Study of the malware SCIRS model with different incidence rates

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Abstract

A study of a SCIRS model for malware propagation with different incidence rates is introduced in this work. This analysis is based on a previous mathematical model to simulate malware spreading in wireless networks where susceptible, carrier, infectious and recovered devices are considered. The notion of incidence is revisited and several types (bilinear, standard and saturated with respect to the infectious and susceptible devices) are studied. Furthermore, the associated basic reproductive numbers are explicitly computed.

Keywords: Malware, wireless networks, mathematical modelling, incidence rate, basic reproductive number.

1 Introduction

Nowadays, malware is one of the major cybersecurity threats in wireless networks. This is mainly because of the extensive use of smartphones and all types of wireless devices, together with the establishment of new paradigms such as the Internet of Things, Bring Your Own Device, Industry 4.0, etc.

In this sense, it is extremely important not only to detect the malware but also to successfully predict its propagation over a wireless network. Furthermore, it is also of interest to simulate the behaviour of possible control measures in order to be implemented in an efficient way.

These tasks (the simulation of malware spreading and the control measures) are achieved by the computational implementation of mathematical models. The great majority of them are deterministic and global and, consequently, they are based on systems of ordinary differential equations (see e.g. [1, 7–9, 11]).

Of special interest is the mathematical model considering carrier devices proposed in [12]. Specifically, it is a compartmental Susceptible-Carrier-Infectious-Recovered-Susceptible (SCIRS) model whose dynamics is governed by means of a system of four ordinary differential equations. From the mathematical study of this system, the equilibrium points and the basic reproductive number are explicitly computed and the stability of the system is stated.

The incidence (i.e. the total number of new infected devices per unit of time) is a key pillar in the design of a mathematical epidemiological model.

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Vol. 27, No. 2, © The Author(s) 2018. Published by Oxford University Press. All rights reserved.
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Advance Access published 12 September 2018 doi:10.1093/jigpal/jzy033

In [12] the incidence is given by the expression $a \cdot S(t) \cdot I(t)$, where a is the transmission coefficient and $S(t)$, $I(t)$ stand for the number of susceptible and infectious devices at time step, t, respectively. As a consequence, it is a general expression for the incidence that follows the mass-action law. The incidence determines the explicit expression of the basic reproductive number, R_0, which is a fundamental epidemiological threshold that provides information about the future behaviour of the malware outbreak.

The main goal of this paper is to analyse the behaviour of the model by considering not only the classical types of incidence, bilinear and standard, but also saturation incidences with respect to the total number of infected devices and the total number of susceptible devices. In addition, the explicit expressions for the basic reproductive numbers associated to each incidence are computed.

The rest of the paper is organized as follows: in Section 2 the description of the SCIRS model for malware propagation over wireless networks is shown; in Section 3 the notion of incidence is revisited and the most important types are introduced; the explicit computation of the basic reproductive numbers associated to each type of incidence is shown in Section 4; and finally, the conclusions and further work are presented in Section 5.

2 Review of the original SCIRS model

2.1 Description of the dynamics of the model

The SCIRS model introduced in [12] to simulate malware spreading in a wireless network is a global model which is governed by means of a system of ordinary differential equations. Thus, it is a compartmental model whose variables are $S(t)$ for susceptible devices at time t, $C(t)$ for carrier devices, $I(t)$ for infectious devices and $R(t)$ for recovered devices. Furthermore, the main assumptions of the model are the following (the flow diagram representing the dynamics of the model is shown in Figure 1):

(i) The infection of susceptible devices depends on the infection rate a and the coefficient δ standing for the fraction of susceptible devices with the same operative system that this targeted by the malware. In this sense, a susceptible device reaches the infectious state at rate δa whereas becomes carrier at rate $(1 - \delta)\, a$.
(ii) Once security software is installed, a susceptible device can acquire temporal immunity (and reaches the recovered state) according to the vaccination rate v.
(iii) When the malicious code is detected and successfully removed, an infectious or carrier device acquires temporal immunity according to the recovered rates b_C and b_I, respectively.
(iv) Finally, a recovered device becomes susceptible again at rate ϵ.

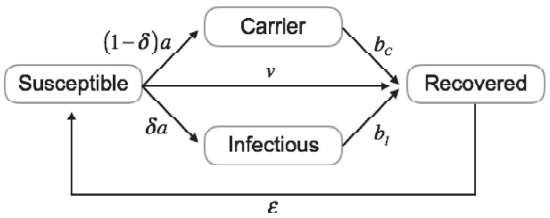

FIGURE 1. Dynamic of the SCIRS model for malware propagation.

TABLE 1. Coefficients of the SCIRS model

Coefficient	Description	Range
a	Transmission coefficient	$[0, 1]$
v	Vaccination coefficient	$[0, 1]$
ϵ	Loss of immunity coefficient	$[0, 1]$
δ	Fraction of mobile devices based on the targeted OS	$[0, 1]$
b_C	Recovered coefficient for carrier devices	$[0, 1], b_C \ll b_I$
b_I	Recovered coefficient for infectious devices	$[0, 1], b_I \gg b_C$

Moreover, the system of ordinary differential equations that describes the dynamics of the model is:

$$S'(t) = -a \cdot S(t) \cdot I(t) - v \cdot S(t) + \epsilon \cdot R(t), \tag{1}$$

$$C'(t) = a \cdot (1 - \delta) S(t) \cdot I(t) - b_C \cdot C(t), \tag{2}$$

$$I'(t) = a \cdot \delta \cdot S(t) I(t) - b_I \cdot I(t), \tag{3}$$

$$R'(t) = b_C \cdot C(t) + b_I \cdot I(t) + v \cdot S(t) - \epsilon \cdot R(t), \tag{4}$$

with the following initial conditions:

$$S(0) = S_0, C(0) = C_0, I(0) = I_0, R(0) = N - S_0 - C_0 - I_0, \tag{5}$$

$$S(t) \geq 0, C(t) \geq 0, I(t) \geq 0, R(t) \geq 0, \tag{6}$$

$$N = S(t) + C(t) + I(t) + R(t). \tag{7}$$

Furthermore, in Table 1 the coefficients involved in the model are shown.

2.2 Equilibrium points and basic reproductive number

A simple calculus shows that there are two equilibrium points of this model, the disease-free equilibrium point

$$E_0^* = \left(S_0^*, C_0^*, I_0^*, R_0^*\right) = \left(\frac{\epsilon N}{\epsilon + v}, 0, 0, \frac{vN}{\epsilon + v}\right), \tag{8}$$

and the endemic equilibrium point $E_1^* = \left(S_1^*, C_1^*, I_1^*, R_1^*\right)$, with

$$S_1^* = \frac{b_I}{a\delta}, \tag{9}$$

$$C_1^* = \frac{b_I (1 - \delta)}{b_C \delta} I_1^*, \tag{10}$$

$$I_1^* = \frac{b_C (aN\delta\epsilon - b_I v - b_I \epsilon)}{a (b_I b_C + b_I \epsilon + \delta\epsilon (b_C - b_I))}, \tag{11}$$

$$R_1^* = \frac{b_I (b_I b_C - b_I v - a b_C N\delta + v\delta (b_I - b_C))}{\delta b_C (b_I v + b_I \epsilon - aN\delta\epsilon)} I_1^*. \tag{12}$$

It is shown that a malware outbreak can evolve to two different scenarios:

1. The number of infected devices does not increase and the malware outbreak dies out (disease-free state). In this case, the final number of susceptible devices decreases to S_0^* whereas the number of recovered devices increases to R_0^* and there are no infectious or carrier devices ($C_0^* = I_0^* = 0$).
2. A growth of the number of infectious devices occurs reaching the endemic steady state. In this case, the final number of infectious devices is I_1^*. Moreover, all susceptible devices will be infected if $b_I = 0$.

Furthermore, a simple computation shows that the basic reproductive number R_0 is the following:

$$R_0 = \frac{a\delta\epsilon N}{b_I\,(\epsilon + v)}. \tag{13}$$

Recall that this is an important epidemiological threshold parameter since it is defined as the expected number of secondary infections produced by an unique infectious device in a completely susceptible network. In fact, it can be considered as a metric of the potential for disease spread within a network: if $R_0 < 1$ the malware disease does not spread, whereas if $R_0 > 1$ the number of infectious devices increases. Furthermore, a simple mathematical analysis of the basic reproductive number yields the following control measures to control the malware outbreak:

(1) Reducing the total number of mobile devices on the network N or the number of devices running under the targeted operative system δ by means of, e.g. isolation.
(2) Reducing the transmission coefficient a by reducing the number of effective contacts between devices or extreme caution when opening suspicious messages.
(3) Reducing the loss of immunity coefficient ϵ by using efficient antivirus software.
(4) Increasing the recovery rate b_I by improving the performance of antivirus software.
(5) Increasing the vaccination coefficient v by sensitizing users to instal security countermeasures.

2.3 Stability analysis of the equilibrium points

From a qualitative study of the behaviour of the solutions of the system (1–4), the following results are obtained (see [12]):

THEOREM 2.1
The disease-free steady state E_0^* is locally and globally asymptotically stable if and only if $R_0 \leq 1$.

THEOREM 2.2
The endemic equilibrium E_1^* exists when

$$N > \left\{ \frac{b_I\,(v+\epsilon)}{a\delta\epsilon}, \frac{b_I\,(b_C - v) + v\delta\,(b_I - b_C)}{a\delta b_C} \right\}, \tag{14}$$

and in this case it is locally and globally asymptotically stable if $R_0 > 1$.

3 The incidence of an epidemic process

3.1 Classical incidence

As is well known, the spreading of malware occurs by means of a direct contact between a susceptible device and an infectious device. A contact between two devices is said to be adequate when such a

contact leads to an infection. The contact rate, k, can be defined as the number of adequate contacts (per unit of time) between a susceptible device and the rest of devices. Usually it depends on the total number of devices N, i.e. $k = k(N)$. An effective contact is an adequate contact that leads to a successful infection; if q is the probability of infection, then $q \cdot k(N)$ stands for the number of effective contacts of each device with the rest of devices per unit of time.

The incidence of a malware epidemic process is the number of new infectious devices per unit of time. Usually, the incidence is proportional to the number of susceptible devices $S(t)$:

$$\text{incidence} = \lambda \cdot S(t), \tag{15}$$

where λ is called the force of infection, and usually it can be mathematically defined as follows:

$$\lambda = q \cdot \frac{k(N)}{N} \cdot I(t). \tag{16}$$

Since $q \cdot \frac{k(N)}{N}$ is the average number of effective contacts of each susceptible device with each device per unit of time, then the incidence stands for the average number of effective contacts of each susceptible device with the infectious devices of the network per unit of time. As a consequence,

$$\text{incidence} = g(N, I(t)) \cdot S(t) = q \cdot \frac{k(N)}{N} \cdot I(t) \cdot S(t), \tag{17}$$

where $a = q \cdot \frac{k(N)}{N}$ is the transmission rate.

3.2 Main types of incidence

As you can see it is very important to determine correctly the contact rate $k(N)$ since it depends on the authenticity of the simulations obtained from the mathematical model.

Different expressions for the contact rate can be considered, and consequently, different expressions for the incidence are derived (see e.g. [3]). Between them we can distinguish the following three: the bilinear incidence (defined by the bilinear contact rate), the standard incidence (given by the standard contact rate) and the saturation incidence (characterized by the use of the saturation contact rate).

3.2.1 Bilinear incidence The bilinear incidence (also called density-dependent incidence or simple mass action) is defined by the bilinear contact rate which is proportional to the total number of devices:

$$k(N) = \alpha_d \cdot N, \quad \alpha_d > 0. \tag{18}$$

As a consequence,

$$\text{bilinear incidence} = q \cdot \alpha_d \cdot I(t) \cdot S(t), \tag{19}$$

where α_d is the average number of adequate contacts between two devices per unit of time. Note that, in this case the transmission rate is $a = q \cdot \alpha_d$. This type of incidence is not very realistic except in the first stages of a malware epidemic process in a moderate network.

3.2.2 Standard incidence When the contact rate does not depend on the population size N and it remains constant, we obtain the standard incidence (also called frequency dependent incidence):

$$k(N) = \alpha_f, \quad \alpha_f > 0, \tag{20}$$

thus

$$\text{standard incidence} = q \cdot \frac{\alpha_f}{N} \cdot I(t) \cdot S(t), \tag{21}$$

where α_f is the average number of adequate contacts of each device with the rest of devices of the network per unit of time. Note that $\alpha_f = \frac{\alpha_d}{N}$ and the transmission coefficient is given by $a = q \cdot \frac{\alpha_f}{N}$.

This type of incidence is more realistic than the previous one when it comes to simulating the behaviour of the malware propagation through a transmission vector as Bluetooth (since in this situation it is more appropriate to suppose that the number of contacts is a non-increasing function in regards with the total number of devices N).

3.2.3 Saturation incidence The bilinear and standard contact rates can be considered as extreme cases of the behaviour of the contacts when population varies: in the bilinear case the contacts increases linearly with the population N, whereas in the standard case, the contacts remain constant (they do not depend on N).

From a mathematical point of view, the saturation contact $k(N)$ is characterized by the following properties:

(i) In the absence of population, the coefficient is null: $k(0) = 0$.
(ii) The saturation contact coefficient does not decreases as the total number of devices increases: $k'(N) \geq 0$.
(iii) As the population increases, the saturation contact coefficient tends to a fixed finite value:

$$\lim_{N \to \infty} k(N) = \alpha_0 \in \mathbb{R}^+. \tag{22}$$

That is, for a certain value of N, the average number of adequate contacts increases minimally or, even, stays constant although the total number of devices increases.
(iv) The average number of adequate contacts between two devices decreases or remains constant although the total number of devices increases:

$$\left(\frac{k(N)}{N}\right)' \leq 0. \tag{23}$$

The most important examples of saturation contact coefficients are the following:

(1) Saturation contact rate due to K. Dietz [4]—also known as Michaelis–Menten coefficient:

$$k(N) = \frac{uN}{1 + vN}, \quad u \geq 0, v \geq 0. \tag{24}$$

(2) Saturation contact rate due to Heesterbeek & Metz [6]:

$$k(N) = \frac{uN}{1 + vN + \sqrt{1 + 2uN}}, \quad u \geq 0. \tag{25}$$

Note that if the size of the population is small, then $k(N) \sim uN$, whereas if N is large, then $k(N) \sim 1$.
(3) Saturation contact rate due to Mena Lorca & Hethcote [13]:

$$k(N) = uN^v, \quad 0 < u, 0 < v < 1. \tag{26}$$

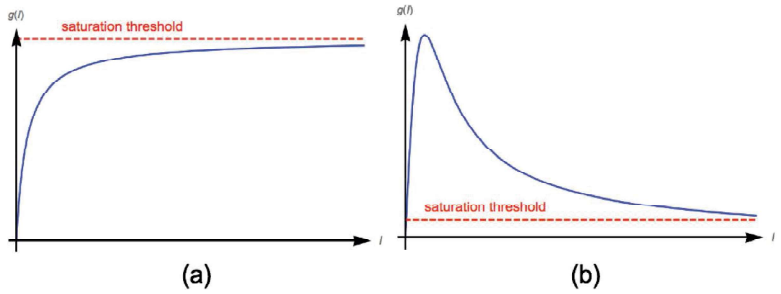

FIGURE 2. (a) Monotone force of infection $g\,(I)$ saturated with respect to I. (b) Non-monotone force of infection $g\,(I)$ saturated with respect to I.

3.3 Other important types of incidence

Note that in the types of incidences described in the Section 3.2, the contact coefficients are saturated with respect to the total number of devices N. Nevertheless, some alternative proposals have appeared where the incidence is saturated taking into account the number of infectious devices or the number of susceptible devices.

In this sense, Capasso & Serio [2] introduced saturation incidences where the force of infection does not depend on N:

$$\text{incidence} = g\,(I\,(t)) \cdot S\,(t)\,, \tag{27}$$

where g is a non-linear function that converges to a certain saturation threshold (see Figure 2a). This is the case of the following function [2]:

$$g\,(I) = \frac{u \cdot I}{1 + \frac{1}{v} \cdot I}, \quad u > 0, v > 0. \tag{28}$$

The use of this type of force of infection, $\lambda = g\,(I)$, makes it possible to take into account psychological aspects: the users of susceptible devices will tend to reduce the number of contacts with the rest of devices if there is a public awareness of malware epidemic. As a consequence, to take into account such effect, an additional condition must be considered: the force of infection $g\,(I)$ must decrease as I increases. Thus, we obtain a non-monotone function $g\,(I)$ that increases for small values of I and that decreases (tending to a certain saturation level) for large values of I (see Figure 2b).

As paradigmatic example of this class of incidence, we can show the incidence due to Xiao & Ruan [15]:

$$\text{incidence} = \frac{u \cdot I\,(t)}{1 + v \cdot I\,(t)^2} \cdot S\,(t)\,, \quad u > 0, v > 0. \tag{29}$$

Several modified versions have also been proposed:

- Ruan & Wang [14] proposed a non-linear incidence saturated with respect to the infectious devices:

$$\text{incidence} = g\,(I) \cdot S\,(t) = q \cdot \frac{I\,(t)^l \cdot S\,(t)}{1 + \alpha \cdot I\,(t)^h}, \quad \alpha \geq 0, l > 0, h > 0. \tag{30}$$

- Incidence introduced by Liu *et al.* [10]:

$$\text{incidence} = g(I,S) = \beta \cdot I(t)^p \cdot S(t)^q, \quad p > 0, q > 0. \tag{31}$$

- Incidence due to Van den Driessche & Watmough [5]:

$$\text{incidence} = g_0(I) + g_1(I) \cdot S(t) = \beta \cdot I(t) \left(1 + v \cdot I(t)^{k-1} \cdot S(t)\right), \tag{32}$$
$$\beta > 0, k > 0, v \geq 0.$$

- Zhang *et al.* [16] proposed a saturated incidence with respect to the susceptible devices and endowed with time-dependent parameters:

$$\text{incidence} = q(t) \cdot \frac{I(t) \cdot S(t)}{1 + \alpha(t) \cdot S(t)}. \tag{33}$$

4 Computing the basic reproductive numbers

4.1 The general case

If we consider the general expression for the incidence, $h(I,S)$, the explicit expression of the system of ordinary differential equations of the SCIRS model introduced in Section 2 is as follows:

$$S'(t) = -h(I(t), S(t)) - v \cdot S(t) + \epsilon \cdot R(t), \tag{34}$$

$$C'(t) = (1-\delta) \cdot h(I(t), S(t)) - b_C \cdot C(t), \tag{35}$$

$$I'(t) = \delta \cdot h(I(t), S(t)) - b_I \cdot I(t), \tag{36}$$

$$R'(t) = b_C \cdot C(t) + b_I \cdot I(t) + v \cdot S(t) - \epsilon \cdot R(t). \tag{37}$$

If we apply the next-generation method to compute the basic reproductive number R_0 from the system of ordinary differential equations (34–37), we obtain that the next-generation matrix is

$$G = F \cdot V^{-1} = \begin{pmatrix} 0 & \frac{1-\delta}{b_I} \cdot \frac{\partial h}{\partial I} \\ 0 & \frac{\delta}{b_I} \cdot \frac{\partial h}{\partial I} \end{pmatrix} \tag{38}$$

where

$$F = \begin{pmatrix} \frac{\partial F_C}{\partial C} & \frac{\partial F_C}{\partial I} \\ \frac{\partial F_I}{\partial C} & \frac{\partial F_I}{\partial I} \end{pmatrix} = \begin{pmatrix} 0 & (1-\delta) \cdot \frac{\partial h}{\partial I} \\ 0 & \delta \cdot \frac{\partial h}{\partial I} \end{pmatrix}, \tag{39}$$

$$V = \begin{pmatrix} \frac{\partial V_C}{\partial C} & \frac{\partial V_C}{\partial I} \\ \frac{\partial V_I}{\partial C} & \frac{\partial V_I}{\partial I} \end{pmatrix} = \begin{pmatrix} b_C & 0 \\ 0 & b_I \end{pmatrix}, \tag{40}$$

and

$$F_C = (1-\delta)\cdot h(I,S), \tag{41}$$

$$F_I = \delta \cdot h(I,S), \tag{42}$$

$$V_C = b_C \cdot C, \tag{43}$$

$$V_I = b_I \cdot I. \tag{44}$$

Consequently, the basic reproductive number is the spectral radius of the matrix G located at the disease-free equilibrium point E_0^*:

$$R_0 = \frac{\delta}{b_I}\cdot\left(\frac{\partial h}{\partial I}\right)_{E_0^*}. \tag{45}$$

Note that if $h(I,S) = g(N,I)\cdot S$ or if $h(I,S) = g(I)\cdot S$, then

$$R_0 = \frac{\delta}{b_I}\cdot S_0^*\cdot\left(\frac{\partial g}{\partial I}\right)_{I=0}. \tag{46}$$

On the other hand, the disease-free equilibrium point can be easily obtained by solving the system:

$$0 = -h(I,S) - v\cdot S + \epsilon R, \tag{47}$$

$$0 = (1-\delta)\cdot h(I,S) - b_C\cdot C, \tag{48}$$

$$0 = \delta\cdot h(I,S) - b_I I, \tag{49}$$

$$0 = b_C\cdot C + b_I\cdot I + v\cdot S - \epsilon\cdot R, \tag{50}$$

when $I = 0$. In this case $h(0,S) = 0$ and consequently

$$E_0^* = \left(S_0^* = \frac{\epsilon N}{v+\epsilon}, C_0^* = 0, I_0^*, R_0^* = \frac{vN}{v+\epsilon}\right). \tag{51}$$

4.2 The R_0 associated to the basic incidences

A simple calculus shows that the explicit expressions of the basic reproductive numbers associated to the system endowed with the incidences introduced in Section 3.2 are shown in Table 2.

TABLE 2. Explicit expressions of the basic reproductive number

Incidence	R_0
Bilinear	$\frac{q\cdot\alpha\cdot N\cdot\alpha\cdot\epsilon}{b_I(\epsilon+v)}$
Standard	$\frac{q\cdot\alpha\cdot f\cdot\delta\cdot\epsilon}{b_I(\epsilon+v)}$
Dietz	$\frac{q\cdot u\cdot N\cdot\delta\cdot\epsilon}{(1+vN)b_I(\epsilon+v)}$
Heesterbeek and Metz	$\frac{q\cdot u\cdot N\cdot\delta\cdot\epsilon}{(1+vn+\sqrt{1+2uN})b_I(\epsilon+v)}$
Mena Lorca and Hethcote	$\frac{q\cdot u\cdot N^v\cdot\delta\cdot\epsilon}{b_I(\epsilon+v)}$

4.3 The R_0 associated to special cases of incidences

Let us consider incidences whose force of infection is given by $\lambda = g(I)$, i.e. it does not depend on N. Recall that, in this case, we obtain

$$R_0 = \frac{\delta \cdot \epsilon \cdot N}{b_I(v+\epsilon)} \cdot \left(\frac{\partial g}{\partial I}\right)_{I=0}. \tag{52}$$

This is the case of Capasso and Serio and Xiao and Ruan incidences for which we obtain

$$R_0 = \frac{\delta \cdot u \cdot \epsilon \cdot N}{b_I(v+\epsilon)}. \tag{53}$$

Moreover, in the case of Ruan and Wang incidence, the basic reproductive number is given by

$$R_0 = \begin{cases} 0, & \text{if } l > 1 \\ \frac{\delta \cdot q \cdot \epsilon \cdot N}{b_I(v+\epsilon)}, & \text{if } l = 1 \\ \text{it does not exist}, & \text{if } l < 1. \end{cases} \tag{54}$$

The basic reproductive number associated to the incidence proposed by Liu, Hethcote and Levin is

$$R_0 = \begin{cases} 0, & \text{if } p > 1 \\ \frac{p \cdot \delta \cdot \beta \cdot \epsilon^q \cdot N^q}{b_I(v+\epsilon)^q}, & \text{if } p = 1 \\ \text{it does not exist}, & \text{if } p < 1. \end{cases} \tag{55}$$

In the case of the incidence due to Van den Driessche and Watmough, we obtain

$$R_0 = \begin{cases} \frac{\delta \cdot \beta}{b_I}, & \text{if } k > 1 \\ \frac{\delta \cdot \beta \cdot (1 + v \cdot k \cdot \frac{\epsilon \cdot N}{v+\epsilon})}{b_I}, & \text{if } k = 1 \\ \text{it does not exists}, & \text{if } 0 < k < 1. \end{cases} \tag{56}$$

Finally, in the case of the incidence saturated with respect to the total number of susceptible devices (Zhang, Gua and Liu incidence), the explicit expression of the basic reproductive number is:

$$R_0 = \frac{\delta \cdot q \cdot \epsilon \cdot N}{b_I(v + \epsilon + \alpha \cdot \epsilon \cdot N)}. \tag{57}$$

5 Conclusions and further work

In this work a study of a SCIRS model with different incidence rates is performed. In the original version of the model, a general expression for the incidence is considered, whereas in this work other different types are studied.

From each incidence, the associated basic reproductive number has been computed, and considering these explicit expressions, we obtain that the basic reproductive number can be expressed as follows:

$$R_0 = \frac{\delta}{b_I} \cdot \omega(v, \epsilon, N, x_1, \ldots, x_n), \tag{58}$$

where $x_1, x_2, \ldots, x_n$ are the parameters associated to the incidence expression.

Taking (58) into account we can ensure that, regardless the type of incidence considered, the basic control measures are the following:

(1) To reduce the numerical value of δ (the proportion of devices endowed with the targeted operative system).
(2) To increase the numerical value of b_I (the recovered coefficient for infectious devices).

Note that both measures affect to infectious devices by reducing its number or increasing its recovery rate.

Further work aimed at analysing the term ω using the mathematical control theory in order to obtain security countermeasures involving the system parameters v, N and ϵ and those specific coefficients of the incidence. In addition to this analytical analysis, some results of experimental work must be obtained.

Acknowledgements

This work has been supported by Ministerio de Economía y Competitividad (Spain) and the European Union through FEDER funds (TIN2014-55325-C2-2-R and MTM2015-69138-REDT). J. D. Hernández Guillén thanks Ministerio de Educación, Cultura y Deporte (Spain) for his departmental grant.

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Received 10 October 2017