^2} + \frac{(A-Z)}{m_n} \sqrt{|\tilde{\mathcal{M}}_n|^2} \right)^2, \end{aligned} \quad (5)$$

with $\int_{-1}^1 2\pi d(\cos \theta_\chi) |\overline{\mathcal{M}}_{\chi N}|^2 \equiv |\tilde{\mathcal{M}}_N|^2$. The Mandelstam variable $s = (p_A + p_\chi)^2 = m_\chi^2 + m_A^2 + 2E_A E_\chi - 2\vec{p}_A \vec{p}_\chi$ can be approximated as $s \simeq (m_\chi + m_A)^2$, neglecting the mildly relativistic nuclei momenta. Thus, we can express the cross section in the center-of-mass frame as

$$\sigma_{A,\chi} = 4\pi a^2 = m_A^2 \frac{\left(\frac{Z}{m_p} \sqrt{|\tilde{\mathcal{M}}_p|^2} + \frac{(A-Z)}{m_n} \sqrt{|\tilde{\mathcal{M}}_n|^2} \right)^2}{16\pi(m_\chi + m_A)^2}. \quad (6)$$

From a zero-order momentum expansion, we recover the usual expression for direct detection spin independent

cross section at low energies [17] for each coupling $\sigma_{A,\chi} \rightarrow \frac{\mu_{\chi A}^2}{\pi} (Zg_{s,p} + (A-Z)g_{s,n})^2$ where $\mu_{\chi A} = \frac{m_\chi m_A}{m_\chi + m_A}$ is the reduced $\chi - A$ mass. Note at this point that the $\sim A^2$ enhancement in the obtained cross section remains as the coherence condition $\lambda \geq R_A$ is fulfilled, with R_A being the nuclear radius and $\lambda = h/|\vec{p}_\chi|$ being the De Broglie wavelength. In addition, the contribution of the nuclear lattice will be described by the summations extended over the lattice sites, or, equivalently, by the inclusion of the structure factor $S(q) \sim |\sum_j e^{-i\vec{q}\vec{r}_j}|^2$, in the full phonon excitation rate expression as will be shown later in the manuscript. Some studies have included form factors $F^2(q)$ to correct a pointlike nucleus nature approach [17]; however, since we will be focusing on $q \rightarrow 0$ limit, we will consider them as unity for the sake of simplicity. In what follows, we will refer to $\vec{p}' \equiv \vec{p}'_\chi$ and $\vec{p} \equiv \vec{p}_\chi$. The single-phonon excitation time rate from the ground state now reads

$$R_{k,\lambda}^{(0)} = \frac{4\pi^2 a^2}{V^2 m_\chi^2} \delta(E_{\vec{p}'} + \omega_{k,\lambda} - E_{\vec{p}}) 2\pi \left| \sum_j \langle 1, \vec{k}\lambda | e^{-i\vec{q}\vec{r}_j} | 0 \rangle \right|^2, \quad (7)$$

where $\vec{r}_j = \vec{x}_j^{(0)} + \vec{u}_j$ with $\vec{x}_j^{(0)}$ the lattice point and \vec{u}_j the displacement vector [18]. We must note at this point that the previous expression includes the squared modulus of the Fourier transform of the periodically arranged lattice sites including thus the usual description in terms of the structure factors $S(q)$. This function provides information on the spatial distribution through a correlation function and presents maxima at the crystal nuclear positions. The contribution of this factor to the global cross section describes coherent scattering from all of the different nuclei as discussed in Ref. [19]. There, the effect of efficient low-energy scattered WIMPs from the interior of the stellar DM distribution was mentioned as an additional factor to prevent DM escaping from the NS once inside. In this way, it thus constitutes a mechanism for *trapping* DM, besides the deep gravitational potential felt by these sub-GeV mass particles.

Beyond this point, we will consider an isotropic medium, and since the Born approximation $|\vec{q}\cdot\vec{r}'| \ll 1$ holds, it is most likely that acoustic modes are excited. It follows that

$$-i\vec{q} \sum_j e^{-i\vec{q}\vec{x}_j^{(0)}} \langle 1, \vec{k}\lambda | \vec{u}_j | 0 \rangle = -in_A \delta^{(3)}(\vec{k} - \vec{q}) \sqrt{\frac{|\vec{k}|}{2m_A c_l}}, \quad (8)$$

where we have used the continuum limit $\sum_j \rightarrow n_A \int d^3x$ and the fact that all \vec{k} have a polarization vector that verifies \vec{e}_l / \vec{k} and the other two vectors are perpendicular to \vec{k} . Finally Eq. (7) can be written as

$$R_k^{(0)} = \frac{4\pi^2 a^2}{m_\chi^2 V} \delta(E_{\vec{p}'} + \omega_{\vec{k},\lambda} - E_{\vec{p}}) 2\pi n_A^2 \frac{\delta^{(3)}(\vec{k} - \vec{q}) |\vec{k}|}{2m_A c_l}. \quad (9)$$

At this point, we must consider the peculiarities of the incoming LDM phase space distribution $f_\chi(\vec{p})$ as it will impact the averaged final phonon excitation rate. Typically, for the Sun or the Earth, the uncertainties have different sources including the orbital speed of the Sun, escape velocity from the DM halo and the form of the phase space distribution itself. About the latter and in local searches, direct and indirect detection are affected in a different manner. For example, direct detection is sensitive to DM with high velocities [20], while for indirect detection, the low-velocity part of the distribution is tested [21,22].

A popular choice is obtained using an approximation based on an isotropic sphere with density profile $\rho_{\text{DM}}(r) \propto r^{-2}$ of collisionless particles, i.e., a Maxwell-Boltzmann type with a local mass density $\rho_{\text{LDM}} = 0.3 \text{ GeV cm}^{-3}$. Uncertainties on the knowledge of the distribution function must be carefully considered as this impacts accuracy when translating event rates to constraints on particle physics models of DM.

In the case we analyze here of a more compact object, it is the high velocity part of the distribution that is tested, as typical values for boosted root-mean-squared velocities are $\langle v^2 \rangle \sim 2GM_{\text{NS}}/R_{\text{NS}} \sim (0.6)^2$. For these relativistic regimes, one must use the Maxwell-Jüttner distribution [23] function and, more properly, take into account the space-time curvature due to the gravitational field created by the NS [24],

$$f_\chi(\vec{p}) = \frac{n_\chi \mu}{4\pi m_\chi^3 K_2(\mu)} e^{-\mu \sqrt{1+g_1(r) \frac{|\vec{p}|^2}{m_\chi^2}}}, \quad (10)$$

where $\mu = \frac{m_\chi}{k_B T}$ and $K_2(\mu)$ is the modified Bessel function of the second kind defined as $K_n(\mu) = \left(\frac{\mu}{2}\right)^n \frac{\Gamma(1/2)}{\Gamma(n+1/2)} \times \int_0^\infty e^{-\mu y} (y^2 - 1)^{n-1/2} dy$. The isotropic Schwarzschild metric for the gravitational field created by the NS source is [24] $ds^2 = g_0(r)(dx^0)^2 - g_1(r)\delta_{ij}x^i x^j$, $i, j = 1, 2, 3$.

Note that, in the close vicinity of the NS where we will be interested in assessing our quantities of interest, $r \sim R_{\text{NS}}$, and it follows that $g_1(R) = (1 + \frac{GM_{\text{NS}}}{2R_{\text{NS}}})^4 \sim 1.42$, $g_0(R) = (1 - \frac{GM_{\text{NS}}}{2R_{\text{NS}}})^2 / (1 + \frac{GM_{\text{NS}}}{2R_{\text{NS}}})^2 \sim 0.69$. The distortion from the flat space with a Minkowski metric effectively sets $g_0(r)$, $g_1(r) \neq 1$ as expected. Furthermore, if we obtain from the above the root-mean-squared $\sqrt{\langle v^2 \rangle} \sim 0.6$, this implies $\mu \approx 6.7$ [25,26].

The normalization condition is such that the particle 4-flow J^α can be defined, and taking the $\alpha = 0$ component, we obtain $\int d^3 \vec{p} f_\chi(\vec{p}) \sqrt{-g}/g_0 = J^0 = n_\chi / \sqrt{g_0}$ with $\sqrt{-g} = \sqrt{g_0 g_1^3}$. n_χ is the DM number density near the NS. Note that at nonrelativistic velocities and flat space we do recover the

Maxwell-Boltzmann distribution as expected. Further, we consider all outgoing χ states are allowed as the net number will be tiny as compared to ordinary matter. The phonon excitation time rate must be weighted with the momenta of the *local* χ phase space that, as mentioned, is shifted to the relativistic values

$$R_k^{(0)} = \frac{4\pi^3 n_A^2 V}{m_\chi^2 m_A c_l} \int \frac{d^3 \vec{p} f_\chi(\vec{p})}{(2\pi)^3} \int \frac{d^3 \vec{p}'}{(2\pi)^3} \delta^{(3)}(\vec{k} - \vec{q}) \times \delta(E_{\vec{p}'} + \omega_{\vec{k},\lambda} - E_{\vec{p}}) |\vec{k}| a^2. \quad (11)$$

Computing the zeros of the delta function and expressing the incoming momentum as $|\vec{p}_0| = \sqrt{\gamma^2 - 1} m_\chi$, we obtain an interval of kinematically allowed $|\vec{k}|$ values $0 \leq |\vec{k}| \leq 2m_\chi (\frac{c_l \gamma}{c_l^2 - 1} + \frac{\sqrt{\gamma^2 - 1}}{|c_l^2 - 1|})$, and Eq. (11) takes the form

$$R_k^{(0)} = \frac{8\pi^4 n_A^2 V}{(2\pi)^6 m_\chi^2 m_A c_l} \int_0^\infty |\vec{p}|^2 d|\vec{p}| f_\chi(\vec{p}) \frac{|\gamma m_\chi - |\vec{k}| c_l|}{m_\chi \sqrt{\gamma^2 - 1}} a^2. \quad (12)$$

In Fig. 1, we show the single phonon excitation rate (per unit volume) from the ground state and averaged over χ phase space as a function of density in the outer crust using the single-nucleus table from Ref. [5]. Curves plotted with solid, dashed and dash-dotted lines correspond to the excitation of phonons with $|\vec{k}| \rightarrow 0$ for $m_\chi = 500, 100$ and 5 MeV and $n_\chi/n_{0,\chi} = 10$. We also plot for the sake of comparison the specific excitation rate at $|\vec{k}| \rightarrow 0$, $R_{l,0}$, for neutrinos with masses $m_\nu = 0.1, 1 \text{ eV}$ with dotted and double-dashed lines, respectively. Note, however, that in this later case there is a strong momentum

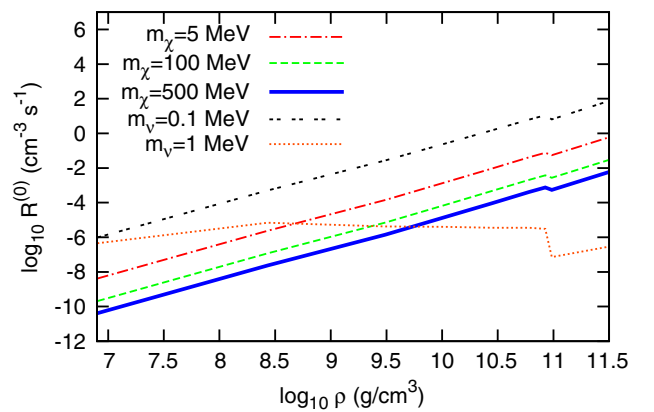


FIG. 1. Averaged single-phonon excitation rate per unit volume as a function of density in the outer crust. DM particle masses $m_\chi = 500, 100$ and 5 MeV are used, and $n_\chi/n_{0,\chi} = 10$. Neutrino contribution at $|\vec{k}| \rightarrow 0$, $R_{l,0}$ is also shown for $m_\nu = 0.1, 1 \text{ eV}$. See the text for details.

dependence that declines rapidly. We can fit this behavior for $m_\nu = 0.1$ eV as $R_\nu^0(|\vec{k}|) = R_{\nu 0} e^{\frac{-1754|\vec{k}|}{1\text{eV}}}$ and for $m_\nu = 1$ eV as $R_\nu^0(|\vec{k}|) = R_{\nu 0} e^{\frac{-2561.3|\vec{k}|}{1\text{eV}}}$.

We have verified that, since, typically, the speed of the thermalized LDM particles far from the star is essentially $v_\chi \sim v_\infty \sim 10^{-3}$, when hitting the NS, it has already acquired a boosted energy. Using an estimate based on a monochromatic value $E_\chi = \gamma_{\text{NS}} m_\chi$, $\gamma_{\text{NS}} = 1.26$, we can straightforwardly integrate and obtain the analytical result

$$R_k^{(0)} = \frac{n_\chi n_A^2 V}{4(2\pi)^3 m_\chi^3 m_A c_l} \frac{|\gamma_{\text{NS}} m_\chi - |\vec{k}| c_l|}{\sqrt{\gamma_{\text{NS}}^2 - 1}} a^2 \quad (13)$$

that underpredicts the exact result by $\sim 20\%$. As deduced from the previous expression, Eq. (13), the rate is indeed constant as a function of momentum as the inequality $\gamma_{\text{NS}} m_\chi \ll |\vec{k}| c_l$ is fulfilled. It seems that the contribution of the phase space distribution of LDM may also have a strong impact on the results, as it happens for the Sun or Earth.

III. ASTROPHYSICAL IMPACT ON THERMAL CONDUCTIVITY

Phonon production can be crucial for determining further transport properties, in particular, thermal conductivity in an ion-electron system such as that in the outer NS crust. As an important contribution to the total ion conductivity, κ_i , partial ion conductivities due to ion-ion, $\kappa_{ii} \equiv \kappa_{ph}$, and ion-electron collisions, κ_{ie} , must be added [27] under the prescription $\kappa_i^{-1} = \kappa_{ii}^{-1} + \kappa_{ie}^{-1}$. Standard mechanisms to produce lattice vibrations include thermal excitations, as analyzed in detail in previous works [28,29]. In a NS, the outer crust can be modelled under the one-component-plasma description. This low density solid phase can be classified according to the Coulomb coupling parameter $\Gamma = Z^2 e^2 / a k_B T$ where $a = (4\pi n_A / 3)^{1/3}$ is the ion sphere radius. It is already known that typically for $\Gamma \geq \Gamma_m \approx 175$, or below melting temperature $T < T_m$, single-ion systems crystallize [30].

There are a number of processes that can affect thermal conductivity in the medium. The so-called U-processes [7] are responsible for modifying the electron conductivity such that for high temperatures, $T > T_U$, electrons move almost freely. Assuming a bcc lattice, $T_U \approx 0.07 T_D$. Thus, in the scenario depicted here, the temperature range must be $T_U < T < T_D < T_m$ for each density considered. According to kinetic theory, the thermal conductivity can be written in the form [7]

$$\kappa_{ii} = \frac{1}{3} k_B C_A n_A c_l L_{ph}, \quad (14)$$

where $C_A = 9 \left(\frac{T}{T_D}\right)^3 \int_0^{T_D/T} \frac{x^4 e^x dx}{(e^x - 1)^2}$ is the phonon (dimensionless) heat capacity *per ion* and L_{ph} is an effective phonon

mean free path that includes all scattering processes considered: U-processes and impurity (I) scattering processes (both dissipative) and the phonon normal (N) scattering, which are nondissipative $L_{ph}^{-1} = L_U^{-1} + L_I^{-1} + L_N^{-1}$. Typically, the thermal conductivity is related to the thermal phonon number at temperature T , $L_{ph} \sim 1/N_{0,k\lambda}$, where $N_{0,k\lambda} = (e^{\omega_{k\lambda}/k_B T} - 1)^{-1}$. The contribution from DM can be obtained by the net number of phonons that results from the competition of thermal and scattering excitation and stimulated emission [9] in a 4-volume $\delta V \delta t$ using the averaged rate per unit volume, and weighting with the incoming distribution providing the frequencies of different values of momenta, we obtain

$$N_{k\lambda} \approx N_{0,k\lambda} + R_k^{(0)} \delta V \delta t - \int \frac{d^3 \vec{p}}{n_\chi} f_\chi(\vec{p}) \tilde{R}_k^{(0)} N_{0,k\lambda} e^{(\omega_{k,\lambda} + \vec{k} \cdot \vec{v})/K_\chi} \delta V \delta t, \quad (15)$$

where $K_\chi = (\gamma - 1)m_\chi$ is the χ kinetic energy and $\tilde{R}_k^{(0)}$ is the single phonon excitation rate for each particular momentum value (not averaged over incoming χ momenta). Since the source (NS) is in relative motion to the LDM flux, there is a Doppler shift characterized by the source velocity $v \equiv v_{\text{NS}} \sim 10^{-2}$, i.e., galactic NS drift velocity. The distribution of NSs in our Galaxy peaks at distances $\langle r \rangle_{\text{max}} \lesssim 4$ Kpc [31]. In this central region DM density is enhanced with respect to the solar neighborhood value $n_{0,\chi} \approx 0.3$ GeV/cm³. Thus we will consider density values $n_\chi \approx (10, 100)n_{0,\chi}$ as prescribed by popular galactic DM distribution profiles. In Fig. 2, we show the phonon thermal conductivity as a function of density (in units of 10^{10} g/cm³) at $T = 510^7$ K, 510^8 K typical for the base of the crust, for $m_\chi = 100$ MeV. Solid lines are the standard thermal result with no DM. Dash-dotted and dashed lines correspond to

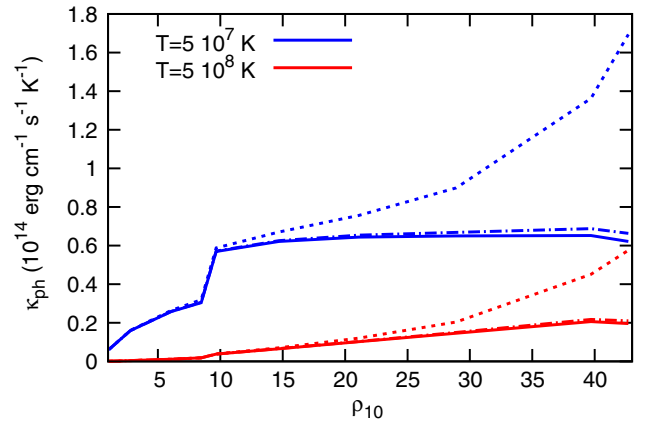


FIG. 2. Phonon thermal conductivity as a function of density (in units of 10^{10} g/cm³) for temperatures $T = 510^7$ K (blue), 510^8 K (red) and $m_\chi = 100$ MeV. Dash-dotted and dashed lines depict the impact of a LDM density $n_\chi/n_{0,\chi} = 10, 100$. Solid lines are the standard thermal result with no DM for each case. See the text for details.

$n_\chi/n_{0,\chi} = 10, 100$, respectively. We see that at the largest LDM local densities considered there is a clear enhancement over the thermal result well inside the outer crust. This corresponds to the site where the DM-induced effects have the most influence [32] as this is the most massive part of the outer crust. Below these densities, there is a negligible change, though. At lower T , the effect of a perturbation over the thermal phonon population is more important. Enhanced (decreased) conductivities at moderate LDM densities are due to a net reduction (increase) of the number of phonons in the lattice as a result of cancellation of modes. As a representative scenario, we have taken $|\vec{k}| = 0.01/a$ at each density since we have verified that this choice verifies the kinematical restrictions on $|\vec{k}|$ when performing the averages over phase space distribution as discussed in Sec. II. Besides, rates are mostly constant at low $|\vec{k}|$. Note that in standard calculations [27] there is no momentum dependence as they replace the frequency mode $\omega_{k\lambda}$ by a constant threshold. We must bear in mind that this result must be compiled with a realistic impurity fraction so that conductivity remains finite. We have considered $L_I \sim 5a$ [27].

In order to understand the significance of our result in the dense stellar context in Fig. 3, we show the phonon thermal conductivity as a function of density (in units of 10^{10} g/cm^3) at $T = 10^8 \text{ K}$ and $m_\chi = 65 \text{ MeV}$ for $|\vec{k}| = 0.01/a$. Solid, dot-dashed and dashed lines correspond to cases with no DM, $n_\chi/n_{0,\chi} = 10, 100$, respectively. Electron thermal conductivity is also shown for magnetized realistic scenarios in the direction perpendicular to a magnetic field B of strength $B = 10^{14} \text{ G}$ (dotted) and $B = 10^{15} \text{ G}$ (doble dotted). Ions are mostly not affected by the presence of a magnetic field. The parallel direction electronic contribution is not

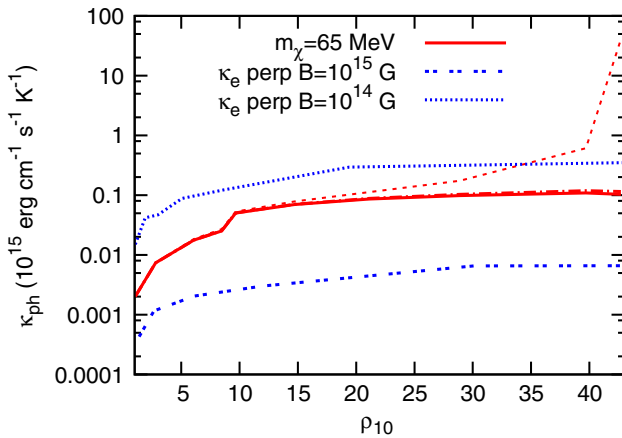


FIG. 3. Phonon thermal conductivity as a function of density (in units of 10^{10} g/cm^3) at $T = 10^8 \text{ K}$ and $m_\chi = 65 \text{ MeV}$. Solid, dot-dashed and dashed lines correspond to cases with no DM, $n_\chi/n_{0,\chi} = 10, 100$. Perpendicular electron thermal conductivity is also shown for $B = 10^{14}, 10^{15} \text{ G}$.

depicted here since it is typically much larger $k_{e\parallel} \sim 10^{17}-10^{19} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-1}$. Since we perform averages over the χ phase space, we again use $|\vec{k}| = 0.01/a$. On the plot, we can see that the electronic contribution in the perpendicular direction falls below the enhanced $n_\chi/n_{0,\chi} = 100$ DM value for densities $\gtrsim 3.5 \cdot 10^{11} \text{ g/cm}^3$. Note that the low value chosen for $|\vec{k}|$ in this plot is to be understood as a compromise value; larger $|\vec{k}|$ values would imply the impossibility of exciting phonons from low-momenta incoming LDM. Since the global conductivity is $\kappa = \kappa_e + \kappa_{ph}$, the obtained result is expected to contribute to the reduction of the difference in heat conduction in both directions and thus to the isotropization of the NS surface temperature pattern as seen in Ref. [32] for standard physics. Temperatures would be smoothly driven toward more isothermal profiles for latitudes among the pole and equator. It is already known [33] that the outer crust plays an important role in regulating the relation among the temperature in the base of it and the surface. The detailed calculation of this implication for surface temperatures remains, however, for future work.

IV. CONCLUSIONS

In conclusion, we have derived for the first time the single-phonon excitation rate in the outer NS crust for relativistic LDM particles in the sub-GeV mass range. We have found that this rate is constant with the phonon momentum and much larger than for cosmological neutrinos at finite $|\vec{k}|$. A non-negligible correction to the local phonon excitation rate of $\sim 20\%$ is obtained when full relativistic phase distribution functions are considered for the incoming χ particles with respect to a monochromatic approximation, that underpredicts the result.

As an astrophysical consequence of the previous, we have calculated the ion thermal conductivity in the dense and hot outer envelope, finding that it can be largely enhanced at LDM densities in the maximum of the NS galactic distribution $n_\chi \sim 100n_{0,\chi}$ due to a net modification of the acoustic phonon population. This effect is non-negligible at densities beyond $\sim 3.5 \cdot 10^{11} \text{ g/cm}^3$ in the base of the outer crust at the level of standard ion-electron or thermal effects [27,28]. We do not expect the degenerate electron contribution to largely modify this result as this would mildly screen nuclear charge in the lattice; however, it remains to be further studied. Although a detailed study of the quantitative effect in the surface temperature pattern remains to be undertaken, it is expected that for magnetized NSs the LDM-enhanced global enhancement of the perpendicular thermal conductivity allows a reduction of the difference of heat transport among parallel and perpendicular directions to the magnetic field. Based on previous works only including standard thermal contributions, we expect that, as a natural consequence, the surface temperature profile would be more isotropic, yielding

flatter profiles for intermediate latitudes, and remains to be calculated in a future contribution.

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- [1] P. Ade *et al.* (Planck Collaboration), *Astron. Astrophys.* **571**, A16 (2014).
- [2] R. Bernabei *et al.* (DAMA and LIBRA Collaborations), *Eur. Phys. J. C* **67**, 39 (2010).
- [3] T. Lin, H.-B. Yu, and K. M. Zurek, *Phys. Rev. D* **85**, 063503 (2012).
- [4] R. Essig, M. Fernández-Serra, J. Mardon, A. Soto, T. Volansky, and T.-T. Yu, *J. High Energy Phys.* **05** (2016) 046.
- [5] S. B. Ruster, M. Hempel, and J. Schaffner-Bielich, *Phys. Rev. C* **73**, 035804 (2006).
- [6] C. J. Horowitz, M. A. Pérez-García, and J. Piekarewicz, *Phys. Rev. C* **69**, 045804 (2004); M. A. Pérez-García, *J. Math. Chem.* **48**, 21 (2010).
- [7] J. M. Ziman, *Electrons and Phonons* (Oxford University Press, Oxford, 1960).
- [8] W. J. Carr, *Phys. Rev.* **122**, 1437 (1961).
- [9] I. Ferreras and I. Wasserman, *Phys. Rev. D* **52**, 5459 (1995); R. Chandrasekharan, Diploma Thesis, Institute for particle physics, ETHZ, 2003.
- [10] M. M. Hedman, *J. Cosmol. Astropart. Phys.* **09** (2013) 029.
- [11] A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, and Y. Xu, *J. Cosmol. Astropart. Phys.* **02** (2013) 004.
- [12] M. Cermeño, M. A. Pérez-García, and J. Silk, *Phys. Rev. D* **94**, 023509 (2016).
- [13] J. Goodman, M. Ibe, A. Rajaraman, W. Spherd, T. M. P. Tait, and H.-B. Yu, *Phys. Rev. D* **82**, 116010 (2010).
- [14] K. Cheung, P. Tseng, Y. S. Tsai, and T. Yuan, *J. Cosmol. Astropart. Phys.* **05** (2012) 001.
- [15] V. Khachatryan *et al.* (CMS Collaboration) *Eur. Phys. J. C* **75**, 235 (2015); G. Aad *et al.*, *Phys. Rev. D* **90**, 012004 (2014); **91**, 012008 (2015).
- [16] J.-M. Zheng, Z.-H. Yu, J.-W. Shao, X.-J. Bi, Z. Li, and H.-H. Zhang *Nucl. Phys.* **B854**, 350 (2012).
- [17] V. Gluscevic, M. I. Gresham, S. D. McDermott, A. H. G. Peter, and K. M. Zurek, *J. Cosmol. Astropart. Phys.* **12** (2015) 057.
- [18] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders College, Philadelphia, 1976).
- [19] C. J. Horowitz, [arXiv:1205.3541v1](https://arxiv.org/abs/1205.3541v1).
- [20] A. M. Green, *Phys. Rev. D* **66**, 083003 (2002); *Mod. Phys. Lett. A* **27**, 1230004 (2012).
- [21] M. Fornasa and A. M. Green, *Phys. Rev. D* **89**, 063531 (2014).
- [22] K. Choi, C. Rott, and Y. Itow, *J. Cosmol. Astropart. Phys.* **05** (2014) 049.
- [23] F. Jüttner, *Ann. Phys. Chem.* **34**, 856 (1911).
- [24] G. M. Kremer, *J. Stat. Mech.* (2013) P04016.
- [25] R. Hakim, *Introduction to Relativistic Statistical Mechanics Classical and Quantum* (World Scientific, Singapore, 2011).
- [26] C. Cercignani *et al.*, *The Relativistic Boltzmann Equation: Theory and Applications* (Birkhäuser Verlag, Berlin, 2002).
- [27] A. I. Chugunov and P. Haensel, *Mon. Not. R. Astron. Soc.* **381**, 1143 (2007).
- [28] J. W. Negele and D. Vautherin, *Nucl. Phys.* **A207**, 298 (1973); A. Y. Potekhin, G. Chabrier, and D. G. Yakovlev, *Astron. Astrophys.* **323**, 415 (1997).
- [29] D. A. Baiko, A. D. Kaminker, A. Y. Potekhin, and D. G. Yakovlev, *Phys. Rev. Lett.* **81**, 5556 (1998).
- [30] H. Nagara, Y. Nagata, and T. Nakamura, *Phys. Rev. A* **36**, 1859 (1987).
- [31] D. R. Lorimer *et al.*, *Mon. Not. R. Astron. Soc.* **372**, 777 (2006).
- [32] J. F. Pérez-Azorín, J. A. Miralles, and J. A. Pons, *Astron. Astrophys.* **451**, 1009 (2006).
- [33] A. D. Kaminker, D. G. Yakovlev, A. Y. Potekhin, N. Shibazaki, P. S. Shternin, and O. Y. Gnedin, *Astrophys. Space Sci.* **308**, 423 (2007).